Pion form factor and reactions $e^+e^- \to \omega\pi^0$ and $e^+e^- \to \pi^+\pi^-\pi^+\pi^-$ at energies up to 2 – 3 GeV in the many-channel approach.

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Outline

Why new expression?

Pion form factor F_π is important characteristics of the low energy phenomena in particle physics. There are a number of expressions for this quantity used in the analysis of experimental data. The simplest approximate vector meson dominance (VDM) expression based on the effective $\gamma - \rho$ coupling $\propto \rho_\mu A_\mu$,

$$F_\pi(s) = \frac{m_\rho^2 g_{\rho\pi\pi}/g_\rho}{m_\rho^2 - s - i\sqrt{s}\Gamma_{\rho\pi\pi}(s)}$$

- does not possess the correct analytical properties to continue to the region $0 \leq s < 4m_\pi^2$ and further to $s \leq 0$
- it does not take into account the mixing among isovector ρ-like resonances.

- Normalization $F_\pi(0) = 1$ is satisfied only within 20% because

$$F_\pi(0) = \frac{g_{\rho\pi\pi}}{g_\rho} = \left(\frac{9m_\rho\Gamma_{\rho\pi\pi}\Gamma_{\rho ee}}{2\alpha^2 q_\pi^3}\right)^{1/2} \approx 1.20.$$

- The expression based on the gauge invariant $\gamma - \rho$ coupling $\propto \rho_{\mu\nu}F_{\mu\nu}$,

$$F_\pi(s) = 1 + \frac{s g_{\rho\pi\pi}/g_\rho}{m_\rho^2 - s - i\sqrt{s}\Gamma_{\rho\pi\pi}(s)}$$

respects the correct normalization, but does not possesses correct analytical properties and breaks unitarity.

- Formula of Gounaris and Sakurai respects normalization but does not include $\rho(770) - \rho(1450) - \rho(1700) - \cdots$ mixings and vector-pseudoscalar, axial pseudoscalar etc intermediate states.
- Our earlier expression for F_π takes into account isovector mixing but normalization condition is satisfied within 20%.

Treatment of the reaction $e^+e^- \to \pi^+\pi^-$ at $\sqrt{s} \leq 1$ GeV where only pseudoscalar-pseudoscalar PP loops are essential is published in part: N. N. Achasov and A. A. Kozhevnikov. Phys. Rev. D**83**, 113005 (2011). The present talk:

- Treatment of $e^+e^- \to \pi^+\pi^-, 2\pi^+2\pi^-$ at $\sqrt{s} \leq 3$ GeV (BaBaR) and $e^+e^- \to \omega\pi^0$ at $\sqrt{s} \leq 2$ GeV (SND2013) taking into account pseudoscalar-pseudoscalar PP, vector-pseudoscalar VP, and axial vector-pseudoscalar AP loops.

Narrow width→ finite width

- Narrow width: $D_R^{(0)} = m_R^2 - s$.
- finite width effects:

$$\frac{1}{D_R(s)} = \frac{1}{D_R^{(0)}} + \frac{1}{D_R^{(0)}}\Pi_{RR}(s)\frac{1}{D_R^{(0)}} + \frac{1}{D_R^{(0)}}\Pi_{RR}(s)\frac{1}{D_R^{(0)}}\Pi_{RR}(s)\frac{1}{D_R^{(0)}} + \cdots = \frac{1}{D_R^{(0)} - \Pi_{RR}(s)},$$

$$D_R(s) = m_R^2 - s - \mathrm{Re}\Pi_{RR}(s) - i\sum_f \sqrt{s}\Gamma_{R\to f}(s).$$

Resonance mixing

Two-resonance case as an example:

$$\frac{1}{D_R} \to \frac{1}{D_R} + \frac{1}{D_R}\Pi_{RR'}\frac{1}{D_{R'}}\Pi_{RR'}\frac{1}{D_R} + \cdots = \frac{D_{R'}}{D_R D_{R'} - \Pi^2_{RR'}} \equiv \left(G^{-1}\right)_{RR},$$

$$\frac{1}{D_{R'}} \to \frac{1}{D_{R'}} + \frac{1}{D_{R'}}\Pi_{RR'}\frac{1}{D_R}\Pi_{RR'}\frac{1}{D_{R'}} + \cdots = \frac{D_R}{D_R D_{R'} - \Pi^2_{RR'}} \equiv \left(G^{-1}\right)_{R'R'},$$

$$\frac{\Pi_{RR'}}{D_R D_{R'}} \to \frac{\Pi_{RR'}}{D_R D_{R'}} + \frac{(\Pi_{RR'})^3}{(D_R D_{R'})^2} + \cdots = \frac{\Pi_{RR'}}{D_R D_{R'} - \Pi^2_{RR'}} \equiv (G^{-1})_{RR'}.$$

Two-resonance case

- The matrix of inverse propagators:

$$G = \begin{pmatrix} D_R & -\Pi_{RR'} \\ -\Pi_{RR'} & D_R \end{pmatrix}$$

- The matrix of propagators:

$$G^{-1} = \frac{1}{D_R D_{R'} - \Pi^2_{RR'}} \begin{pmatrix} D_{R'} & \Pi_{RR'} \\ \Pi_{RR'} & D_R \end{pmatrix}$$

- The amplitude:

$$A(i \to R + R' \to f) = \begin{pmatrix} g_{i\to R} & g_{i\to R'} \end{pmatrix} G^{-1} \begin{pmatrix} g_{R\to f} \\ g_{R'\to f} \end{pmatrix}$$

Generalization to arbitrary number of mixed resonances

Generalization to three (and any number of) resonance mixing:

$$G=\begin{pmatrix} D_1 & -\Pi_{12} & -\Pi_{13} & \cdots \\ -\Pi_{12} & D_2 & -\Pi_{23} & \cdots \\ -\Pi_{13} & -\Pi_{23} & D_3 & \cdots \\ \cdots & \cdots & \cdots & \end{pmatrix},$$

$$G^{-1}=\frac{1}{\Delta}\begin{pmatrix} g_{11} & g_{12} & g_{13} & \cdots \\ g_{12} & g_{22} & g_{23} & \cdots \\ g_{13} & g_{23} & g_{33} & \cdots \\ \cdots & \cdots & \cdots & \end{pmatrix}, \Delta=\det G.$$

Dispersion representation for polarization operators

- Dispersion relation for polarization operators are used under assumption of quasi-two body intermediate hadronic states:

$$\frac{\Pi^{(ab)}_{\rho_i\rho_i}(s)}{s} = \frac{1}{\pi}\int_{(m_a+m_b)^2}^{\infty}\frac{\sqrt{s'}\Gamma_{\rho_i\to ab}(s')}{s'(s'-s-i\varepsilon)}ds',$$
$$\mathrm{Im}\Pi^{(ab)}_{\rho_i\rho_i}(s) = \sqrt{s}\Gamma_{\rho_i\to ab}(s).$$

- This form automatically provides the correct normalization of the form factor $F_\pi(0)=1$.

Loops contributing to polarization operators

We take into account analytically calculated

- pseudoscalar (PP) $\pi^+\pi^-$ and $K^+K^- + K^0\bar{K}^0$ loops:

$$\Pi^{(PP)}_{\rho_i\rho_j} = g_{\rho_i\pi\pi} g_{\rho_j\pi\pi} \left[\Pi^{(PP)}(s, m_{\rho_i}, m_\pi) + \frac{1}{2}\Pi^{(PP)}(s, m_{\rho_i}, m_K)\right]$$

- vector-pseudoscalar (VP) $\omega\pi^0$ and $K^*\bar{K} + \bar{K}^*K$ loops:

$$\Pi^{(VP)}_{\rho_i\rho_j} = g_{\rho_i\omega\pi} g_{\rho_j\omega\pi} \Big[\Pi^{(VP)}(s, m_{\rho_i}, m_\omega, m_\pi) + \Pi^{(VP)}(s, m_{\rho_i}, m_{K^*}, m_K)\Big],$$

- the axial vector-pseudoscalar (AP) $a_1^{\pm}\pi^{\mp}$, $K_1(1270)\bar{K}+$ c.c. loops:

$$\Pi^{(AP)}_{\rho_i\rho_j} = 2g_{\rho_i a_1\pi}g_{\rho_j a_1\pi}\left[\Pi^{(AP)}(s,m_{\rho_i},m_{a_1},m_\pi)+ \frac{1}{2}\Pi^{(AP)}(s,m_{\rho_i},m_{K_1(1270)},m_K)\right]$$

Relations among coupling constants

- $q\bar{q}$ quark model relations among the coupling constants are assumed for simplicity:

$$
\begin{aligned}
g_{\rho_i KK} &= \frac{1}{2} g_{\rho_i \pi\pi}, \\
g_{\rho_i K^* K} &= \frac{1}{2} g_{\rho_i \omega\pi}, \\
g_{\rho_i K_1(1270) K} &= \frac{1}{2} g_{\rho_i a_1(1260)\pi}.
\end{aligned}
$$

Polarization operators

- Diagonal polarization operators:

$$\Pi_{\rho_i\rho_i} = \Pi^{(PP)}_{\rho_i\rho_i} + \Pi^{(VP)}_{\rho_i\rho_i} + \Pi^{(AP)}_{\rho_i\rho_i}$$

- Pseudoscalar-Pseudoscalar

$$\Pi^{(PP)}_{\rho_1\rho_i} = \frac{g_{\rho_i\pi\pi}}{g_{\rho_1\pi\pi}}\Pi^{(PP)}_{\rho_1\rho_1},\ \Pi^{(PP)}_{\rho_i\rho_j} = \frac{g_{\rho_i\pi\pi}g_{\rho_j\pi\pi}}{g^2_{\rho_1\pi\pi}}\Pi^{(PP)}_{\rho_1\rho_1},$$

  VP and AP analogously to PP
- Total non-diagonal $(i, j \neq 1)$

$$\Pi_{\rho_1\rho_i} = \Pi^{(PP)}_{\rho_1\rho_i} + \Pi^{(VP)}_{\rho_1\rho_i} + \Pi^{(AP)}_{\rho_1\rho_i},$$
$$\Pi_{\rho_i\rho_j} = \Pi^{(PP)}_{\rho_i\rho_j} + \Pi^{(VP)}_{\rho_i\rho_j} + \Pi^{(AP)}_{\rho_i\rho_j} + sa_{ij}$$

Dispersion representations

- Dispersion representations (remaining logarithmic divergencies cancel after subtracting the real parts at the $\rho(770)$ mass):

$$\frac{\Pi^{(PP)}_{RR}(s)}{s} = \frac{g^2_{RPP}}{6\pi^2}\int_{(2m_P)^2}^{\infty}\frac{q^3_{PP}(s')}{s'^{3/2}(s'-s-i\varepsilon)}ds',$$

$$\frac{\Pi^{(VP)}_{RR}(s)}{s} = \frac{g^2_{RVP}}{12\pi^2}\int_{(m_V+m_P)^2}^{\infty}\frac{q^3_{VP}(s')}{\sqrt{s'}(s'-s-i\varepsilon)}\frac{s_0+m^2_R}{s_0+s'}ds',$$

$$\frac{\Pi^{(AP)}_{RR}(s)}{s} = \frac{g^2_{RAP}}{48\pi^2}\int_{(m_A+m_P)^2}^{\infty}\frac{[(s'+m^2_A-m^2_P)^2+2s'm^2_A]}{(s'-s-i\varepsilon)s'^{3/2}}\times \frac{s_0+m^2_R}{s_0+s'}q_{AP}(s')ds'.$$

$\mathrm{Re}\Pi^{(VP)}_{\rho_1\rho_1}$

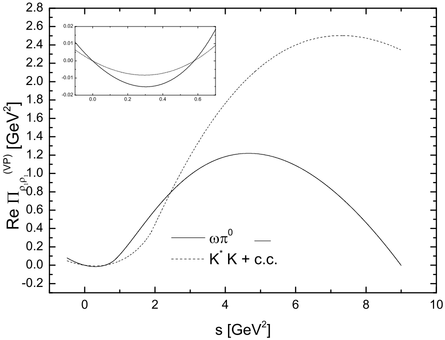


Figure : Energy dependence of $\mathrm{Re}\Pi^{(VP)}_{\rho_1\rho_1}(s)$.

$\mathrm{Re}\Pi^{(AP)}_{\rho_1\rho_1}$

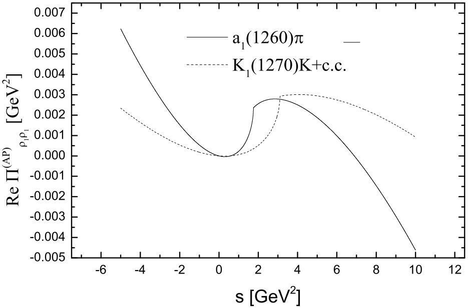


Figure : Energy dependence of $\mathrm{Re}\Pi^{(AP)}_{\rho_1\rho_1}(s)$.

The pion form factor

- The expression for the pion form factor:

$$F_\pi(s) = (g_{\gamma\rho_1}, g_{\gamma\rho_2}, g_{\gamma\rho_3}, \cdots)G^{-1}\begin{pmatrix} g_{\rho_1\pi\pi} \\ g_{\rho_2\pi\pi} \\ g_{\rho_3\pi\pi} \\ \cdots \end{pmatrix} + \frac{g_{\gamma\omega}\Pi_{\rho_1\omega}}{D_\omega\Delta}(g_{11}g_{\rho_1\pi\pi} + g_{12}g_{\rho_2\pi\pi} + g_{13}g_{\rho_3\pi\pi}) + \cdots .$$

  $\rho_1 - \omega(782)$ mixing is essential because of the mass proximity, $\rho_{2,3,\ldots}\omega$, mixings are negligible.

- Automatically respects the current conservation condition $F_\pi(0) = 1$ and possesses correct analytical properties over entire s axis

The cross section of $e^+e^- \to \omega\pi^0$

$$\sigma_{e^+e^-\to\omega\pi^0} = \frac{4\pi\alpha^2}{3s^{3/2}} \left|A_{e^+e^-\to\omega\pi^0}\right|^2 q^3_{\omega\pi},$$

$$A_{e^+e^-\to\omega\pi^0} = (g_{\gamma\rho_1}, g_{\gamma\rho_2}, g_{\gamma\rho_3}, \cdots)G^{-1}\begin{pmatrix} g_{\rho_1\omega\pi} \\ g_{\rho_2\omega\pi} \\ g_{\rho_3\omega\pi} \\ \cdots \end{pmatrix}$$

Model for $e^+e^- \to \pi^+\pi^-\pi^+\pi^-$

The model: $a_1^\pm\pi^\mp$ dominance of the $\pi^+\pi^-\pi^+\pi^-$ final state.

- $A(\rho_{iq} \to a_{1k}\pi_p) = g_{\rho_i a_1\pi}[(\epsilon_{a_1}\epsilon_{\rho_i})(kq) - (\epsilon_{a_1}q)(\epsilon_{\rho_i}k)]$
- $\Gamma_{\rho_i\to 2\pi^+2\pi^-}(s) = \frac{g^2_{\rho_i a_1\pi}}{12\pi}\int_{(3m_\pi)^2}^{(\sqrt{s}-m_\pi)^2}\rho_{a_1}(m^2)\left[\frac{(s+m^2-m_\pi^2)^2}{2s}+m^2\right]q_{a_1\pi}dm^2$
- $\rho_{a_1}(m^2) = \frac{m_{a_1}\Gamma_{a_1}/\pi}{(m^2-m^2_{a_1})^2+m^2_{a_1}\Gamma^2_{a_1}}$

The cross section of $e^+e^- \to \pi^+\pi^-\pi^+\pi^-$

$$\sigma_{\pi^+\pi^-\pi^+\pi^-} = \frac{(4\pi\alpha)^2}{3s^{3/2}} \left| (g_{\gamma\rho_1}, g_{\gamma\rho_2}, g_{\gamma\rho_3}, \cdots) G^{-1} \begin{pmatrix} g_{\rho_1 a_1 \pi} \\ g_{\rho_2 a_1 \pi} \\ g_{\rho_3 a_1 \pi} \\ \cdots \end{pmatrix} \right|^2 \times W_{\pi^+\pi^-\pi^+\pi^-}(s)$$

$$W_{\pi^+\pi^-\pi^+\pi^-}(s) = \frac{\Gamma_{\rho_i \to 2\pi^+ 2\pi^-}(s)}{g^2_{\rho_i a_1 \pi}}$$

The quantities to fit

The quantities to fit are

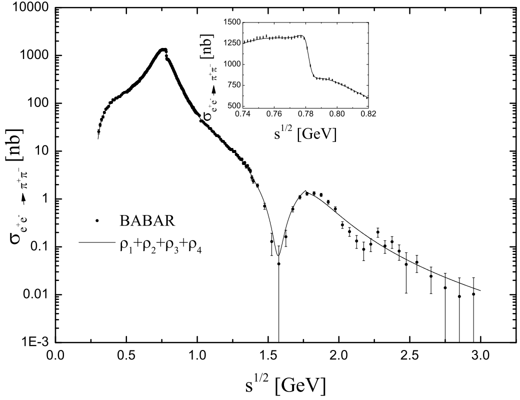
- The bare cross section $e^+e^- \to \pi^+\pi^-$undressed from vacuum polarization of the photon but the final state radiation of charged pion included via function $a(s)$:
  $\sigma_{\text{bare}} = \frac{8\pi\alpha^2}{3s^{5/2}}|F_\pi(s)|^2 q_\pi^3(s)\left[1 + \frac{\alpha}{\pi}a(s)\right]$
  $a(s)$ in the point-like pion approximation.
- The cross section of the reaction $e^+e^- \to \omega\pi^0$
- The cross section of the reaction $e^+e^- \to \pi^+\pi^-\pi^+\pi^-$

What's fitted

Two fitting schemes:

- (Scheme I) The low-energy ($\sqrt{s} \leq 1$ GeV) SND, CMD2, and KLOE2010 data on $e^+e^- \to \pi^+\pi^-$ fitted upon neglecting VP,AP intermediate states inessential in this region. Taken into account are $\rho(770) + \rho(1450) + \rho(1700)$.
- (Scheme II) BaBaR data on $e^+e^- \to \pi^+\pi^-, 2\pi^+2\pi^-$ at $\sqrt{s} \leq 3$ GeV and SND2013 data on $e^+e^- \to \omega\pi^0$. Taken into account are the PP, VP, AP intermediate states and $\rho(770) + \rho(1450) + \rho(1700) + \rho(2100)$ whose masses, coupling constants are free but subjected to the constrain $\frac{g_{\rho_1\pi\pi}}{g_{\rho_1}} + \frac{g_{\rho_2\pi\pi}}{g_{\rho_2}} + \frac{g_{\rho_3\pi\pi}}{g_{\rho_3}} + \frac{g_{\rho_4\pi\pi}}{g_{\rho_4}} = 1$ necessary for correct normalization $F_\pi(0) = 1$

Fitting BaBaR on $e^+e^- \to \pi^+\pi^-$ in Scheme II



Resonance parameters from fitting BABAR $\pi^+\pi^-$ data

parameter	Scheme I	Scheme II
m_{ρ_1} [MeV]	773.92 ± 0.10	766.9 ± 0.2
$g_{\rho_1\pi\pi}$	5.785 ± 0.004	6.345 ± 0.003
g_{ρ_1}	5.167 ± 0.002	4.666 ± 0.002
m_ω [MeV]	782.04 ± 0.10	782.01 ± 0.09
g_ω	17.05 ± 0.29	$\equiv 17.06$ (PDG)
$10^3\Pi'_{\rho_1\omega}$ [GeV^2]	4.00 ± 0.06	4.41 ± 0.10
$\chi^2/N_{\mathrm{d.o.f.}}$	216/260	320/313

Renormalization

Renormalization in the single-resonance approximation

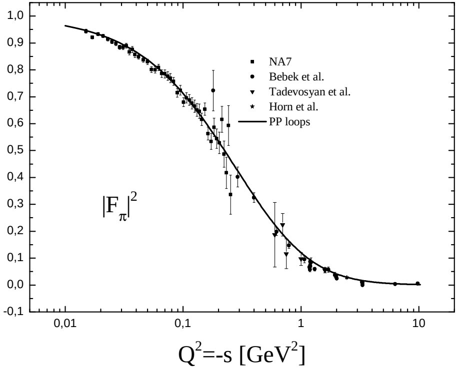
- Inverse propagator

$$D_{\rho_1}=m^2_{\rho_1}-s+(m^2_{\rho_1}-s)\frac{d\mathrm{Re}\Pi_{\rho_1\rho_1}(s)}{ds}\Big|_{s=m^2_{\rho_1}}-i\sqrt{s}\Gamma_{\rho_1\pi\pi}(s)$$

- Physical coupling constants determined from visible peak are expressed via bare ones as $g^{\mathrm{phys}}_{\rho_1\pi\pi}=Z_\rho^{-1/2}g_{\rho_1\pi\pi}$, $g^{\mathrm{phys}}_{\rho_1}=Z_\rho^{1/2}g_{\rho_1}$, where $Z_\rho=1+\frac{d\mathrm{Re}\Pi_{\rho_1\rho_1}(s)}{ds}\Big|_{s=m^2_{\rho_1}}$
- The contributions of the VP loop to $d\mathrm{Re}\Pi_{\rho_1\rho_1}/ds$ near $s=m^2_{\rho_1}$ is positive and exceed the negative contribution from the PP loop. The same is true for the AP loop. As a result, $Z_\rho>1$.

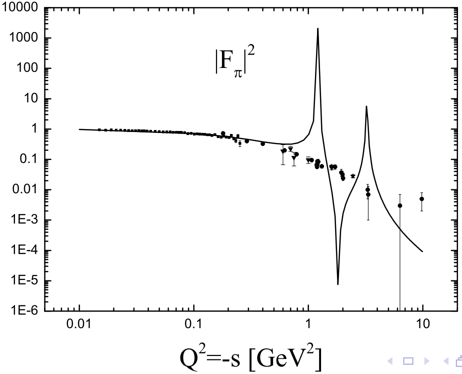
Continuation to space-like region in the scheme I

Continuation to the space-like domain using the resonance parameters obtained from fitting Scheme I:



Continuation to space-like region in the scheme II

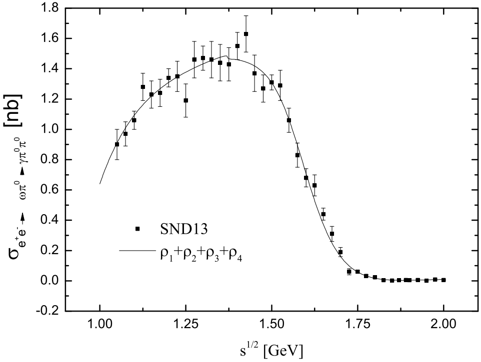
Continuation to the space-like domain using the resonance parameters obtained from fitting Scheme II:



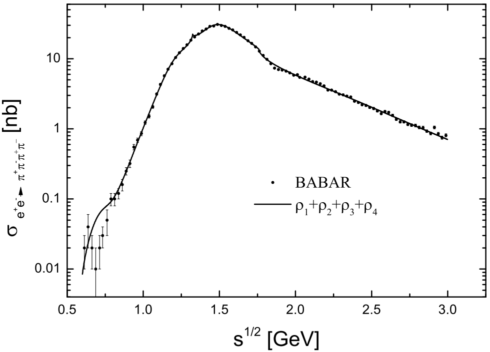
The Landau pole

- Deeply in the space-like domain: $\Pi^{(\pi\pi)}_{\rho\rho} \sim \frac{g^2_{\rho\pi\pi}|s|}{48\pi^2} \ln \frac{|s|}{m^2_\rho}$
- Inverse propagator $m^2_\rho + |s| - \Pi^{(\pi\pi)}_{\rho\rho} = 0$ has zero at some $|s_L|$ – the Landau pole $\sqrt{|s_L|} \sim m_\rho \exp \frac{24\pi^2}{g^2_{\rho\pi\pi}}$
- Numerically, for the $\pi^+\pi^-$ loop, with the parameters from different fits, $\sqrt{|s_L|} = 80 - 90$ GeV.
- VP and AP loops bring the pole to the observable region.
- Stronger suppression factors for VP and AP partial widths are required?

Fitting SND2013 on $e^+e^- \to \omega\pi^0$ in Scheme II



Fitting BaBaR on $e^+e^- \to \pi^+\pi^-\pi^+\pi^-$ in Scheme II



- Masses of heavier resonances:

mass	$e^+e^- \to \pi^+\pi^-$	$e^+e^- \to 2\pi^+2\pi^-$	$e^+e^- \to \omega\pi^0$
m_{ρ_2}[MeV]	1307 ± 4	1323 ± 3	1747 ± 4
m_{ρ_3}[MeV]	1491 ± 2	1506 ± 1	1880 ± 160
m_{ρ_4}[MeV]	2008 ± 8	1875 ± 5	2060 ± 40

- Coupling constants determined from fitting three channels, are not consistent with each other. Perhaps, this is the consequence of the oversimplified model for four-pion production amplitude.
- Possible way out: a different subtraction scheme with three subtractions for the non-diagonal polarization operators, $\mathrm{Re}\Pi_{\rho_i\rho_j}(0) = \mathrm{Re}\Pi_{\rho_i\rho_j}(m^2_{\rho_i}) = \mathrm{Re}\Pi_{\rho_i\rho_j}(m^2_{\rho_j}) = 0$ is now under study

Conclusion

New expression for $F_{\pi}(s)$:

- gives a good description of the data of SND, CMD-2, KLOE, BaBaR on $\pi^+\pi^-$ production in e^+e^- at $\sqrt{s} < 1$ GeV
- describes the BaBaR data on $e^+e^- \to \pi^+\pi^-$ in a wider energy range $\sqrt{s} \leq 3$ GeV upon introducing necessary coupling constants characterizing other two channels VP,AP observed in $e^+e^- \to \omega\pi^0$ and $e^+e^- \to \pi^+\pi^-\pi^+\pi^-$
- does not contradict the measured δ_1^1
- does not require the commonly accepted Blatt – Weisskopf centrifugal factor $(1 + R_{\pi}^2 k_R^2)/(1 + R_{\pi}^2 k^2)$

Conclusion

- Loops of PP, VP, AP intermediate states and the contributions of higher energy resonances $\rho(1450)$, $\rho(1700)$, $\rho(2100)$ affect the values of $m_{\rho(770)}$, $g_{\rho_1\pi\pi}$, g_{ρ_1} extracted from the data
- Resonance contributions restricted to the PP loops reproduce the spacelike pion form factor up to -10 GeV^2. Adding VP, AP intermediate states spoils form factor due to the Landau zeros. Some work is required to push the Landau zeros to higher spacelike momenta.

Thank You!