



# Gamma gamma physics at Belle



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***PHI******PSI*****13**

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**PHI**  
**PSI****13**



# KEKB Accelerator and Belle Detector

- Asymmetric  $e^- e^+$  collider  
8 GeV  $e^-$  (HER) x 3.5 GeV  $e^+$  (LER)

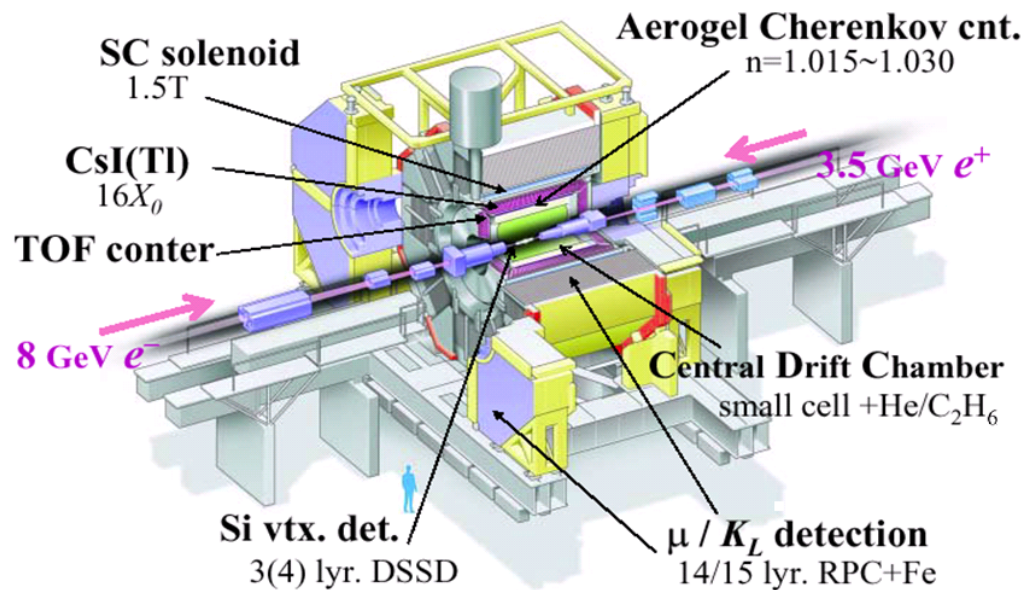
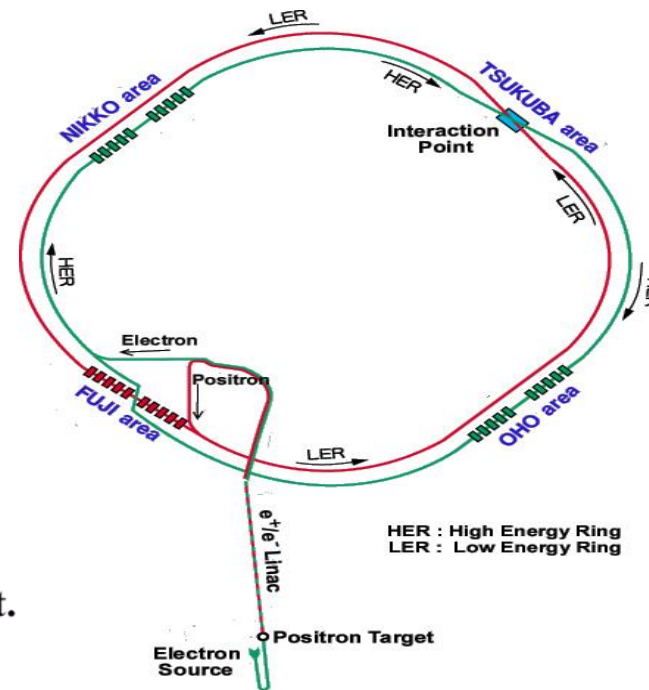
$$\sqrt{s} = 10.58 \text{ GeV} \Leftrightarrow \Upsilon(4S)$$

Beam crossing angle: 22mrad

- Continuous injection
- Luminosity

$$L_{\text{max}} = 2.1 \times 10^{34} \text{ cm}^{-2} \text{ s}^{-1}$$

$$\int L dt \sim 1040 \text{ fb}^{-1}$$



High momentum/energy resolutions

CDC+Solenoid, CsI

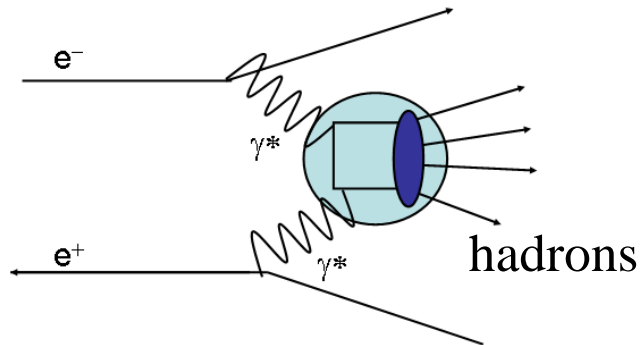
Vertex measurement – Si strips

Particle identification

TOF, Si-aerogel, CDC-dE/dx,

RPC for  $K_L/\mu$ on

# Two-Photon Collisions and Hadron/QCD Physics



## Hadron production from collisions of virtual or quasi-real photons

- Perturbative/Non-perturbative QCD
- Resonances
- Hadron/photon form factors

## Single resonance formation in $\gamma\gamma^{(*)}$ collisions

Zero-tag with  $p_t$ -balance requirement for the hadron system

$Q^2 \ll W^2$  ( $\gamma\gamma$  c.m. energy),  $Q^2 \ll E_{\text{QCD}}^2$  (Energy scale of QCD)

Measurement of  $\Gamma_{\gamma\gamma} B(\rightarrow \text{final state})$

Single-tag process ( $Q^2$  dependence in  $\gamma\gamma^*$  collisions)

Measurement of transition form factor



$$\gamma\gamma \rightarrow K_S^0 K_S^0$$

The first measurement of the differential cross section in  
 $W = 1.05 - 2.4 \text{ GeV}$  with  $972 \text{ fb}^{-1}$  Belle data

$W$ : c.m. energy of  $\gamma\gamma$  collisions

arXiv:1307.7457[hep-ex], submitted to PTEP  
(Progress of Theoretical and Experimental Physics)

Study of **resonances** including

exotic candidates (e.g. glueball state)

This process is dominated by resonances in  $W < \sim 2.4 \text{ GeV}$

$W > 2.4 \text{ GeV}$  -- Update of the previous Belle publication

(W.T.Chen et al., PLB651, 15 (2007),  $397.6 \text{ fb}^{-1}$ )

**QCD study** – Angular and  $W$  dependences

**Charmonia**: Partial decay widths for  $\chi_{c0}$  and  $\chi_{c2}$

Search for  $\chi_{c0}(2P)$  etc., which is to be  $3.80 - 3.93 \text{ GeV}$

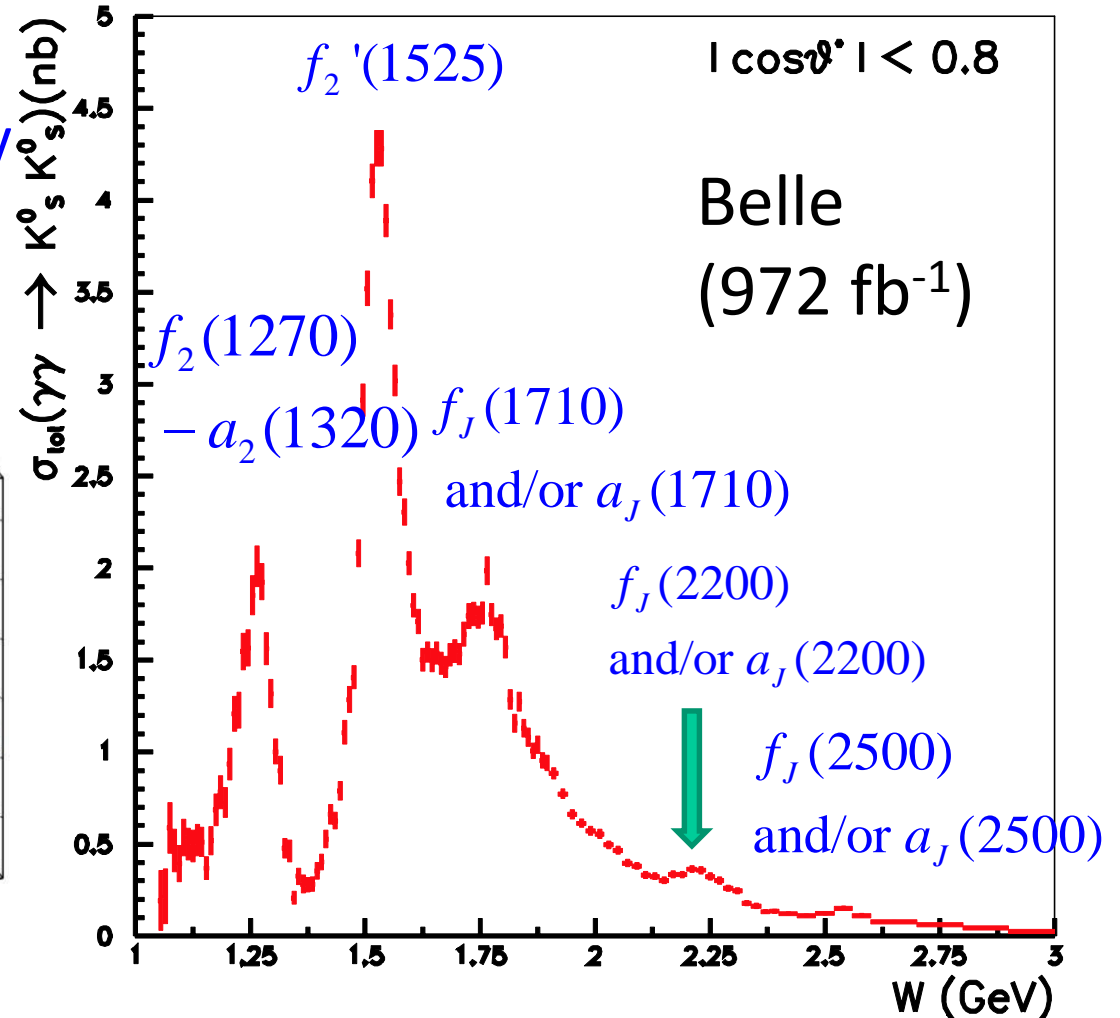
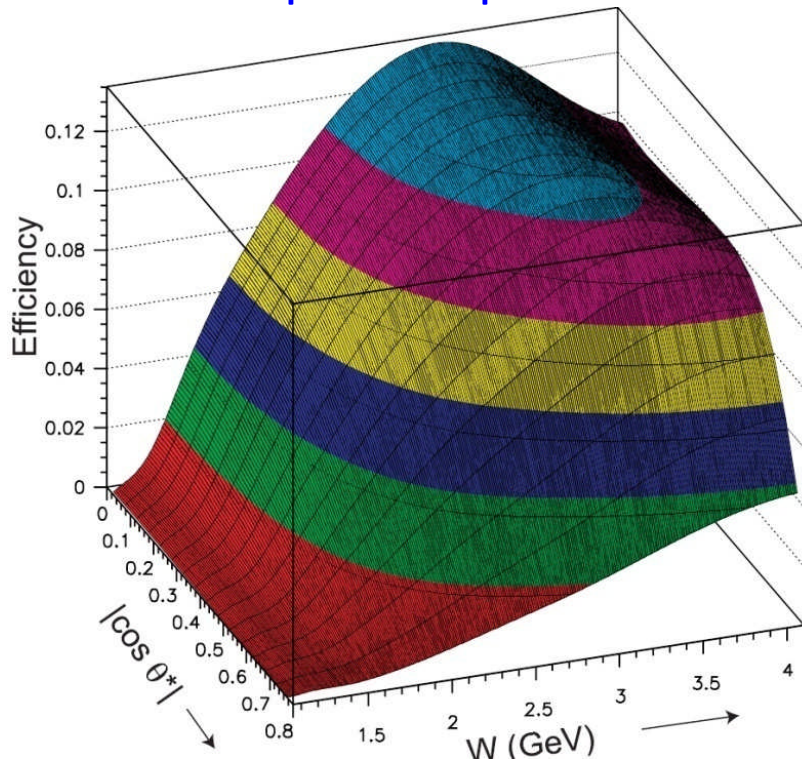


# Cross section integrated over the angle

**Five resonance-like peaks visible below 3 GeV**

W and angular dependence  
of the efficiency

Covering  $W = 1.05 - 4.0$  GeV  
 $|\cos \theta^*| < 0.8$



# Formula for differential cross section

- At low energy ( $W < 3$  GeV)

$$\frac{d\sigma}{d\Omega} = \left| S Y_0^0 + D_0 Y_2^0 + G_0 Y_4^0 \right|^2 + \left| D_2 Y_2^2 + G_2 Y_4^2 \right|^2$$

- $S, D_0, G_0, D_2, G_2$  Partial wave amplitudes

$J = L = 0, 2, 4$  (even only) and total two-photon helicity = 0 or 2

– give  $W$  dependence of each partial wave

assuming resonance and continuum components

- $Y_J^m$  : spherical harmonics

– Each determines the angular dependence of the wave

– But, not mutually independent

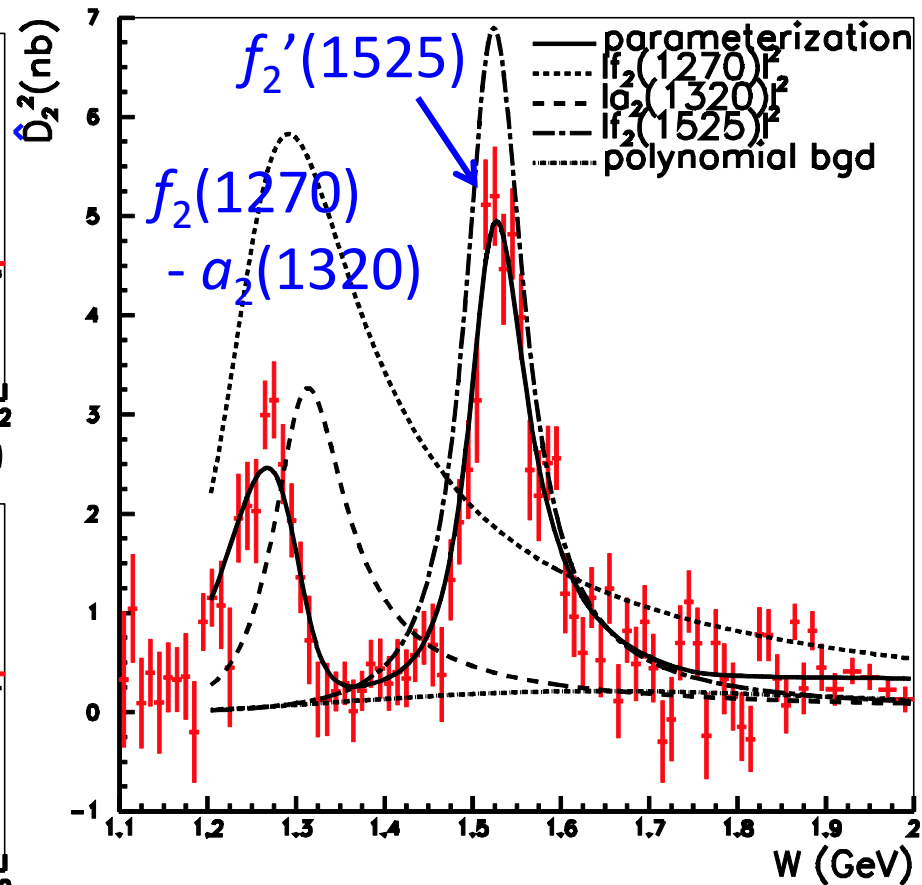
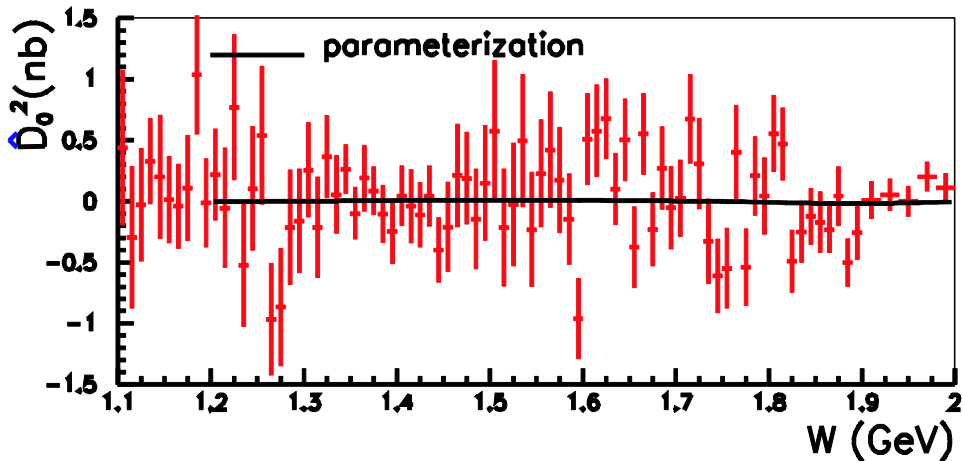
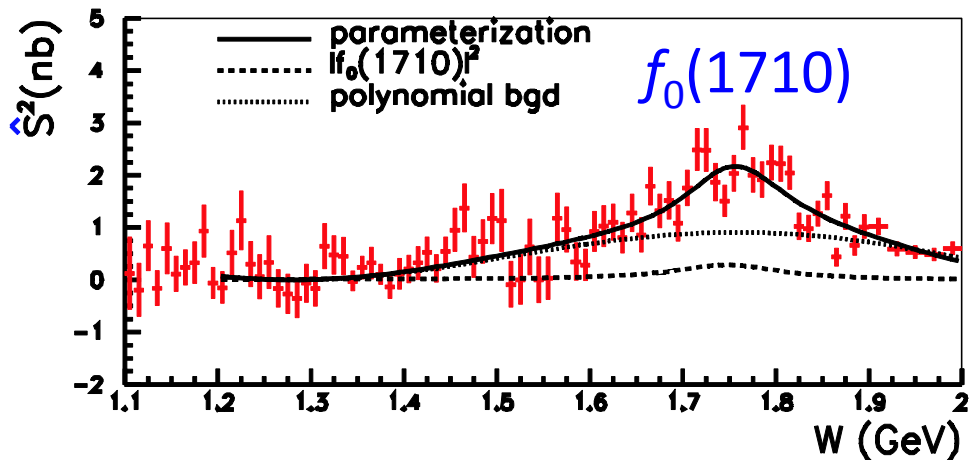


# Fit with S and D waves for $W < 2.0$ GeV

Hat amplitudes

to visualize, model independent

$$\frac{d\sigma}{d\Omega} = \hat{S}^2 |Y_0^0|^2 + \hat{D}_0^2 |Y_2^0|^2 + \hat{D}_2^2 |Y_2^2|^2$$





# Fit results for $W < 2.0$ GeV

$f_2(1270)$  -  $a_2(1320)$  interference and  $f_2'(1525)$

Parameter	Sol. H	Sol. L	H,L combined	Incoh. fit	PDG [23]
$\chi^2/ndf$	375.09/387	375.22/387	–	406.6/388	–
$\phi_{a_2(1320)}$ (deg.)	$178.1^{+1.7+6.7}_{-1.3-12.5}$	$172.6^{+1.3+6.7}_{-1.0-3.1}$	$172.6^{+6.0+12.2}_{-0.7-7.0}$	$173.6^{+1.3}_{-1.4}$	–
Mass( $f_2'(1525)$ ) (MeV/ $c^2$ )	$1526.1^{+0.9+2.9}_{-1.0-2.8}$	$1524.3^{+1.0+1.6}_{-0.9-1.1}$	$1525.3^{+1.2+3.7}_{-1.4-2.1}$	$1530.7 \pm 0.4$	$1525 \pm 5$
$\Gamma_{\text{tot}}(f_2'(1525))$ (MeV)	$83.4^{+1.9+2.0}_{-1.7-3.4}$	$81.8^{+2.3+4.4}_{-2.0-0.9}$	$82.9^{+2.1+3.3}_{-2.2-2.0}$	$82.7 \pm 1.4$	$73^{+6}_{-5}$
$\Gamma_{\gamma\gamma} \mathcal{B}(K\bar{K})(f_2'(1525))$ (eV)	$113^{+25+43}_{-28-77}$	$48 \pm 4^{+33}_{-10}$	$48^{+67+108}_{-8-12}$	$79.1 \pm 1.4$	$72 \pm 7$

Two solutions are found, and they are combined

- **Destructive interference** btw.  $f_2(1270)$  and  $a_2(1320)$  confirmed
- **First attempt to include interference** effect in measuring  $\Gamma_{\gamma\gamma} \mathcal{B}(K\bar{K})$  of  $f_2'(1525)$ .





## Fit results for $W < 2.0$ GeV (cont.)

$f_J(1710)$

Parameter	$f_0(1710)$ fit				$f_2(1710)$ fit	
	fit-H	fit-L	H,L combined	PDG	fit-H	fit-L
$\chi^2/ndf$	694.2/585	701.6/585	–	–	796.3/585	831.5/585
Mass( $f_J$ ) (MeV/ $c^2$ )	$1750_{-6-18}^{+5+29}$	$1749_{-6-42}^{+5+31}$	$1750_{-7-18}^{+6+29}$	$1720 \pm 6$	$1750_{-7}^{+6}$	$1729_{-7}^{+6}$
$\Gamma_{\text{tot}}(f_J)$ (MeV)	$138_{-11-50}^{+12+96}$	$145_{-10-54}^{+11+31}$	$139_{-12-50}^{+11+96}$	$135 \pm 6$	$132_{-11}^{+12}$	$150 \pm 10$
$\Gamma_{\gamma\gamma}\mathcal{B}(K\bar{K})_{f_J}$ (eV)	$12_{-2-8}^{+3+227}$	$21_{-4-26}^{+6+38}$	$12_{-2-8}^{+3+227}$	unknown	$2.1_{-0.3}^{+0.5}$	$1.6 \pm 0.2$

Scalar rather than tensor! (in contrast to L3)

$f_0(1710)$  :  $\Gamma_{\gamma\gamma} > 0(10 \text{ eV})$  indicates not likely a pure glueball



# Fit Results for resonances in $W > 2.0$ GeV

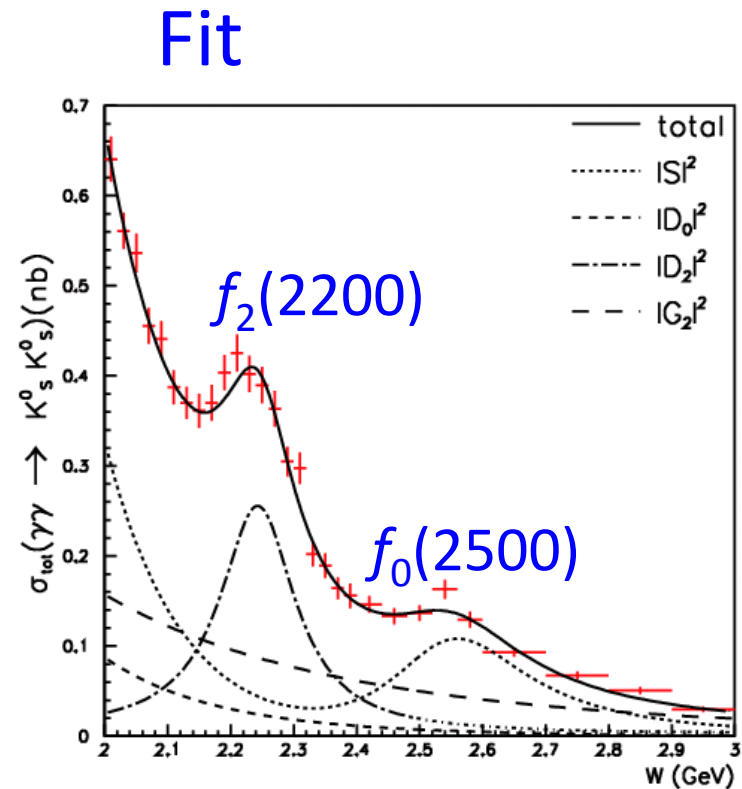
$f_2(2200)$ - $f_0(2500)$  is the best solution (in all trials of  $J = 0, 2, 4$ )

- The resonance parameters

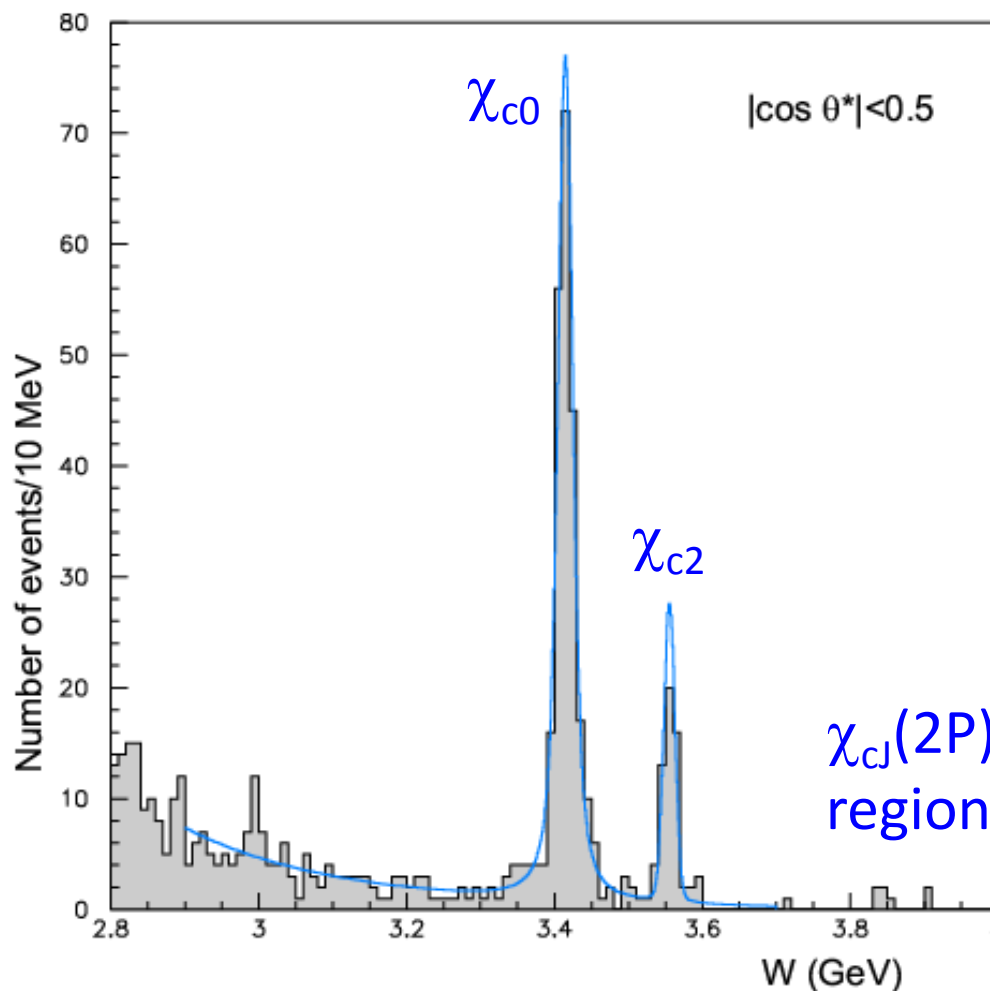
Parameter	$f_2(2200)$	$f_0(2500)$
Mass (MeV/ $c^2$ )	$2243_{-6-29}^{+7+3}$	$2539 \pm 14_{-14}^{+38}$
$\Gamma_{\text{tot}}$ (MeV)	$145 \pm 12_{-34}^{+27}$	$274_{-61-163}^{+77+126}$
$\Gamma_{\gamma\gamma} \mathcal{B}(K\bar{K})$ (eV)	$3.2_{-0.4-2.2}^{+0.5+1.3}$	$40_{-7-40}^{+9+17}$

- Significances

- $3.4\sigma$  for  $f_2(2200)$  over  $f_0(2200)$
  - $4.3\sigma$  for  $f_0(2500)$  over  $f_2(2500)$
- evaluated from  $\min.(\Delta\chi^2)$  for every sys. source



# Charmonia $\chi_{c0}$ and $\chi_{c2}$



## Yield

Interference	$N_{\chi_{c0}}$	$N_{\chi_{c2}}$	$-2 \ln \mathcal{L}/ndf$
not included	$248.3^{+17.9}_{-17.2}$	$53.0^{+8.1}_{-7.4}$	57.34/73
included	$266 \pm 53$	$53^{+14}_{-12}$	57.22/71

Interference between  $\chi_{c0}$  and continuum

Product of two-photon decay width and  $B(K_S^0 K_S^0)$

Interference	$\Gamma_{\gamma\gamma} \mathcal{B}(\chi_{c0})$ (eV)	$\Gamma_{\gamma\gamma} \mathcal{B}(\chi_{c2})$ (eV)
not included	$8.09 \pm 0.58 \pm 0.83$	$0.268^{+0.041}_{-0.037} \pm 0.028$
included	$8.7 \pm 1.7 \pm 0.9$	$0.27^{+0.07}_{-0.06} \pm 0.03$
Belle 2007	$7.00 \pm 0.65 \pm 0.71$	$0.31 \pm 0.05 \pm 0.03$
PDG 2012	$7.3 \pm 0.5$	$0.297 \pm 0.026$



# QCD Studies: Angular dependence

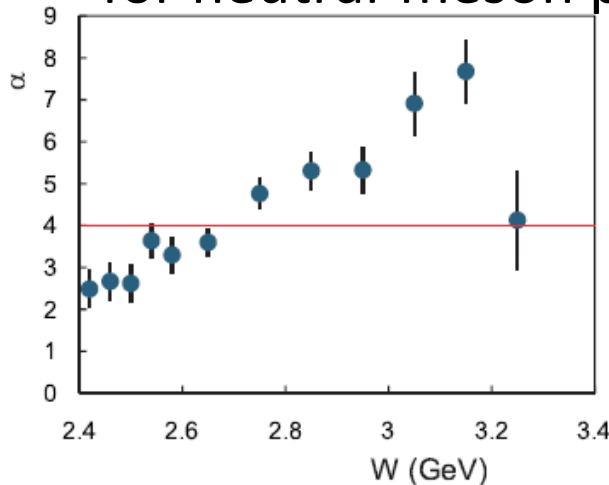
Assume non-resonant effect in  $2.6 < W < 3.3$  GeV ;  
 we know there is a resonance near 2.5 GeV

$$d\sigma/d|\cos\theta^*| \propto 1/\sin^\alpha \theta^*$$

Handbag:  $\alpha = \text{const.} = 4$

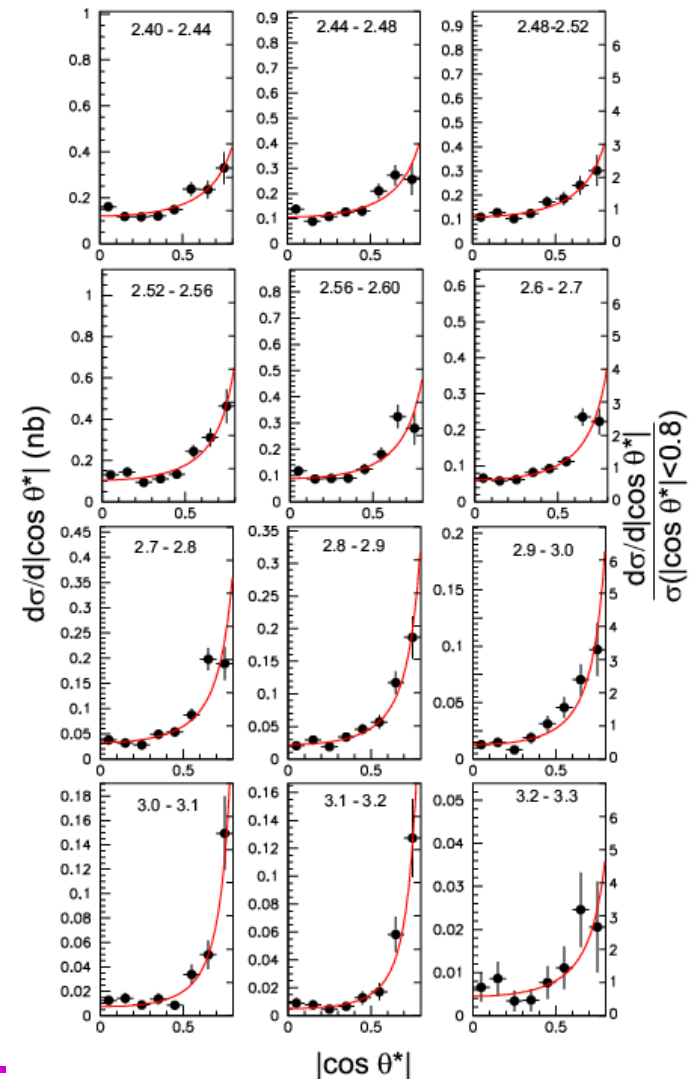
M.Diehl, P.Kroll, and C. Vogt, PLB 532, 99 (2002)  
 M.Diehl, P.Kroll, PLB 683, 165 (2010)

pQCD:  $\alpha$  not const, depend on  $W$   
 for neutral-meson pair



No tendency is seen to converge to 4 at high energies

$W = 2.4 - 3.3$  GeV



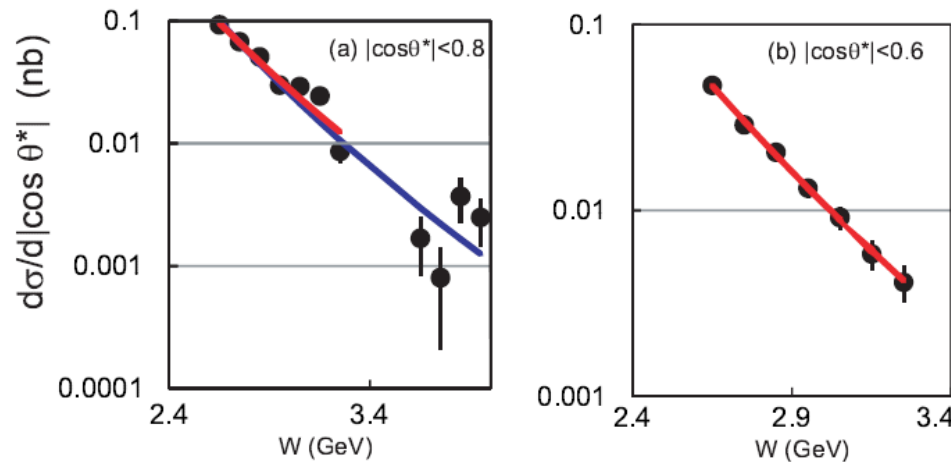
# W-dependence

$\sigma \propto W^{-n}$  pQCD predictions

for charged-meson pair  $n = 6$  ( $\pi^+\pi^-, K^+K^-$  etc.) = Dimensional counting rule

for neutral-meson pair  $n=10$  ( $K^0sK^0s$  etc.) by Chernyak

PLB 640, 246 (2006)  
arXiv 1212.1304 [hep-ph]




$W$ range (GeV)	$ \cos\theta^* $ range	$n$	note
2.6 – 4.0 (except 3.3 – 3.6)	$< 0.8$	$11.0 \pm 0.4 \pm 0.4$	
2.6 – 3.3	$< 0.8$	$10.0 \pm 0.5 \pm 0.4$	
2.6 – 3.3	$< 0.6$	$11.8 \pm 0.6 \pm 0.4$	
2.4 – 4.0 (except 3.3 – 3.6)	$< 0.6$	$10.5 \pm 0.6 \pm 0.5$	Belle 2007

Close to  $n=10$ , agree with pQCD prediction



# “ $\gamma\gamma \rightarrow$ meson pair” (six final states) from Belle

QCD Test in 2.4 – 4.1 GeV energy region

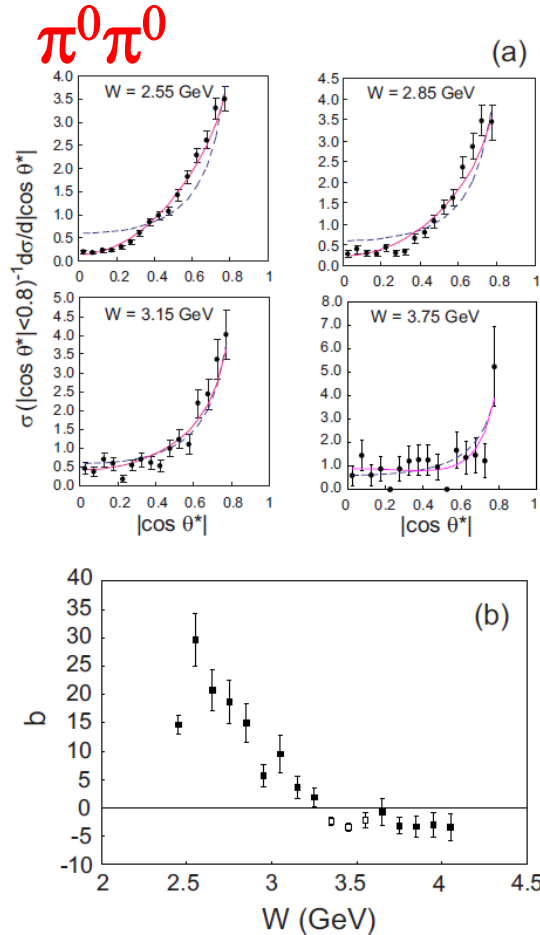
Process	Reference		Int.Lum. (fb <sup>-1</sup> )	$\gamma\gamma$ c.m. Energy (GeV)	Physics covered		
					Light Mesons	QCD	Char- monia
$\pi^+\pi^-$	● PLB 615, 39 (2005)		87.7	2.4 - 4.1		✓	✓
	PRD 75, 051101(R) (2007)		85.9	0.8 - 1.5	✓		
	J. Phys. Soc. Jpn. 76, 074102 (2007)		85.9	0.8 - 1.5	✓		
$K^+K^-$	EPJC 32, 323 (2003)		67	1.4 - 2.4	✓		
	● PLB 615, 39 (2005)		87.7	2.4 - 4.1		✓	✓
$\pi^0\pi^0$	PRD 78, 052004 (2008)		95	0.6 - 4.0	✓		
	● PRD 79, 052009 (2009)		223	0.6 - 4.0	✓	✓	✓
$K^0_S K^0_S$	PLB 651, 15 (2007)		397.1	2.4 - 4.0		✓	✓
	● arXiv:1307.7457[hep-ex] <b>NEW</b>		972	1.05 - 4.0	✓	✓	✓
$\eta\pi^0$	● PRD 80, 032001 (2009)		223	0.84 - 4.0	✓	✓	
$\eta\eta$	● PRD 82, 114031 (2010)		393	1.1 - 4.0	✓	✓	✓

Differential cross section  $d\sigma/d|\cos \theta^*|$  for these reaction processes are measured.



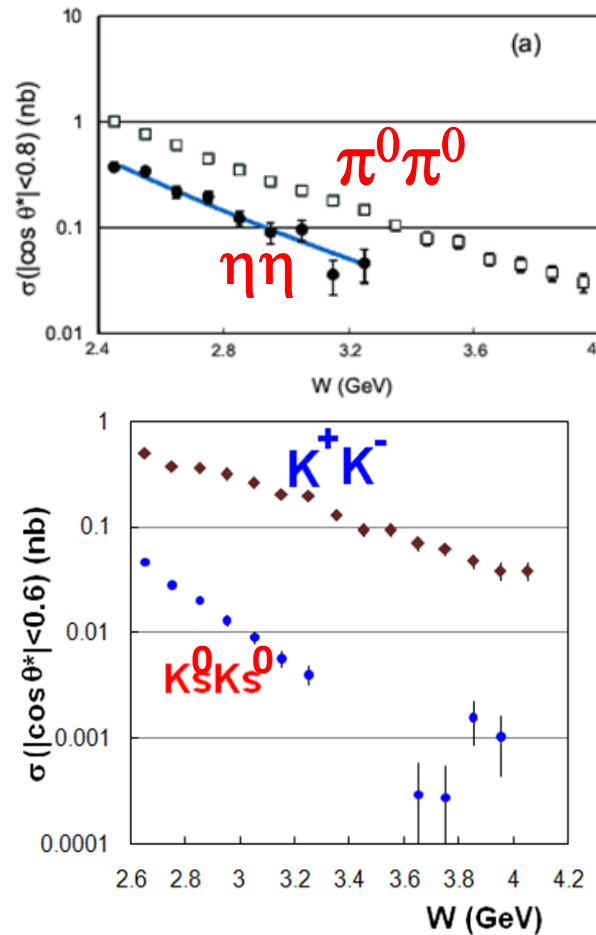
# Three kinds of QCD Studies for 2.4 – 4.1 GeV Region

## Angular dependence



$$d\sigma/d|\cos\theta^*| = a(\sin^{-4}\theta^* + b\cos^2\theta^*).$$

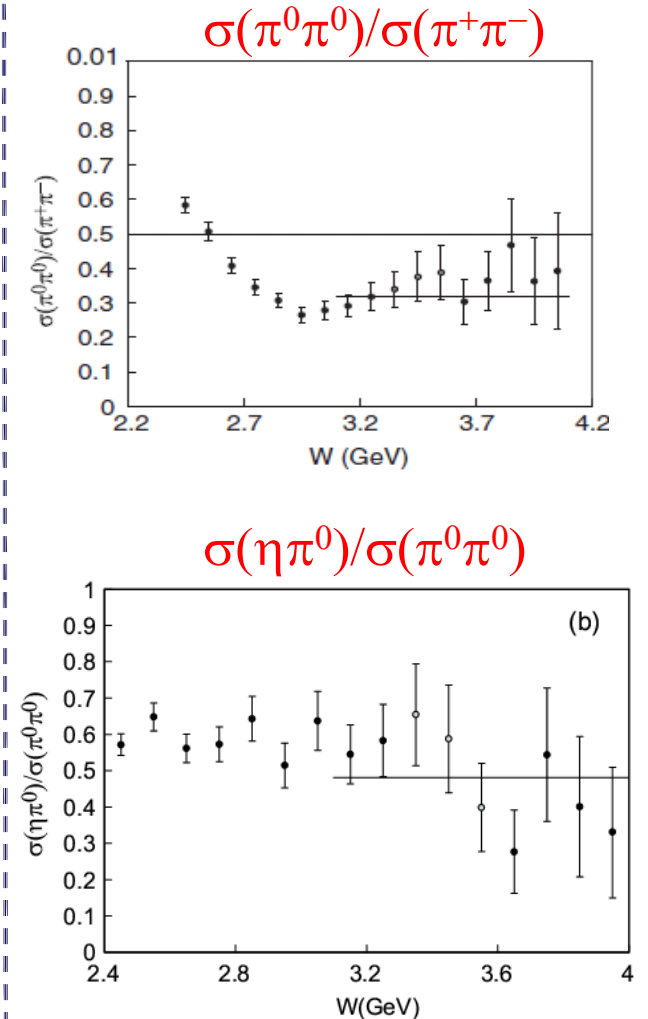
## Energy dependence



## Difference of the slopes

$$\sigma \sim W^{-n}$$

## Cross-section ratio





# Summary of the six channels

mode	$\sin^{-4} \theta^*$	$W$ [GeV]	$ \cos \theta^* $
$\pi^+ \pi^-$	Well matched	3.0 - 4.1	< 0.6
$K^+ K^-$	Well matched	3.0 - 4.1	< 0.6
$K_S^0 K_S^0$	$\sin^{-\alpha} \theta^*$ $\alpha = 3 - 8$	<b>2.6 - 3.3</b>	<b>&lt; 0.8</b>
$\pi^0 \pi^0$	Better agreement with $\sin^{-4} \theta^* + b \cos \theta^*$ Approaches $\sin^{-4} \theta^*$ above 3.1 GeV	2.4 - 4.1	< 0.8
$\eta \pi^0$	Good agreement above 2.7 GeV	3.1 - 4.1	< 0.8
$\eta \eta$	Poor agreement Close to $\sin^{-6} \theta^*$ above 3.0 GeV	2.4 - 3.3	< 0.9

$\sim \sin^{-4} \theta^*$  for charged meson pairs  
is predicted by pQCD

S.J.Brodsky, G.P.Lepage, PRD 24, 1808 (1981)

↓ Cross-section ratio

↑ Angular dependence

↓ Slope parameter

	$\sigma(\pi^0 \pi^0) : \sigma(\eta \pi^0) : \sigma(\eta \eta)$
Theory (Brodsky and Lepage) $\eta$ in SU(3) octet	1 : 0.24 $R_f$ : 0.36 $R_f^2$
Theory $V_p = -18$ deg	1 : 0.46 $R_f$ : 0.62 $R_f^2$
<b>Belle</b>	<b>1 : (0.48 ± 0.06) : (0.37 ± 0.04)</b>

$R_f$ : Squared form factor ratio of  $\eta/\pi^0$

Process	$n$	$W$ range (GeV)	$ \cos \theta^* $ range
$\eta \eta$	$7.8 \pm 0.6 \pm 0.4$	2.4 - 3.3	< 0.8
$\eta \pi^0$	$10.5 \pm 1.2 \pm 0.5$	3.1 - 4.1	< 0.8
$\pi^0 \pi^0$	$8.0 \pm 0.5 \pm 0.4$	3.1 - 4.1 (3.3 - 3.6 excluded)	< 0.8
$K_S^0 K_S^0$	<b><math>11.0 \pm 0.4 \pm 0.4</math></b>	<b>2.6 - 4.0 (3.3 - 3.6 excluded)</b>	<b>&lt; 0.8</b>
$\pi^+ \pi^-$	$7.9 \pm 0.4 \pm 1.5$	3.0 - 4.1	< 0.6
$K^+ K^-$	$7.3 \pm 0.3 \pm 1.5$	3.0 - 4.1	< 0.6

$\sigma(K^+ K^-)/\sigma(\pi^+ \pi^-)$

=  **$0.89 \pm 0.04$ (stat.)  $\pm 0.15$ (syst.)**

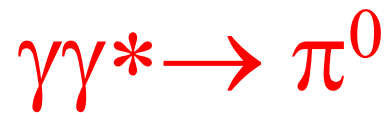
$\sigma(\pi^0 \pi^0)/\sigma(\pi^+ \pi^-) = \mathbf{0.32 \pm 0.03 \pm 0.05}$ ,

$\sigma(K_S^0 K_S^0)/\sigma(K^+ K^-)$

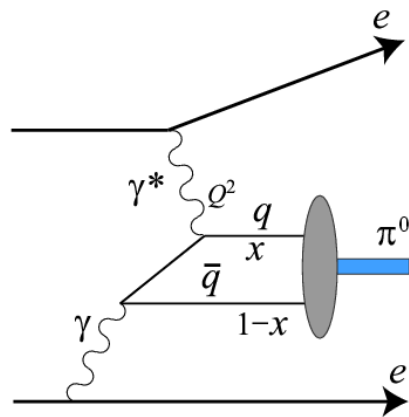
**changes  $\sim 0.10$  to  $\sim 0.03$**



# $\pi^0$ Transition Form Factor



Coupling of neutral pion with two photons  
Good test for QCD at high  $Q^2$



Single-tag  $\pi^0$  production in two-photon process with a large- $Q^2$  and a small- $Q^2$  photon

Theoretically calculated from pion distribution amplitude and decay constant

$$F(Q^2) = \frac{\sqrt{2}f_\pi}{3} \int T_H(x, Q^2, \mu) \phi_\pi(x, \mu) dx$$

Measurement:

$$|F(Q^2)|^2 = |F(Q^2, 0)|^2 = (d\sigma/dQ^2)/(2A(Q^2)) \quad A(Q^2) \text{ is calculated by QED}$$

$$|F(0, 0)|^2 = 64\pi\Gamma_{\gamma\gamma}/\{(4\pi\alpha)^2 m_R^3\}$$

Detects  $e$  (tag side) and  $\pi^0$

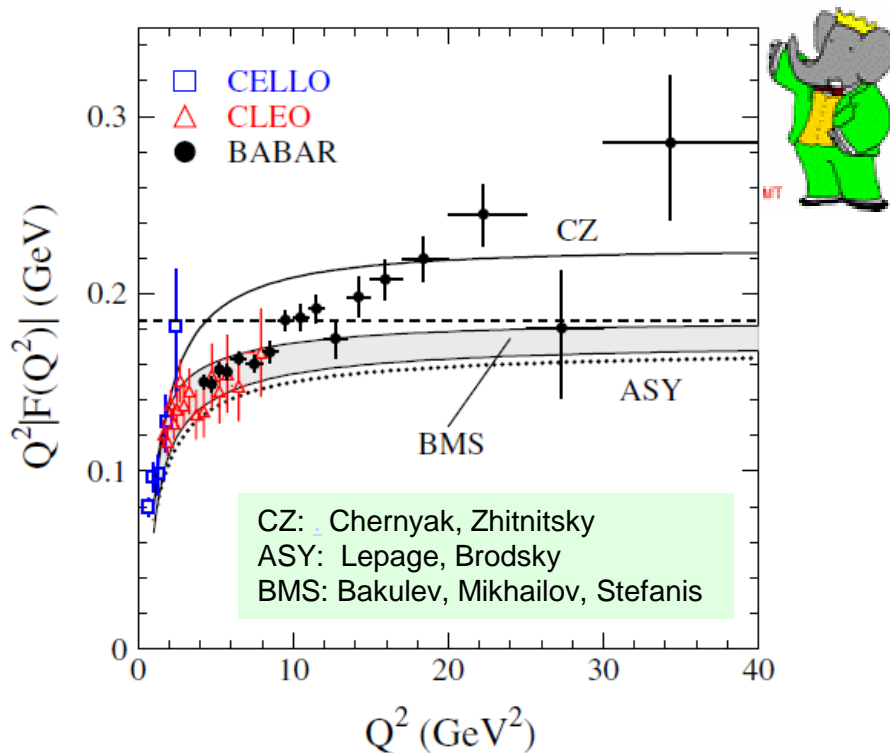
$Q^2 = 2EE'(1 - \cos \theta)$  from energy and polar angle of the tagged electron



# BaBar's Measurement

$\pi^0$  transition form factor (TFF) measured by BaBar is larger than the asymptotic pQCD prediction above  $Q^2 > 10 \text{ GeV}^2$

BaBar, PRD 80, 052009 (2009)  $442 \text{ fb}^{-1}$



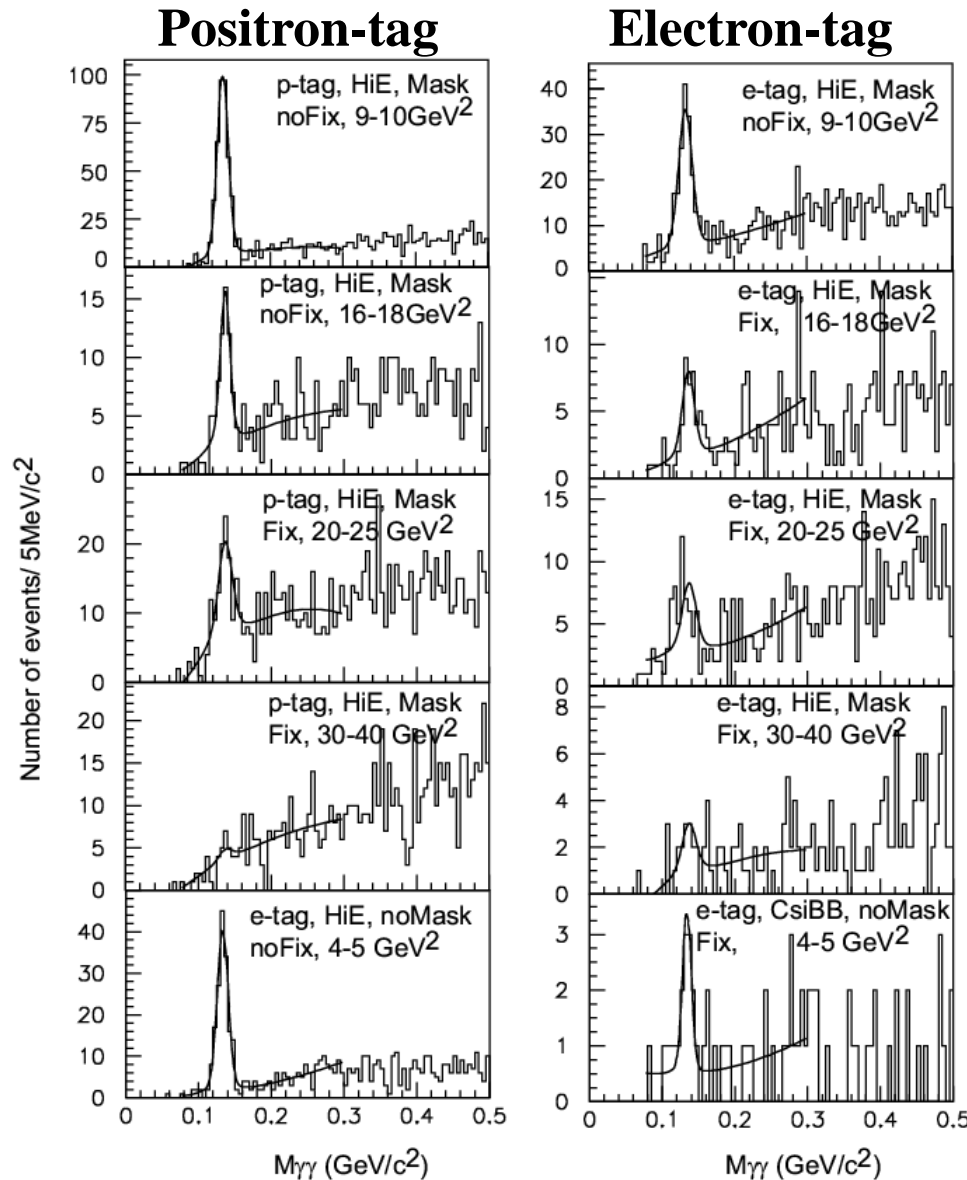
Below  $Q^2 < 8 \text{ GeV}^2$ , the BaBar result supports the CLEO result.

$\eta$  and  $\eta'$  TFFs from BaBar PRD 84, 052001(2011) are consistent with pQCD predictions.

Explanation of this situation for the  $(\pi^0, \eta, \eta')$ -TFF's within standard QCD calculations is difficult.



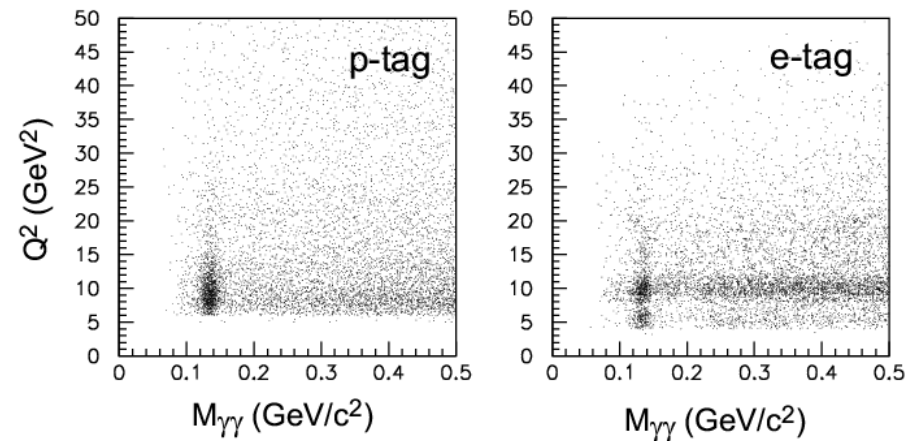
# Belle measurement: Extraction of $\pi^0$ Yield



Int. Luminosity : 759 fb<sup>-1</sup>  
(Larger than BaBar's)

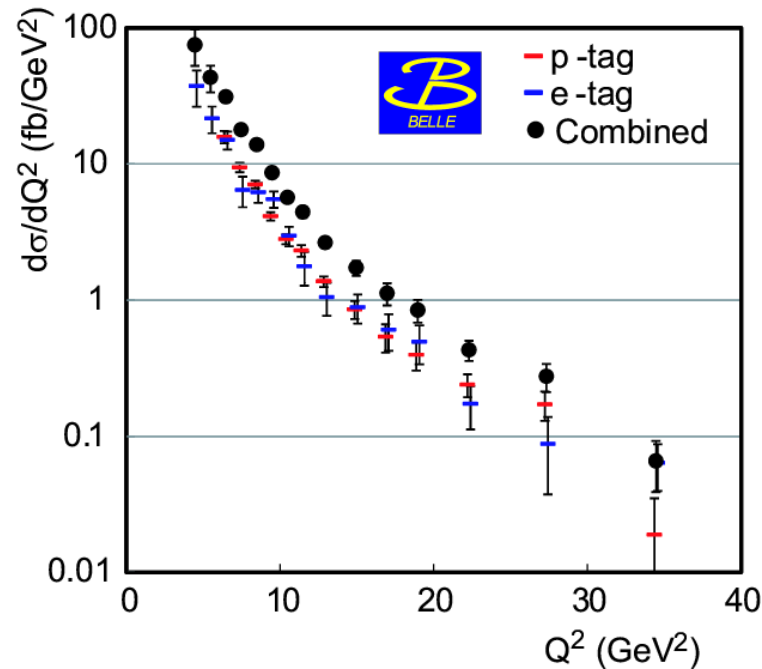
PRD 86, 092007 (2012)

Fit  $M_{\gamma\gamma}$  distribution by  
Double Gaussian (for signal)  
+ 2<sup>nd</sup>-Order Polynomial (for background)  
in each  $Q^2$  bin



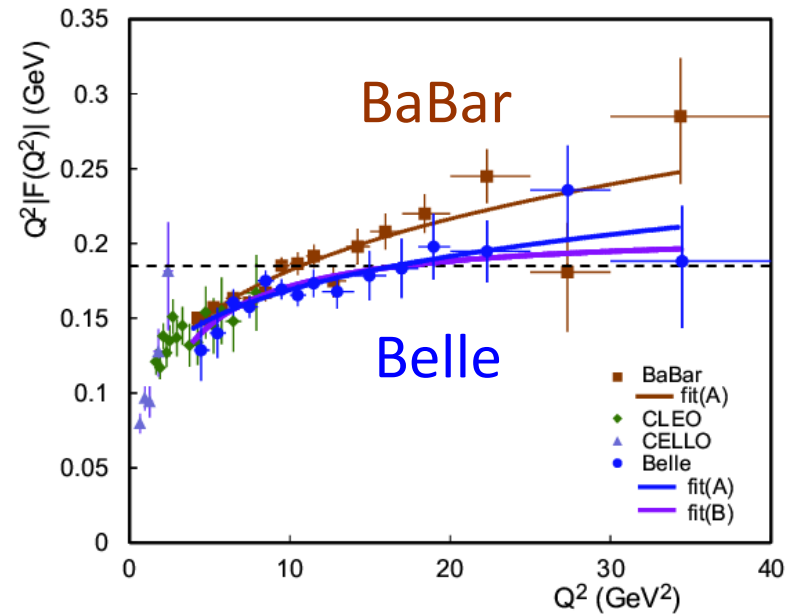
# Belle result

The cross sections from p-tag and e-tag are evaluated, separately, and then combined.



$Q^2_{\max} = 1.0 \text{ GeV}^2$  for the less-virtual photon  
Corrected for  $\sqrt{s} = 10.58 \text{ GeV}$

## $\pi^0$ Transition Form Factor



No rapid growth above  $Q^2 > 9 \text{ GeV}^2$  is seen in Belle result.

$\sim 2.3\sigma$  difference between Belle and BaBar in 9 – 20  $\text{GeV}^2$

Fit with an asymptotic parameter

$$Q^2 |F(Q^2)| = B Q^2 / (Q^2 + C) \quad B = 0.209 \pm 0.016 \text{ GeV}$$

Consistent with the QCD value (0.185 GeV)



# Summary

- **$d\sigma/d|\cos\theta^*|$  of  $\gamma\gamma \rightarrow K_S^0 K_S^0$  is measured for the first time for  $1.05 < W < 2.4$  GeV**
- $f_2(1270)$  and  $a_2(1320)$  interfere indeed destructively
- $f_0(1710)$  is favored over  $f_2(1710)$ ,  $\Gamma_{\gamma\gamma} > O(10 \text{ eV})$  Not likely a pure glueball
- $f_2(2200)$  and  $f_0(2500)$  favored
- **QCD test using measurements of six processes of  $\gamma\gamma \rightarrow \text{meson pair}$**   
(  $\pi^+\pi^-$ ,  $K^+K^-$ ,  $\pi^0\pi^0$ ,  $K_S^0 K_S^0$ ,  $\eta\pi^0$ ,  $\eta\eta$ , for  $W = 2.4 - 4.1$  GeV)
- W-dependence of  $K_S^0 K_S^0$ ,  $n \sim 10$  predicted by pQCD, is confirmed  
 $\pi^+\pi^-$ ,  $K^+K^-$  (  $n=6$  predicted,  $n=7 - 8$  measured)
- Systematic QCD studies using W and angular dependences and cross section ratio of these exclusive processes are now possible
- **Measurement of  $\gamma\gamma^* \rightarrow \pi^0$  transition form factor**
- Steep increase in  $Q^2 > \sim 9 \text{ GeV}^2$  observed by BaBar is not seen by Belle
- Belle result is consistent with the QCD asymptotic value



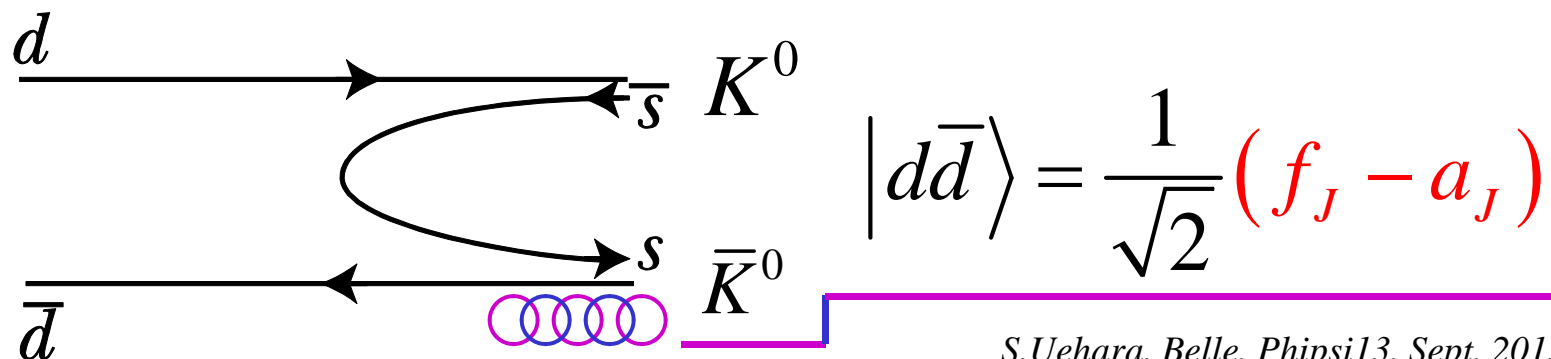
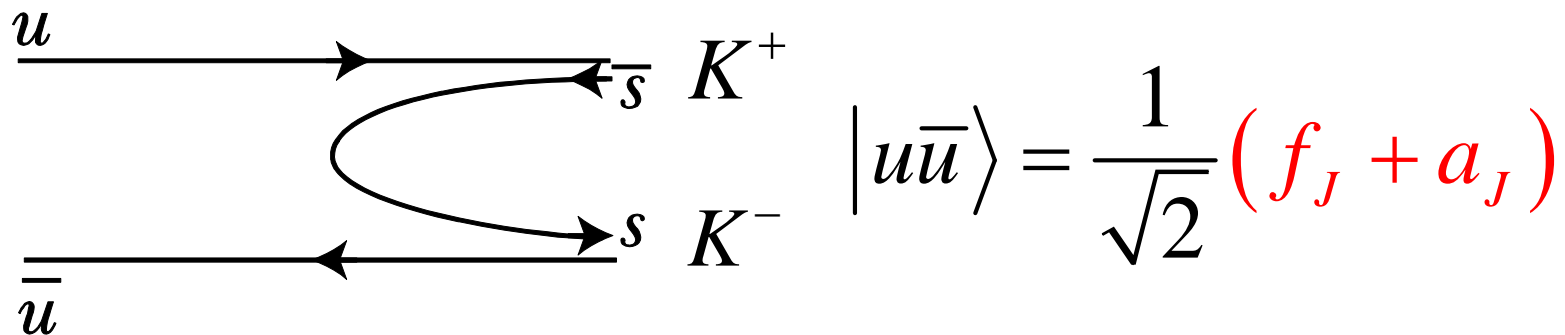
backup





# Nature of $\gamma \gamma \rightarrow R \rightarrow K_s^0 K_s^0$

- $R = f_J$  or  $a_J$  ( $J = \text{even}$ )
- Destructive interference between  $f_J$  and  $a_J$   
 $(|\phi_{a_2} - \phi_{f_2}| = 180^\circ)$ 
  - (D. Faiman, H.J. Lipkin and H.R. Rubinstein, PL 59B,269 (1975))  
 based on OZI (Okubo-Zweig-Iizuka) rule and isospin



# Selection Criteria

## 4 Pions from 2 Ks's

- L4 (filtering) brings non-negligible inefficiency  
(At least 1 track with  $p_t > 0.3\text{GeV}$ ,  $dr < 1\text{cm}$  and  $|dz| < 4\text{cm}$ )
- Trigger restricted in bit#3(ff\_t2oc, **Trigger A**)  
#27(loe\_fs\_o, **Trigger B**)  
#24(hadron\_a=loe\_sss\_tc, **Trigger C**)
- LowMult – 4track (previous page)
- 4 charged pions ( $L(K)/(L(K)+L(\pi)) < 0.8$ ) with  $|\Sigma \mathbf{p}_t| < 0.2 \text{ GeV}/c$
- No neutral pion candidate with  $p_t > 0.1 \text{ GeV}/c$
- Just two Ks candidates with  
z-matching @vertex  $|\Delta z| < p_K[\text{cm}/\text{GeV}/c] + 1.6 \text{ cm}$   
 $\pi\pi$  invariant mass@vertex  $|M_{\pi\pi} - m_K| < 20 \text{ MeV}/c^2$
- Two  $M_{\pi\pi}$  mass conditions:  $|M_{K1} - M_{K2}| < 10 \text{ MeV}/c^2$
- Vertices off IP (**only for  $W > 2\text{GeV}$** ):  
 $r_{vi} > (W - 2\text{GeV}) \times 0.1 \text{ cm}/\text{GeV}$



# Selection Criteria (continued)

---

## The 2 Ks-vertex distances and tr.-momentum relations etc.

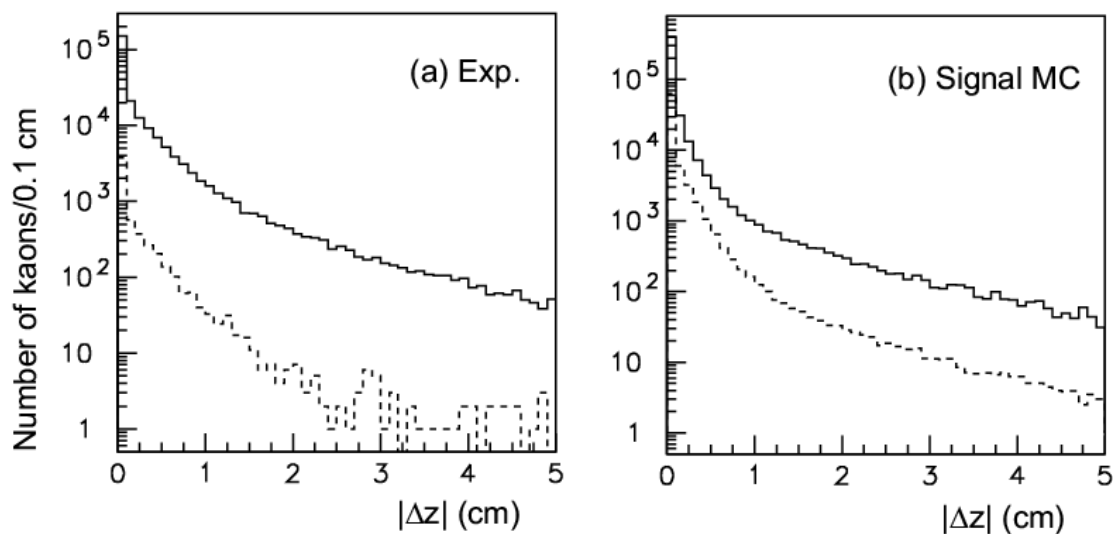
- Distance between the vertices in the  $r\phi$ ,  $dVr > +0.5$  cm  
( $dVr$  has a sign according to the relative momentum of the 2 Ks's)
- 3D distance  $dV > 0.7$  cm OR 2D distance  $dVr > +0.3$  cm
- Projected vertex distance on the relative momentum  $\delta v < 0.7$  cm  
$$\delta_V = |(r_{V2} - r_{V1}) \times (p_{t2} - p_{t1})| / |p_{t2} - p_{t1}| = |r_{V2} - r_{V1}| \sin \Delta\varphi$$
- $|\Sigma p_t(Ks)| < 0.1$  GeV/c
- Refined cut for the Ks mass  
 $|\langle M_K \rangle - m_K| < 5$  MeV/c<sup>2</sup>,  $\langle M_K \rangle = (M_{K1} + M_{K2})/2$
- ECL total energy cut  
 $E_{ECL} < E_{K1} + E_{K2} - 0.3$  GeV,  $E_{Ki}$  – Ks's total energy calculated from its lab. momentum



# Ks Selection

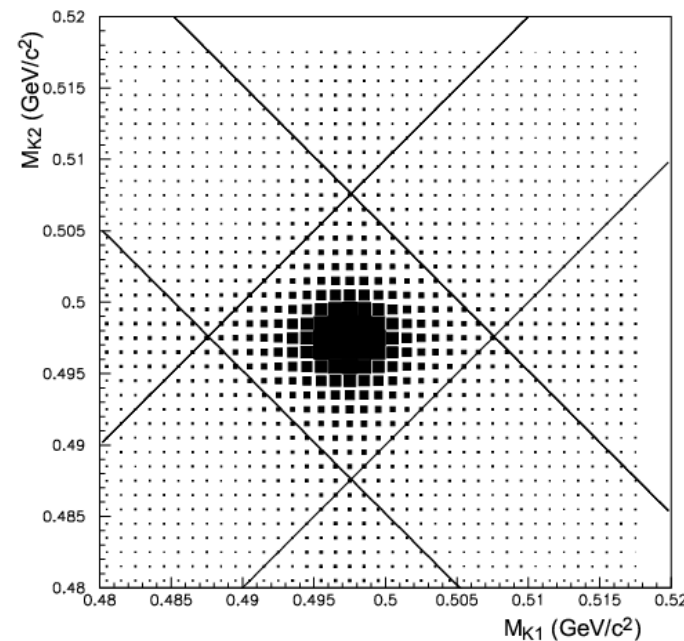
## Final KsKs mass-cut

z-matching @vertex



Solid :  $W < 2.5$  GeV

Dashed:  $W > 2.5$  GeV



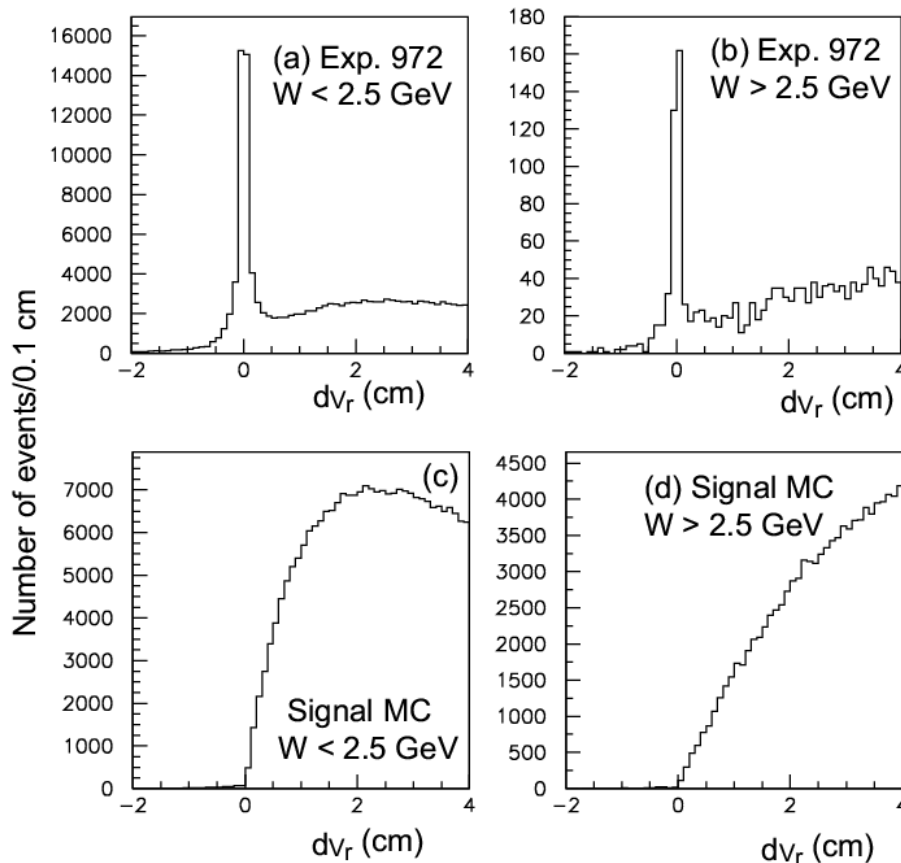
Ks-mass vs Ks-mass

Cut lines for  
 $[M(Ks1) + M(Ks2)]/2$   
and  
 $|M(Ks1) - M(Ks2)|$



# Ks Ks vertex distances

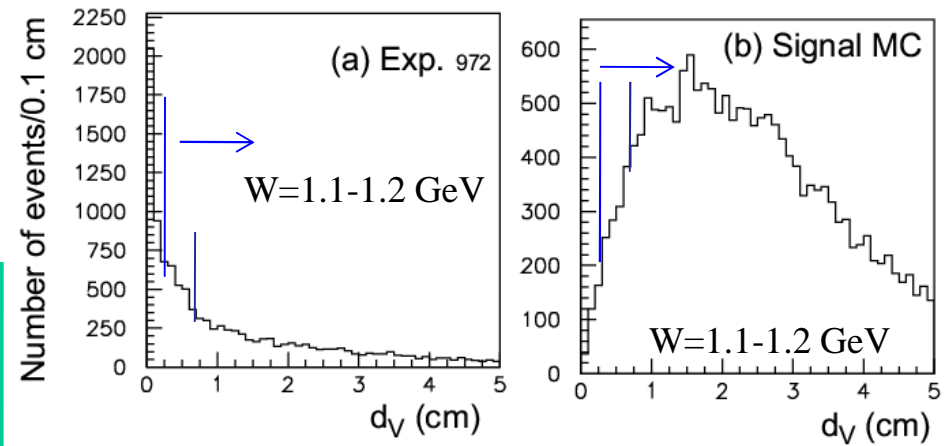
## 2D vertex distance



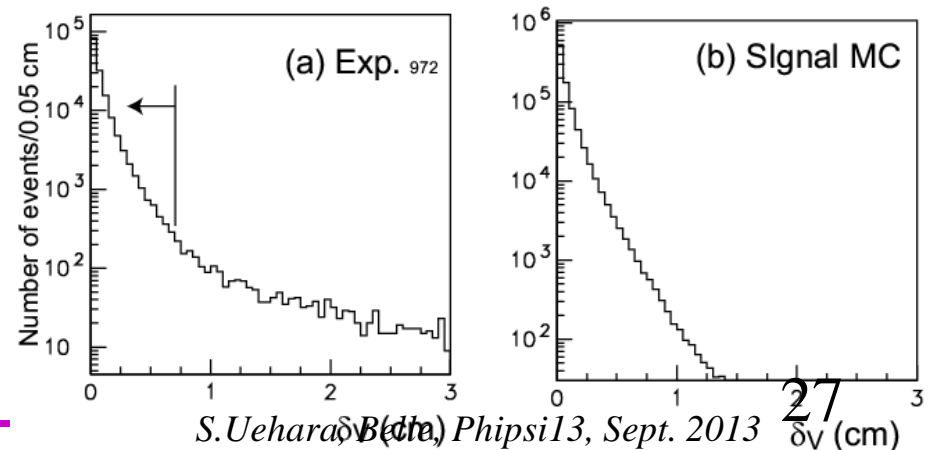
Sharp peaks near 0cm seen only in Exp. are from Direct  $4\pi$  ( $\pi^+\pi^-\pi^+\pi^-$ ) production backgrounds.



## 3D vertex distance



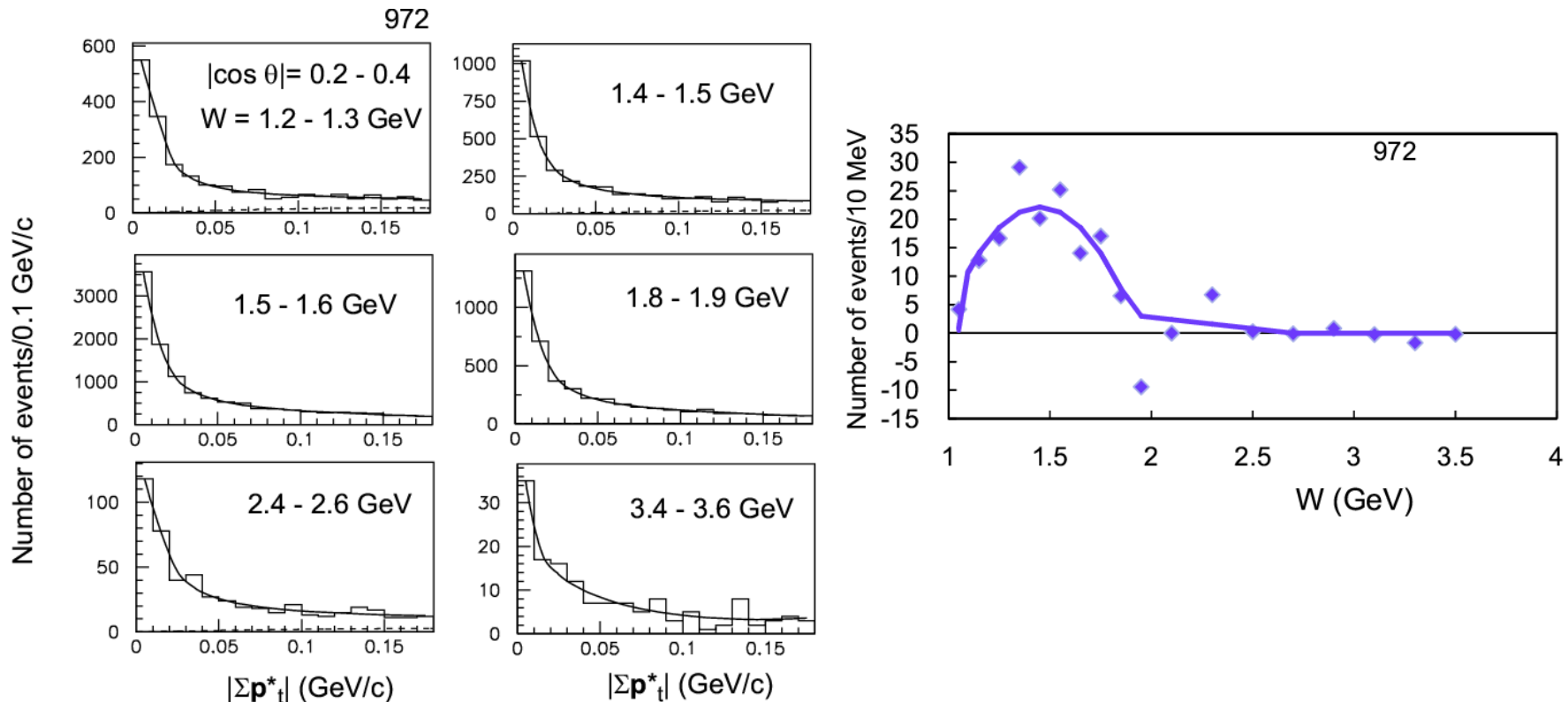
Tr. momentum diff. and vertex position diff. must be in parallel



# Background Subtractions (1)

## - Non-exclusive (KsKsX) backgrounds

Estimated from a fit of  $|\Sigma \mathbf{p}_t^*|$  distributions



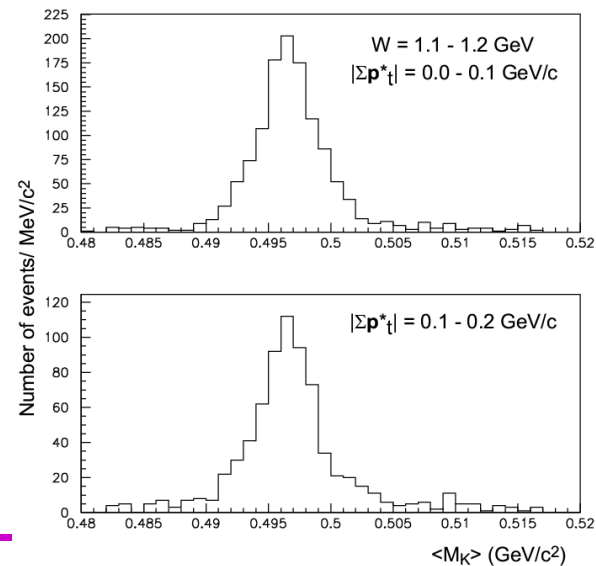
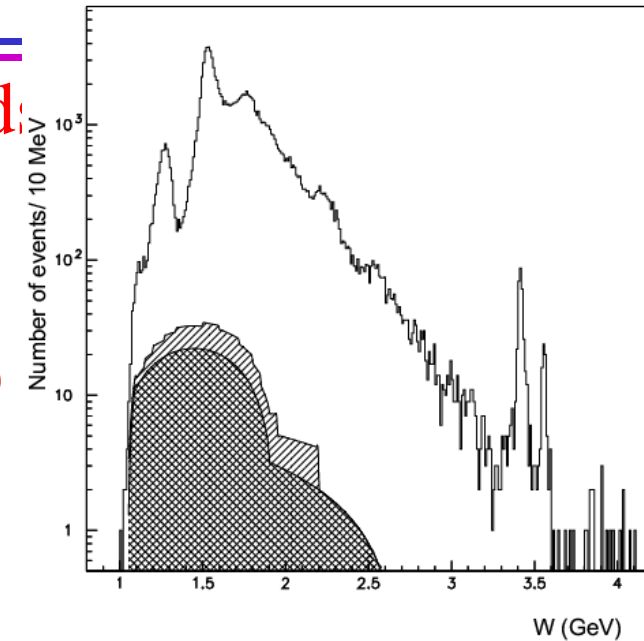
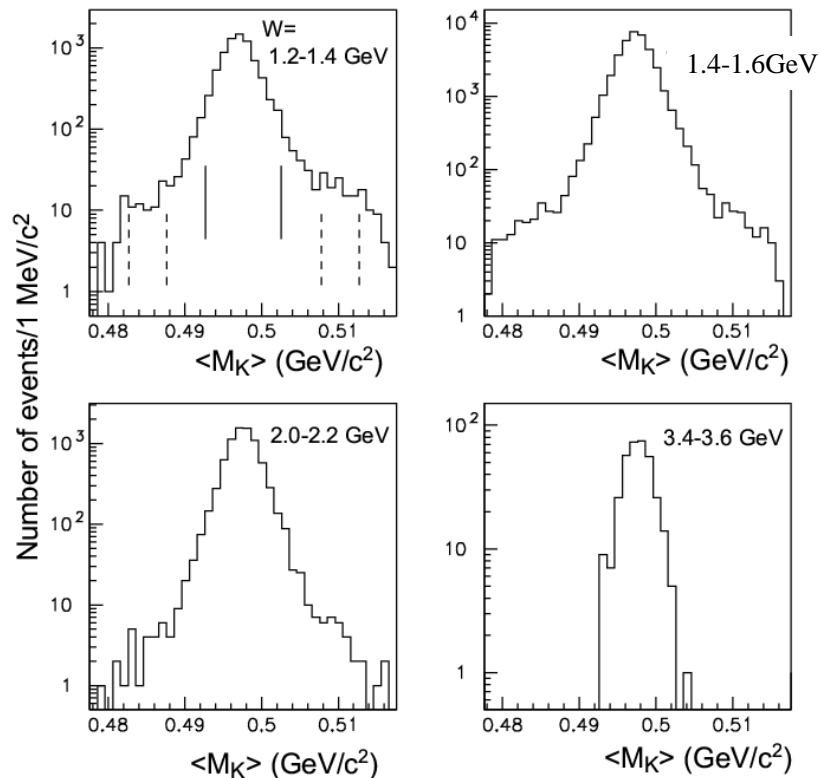
# Background Subtractions (2)

## - Non-Ks Ks(4 $\pi$ -process) background

Estimated from  $\langle M_K \rangle$  sideband

Non-exclusive and non-Ks

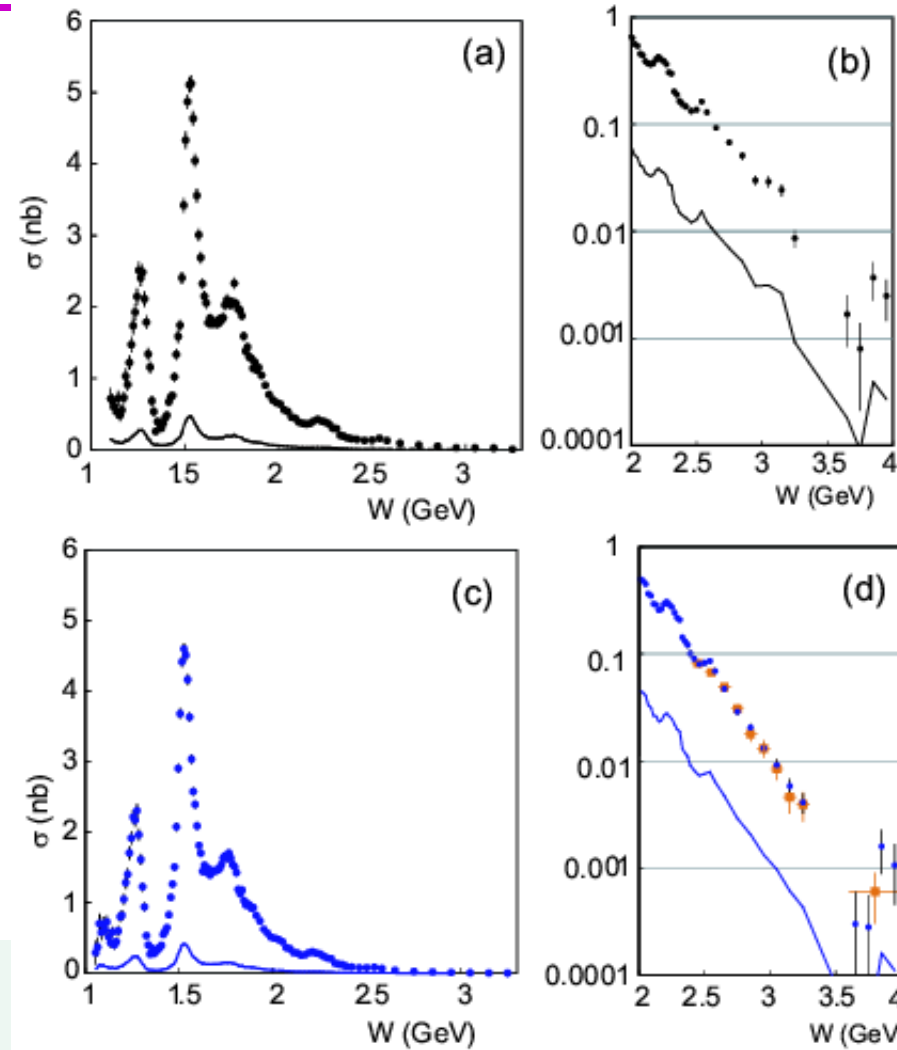
--- very small (typically  $\sim 1\%$  level)





# Cross section integrated over the angle

$$\int \frac{d\sigma}{d|\cos\theta^*|} d|\cos\theta^*|$$



Black ---  $|\cos\theta^*| < 0.8$

Blue ---  $|\cos\theta^*| < 0.6$

Orange ---  $|\cos\theta^*| < 0.6$  (previous, Belle 2007)



# Systematic errors

Source	Uncertainty(%)
Tracking efficiency (for 4 tracks)	2
Beam background effect	1
Pion identification (for 4 tracks)	2
Non-exclusive and four-pion backgrounds	2 – 19
Geometrical coverage and fit uncertainty	4
$K_S^0 K \pi$ background subtraction	1 – 2
$K_S^0$ -pair reconstruction	5 – 3
Trigger efficiency	5 – 7
$E_{ECL}$ cut	1
Integrated luminosity and luminosity function	5 – 4
L4 efficiency	1 – 10
Total	9 – 25, typically 10

From correlation study of different Exp# settings in data and signal MC

A Half of the subtraction + 2% from pt-fit (quad.sum)

Loose-cut sample

Correlation of the two triggers

About 10% of the inefficiency



# Hat amplitudes

---

- We rewrite

$$\frac{d\sigma}{d\Omega} = \hat{S}^2 |Y_0^0|^2 + \hat{D}_0^2 |Y_2^0|^2 + \hat{G}_0^2 |Y_4^0|^2 + \hat{D}_2^2 |Y_2^2|^2 + \hat{G}_2^2 |Y_4^2|^2$$

- $|Y_J^m|^2$  are mutually independent

→ obtain “hat amplitudes”:

$$\hat{S}^2, \hat{D}_0^2, \hat{G}_0^2, \hat{D}_2^2, \hat{G}_2^2 \text{ and } \hat{D}_2^2$$

through fitting  $d\sigma/d\Omega$

- They contain interference terms
- Yet, they convey useful information on partial waves



# $W < 2 \text{ GeV} : f_0(1710)$ assumption

---

- Parameterization

$$S = f_0(1710)e^{i\phi_{f_0}} + B_S$$

$$D_0 = B_{D_0}e^{i\phi_{D_0}}$$

$$D_2 = f_2(1270) + a_2(1320)e^{i\phi_{a_2}} + f_2'(1525)e^{i\phi_{f_2'}} \\ + B_{D_2}e^{i\phi_{D_2}}$$

$$B_L = \beta^{2L+1}(aW'^2 + bW' + c) \text{ (bgd amplitude)}$$

$$\beta = \sqrt{1 - \frac{4m_K^2}{W^2}}, \quad W' = W - W_{\text{th}}$$

- Fix param. of  $f_2(1270)$  and  $a_2(1320)$ . Free  $f_2'(1525)$
- Then fit  $d\sigma/d\Omega$  (20 free param.)
- phases in  $D_2$  are relative to  $f_2(1270)$



# Systematic uncertainties

+  $\sigma$  and - $\sigma$

Source	Fit-H						Fit-L					
	Mass (MeV/ $c^2$ )		$\Gamma_{\text{tot}}$ (MeV)		$\Gamma_{\gamma\gamma}\mathcal{B}(KK)$ (eV)		Mass (MeV/ $c^2$ )		$\Gamma_{\text{tot}}$ (MeV)		$\Gamma_{\gamma\gamma}\mathcal{B}(KK)$ (eV)	
$W$ -range	21	0	0	-15	0	-1	16	0	5	-13	6	-4
$W$ bias	2	-2	6	-5	2	-1	4	0	2	-7	2	-4
Efficiency	8	-4	25	-30	209	0	11	-5	0	-28	2	-12
Overall normalization	4	-2	9	-11	1	-2	7	-2	4	-16	5	-8
$\cos\theta^*$ bias	0	-1	3	-1	1	0	4	0	2	-7	2	-4
$B_S$	5	-7	84	-9	87	-2	7	0	26	-3	35	-2
$B_{D0}$	0	0	1	0	0	0	1	0	0	-2	0	-1
$B_{D2}$	1	0	0	-1	0	0	11	-37	4	-11	1	-2
Mass( $f_2(1270)$ )	3	-1	6	-6	1	-1	3	0	0	-4	2	-3
$\Gamma_{\text{tot}}(f_2(1270))$	0	0	1	0	0	0	2	-2	5	-4	4	-2
$\mathcal{B}(\gamma\gamma)(f_2(1270))$	7	0	12	-18	2	-4	6	-1	0	-17	5	-10
$r_R$	0	0	0	0	0	0	1	-1	1	0	1	0
Mass( $a_2(1320)$ )	1	0	0	-2	0	0	2	0	0	-2	0	-1
$\Gamma_{\text{tot}}(a_2(1320))$	2	-2	7	-5	2	-1	3	0	2	-9	3	-6
$\mathcal{B}(\gamma\gamma)(a_2(1320))$	1	-1	2	0	1	0	2	0	0	-2	0	-1
Mass( $f'_2(1525)$ )	2	-2	1	0	1	0	1	-1	3	-4	3	-3
$\Gamma_{\text{tot}}(f'_2(155))$	2	-2	4	-3	2	-1	4	0	0	-4	0	-2
$\mathcal{B}(\gamma\gamma)(f'_2(1525))$	14	0	0	-24	0	-4	14	-18	14	-27	9	-12
$\phi_{f'_2(1525)}$	4	-15	33	-12	22	-3	4	-5	0	-17	3	-11
$\phi_{a_2(1320)}$	4	-1	5	-8	1	-2	3	0	0	-4	0	-2
Total	29	-18	96	-50	227	-8	31	-42	31	-54	38	-26

# Resonances in M=1.7 – 2.4 GeV (from PDG2012)

Parameter	$f_0(1710)$	$a_2(1700)$	$f_2(1810)$
Mass (MeV/ $c^2$ )	$1720 \pm 6$	$1732 \pm 16$	$1815 \pm 12$
$\Gamma_{\text{tot}}$ (MeV)	$135 \pm 8$	$194 \pm 40$	$197 \pm 22$
$f_J/a_J \rightarrow K\bar{K}$	seen	seen	unknown
$f_J/a_J \rightarrow \gamma\gamma$	unknown	unknown	seen

Parameter	$f_0(2200)$	$f_2(2300)$	$f_4(2300)$
Mass (MeV/ $c^2$ )	$2189 \pm 13$	$2297 \pm 28$	$\sim 2300$
$\Gamma_{\text{tot}}$ (MeV)	$238 \pm 50$	$149 \pm 41$	$250 \pm 80$
$f_J \rightarrow K\bar{K}$	seen	seen	seen
$f_J \rightarrow \gamma\gamma$	unknown	seen	unknown

**$f_J(2220)$**

$$J^{G(J^{PC})} = 0^{+}(2^{++} \text{ or } 4^{++})$$

OMITTED FROM SUMMARY TABLE

Needs confirmation. See our mini-review in the 2004 edition of this Review, PDG 04.

### $f_J(2220)$ MASS

VALUE (MeV)	EVTS	DOCUMENT ID	TECN	COMMENT
<b><math>2231.1 \pm 3.5</math></b>	<b>OUR AVERAGE</b>			

### $f_J(2220)$ WIDTH

VALUE (MeV)	CL%	EVTS	DOCUMENT ID	TECN	COMMENT
<b><math>23_{-7}^{+8}</math></b>		<b>8</b>			<b>OUR AVERAGE</b>

# Fitting the region $W > 2 \text{ GeV}$

---

- Parameterization

$$i\text{-wave} = B.W. e^{i\phi_i} + B_i$$

$$B_i = b_i \left( \frac{W}{W_0} \right)^{-c_i} e^{i\phi_i}$$

(assume power behavior

for non-resonant background:

$i = S, D_0, D_2$  and  $G_2$ ; (we assume  $G_0=0$ ))

B.W.=  $f_J(2200)$  and/or  $f_J(2500)$  with  $J=0, 2$  and  $4$

- Then fit  $d\sigma/d\Omega$  (typically 16 free parameters)

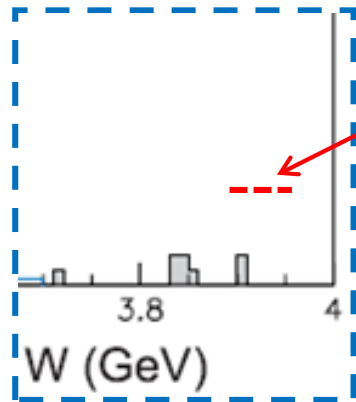




# Fit results for 13 assumptions

Assumption	No. of sol.	$\chi^2$	<i>ndf</i>
$f_0-f_0$	2	293.3, 293.9	214
$f_0-f_2$	4	320.9, 321.9, 324.5, 327.6	214
$f_0-f_4$	1	291.4	214
$f_2-f_0$	1	228.3	214
$f_2-f_2$	1	260.4	214
$f_2-f_4$	1	323.6, 306.7	214
$f_4-f_0$	1	411.6	214
$f_4-f_2$	2	468.6, 472.1	214
$f_4-f_4$	4	459.6, 464.1, 466.4, 467.5	214
Only- $f_0$	1	390.0	218
Only- $f_2$	1	323.6	218
Only- $f_4$	1	518.7	218
No resonances	1	659.32	222

# Upper limit for $\chi_{c2}(2P) \rightarrow K_S K_S$



$\chi_{c2}(2P)$

$$1^G(J^{PC}) = 0^+(2^{++})$$

Mass  $m = 3927.2 \pm 2.6$  MeV

Full width  $\Gamma = 24 \pm 6$  MeV

$\chi_{c2}(2P)$ DECAY MODES	Fraction ( $\Gamma_i/\Gamma$ )	$p$ (MeV/c)
$\gamma\gamma$	seen	1964
$D\bar{D}$	seen	615
$D^+D^-$	seen	600
$D^0\bar{D}^0$	seen	615

We use a counting method

No knowledge for  $\chi_{c0}(2P)$   $\Gamma_{\gamma\gamma}(\chi_{c2}(2P))\mathcal{B}(\chi_{c2}(2P) \rightarrow K_S^0 K_S^0) < 0.064$  eV

2 events in  $M \pm 2\Gamma$

90% CL UL,  $1\sigma$  of syst.err. shifted

$N^{\text{UL}} = 5.32$  for 90%CL  
without assuming interference

$\Gamma_{\gamma\gamma}(\chi_{c2}(2P))$  is not known, but conjectured  
to be around 500 eV

Poisson( $\mu=5.32$ ;  $n \leq 2$ ) = 0.10



# Upper limit for $X(3915) \rightarrow K_S K_S$

PDG2013  $X(3915) = \chi_{c0}(2P)$

$\chi_{c0}(2P)$   
was  $X(3915)$

$$J^{PC} = 0^{+}(0^{++})$$

Same counting as that in  $\chi_{c2}(2P)$

2 events in  $M \pm 2\Gamma$

$N^{UL} = 5.32$  for 90% CL

without assuming interference

Same method, the same events

Almost same  $M$  and  $\Gamma$

Spin and angular distribution are different.

$$\Gamma_{\gamma\gamma}(\chi_{c0}(2P))\mathcal{B}(\chi_{c0}(2P) \rightarrow K_S^0 K_S^0) < 0.49 \text{ eV} \quad \text{90\% CL UL, } 1\sigma \text{ of syst.err. shifted}$$

## $\chi_{c0}(2P)$ MASS

VALUE (MeV)	EVTS	DOCUMENT ID	TECN	COMMENT
<b>3918.4 ± 1.9 OUR AVERAGE</b>				
[3917.5 ± 2.7 MeV OUR 2012 AVERAGE]				
3919.4 ± 2.2 ± 1.6	59 ± 10	LEES	12AD BABR	$e^+e^- \rightarrow e^+e^-\omega J/\psi$
3919.1 <sup>+</sup> <sub>−</sub> 3.8 ± 2.0		DEL-AMO-SA..10B	BABR	$B \rightarrow \omega J/\psi K$
3915 ± 3 ± 2	49 ± 15	UEHARA	10 BELL	10.6 $e^+e^- \rightarrow e^+e^-\omega J/\psi$
3943 ± 11 ± 13	58 ± 11	<sup>1</sup> CHOI	05 BELL	$B \rightarrow \omega J/\psi K$
••• We do not use the following data for averages, fits, limits, etc. •••				
3914.6 <sup>+</sup> <sub>−</sub> 3.8 ± 2.0		<sup>1</sup> AUBERT	08W BABR	Superseded by DEL-AMO-SANCHEZ 10B

<sup>1</sup> $\omega J/\psi$  threshold enhancement fitted as an S-wave Breit-Wigner resonance.

## $\chi_{c0}(2P)$ WIDTH

VALUE (MeV)	EVTS	DOCUMENT ID	TECN	COMMENT
<b>20 ± 5 OUR AVERAGE</b>				
Error includes scale factor of 1.1. [27 ± 10 MeV OUR 2012 AVERAGE Scale factor = 1.4]				
13 ± 6 ± 3	59 ± 10	LEES	12ADBABR	$e^+e^- \rightarrow e^+e^-\omega J/\psi$
31 <sup>+</sup> <sub>−</sub> 10 ± 5		DEL-AMO-SA..10B	BABR	$B \rightarrow \omega J/\psi K$
17 ± 10 ± 3	49 ± 15	UEHARA	10 BELL	10.6 $e^+e^- \rightarrow e^+e^-\omega J/\psi$
87 ± 22 ± 26	58 ± 11	<sup>2</sup> CHOI	05 BELL	$B \rightarrow \omega J/\psi K$
••• We do not use the following data for averages, fits, limits, etc. •••				



# Upper limit for $\eta_c \rightarrow K_S K_S$

P and CP violating process

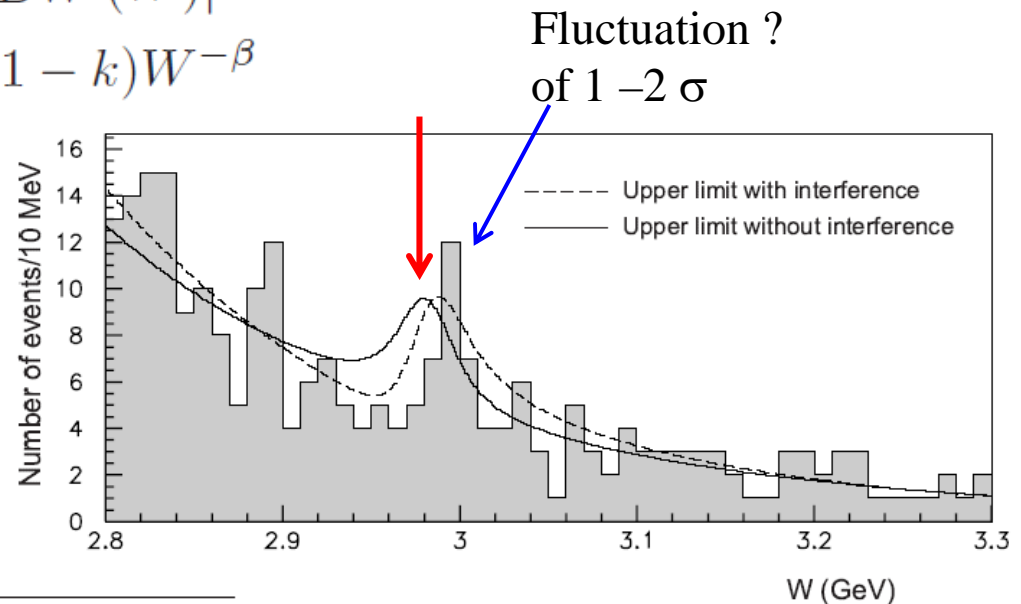
(PDG2012  $BF < 4.2 \times 10^{-4}$ )

Fit function

$$Y(W) = |\sqrt{\alpha k} W^{-\beta} + e^{i\phi} \sqrt{N} BW(W)|^2 + \alpha(1-k)W^{-\beta}$$

$N^{UL} = 15 (= 85)$  for 90%CL  
without (with) interference  
 $k=0 (0 < k \leq 1)$

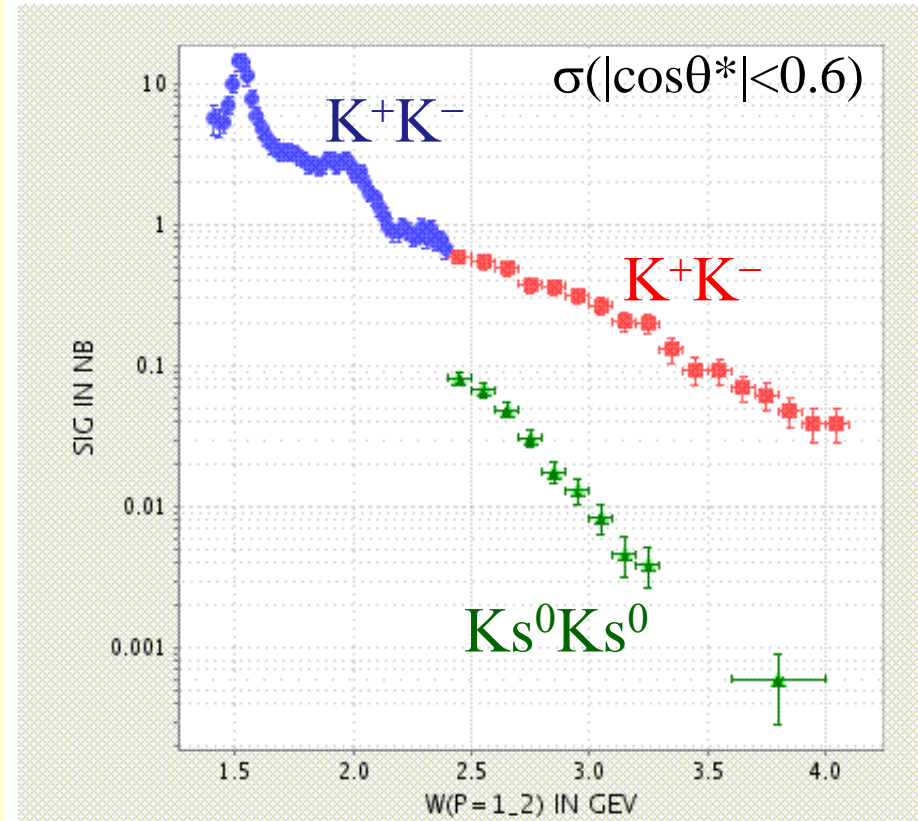
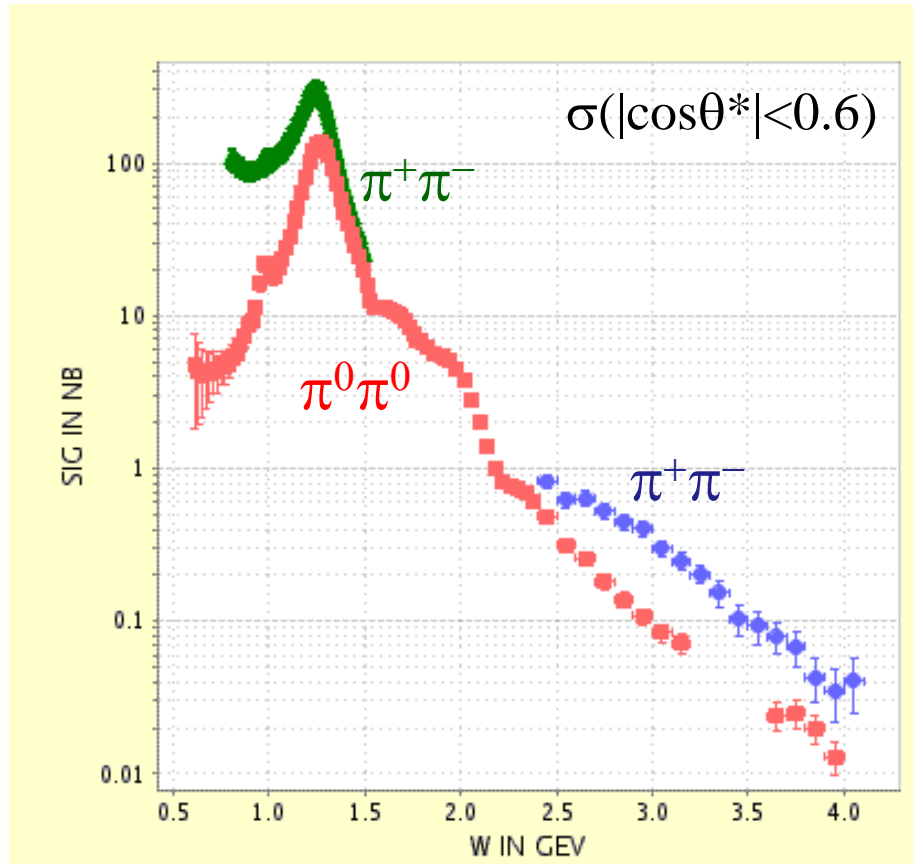
$k$  cannot be determined by the fit



Interference	$\Gamma_{\gamma\gamma} \mathcal{B}(\eta_c)$	$\mathcal{B}(\eta_c \rightarrow K_S^0 K_S^0)$	
Without	$< 0.29 \text{ eV}$	$< 5.6 \times 10^{-5}$	90% CL
With	$< 1.6 \text{ eV}$	$< 3.2 \times 10^{-4}$	90% CL

90% CL:  $UL \Delta(2\ln L) = (1.64)^2$   
floating the other parameters  
 $1\sigma$  of syst.err. shifted  
 $\rightarrow$ World severest upper limits

# Cross sections integrated over angle

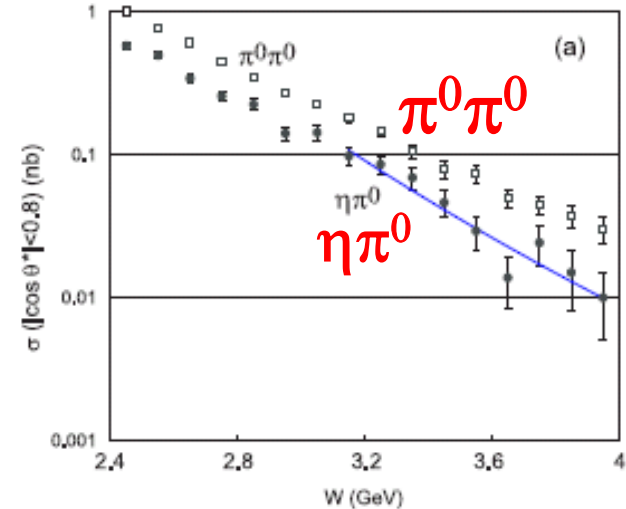
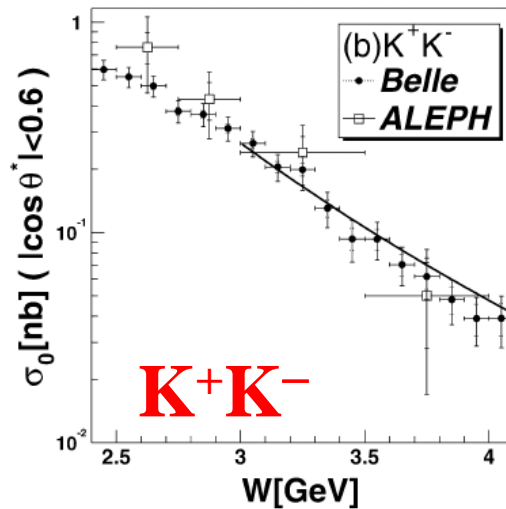
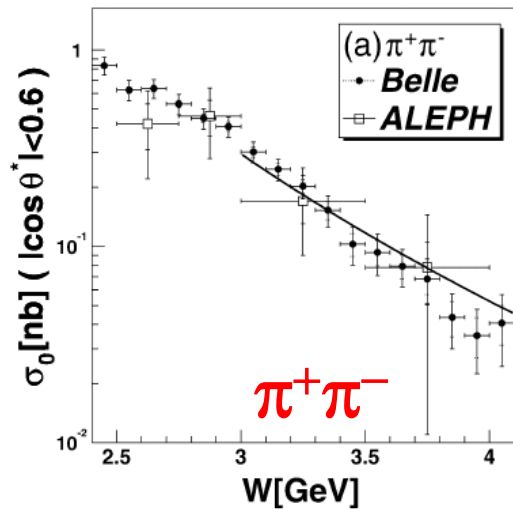


♠ Those for  $\eta\pi^0$  and  $\eta\eta$  are shown in other slides



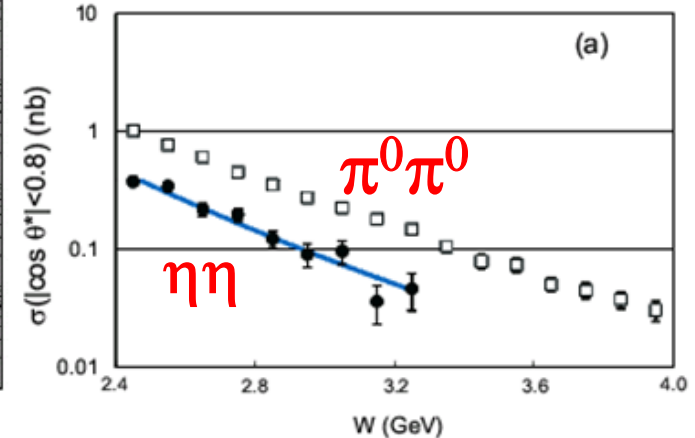
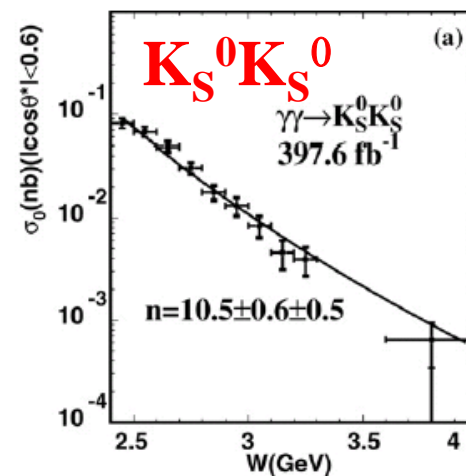
# W-dependences at high energies

Assume or expect  $\sigma(W) \sim W^{-n}$



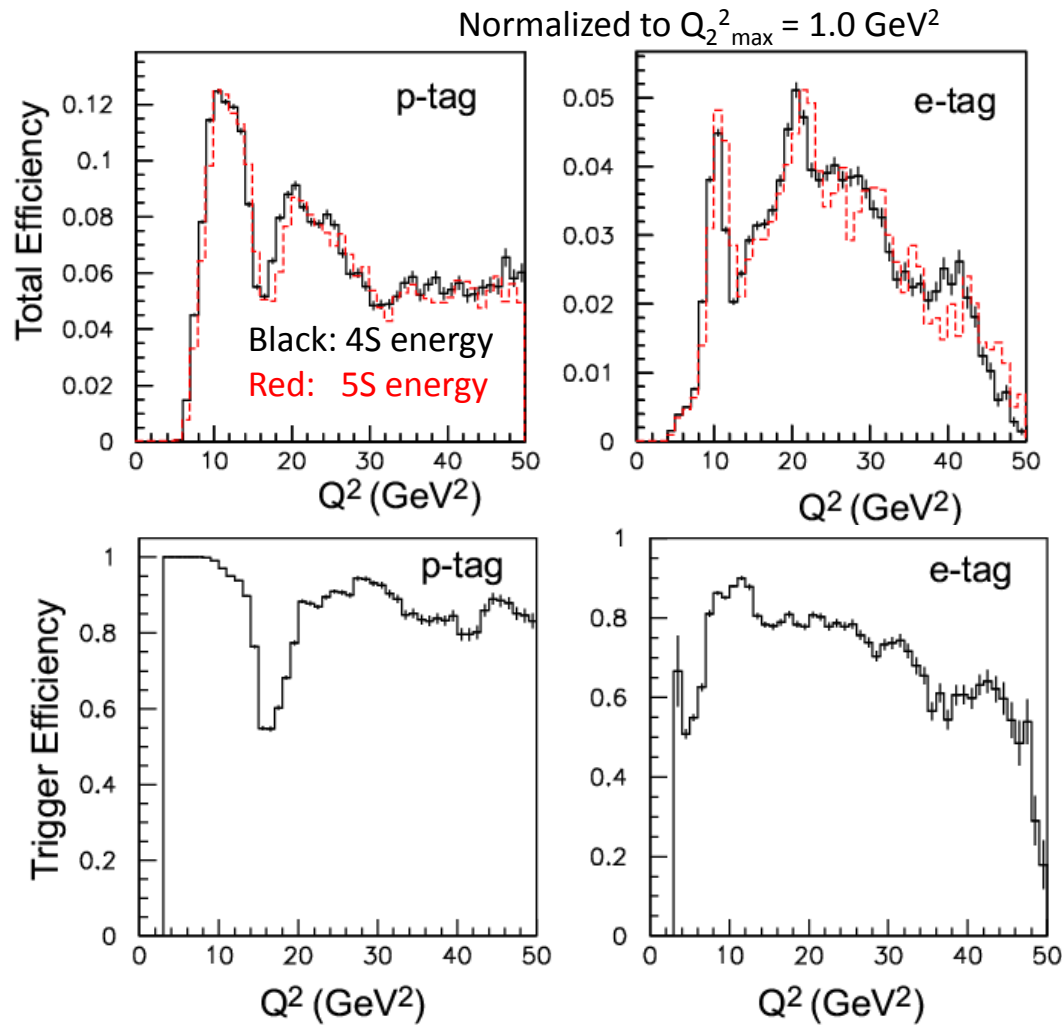
Fitted and reproduced  
Slope parameter **n** different  
among the reactions

Charmonium contributions  
not included/removed



# Efficiency for the Signal Process at Belle

Efficiency determined by MC  
(twice of BaBar's definition)



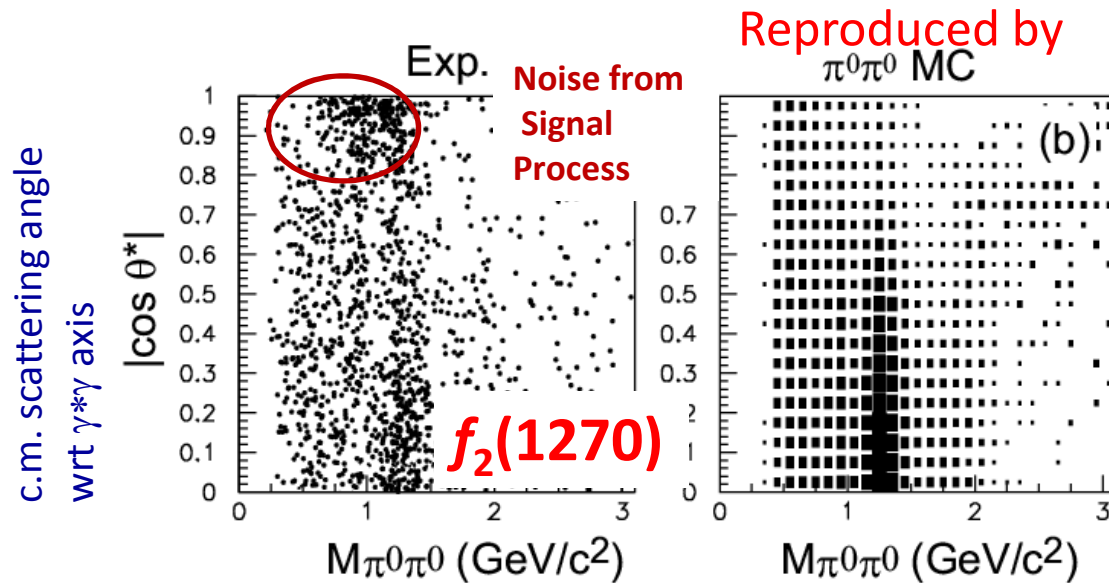
Up-down structures in the efficiencies are due to Bhabha-veto trigger condition correlated in the  $(\cos\theta_e, \cos\theta_{\gamma\gamma})$  plane

The trigger efficiency is defined for the acceptance after the selection



# $\pi^0\pi^0$ background MC

Experimentally identified  $\gamma\gamma^* \rightarrow \pi^0\pi^0$



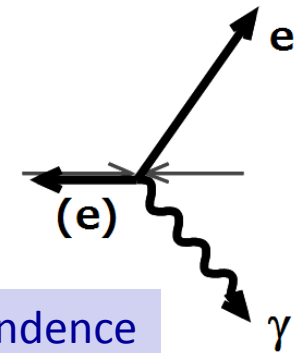
Background contamination in signal is estimated by the  $\pi^0\pi^0$  background MC which is normalized to the observation, as 2%



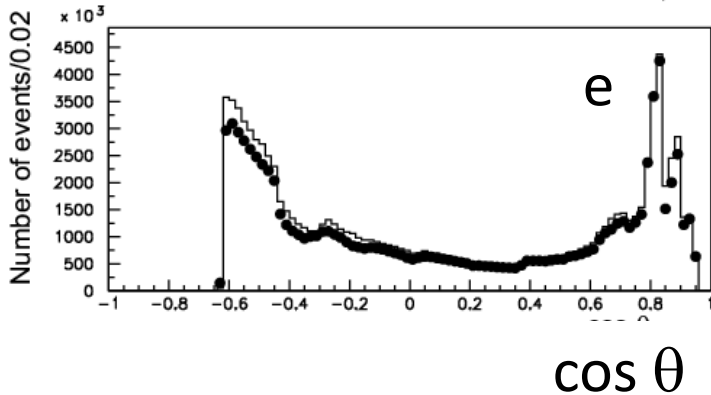
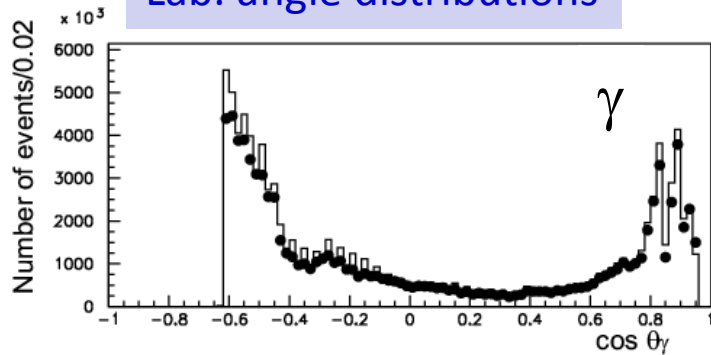


# Calibrations using Radiative-Bhabha (VC) Events

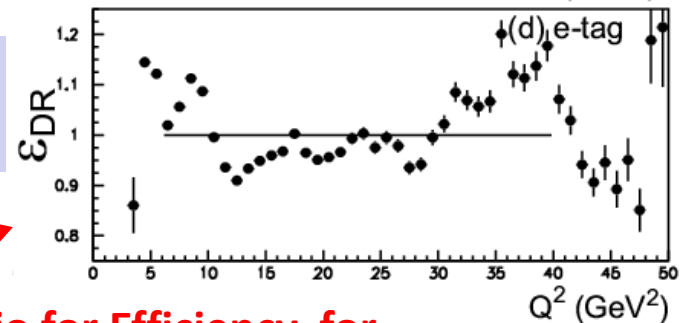
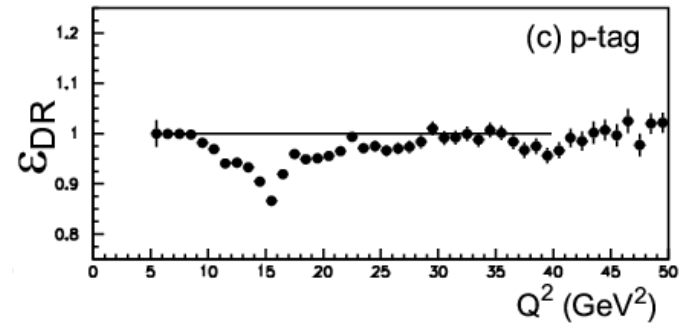
Bhabha-veto threshold is measured in real data  
of **Virtual-Compton process of (e)ey**  
and is tuned in Trigger Simulator



Lab. angle distributions



Q<sup>2</sup> dependence

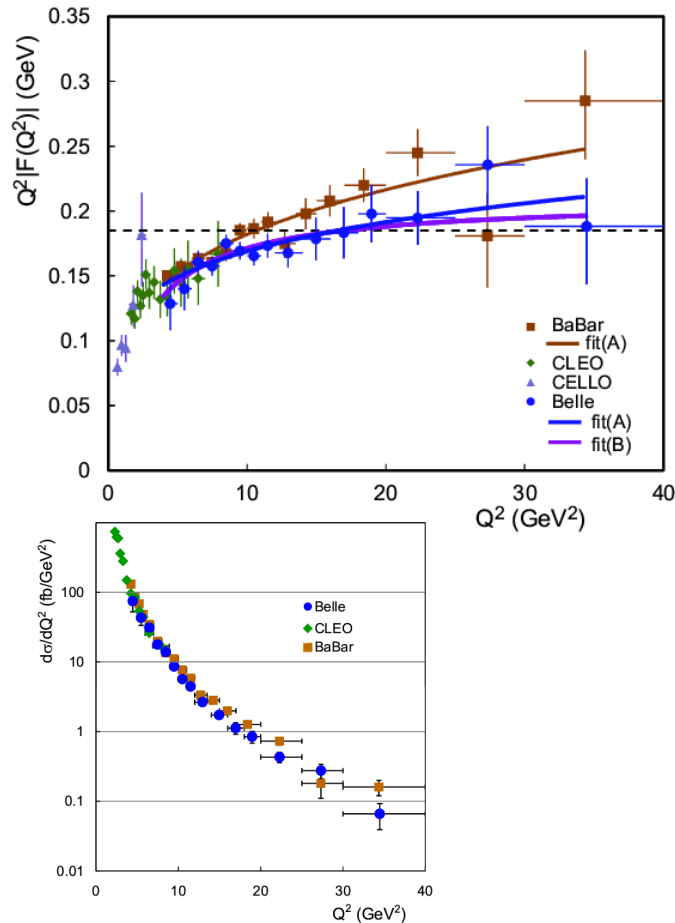


dots: Exp.  
histograms: MC

$\epsilon_{DR}$ :  
Exp./MC Ratio for Efficiency for  
“Bhabha-Mask” × “Bhabha-veto”



# Comparisons with Previous Measurements and Fits



No rapid growth above  $Q^2 > 9 \text{ GeV}^2$  is seen in Belle result.

$\sim 2.3\sigma$  difference between Belle and BaBar in  $9 - 20 \text{ GeV}^2$

## Fit A (suggested by BaBar)

$$Q^2 |F(Q^2)| = A (Q^2/10 \text{ GeV}^2)^\beta$$

BaBar:           

$$A = 0.182 \pm 0.002 (\pm 0.004) \text{ GeV}$$

$$\beta = 0.25 \pm 0.02$$

Belle:           

$$A = 0.169 \pm 0.006 \text{ GeV}$$

$$\beta = 0.18 \pm 0.05$$

$$\chi^2/\text{ndf} = 6.90/13 \quad \sim 1.5\sigma \text{ difference from BaBar}$$

## Fit B (with an asymptotic parameter)

$$Q^2 |F(Q^2)| = BQ^2/(Q^2+C)$$

Belle:           

$$B = 0.209 \pm 0.016 \text{ GeV}$$

$$C = 2.2 \pm 0.8 \text{ GeV}^2$$

$$\chi^2/\text{ndf} = 7.07/13$$

B is consistent with the QCD value (0.185 GeV)

