η and η’ transition form factors from rational approximants

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Purpose:

To present an analysis of the η and η' transition form factors in the space-like region at low and intermediate energies in a model-independent way through the use of rational approximants

Motivations:

- To extract the slope and curvature parameters of the TFFs as well as their values at zero and infinity from experimental data

- To discuss the impact of these results on the mixing parameters of the η and η' system and the pseudoscalar-exchange contributions to the HLBL scattering part of the muon anomalous magnetic moment

Outline:

- *Pseudoscalar transition form factors*
- *Padé approximants*
- *Application to η and η'TFFs*
- *Results*
- *Impact on η-η' mixing parameters*
- *Conclusions*

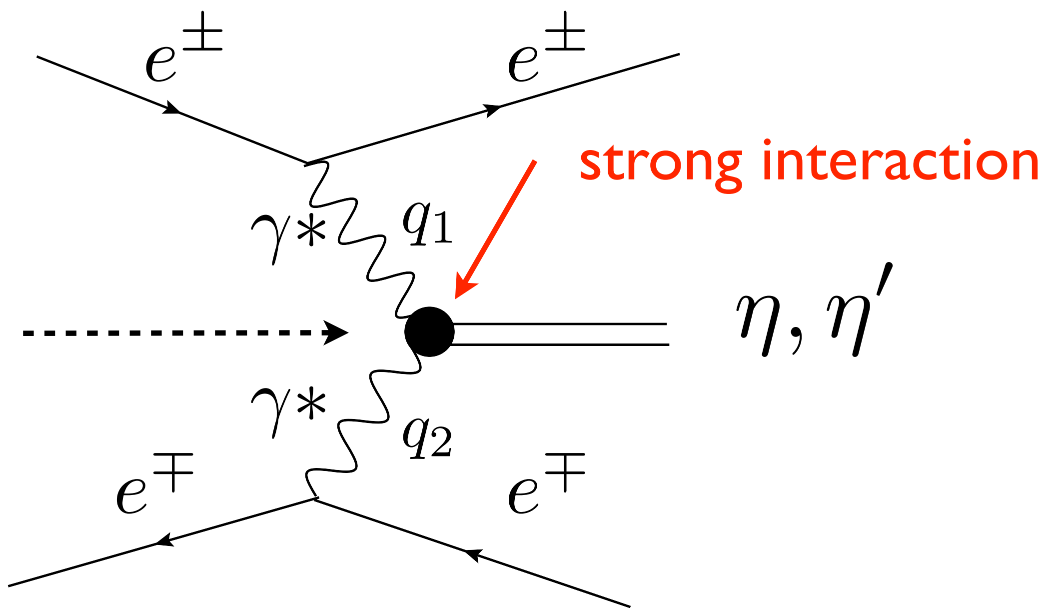
In collab. with P. Masjuan and P. Sánchez-Puertas (Mainz)

arXiv:1307.2061 [hep-ph]

• *Pseudoscalar transition form factors*

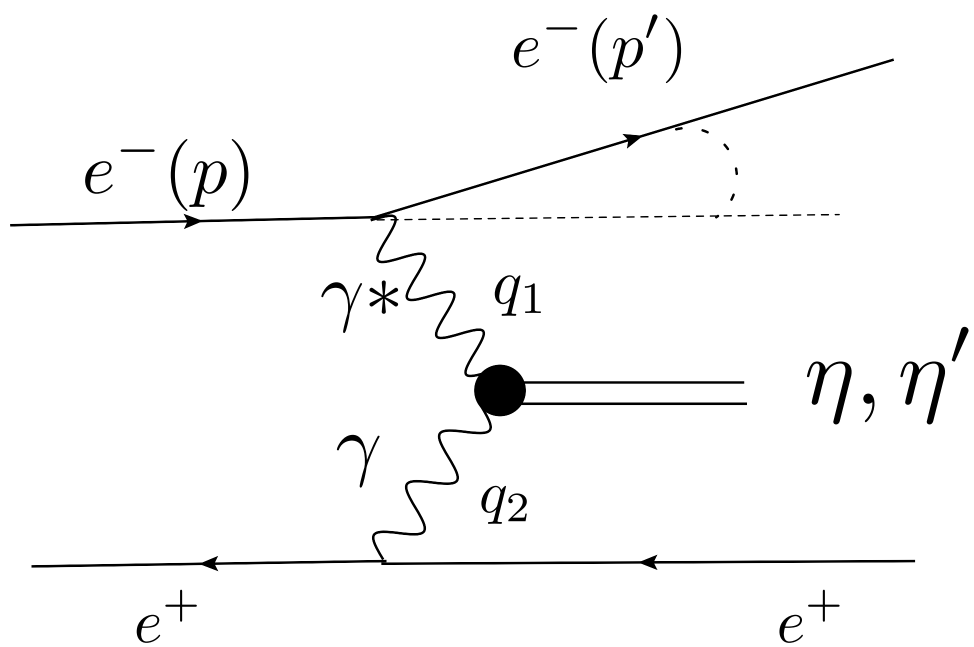
$F(q_1^2, q_2^2)$

TFF



not exp. accesible

Single Tag Method



$$F_{P\gamma^*\gamma}(Q^2) \equiv F_{P\gamma^*\gamma^*}(-Q^2, 0)$$

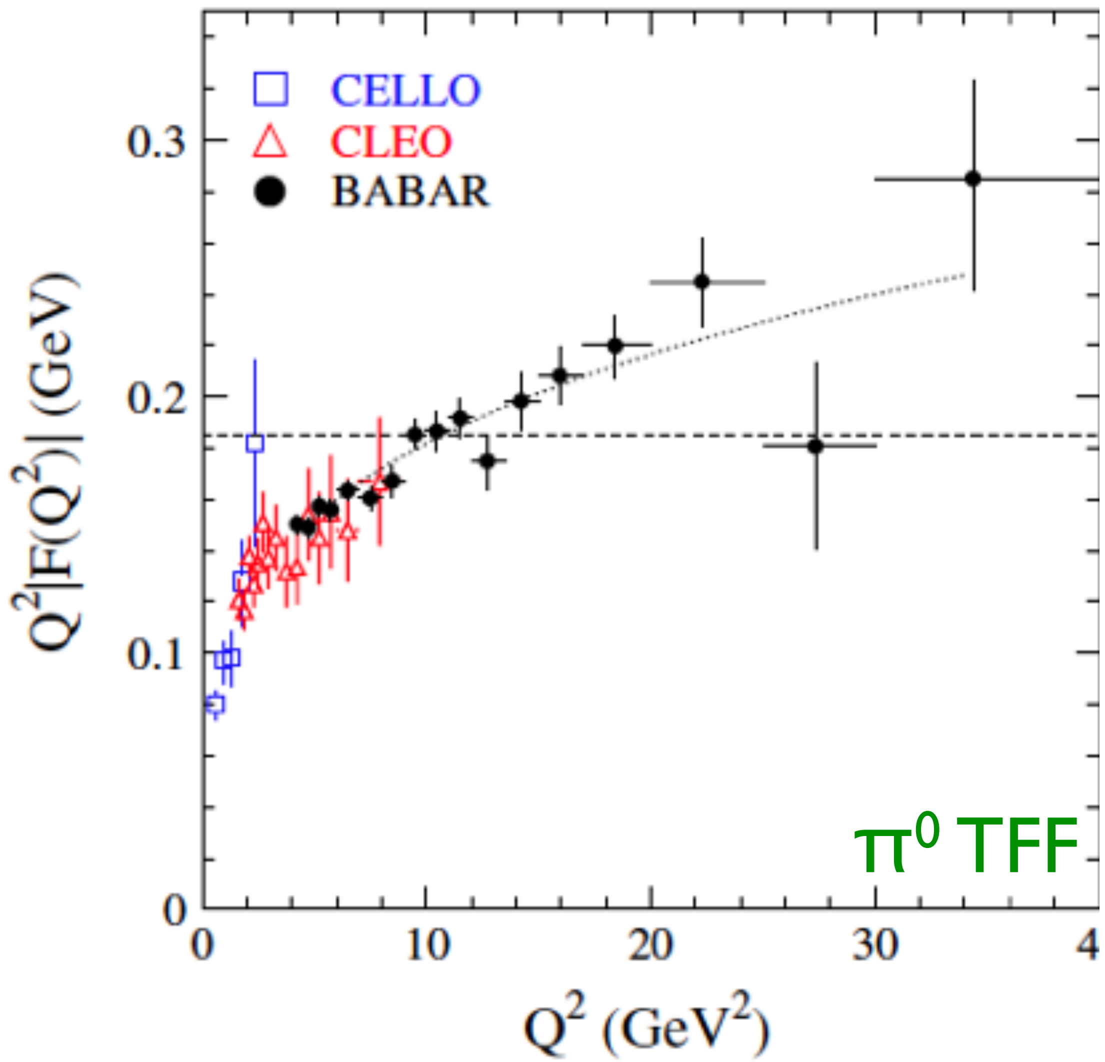
Momentum transfer

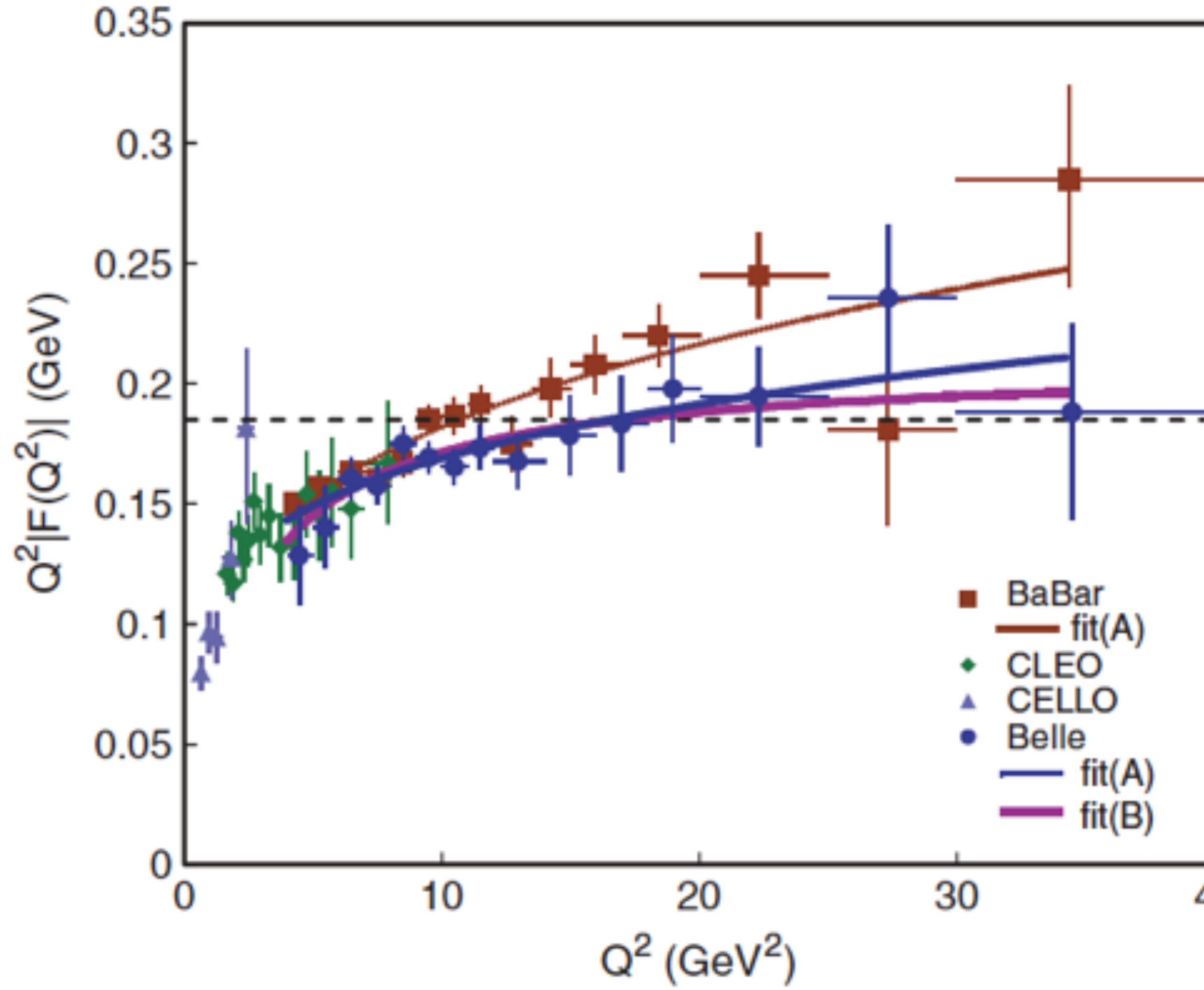
- highly virtual photon ⇒ tagged
- quasi-real photon ⇒ untagged

Selection criteria

- 1 e^- detected
- 1 e^+ along beam axis
- Meson full reconstructed

- *Pseudoscalar transition form factors*





S. Uehara et al. (BABAR Collaboration), PRD 96 (2012) 092007

FIG. 22 (color online). The $\gamma\gamma^* \rightarrow \pi^0$ transition form factor multiplied by Q^2. The dashed line indicates the asymptotic limit for the form factor. The dotted curve shows the interpolation given by Eq. (9).

B. Aubert et al. (BABAR Collaboration), PRD 80 (2009) 052002

• *Pseudoscalar transition form factors*

@ low-momentum transfer:

slope (related to charge radius)

$$F_{P\gamma^*\gamma}(Q^2) = F_{P\gamma\gamma}(0)\left(1 - b_P\frac{Q^2}{m_P^2} + c_P\frac{Q^4}{m_P^4} + \cdots\right)$$

curvature

$$|F_{P\gamma\gamma}(0)|^2 = \frac{64\pi}{(4\pi\alpha)^2}\frac{\Gamma(P\to\gamma\gamma)}{m_P^3}$$

exp. decay width

or

$$F_{\pi^0\gamma\gamma}(0) = 1/(4\pi^2 F_\pi)$$

axial anomaly
(not for η and η')

@ large-momentum transfer:

$$F(Q^2) = \int T_H(x,Q^2)\Phi_P(x,\mu_F)\mathrm{d}x$$

$T_H(\gamma^*\gamma\to q\bar{q})$ $\quad\Phi_P(q\bar{q}\to P)$

convolution of perturbative and
non-perturbative regimes

@ lowest order in pQCD

$$Q^2F(Q^2) = \frac{\sqrt{2}f_\pi}{3}\int_0^1\frac{dx}{x}\phi_\pi(x,Q^2) + O(\alpha_s) + O\left(\frac{\Lambda^2_{QCD}}{Q^2}\right),$$

$$Q^2F(Q^2) = \sqrt{2}f_\pi$$

- *Padé Approximants*

$$Q^2 F_{\eta^{(\prime)}\gamma*\gamma}(Q^2,0) = a_0 Q^2 + a_1 Q^4 + a_2 Q^6 + \dots$$

$$P^N_M(Q^2) = \frac{T_N(Q^2)}{R_M(Q^2)} = a_0 Q^2 + a_1 Q^4 + a_2 + Q^6 + \dots + \mathcal{O}((Q^2)^{N+M+1})$$

→ simple, systematic and model-independent parametrization of experimental data in the whole energy range (better convergence)

Fitting method: use of different sequences of PAs

- *How many sequences?*
  depends on the analytic structure of the exact function
- *How many elements per sequence?*
  limited by exp. data points and statistical errors

• *Padé Approximants*

P. Masjuan, S. Peris and J.J. Sanz-Cillero, PRD 78 (2008) 074028

P. Masjuan, PRD 86 (2012) 094021

How to ascribe a systematic error to the results?

test the method with a model → try different models

• *Log model:* $$F_{\pi^0\gamma^*\gamma}(Q^2) = \frac{M^2}{4\pi^2 f_\pi Q^2}\log\left(1+\frac{Q^2}{M^2}\right),$$

TABLE I. a_0, a_1, and a_2 low-energy coefficients of the log model in Eq. (3), fitted with a $P_1^L(Q^2)$ and its exact values (last column). We also include the prediction for the pole of each $P_1^L(Q^2)$ (s_p) to be compared with the lowest-lying meson in the model.

		P_1^0	P_1^1	P_1^2	P_1^3	P_1^4	P_1^5	$F_{\pi^0\gamma^*\gamma}$ (exact)	
	a_0 (GeV^{-1})	0.2556	0.2694	0.2734	0.2746	0.2751	0.2752	0.2753	
slope	a_1 (GeV^{-3})	0.1290	0.1716	0.1935	0.2051	0.2124	0.2166	0.2294	5.6% of sys. error
curvature	a_2 (GeV^{-5})	0.0651	0.1147	0.1492	0.1725	0.1898	0.2013	0.2549	21% of sys. error
	$\sqrt{s_p}$ (GeV)	1.41	1.22	1.14	1.09	1.05	1.03	0.77	

• *Regge model:*

$$F_{\pi^0\gamma^*\gamma^*}(q_1^2, q_2^2) = \sum_{V_\rho, V_\omega} \frac{F_{V_\rho}(q_1^2)F_{V_\omega}(q_2^2)G_{\pi V_\rho V_\omega}(q_1^2, q_2^2)}{(q_1^2 - M_{V_\rho}^2)(q_2^2 - M_{V_\omega}^2)} + (q_1 \leftrightarrow q_2),$$

slope	a_1 (GeV^{-3})	0.2662	0.3121	0.3338	0.3457	0.3529	0.3571	0.3678	2.9% of sys. error
curvature	a_2 (GeV^{-5})	0.2652	0.3600	0.4244	0.4616	0.4868	0.5030	0.5550	9.4% of sys. error

- *Application to η and η'TFFs*

asymptotic behaviour

To use the P[N,1](Q^2) and P[N,N](Q^2) sequences of PAs

single resonance dominance

η TFF η' TFF

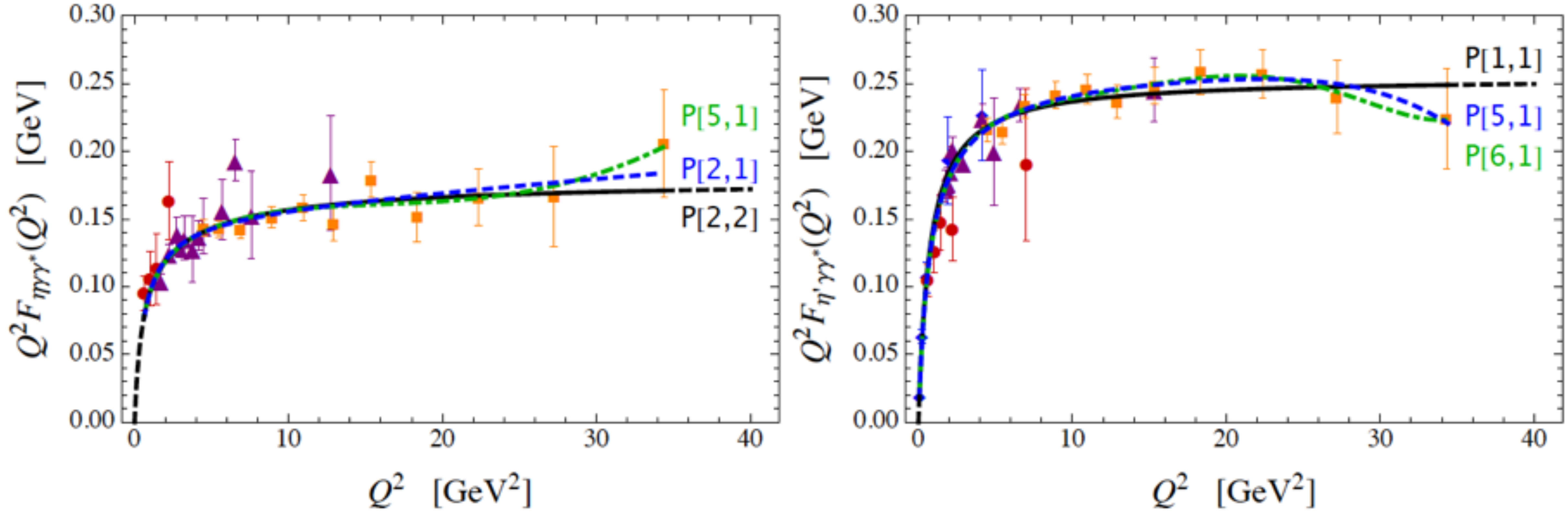


FIG. 1. η- and η'-TFFs best fits (left and right panels reps.). Blue dashed line shows our best $P_1^L(Q^2)$ when the two-photon partial decay width is *not* included in our set of data to be fitted. When the two-photon partial decay width *is* included, dark-green dot-dashed line shows our best $P_1^L(Q^2)$, and black solid line shows our best $P_N^N(Q^2)$. Black dashed lines are the extrapolation of such approximant at $Q^2 = 0$ and at $Q^2 \to \infty$. Data points are from CELLO (red circles) [28], CLEO (purple triangles) [36], L3 (blue diamonds) [31], and *BABAR* (orange squares) [30] Collaborations. See main text for details.

- *Application to η and η’TFFs*

Slope:

η TFF

η’ TFF

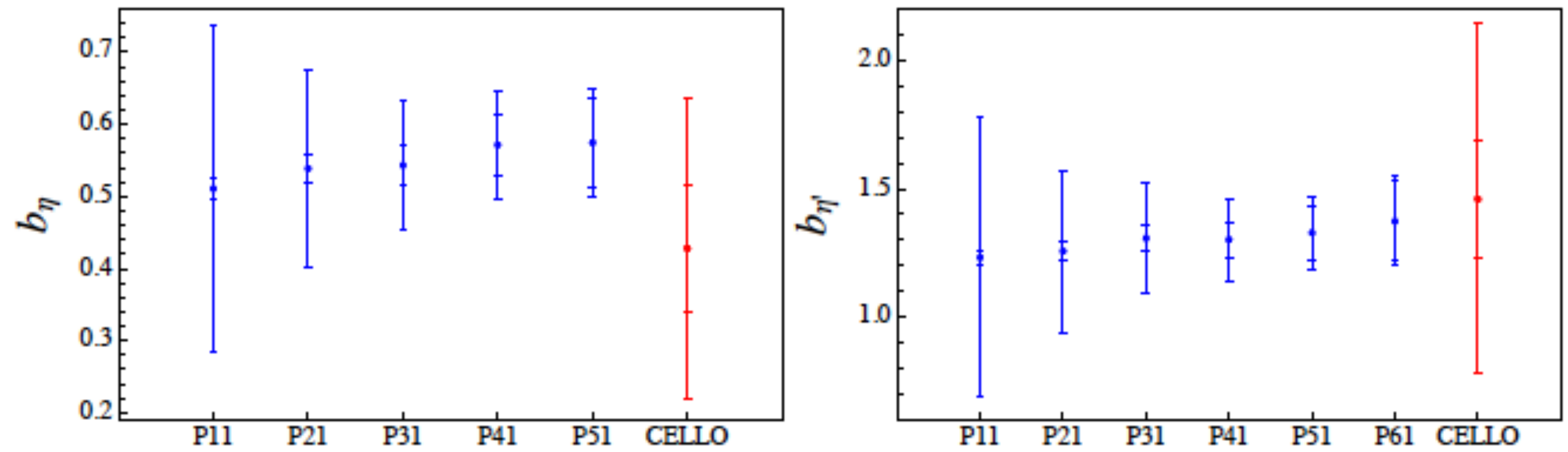


FIG. 2. Slope predictions with the $P_1^L(Q^2)$ up to $L = 5$ and $L = 6$ for the η-TFF and the η'-TFF (left and right panels respectively). The internal band is the statistical error from the fit and the external one is the combination of statistical and systematic errors determined in the previous section.

Curvature:

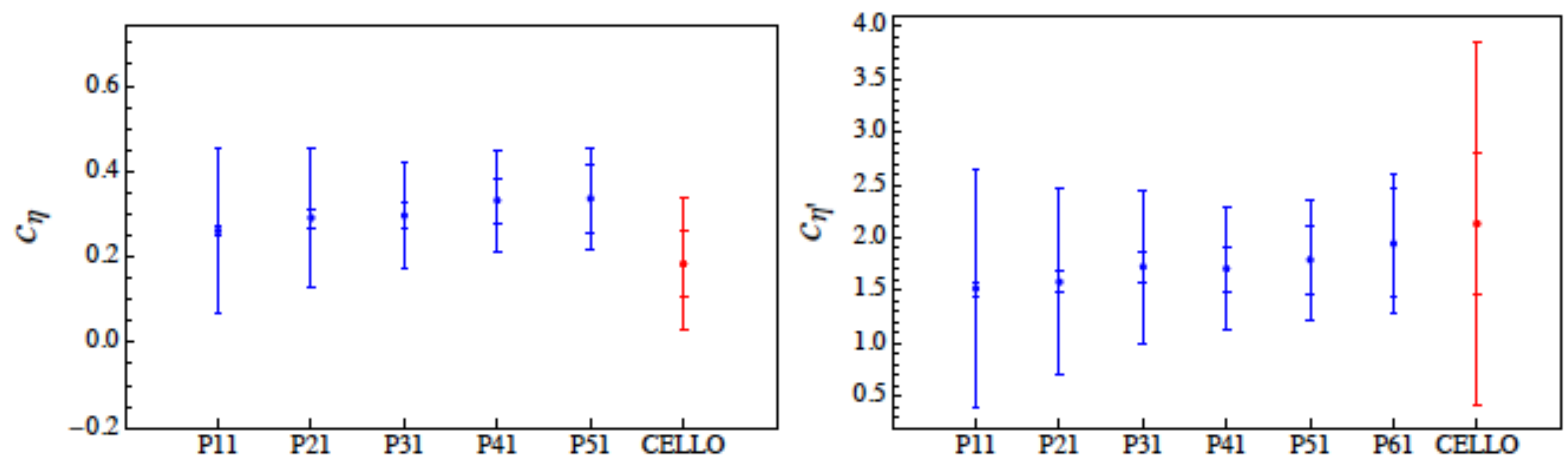


FIG. 3. Curvature predictions with the $P_1^L(Q^2)$ up to $L = 5$ and $L = 6$ for the η-TFF and the η'-TFF (left and right panels respectively). The internal band is the statistical error from the fit and the external one is the combination of statistical and systematic errors determined in the previous section.

- *Results*

Slope and curvature:

$$b_\eta = 0.596(48)_{stat}(33)_{sys}$$
$$c_\eta = 0.362(66)_{stat}(76)_{sys} \times 10^{-3}$$

$$b_{\eta'} = 1.37(16)_{stat}(8)_{sys}$$
$$c_{\eta'} = 1.94(52)_{stat}(41)_{sys} \times 10^{-3}$$

Comparison with other results:

$$F_{P\gamma^*\gamma}(Q^2) = \frac{F_{P\gamma\gamma}(0)}{1+Q^2/\Lambda_P^2}$$

ChPT: b_η=0.51, $b_{\eta'}$=1.47

VMD: b_η=0.53, $b_{\eta'}$=1.33

cQL: b_η=0.51, $b_{\eta'}$=1.30

BL: b_η=0.36, $b_{\eta'}$=2.11

$$\mathcal{F}_{\gamma^*\gamma\mathcal{R}}(Q^2) \sim \frac{1}{4\pi^2 f_{\mathcal{R}}} \frac{1}{1+(Q^2/8\pi^2 f_{\mathcal{R}}^2)}$$

CELLO: b_η=0.428(89), $b_{\eta'}$=1.46(23)

CLEO: b_η=0.501(38), $b_{\eta'}$=1.24(8)

Lepton-G: b_η=0.57(12), $b_{\eta'}$=1.6(4)

NA60: b_η=0.585(51)

MAMI: b_η=0.58(11), WASA: b_η=0.68(26)

Disp: b_η=2.05(+0.22)(-0.10), $b_{\eta'}$=1.58(+0.18)(-0.13)

$\eta,\eta' \to \gamma^*\gamma$

- *Results*

$\eta,\eta' \to \gamma\gamma$ decay widths (TFFs @ $Q^2=0$):

$$\Gamma^{pred}_{\eta\to\gamma\gamma} = (0.41 \pm 0.18) keV \qquad \Gamma^{pred}_{\eta'\to\gamma\gamma} = (4.21 \pm 0.43) keV$$

$$\Gamma^{PDG}_{\eta\to\gamma\gamma} = (0.51 \pm 0.03) keV \qquad \Gamma^{PDG}_{\eta'\to\gamma\gamma} = (4.34 \pm 0.14) keV$$

Asymptotic values (TFFs @ $Q^2\to\infty$):

$$\lim_{Q^2\to\infty} Q^2 F_{\eta\gamma^*\gamma}(Q^2) = 0.164(21)\ \mathrm{GeV}$$

$$\lim_{Q^2\to\infty} Q^2 F_{\eta'\gamma^*\gamma}(Q^2) = 0.254(4)\ \mathrm{GeV}$$

→ determination of η-η' mixing parameters

- *Impact on η-η' mixing parameters*

Quark-flavour basis:

large-N_c limit

$$\begin{pmatrix} f_\eta^q & f_\eta^s \\ f_{\eta'}^q & f_{\eta'}^s \end{pmatrix} = \begin{pmatrix} f_q \cos[\phi] & -f_s \sin[\phi] \\ f_q \sin[\phi] & f_s \cos[\phi] \end{pmatrix}$$

pseudoscalar decay constants

Decay widths:

$$\Gamma_{\eta\gamma\gamma} = \frac{\alpha^2}{32\pi^3} m_\eta^3 \left(\frac{f_{\eta'}^s \left(\frac{5}{3\sqrt{2}}\right) - f_{\eta'}^q \left(\frac{1}{3}\right)}{f_{\eta'}^s f_\eta^q - f_{\eta'}^q f_\eta^s} \right)^2$$

$$\Gamma_{\eta'\gamma\gamma} = \frac{\alpha^2}{32\pi^3} m_{\eta'}^3 \left(\frac{f_\eta^s \left(\frac{5}{3\sqrt{2}}\right) - f_\eta^q \left(\frac{1}{3}\right)}{f_{\eta'}^s f_\eta^q - f_{\eta'}^q f_\eta^s} \right)^2$$

Asymptotic expressions:

$$\lim_{Q^2 \to \infty} Q^2 F_{\eta\gamma\gamma^*}(Q^2) = f_\eta^q \frac{10}{3} + f_\eta^s \frac{2\sqrt{2}}{3}$$

$$\lim_{Q^2 \to \infty} Q^2 F_{\eta'\gamma\gamma^*}(Q^2) = f_{\eta'}^q \frac{10}{3} + f_{\eta'}^s \frac{2\sqrt{2}}{3}$$

- *Impact on η-η' mixing parameters*

Results:

$$f_q = 1.21(7)\text{GeV}\,, \quad f_s = 1.5(2)\text{GeV}\,, \phi = 45(3)^\circ$$

$\eta,\eta' \to \gamma\gamma$ not included

$$f_q = 1.07(1)\text{GeV}\,, f_s = 1.53(23)\text{GeV}\,, \phi = 40.2(1.6)^\circ$$

$\eta,\eta' \to \gamma\gamma$ included

$$f_q = 1.01(2)\text{GeV}\,, f_s = 0.95(4)\text{GeV}\,, \phi = 33.2(0.7)^\circ$$

η' TFF used

to compare with:

$$f_q = 1.07(1)\text{GeV}\,, f_s = 1.63(3)\text{GeV}\,, \phi = 39.6(0.4)^\circ$$

Update'13 of R. Escribano and J.M.-Frère, JHEP0506 (2005) 029

• *Summary and Conclusions*

We have analyzed the experimental data on the η and η' TFF at low and intermediate energies with a model independent approach based on Padé approximants (extending the analysis for the π^0-TFF) P. Masjuan, PRD 86 (2012) 094021

We have obtained accurate values of the corresponding slope and curvature parameters as well as the values of the TFFs at zero and infinity

We have quantified the impact of these results on the η and η' mixing parameters

More experimental data would be desirable (BELLE?)
Forthcoming KLOE-2 and BES-III measurements will be helpful in order to build up a solid MonteCarlo generator for data analysis and feasibility studies