Virtual photon–photon scattering

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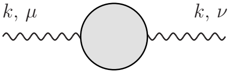
Outline

1. Hadronic vacuum polarization
2. Towards a data-driven analysis of light-by-light scattering?
3. $\gamma^*\gamma^* \to \pi\pi$: analytic structure, anomalous thresholds
4. Conclusions

Hadronic vacuum polarization

- General principles yield **direct connection with experiment**
  - **Gauge invariance**



$$= -i\left(k^2 g^{\mu\nu} - k^\mu k^\nu\right)\Pi\left(k^2\right)$$

  - **Analyticity**

$$\Pi_{\text{ren}} = \Pi\left(k^2\right) - \Pi(0) = \frac{k^2}{\pi}\int_{4M_\pi^2}^{\infty} ds \frac{\text{Im}\,\Pi(s)}{s\left(s-k^2\right)}$$

  - **Unitarity**

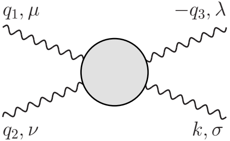
$$\text{Im}\,\Pi(s) = \frac{s}{4\pi\alpha}\sigma_{\text{tot}}\left(e^+e^- \to \text{anything}\right) = \frac{\alpha}{3}R(s)$$

- 1 Lorentz structure, 1 kinematic variable, no subtraction constant
- Can one find a **data-driven** approach also for **light-by-light scattering**?

Light-by-light scattering: Lorentz structure

- **Light-by-light tensor**

  $\gamma^*(q_1,\mu)\gamma^*(q_2,\nu) \to \gamma^*(-q_3,\lambda)\gamma(k,\sigma)$ $i\Pi^{\mu\nu\lambda\sigma} =$



  - **Gauge invariance**: 29 independent gauge-invariant structures cf. Bijnens et al. 1995

$$\Pi^{\mu\nu\lambda\sigma} = \sum_{i=1}^{29} A_i^{\mu\nu\lambda\sigma} \Pi_i$$

  - 5 kinematic variables: $s = (q_1+q_2)^2$, $t = (q_1+q_3)^2$, q_1^2, q_2^2, q_3^2
  - Need only the derivative Aldins et al. 1970

$$\frac{\partial}{\partial k_\rho} \Pi^{\mu\nu\lambda\sigma}\bigg|_{k=0}$$

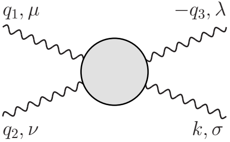
  - **Crossing symmetry**: $(q_1,\mu) \leftrightarrow (q_2,\nu)$ etc.
  - **Analyticity**: find **suitable** basis functions $A_i^{\mu\nu\lambda\sigma}$, write dispersion relations for Π_i

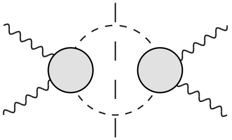
    $\hookrightarrow$ crossing symmetry, kinematic singularities, …

- **Light-by-light tensor**

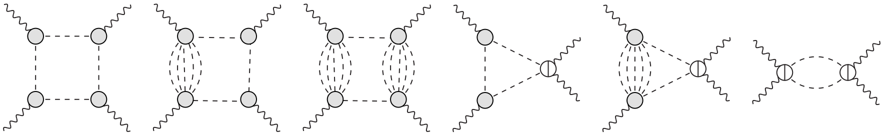
$\gamma^*(q_1,\mu)\gamma^*(q_2,\nu) \to \gamma^*(-q_3,\lambda)\gamma(k,\sigma)$

$i\Pi^{\mu\nu\lambda\sigma} =$



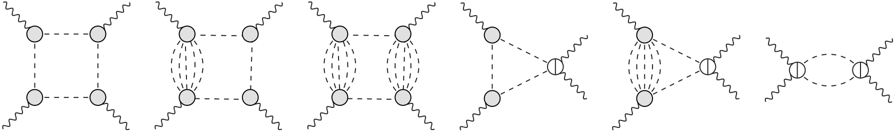


- **Unitarity**: restrict to $\pi\pi$ (and $K\bar{K}$) intermediate states
  $\hookrightarrow$ **two-meson reducible** contributions
- **Unitarity diagrams**: box, triangle, and bulb topologies



$\hookrightarrow$ sorted by analytic structure in the crossed channel

Light-by-light scattering: topologies



- Approximate **multi-pion states** by a **resonance** description, e.g. for the ρ

$$\int_{4M_\pi^2}^{\infty} \mathrm{d}s' \frac{\rho(s')}{s'-s} \qquad\qquad \rho(s) = \frac{\left(s-4M_\pi^2\right)^{3/2}}{192\pi\sqrt{s}} f_{\gamma^*\pi\to\pi\pi}(s,q_1^2) f^*_{\gamma^*\pi\to\pi\pi}(s,q_2^2)$$

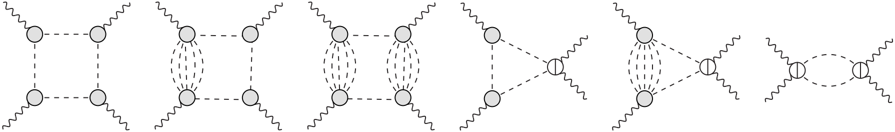
  and NWA for ω and ϕ, **symmetrize** in the end

- Ingredients

  1. Pion vector form factor
  2. Transition form factors for $\omega/\phi \to \pi^0\gamma^*$ see talk by B. Kubis
  3. Amplitude for $\gamma^*\pi \to \pi\pi$ see poster by S. P. Schneider
  4. Amplitude for $\gamma^*\gamma^* \to \pi\pi$ see talk by B. Moussallam

     $\hookrightarrow$ $\pi\pi$ scattering amplitude and 1–3
  5. Pion transition form factor for pole contribution

Light-by-light scattering: toy example



- Scalar toy example

$$\Pi(s,t,q_1^2,q_2^2,q_3^2) = \frac{F_{\pi^0\gamma^*\gamma^*}(q_1^2,q_2^2)F_{\pi^0\gamma^*\gamma^*}(q_3^2,0)}{M_\pi^2 - s} + \text{crossed}$$

$$+\frac{1}{\pi}\int\limits_{4M_\pi^2}^{\infty} ds' \frac{16\pi\sqrt{1-\frac{4M_\pi^2}{s'}} f_0(s';q_1^2,q_2^2) f_0(s';q_3^2,0)}{s'-s} + \text{crossed} + \text{triangle} + \text{box} + D\text{-waves}$$

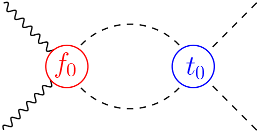
- Transition form factor $F_{\pi^0\gamma^*\gamma^*}$ **on-shell** by definition
- Further: choice of basis functions Π_i, subtractions, symmetrization for D-waves
- **Crucial input**: pole-subtracted, doubly-virtual $\gamma^*\gamma^* \to \pi\pi$ partial waves $f_\ell(s;q_1^2,q_2^2)$

$\gamma^*\gamma^* \to \pi\pi$

- **Left-hand cut** approximated by **pion pole** + **resonances** (as before)
- **Unitarity** for $\gamma^*\gamma^* \to \pi\pi$ system: Watson's theorem

$$\text{disc}\, f_0(s;q_1^2,q_2^2) = 2i\sigma_s f_0(s;q_1^2,q_2^2)\, t_0^*(s)$$

$$t_0(s) = \frac{1}{\sigma_s} e^{i\delta_0(s)} \sin\delta_0(s) \qquad \sigma_s = \sqrt{1-\frac{4M_\pi^2}{s}}$$



$\hookrightarrow$ solution in terms of **Omnès function**, e.g. for pion pole only

$$f_0(s;q_1^2,q_2^2) = \frac{\Omega_0(s)}{\pi} \int\limits_{4M_\pi^2}^{\infty} ds' \frac{N_0(s';q_1^2,q_2^2)\sin\delta_0(s')}{(s'-s)|\Omega_0(s')|} \qquad N_0(s;q_1^2,q_2^2) = \frac{2L}{\sigma_s\sqrt{\lambda(s,q_1^2,q_2^2)}}$$

$$L = \log\frac{s-q_1^2-q_2^2+\sigma_s\sqrt{\lambda(s,q_1^2,q_2^2)}}{s-q_1^2-q_2^2-\sigma_s\sqrt{\lambda(s,q_1^2,q_2^2)}} \qquad \lambda(x,y,z) = x^2+y^2+z^2-2(xy+xz+yz)$$

- **Analytic continuation** in q_i^2? see talk by B. Moussallam for one off-shell photon

$\gamma^*\gamma^* \to \pi\pi$: analytic continuation

$$L = \log \frac{s-q_1^2-q_2^2+\sigma_s\sqrt{\lambda(s,q_1^2,q_2^2)}}{s-q_1^2-q_2^2-\sigma_s\sqrt{\lambda(s,q_1^2,q_2^2)}} \xrightarrow{q_2^2\to 0} \pm\log\frac{1+\sigma_s}{1-\sigma_s}$$

- Singularities of the log: **anomalous thresholds**

$$s_\pm = q_1^2+q_2^2-\frac{q_1^2q_2^2}{2M_\pi^2}\pm\frac{1}{2M_\pi^2}\sqrt{q_1^2(q_1^2-4M_\pi^2)q_2^2(q_2^2-4M_\pi^2)}$$

$\hookrightarrow$ usual Omnès derivation breaks down

- Idea: consider first the **scalar loop function**

$$C_0(s) \equiv C_0\big((q_1+q_2)^2;q_1^2,q_2^2\big) = \frac{1}{i\pi^2}\int\frac{\mathrm{d}^4k}{(k^2-M_\pi^2)\big((k+q_1)^2-M_\pi^2\big)\big((k-q_2)^2-M_\pi^2\big)}$$

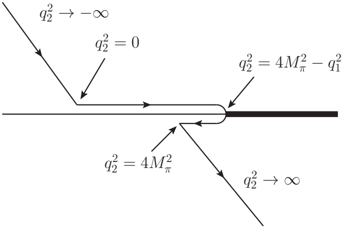
$$\operatorname{disc} C_0(s) = -\frac{2\pi i}{\sqrt{\lambda\big(s,q_1^2,q_2^2\big)}}L = -\pi i\,\sigma_s N_0\big(s;q_1^2,q_2^2\big)$$

$\gamma^*\gamma^* \to \pi\pi$: anomalous thresholds

$$s_+ = q_1^2 + q_2^2 - \frac{q_1^2 q_2^2}{2M_\pi^2} + \frac{1}{2M_\pi^2}\sqrt{q_1^2(q_1^2 - 4M_\pi^2)q_2^2(q_2^2 - 4M_\pi^2)}$$

- **Anomalous threshold** usually on the **second sheet**
- Trajectory of $s_+(q_2^2)$ for $0 \leq q_1^2 \leq 4M_\pi^2$

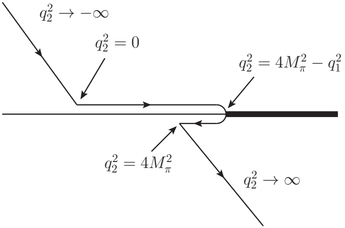
  $\hookrightarrow$ moves through unitarity cut onto first sheet



$\gamma^*\gamma^* \to \pi\pi$: anomalous thresholds

$$s_+ = q_1^2 + q_2^2 - \frac{q_1^2 q_2^2}{2M_\pi^2} + \frac{1}{2M_\pi^2}\sqrt{q_1^2(q_1^2-4M_\pi^2)q_2^2(q_2^2-4M_\pi^2)}$$

- **Anomalous threshold** usually on the **second sheet**
- Trajectory of $s_+(q_2^2)$ for $0 \leq q_1^2 \leq 4M_\pi^2$
  $\hookrightarrow$ moves through unitarity cut onto first sheet
- Need to deform the contour

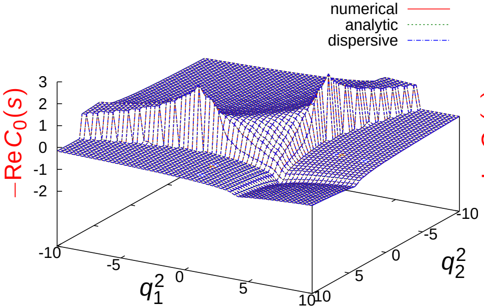


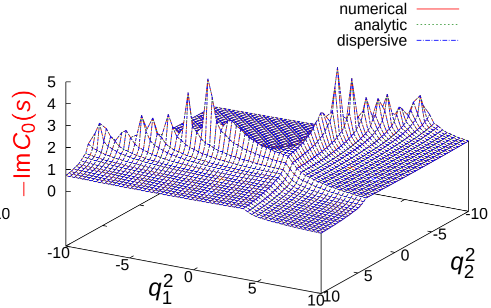
$$C_0(s) = \frac{1}{2\pi i}\int_{4M_\pi^2}^{\infty} \text{d}s' \frac{\text{disc}\, C_0(s')}{s'-s}$$

$$+\,\theta\left(q_1^2+q_2^2-4M_\pi^2\right)\frac{1}{2\pi i}\int_0^1 \text{d}x\, \frac{\partial s_x}{\partial x}\frac{\text{disc}_{\text{an}}\, C_0(s_x)}{s_x - s}$$

$$s_x = x4M_\pi^2 + (1-x)s_+ \qquad \text{disc}_{\text{an}}\, C_0(s) = \frac{4\pi^2}{\sqrt{\lambda\left(s,q_1^2,q_2^2\right)}}$$

$\gamma^*\gamma^* \to \pi\pi$: numerical check

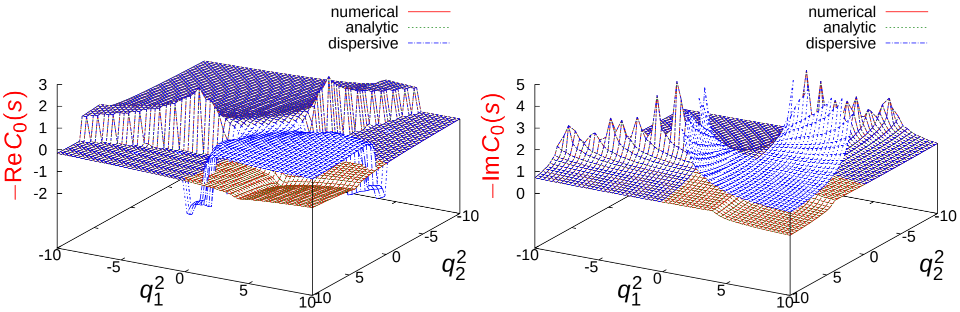




- Comparison for $s = 5$, $M_\pi = 1$

$\hookrightarrow$ **Dispersive reconstruction** of $C_0(s)$ works!

$\gamma^*\gamma^* \to \pi\pi$: switching off anomalous contributions



- Ignore anomalous piece

  $\hookrightarrow$ Substantial deviations for **large virtualities**!

$\gamma^*\gamma^* \to \pi\pi$: back to the Omnès representation

$$f_0(s;q_1^2,q_2^2) = \frac{\Omega_0(s)}{\pi}\int\limits_{4M_\pi^2}^{\infty} ds' \frac{N_0(s';q_1^2,q_2^2)\sin\delta_0(s')}{(s'-s)|\Omega_0(s')|}$$

- Integrand similar to the scalar-loop example

$$\frac{N_0(s;q_1^2,q_2^2)\sin\delta_0(s)}{|\Omega_0(s)|} = -\frac{\text{disc}\,C_0(s)}{\pi i\,\sigma_s}\,\frac{\sin\delta_0(s)}{|\Omega_0(s)|} = -\frac{\text{disc}\,C_0(s)}{\pi i}\,\frac{t_0(s)}{\Omega_0(s)}$$

$\hookrightarrow$ Additional factor **independent of q_i^2** and **well-defined in the whole s-plane**

$\gamma^*\gamma^* \to \pi\pi$: back to the Omnès representation

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$\hookrightarrow$ Additional factor **independent of q_i^2** and **well-defined in the whole s-plane**

Omnès representation for $\gamma^*\gamma^* \to \pi\pi$

$$f_0(s;q_1^2,q_2^2) = \frac{\Omega_0(s)}{\pi}\int\limits_{4M_\pi^2}^{\infty} ds' \frac{N_0(s';q_1^2,q_2^2)\sin\delta_0(s')}{(s'-s)|\Omega_0(s')|}$$

$$+\theta(q_1^2+q_2^2-4M_\pi^2)\frac{\Omega_0(s)}{2\pi i}\int\limits_0^1 dx\,\frac{\partial s_x}{\partial x}\,\frac{\text{disc}_{\text{an}}\, f_0(s_x;q_1^2,q_2^2)}{s_x-s}$$

$$s_x = x4M_\pi^2+(1-x)s_+ \qquad \text{disc}_{\text{an}}\, f_0(s;q_1^2,q_2^2) = -\frac{8\pi}{\sqrt{\lambda(s,q_1^2,q_2^2)}}\frac{t_0(s)}{\Omega_0(s)}$$

Conclusions

- **Goal**: find a representation for light-by-light scattering **connected to data** as closely as possible
  - **(Transition) form factors**: pion, vector mesons
  - **Scattering amplitudes**: $\pi\pi \to \pi\pi$, $\gamma^*\pi \to \pi\pi$, $\gamma^*\gamma^* \to \pi\pi$
- $\gamma^*\gamma^* \to \pi\pi$ amplitudes most critical, but **anomalous thresholds** appear manageable
- Omissions:
  - Optimal basis
  - Implementation of $\gamma^*\gamma^* \to \pi\pi$ amplitudes
  - Convergence/subtractions
  - ...