Final state interaction and asymmetry of pair production close to threshold in e^+e^- annihilation

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Outline

1. Coulomb final state interaction
   - A lepton pair production

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2. Strong Final State Interaction
    - $N\bar{N}$ pair production

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Cross section for pair production

A general form of the the cross section is well known The standard equation for the cross section

$$\frac{d\sigma}{d\Omega} = \frac{\beta\alpha^2}{4Q^2}\frac{2\pi\eta}{1-e^{-2\pi\eta}}\left[|G_M(Q^2)|^2(1+\cos^2\theta)+\frac{4M^2}{Q^2}|G_E(Q^2)|^2\sin^2\theta\right].$$

Near threshold, it is usually assumed $G_E = G_M$, and the cross section is isotropic. At the same time, in the presence of long range Coulomb final state interaction (FSI), all partial waves give a nonzero contribution at the threshold, and $G_E \neq G_M$, although the difference is small.

Coulomb wave function at the origin

Let us consider the attractive Coulomb potential $U(r) = -e^2/r$ and the corresponding radial wave function $R^{(c)}_{k\,l}(r)$, where e is the electric charge, k is the particle momentum, and l is the angular momentum. Then the ratio $C_{k\,l} = |R^{(c)}_{k\,l}(r)/R^{(0)}_{k\,l}(r)|^2$ at $kr \longrightarrow 0$, where $R^{(0)}_{k\,l}(r)$ is the radial wave function for plain wave, has the form,

$$C_{k\,0} = \frac{2\pi\eta}{1 - e^{-2\pi\eta}}\,, \quad C_{k\,l\neq 0} = C_{k\,0}\prod_{s=1}^{l}\left(1 + \frac{\eta^2}{s^2}\right)\,, \quad \eta = \frac{\mu\alpha}{k}\,.$$

Here μ is the reduced mass of the system, $\alpha = e^2$ is the fine structure constant, units $\hbar = c = 1$ are used.
Note, $\eta \to \infty$ at threshold.

Born amplitude in nonrelativistic form

The Born amplitude of the process $e^+e^- \longrightarrow L\bar{L}$ near threshold in units $4\pi\alpha/Q^2$

$$T^B_{\lambda\mu} = \sqrt{2}\boldsymbol{\epsilon}^*_\lambda \cdot \left[G_s \boldsymbol{e}_\mu + G_d \frac{\mathbf{k}^2\mathbf{e}_\mu - 3(\mathbf{k}\cdot\mathbf{e}_\mu)\mathbf{k}}{6M^2} \right].$$

Here $\mathbf{e}_\mu$ is a virtual photon polarization vector, $\boldsymbol{\epsilon}_\lambda$ is the spin-1 function of $L\bar{L}$ pair, $G_s = F_1(Q^2) + F_2(Q^2)$, $G_d = F_1(Q^2) - F_2(Q^2)$, $F_1(Q^2)$ and $F_2(Q^2)$ are the Dirac form factors of lepton. Two tensor structures in Eq.(1) correspond to the s-wave and d-wave production amplitudes. The total angular momentum of the $L\bar{L}$ pair is fixed by a production mechanism.

Exacte amplitude with Coulomb FSI

$$T_{\lambda\mu} = \sqrt{2}\epsilon^*_\lambda \cdot \int \frac{d^3p}{(2\pi)^3}\Phi^{(-)*}_{\mathbf{k}}(\mathbf{p})\left[G_s \boldsymbol{e}_\mu + G_d \frac{\mathbf{p}^2\mathbf{e}_\mu - 3(\mathbf{p}\cdot\mathbf{e}_\mu)\mathbf{p}}{6M^2}\right],$$

where $\Phi^{(-)}_{\mathbf{k}}(\mathbf{p})$ is the Fourier transform of the function $\psi^{(-)}_{\mathbf{k}}(\mathbf{r})$, which is the wave function of the $L\bar{L}$ pair in coordinate space:

$$\psi^{(-)}_{\mathbf{k}}(\mathbf{r}) = \frac{1}{2k}\sum_{l=0}^{\infty}\imath^l\,\mathrm{e}^{-\mathrm{i}\delta_l}\,(2l+1)\,R_{k\,l}(r)\,P_l(\boldsymbol{n}\cdot\hat{\boldsymbol{k}}),$$
$$\delta_l = \arg\Gamma(l+1-\imath\eta).$$

Here $\boldsymbol{n} = \boldsymbol{r}/r$, $\hat{\boldsymbol{k}} = \boldsymbol{k}/k$, P_l are the Legendre polynomials

$$R_{k\,l}(r) = \frac{2k\,l!\,\sqrt{C_{kl}}}{(2l+1)!}(2kr)^l\,e^{-\imath kr}\,F(\imath\eta + l + 1, 2l+2, 2\imath kr),$$

Calculated exact amplitude

$$T_{\lambda\mu} = \sqrt{2}e^{i\delta_0}\sqrt{C_{k0}}\epsilon^*_\lambda \cdot \left[G_s \boldsymbol{e}_\mu + \left(1 - \frac{\eta^2}{2} - \frac{3i\eta}{2}\right) G_d \, \frac{\mathbf{k}^2\mathbf{e}_\mu - 3(\mathbf{k}\cdot\mathbf{e}_\mu)\mathbf{k}}{6M^2}\right].$$

The "dressed" form factor G_2

$$G_2 = \left(1 - \frac{\eta^2}{2} - \frac{3\imath\eta}{2}\right) G_d \, ,$$

and $G_0 = G_s$.

Differential cross section

The cross section

$$\frac{d\sigma}{d\Omega} = \frac{\beta\alpha^2}{2Q^2}C_{k0}\Bigg[|G_0(Q^2)|^2 + \frac{\beta^2}{3}\mathrm{Re}(G_0^* G_2)P_2(\cos\theta) + \beta^4|G_2(Q^2)|^2[2 - P_2(\cos\theta)]\Bigg]$$

where $\beta = k/M$. Omitting the form factor $F_2(Q^2)$ for lepton production, we obtain from Eq.(1)

$$\frac{d\sigma}{d\Omega} = \frac{\beta\alpha^2}{2Q^2}C_{k0}|F_1(Q^2)|^2\left[1 + \frac{1}{3}(\beta^2 - \frac{\alpha^2}{8})P_2(\cos\theta)\right]$$

Sachs's form factors

The standard equation for the cross section

$$\frac{d\sigma}{d\Omega} = \frac{\beta\alpha^2}{4Q^2} C_{k\,0} \left[|G_M(Q^2)|^2 (1+\cos^2\theta) + \frac{4M^2}{Q^2} |G_E(Q^2)|^2 \sin^2\theta \right].$$

In terms of the form factor G_0 and G_2 the electromagnetic Sachs form factors have the form

$$G_M = G_0 + \frac{\beta^2}{6} G_2 , \quad \frac{2M}{Q} G_E = G_0 - \frac{\beta^2}{3} G_2 .$$

$\beta^2 G_2$ does not vanishes in the limit $\beta \to 0$, we find that $G_E \neq G_M$ at threshold. Unlike the form factors G_0 and G_2, the electromagnetic Sachs form factors G_E and G_M are not singular in the limit $\beta \to 0$.

$N\bar{N}$ production near threshold

For $N\bar{N}$ production near threshold there is a strong FSI between slowly moving nucleon and antinucleon. At small distances, where strong interaction dominates, we can forget about Coulomb interaction. The electromagnetic interaction is important only in the narrow region where $\beta \sim \pi\alpha$ and the nucleon energy is $E = M\beta^2/2 \sim 0.3$ MeV.
Thus, we can work with the isospin amplitudes $T^1_{\lambda\mu}$ and $T^0_{\lambda\mu}$ related to $p\bar{p}$ and $n\bar{n}$ production amplitudes by

$$T^{(p)}_{\lambda\mu} = \frac{T^1_{\lambda\mu} + T^0_{\lambda\mu}}{\sqrt{2}}, \quad T^{(n)}_{\lambda\mu} = \frac{T^1_{\lambda\mu} - T^0_{\lambda\mu}}{\sqrt{2}}.$$

$N\bar{N}$ production amplitude

The amplitude of $N\bar{N}$ pair production in a certain isospin channel $I = 0, 1$ near threshold in units $4\pi\alpha/Q^2$:

$$T^I_{\lambda\mu} = \sqrt{2}\epsilon^*_\lambda \left[G^I_s \boldsymbol{e}_\mu + G^I_d \frac{\boldsymbol{k}^2 \boldsymbol{e}_\mu - 3(\boldsymbol{k}\cdot\boldsymbol{e}_\mu)\boldsymbol{k}}{6M^2} \right],$$

$$G^I_s = F^I_1(Q^2) + F^I_2(Q^2)$$

$$G^I_d = F^I_1(Q^2) - F^I_2(Q^2),$$

$\mathbf{e}_\mu$ is a virtual photon polarization vector, and ϵ_λ is the spin-1 function of $N\bar{N}$ pair, $\lambda = \pm 1, 0$ is the projection of spin on the vector $\boldsymbol{k}$. Here we omitted terms $\sim \beta^2$,where $\beta = k/M \ll 1$.

Full $N\bar{N}$ production amplitude

$$T^I_{\lambda\mu} \;=\; \sqrt{2}\int\frac{d^3p}{(2\pi)^3}\,\mathbf{\Phi}^{I(-)*}_{\boldsymbol{k}\lambda}(\boldsymbol{p})\cdot\left[G^I_s\boldsymbol{e}_\mu + G^I_d\,\frac{\boldsymbol{p}^2\boldsymbol{e}_\mu - 3(\boldsymbol{p}\cdot\boldsymbol{e}_\mu)\boldsymbol{p}}{6M^2}\right],$$

where $\mathbf{\Phi}^{I(-)}_{\boldsymbol{k}\lambda}(\boldsymbol{p})$ is the Fourier transform of the function $\mathbf{\Psi}^{I(-)}_{\boldsymbol{k}\lambda}(\boldsymbol{r})$, the wave function of the $N\bar{N}$ pair in coordinate space. This wave function is the solution of the Schrödinger equation

$$\mathbf{\Psi}^{I(-)*}_{\boldsymbol{k}\lambda}(\boldsymbol{r})\hat{H} = M\beta^2\mathbf{\Psi}^{I(-)*}_{\boldsymbol{k}\lambda}(\boldsymbol{r})\,,\quad \hat{H} = \frac{\boldsymbol{p}^2}{M} + V_{N\bar{N}}\,,$$

where $V_{N\bar{N}}$ is the optical potential.

Full $N\bar{N}$ production amplitude

$$T^I_{\lambda\mu} = \sqrt{2}\lim_{r\to 0}\left[G^I_s \boldsymbol{e}_\mu - G^I_d \frac{\boldsymbol{e}_\mu\triangle - 3(\boldsymbol{\nabla}\cdot\boldsymbol{e}_\mu)\boldsymbol{\nabla}}{6M^2}\right]\psi^{I*}_{\boldsymbol{k}\lambda}(\boldsymbol{r})\,,$$

$$\psi^I_{\boldsymbol{k}\lambda}(\boldsymbol{r}) = [u^{I*}_{11}(r)\epsilon_\lambda + w^{I*}_{11}(r)\sqrt{4\pi}\,\boldsymbol{Y}^2_{1\lambda}(\boldsymbol{n})]$$
$$+\sqrt{5}\,C^{1\lambda}_{20,1\lambda}[u^{I*}_{21}(r)\epsilon_\lambda + w^{I*}_{21}(r)\sqrt{4\pi}\,\boldsymbol{Y}^2_{1\lambda}(\boldsymbol{n})]\,.$$

The radial wave functions u^I_{n1} and w^I_{n1} , $n = 1, 2$, satisfy the system of equations

$$\frac{p_r^2}{M}\chi + \mathcal{V}\chi = 2E\chi\,,$$

$$\mathcal{V} = \begin{pmatrix} V^I_0 & -2\sqrt{2}\,V^I_3 \\ -2\sqrt{2}\,V^I_3 & V^I_2 - 2V^I_3 \end{pmatrix}\,,\quad \chi = \begin{pmatrix} u^I_{n1} \\ w^I_{n1} \end{pmatrix}\,.$$

Radial wave functions

The asymptotic form of the solutions at large distances is,

$$
\begin{aligned}
u_{11}^{I}(r) &= \frac{1}{2ikr}\left[S_{11}^{I1}\,e^{ikr} - e^{-ikr}\right],\\
w_{11}^{I}(r) &= -\frac{1}{2ikr}S_{12}^{I1}\,e^{ikr},\\
u_{21}^{I}(r) &= \frac{1}{2ikr}S_{21}^{I1}\,e^{ikr},\\
w_{21}^{I}(r) &= \frac{1}{2ikr}\left[-S_{22}^{I1}e^{ikr} + e^{-ikr}\right].
\end{aligned}
$$

If $V_3^I = 0$, then $w_{11}^I(r) = 0$ and $u_{21}^I(r) = 0$

Full $N\bar{N}$ amplitude

Performing differentiation and putting $r = 0$ we obtain

$$T^{I}_{\lambda\mu} = \sqrt{2}\epsilon^{*}_{\lambda}\left\{\mathcal{G}^{I}_{s}\boldsymbol{e}_{\mu} + \mathcal{G}^{I}_{d}\frac{\boldsymbol{k}^{2}\boldsymbol{e}_{\mu} - 3(\boldsymbol{k}\cdot\boldsymbol{e}_{\mu})\boldsymbol{k}}{6M^{2}}\right\},$$

where dressed s-wave and d-wave form factors are

$$\mathcal{G}^{I}_{s} = G^{I}_{s}\,u^{I}_{11}(0) + \frac{5\,G^{I}_{d}}{\sqrt{2}M^{2}}\lim_{r\to 0}\left(\frac{w^{I}_{11}(r)}{r^{2}}\right),$$

$$\mathcal{G}^{I}_{d} = \frac{6G^{I}_{s}}{\sqrt{2}\beta^{2}}\,u^{I}_{21}(0) + 15\,G^{I}_{d}\lim_{r\to 0}\left(\frac{w^{I}_{21}(r)}{k^{2}r^{2}}\right).$$

Final $N\bar{N}$ amplitude

$$\mathcal{G}^I_s = G^I_s\, u^I_{11}(0), \quad \mathcal{G}^I_d = \frac{6G^I_s}{\sqrt{2}\beta^2}\, u^I_{21}(0)$$

Final form of the amplitude

$$T^I_{\lambda\mu} = G^I_s \left\{ \sqrt{2} u^I_{11}(0)(\boldsymbol{e}_\mu \cdot \boldsymbol{\epsilon}^*_\lambda) + u^I_{21}(0)[(\boldsymbol{e}_\mu \cdot \boldsymbol{\epsilon}^*_\lambda) - 3(\hat{\boldsymbol{k}} \cdot \boldsymbol{e}_\mu)(\hat{\boldsymbol{k}} \cdot \boldsymbol{\epsilon}^*_\lambda)] \right\},$$

where $\hat{\boldsymbol{k}} = \boldsymbol{k}/k$, and the only parameter $G^I_s = F^I_1 + F^I_2$

Sachs form factors and cross section

$$G_M^I = \mathcal{G}_s^I + \frac{\beta^2}{6}\mathcal{G}_d^I = G_s^I[u_{11}^I(0) + \frac{1}{\sqrt{2}}u_{21}^I(0)]\,,$$

$$\frac{2M}{Q}G_E^I = \mathcal{G}_s^I - \frac{\beta^2}{3}\mathcal{G}_d^I = G_s^I[u_{11}^I(0) - \sqrt{2}u_{21}^I(0)]\,.$$

The ratio G_E^I/G_M^I is independent on the parameter G_s^I

$$\frac{G_E^I}{G_M^I} = \frac{Q}{2M}\frac{u_{11}^I(0) - \sqrt{2}u_{21}^I(0)}{u_{11}^I(0) + \frac{1}{\sqrt{2}}u_{21}^I(0)}\,.$$

The cross section has a standard form

$$\frac{d\sigma}{d\Omega} = \frac{\beta\alpha^2}{4Q^2}\left[|G_M(Q^2)|^2(1+\cos^2\theta) + \frac{4M^2}{Q^2}|G_E(Q^2)|^2\sin^2\theta\right].$$

Cross section data

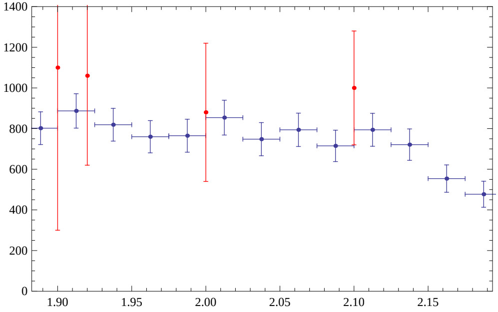


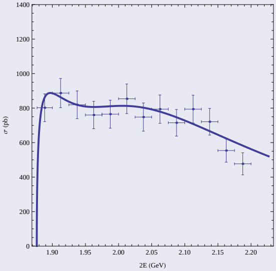
Figure : Cross sections for $e^+e^- \to p\bar{p}$ (blue) and $e^+e^- \to n\bar{n}$ (red)

$$T^{(p)}_{\lambda\mu} = \frac{T^1_{\lambda\mu} + T^0_{\lambda\mu}}{\sqrt{2}}, \quad T^{(n)}_{\lambda\mu} = \frac{T^1_{\lambda\mu} - T^0_{\lambda\mu}}{\sqrt{2}}.$$

Eather $|T^0| \gg |T^1|$ or $|T^1| \gg |T^0|$.

Calculations with Paris $N\bar{N}$ potential

Cross section for $I = 0$



Calculations with Paris $N\bar{N}$ potential

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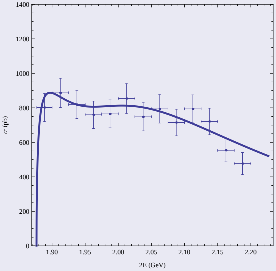
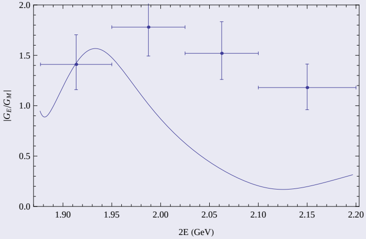


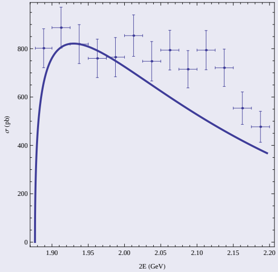
$|G_E/G_M|$ for $I = 0$



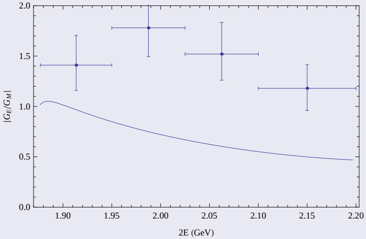
There is only one parameter $|G_s^0|$ in the fit of the cross section and $|G_E/G_M|$ is free of parameters. Here $|G_s^0|^2 = 50.9$

Isovector channel

Cross section for $I = 1$



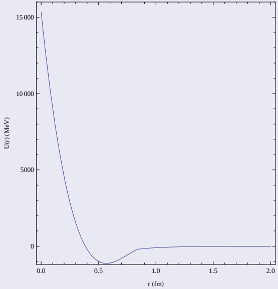
$|G_E/G_M|$ for $I = 0$



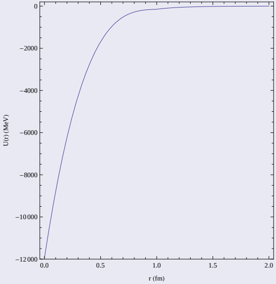
Here $|G_s^1|^2 = 5092$; it does not look reasonable.

Paris ptentials for triplet $N\bar{N}$ states

Paris triplet potential for $I = 1$



Paris triplet potential for $I = 0$



For $I = 1$ the wave function at the origin suppressed exponentially under the barrier, while for $I = 0$ it is enhanced due to attractive potential.

Summary

1. For lepton pair production the long range Coulomb final state interaction provides a nonzero contribution of all partial waves, $l \geq 0$, at the threshold. Interference of partial waves results in small asymmetry, $\sim \alpha^2$ in angular distribution of the produced lepton pair.

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2. For $N\bar{N}$-pair production near threshold the asymmetry in angular distribution owes to the tensor component of the strong final state interaction. The ratio G_E/G_M is fully determined by the $N\bar{N}$ optical potential without any free parameters.

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1. For lepton pair production the long range Coulomb final state interaction provides a nonzero contribution of all partial waves, $l \geq 0$, at the threshold. Interference of partial waves results in small asymmetry, $\sim \alpha^2$ in angular distribution of the produced lepton pair.
2. For $N\bar{N}$-pair production near threshold the asymmetry in angular distribution owes to the tensor component of the strong final state interaction. The ratio G_E/G_M is fully determined by the $N\bar{N}$ optical potential without any free parameters.
3. Due to strong repulsion at very short distances in the isovector part of the Paris $N\bar{N}$-potential, the isoscalar amplitude dominates near threshold in $e^+e^- \to N\bar{N}$ process.