Technische Universität München

Alexander von Humboldt Stiftung / Foundation

τ hadronic spectral function moments: perturbative expansions and α_s

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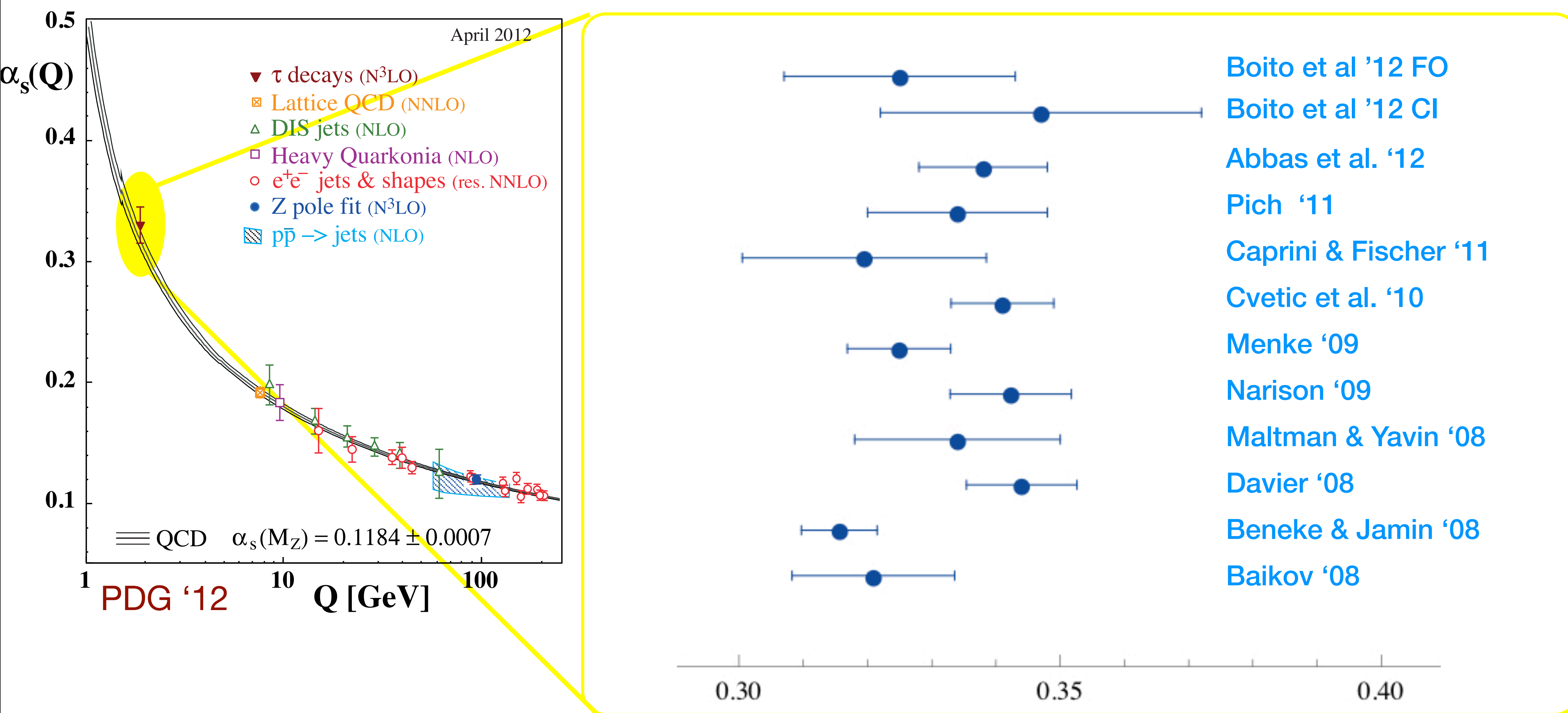
Work done in collaboration with M. Beneke and M. Jamin

--- M Beneke, DB, M Jamin, JHEP 01 125 (2013)

Phi to Psi 2013
Roma, 09-12 Sep. 2013

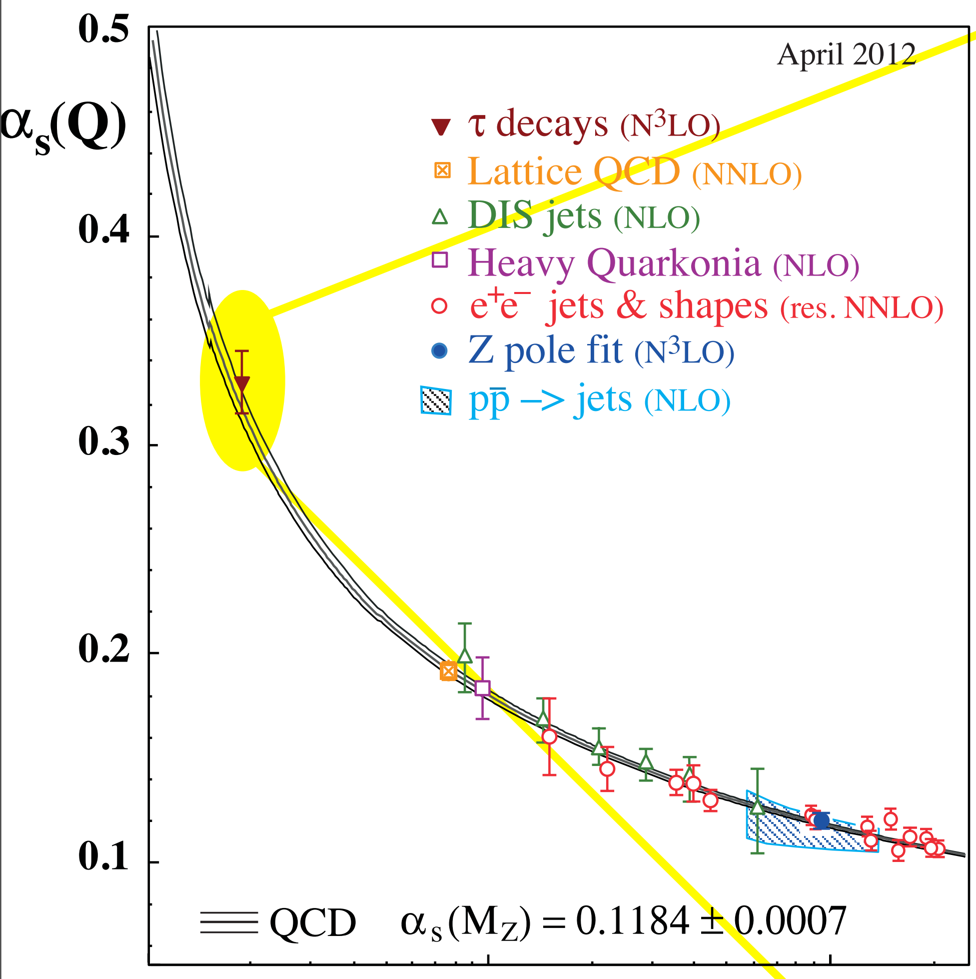
introduction

α_s from τ decays



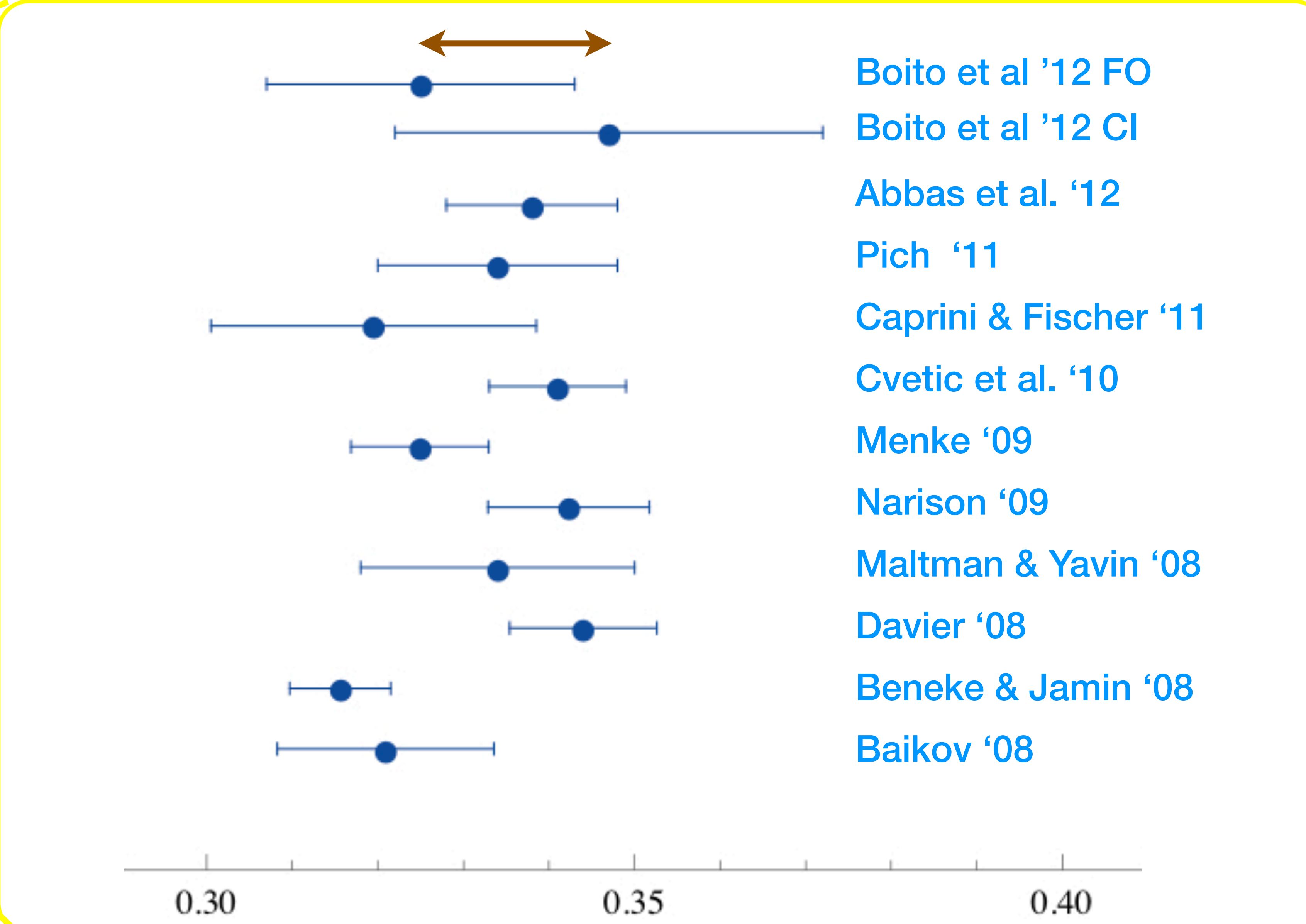
$$\alpha_s(m_\tau^2) = 0.36 \pm 0.04$$ Braaten, Narison, and Pich '92

- Spread in the results reflect (mainly) details of the theoretical input.



PDG '12

Prescription for the RG improvement



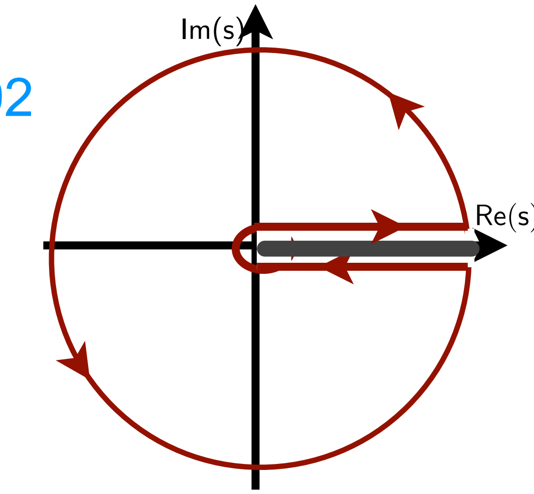
$\alpha_s(m_\tau^2) = 0.36 \pm 0.04$ Braaten, Narison, and Pich '92

- Spread in the results reflect (mainly) details of the theoretical input.
- There are still open questions (Renormalization Group Improvement, duality violations, ...)

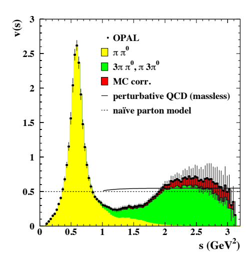
see talk by Peris

Sum rules for the spectral functions

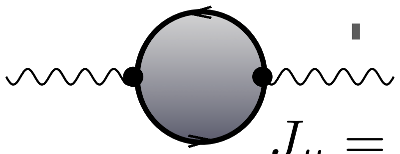
(in tau decays) Braaten, Narison, and Pich, 1992



$$\int_0^{s_0} ds\, w(s)\, \frac{1}{\pi} \mathrm{Im}\tilde{\Pi}(s) = \frac{-1}{2\pi i} \oint_{|z|=s_0} dz\, w(z)\, \tilde{\Pi}(z)$$



experiment
(OPAL and ALEPH)



theory

$J_\mu = \bar{u}\gamma_\mu(\gamma_5)q(x)$

Sum rules for the spectral functions

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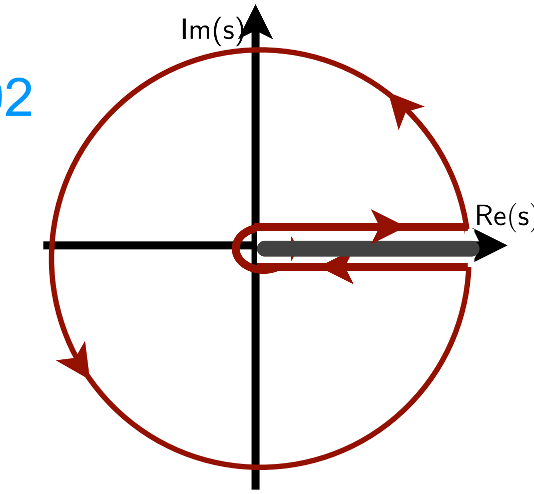

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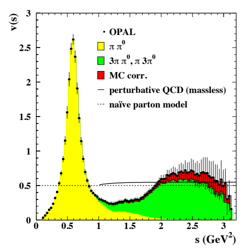

experiment
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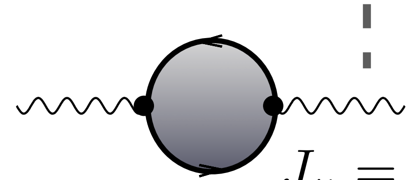


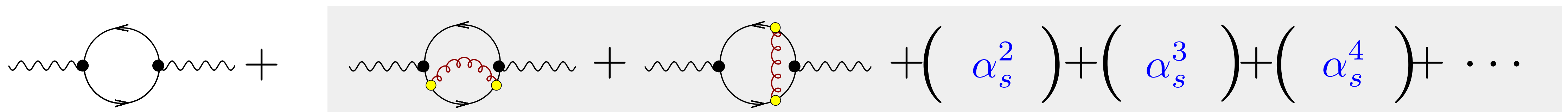
theory

$J_\mu = \bar{u}\gamma_\mu(\gamma_5)q(x)$

- Contributions to the sum rule (theory side)

$$R^{w_i}_{V/A}(s_0) = \frac{N_c}{2} S_{EW} |V_{ud}|^2 \left[\delta^{\text{tree}}_{w_i} + \delta^{(0)}_{w_i}(s_0) + \sum_{D\geq 2} \delta^{(D)}_{w_i,V/A}(s_0) + \delta^{\text{DV}}_{w_i,V/A}(s_0) \right]$$

OPE DVs

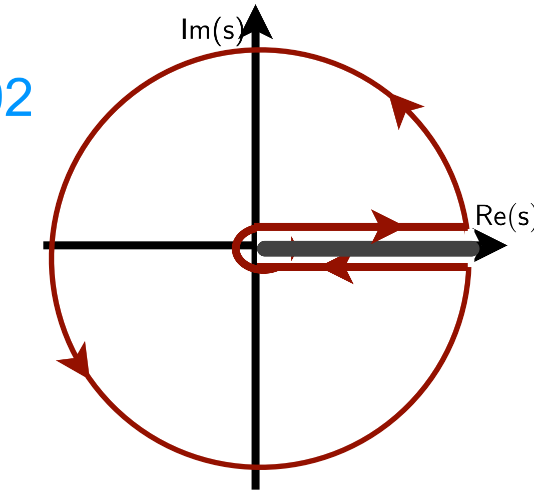


α_s^4 : Baikov, Chetyrkin, Kühn 2008

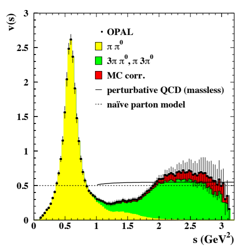
our focus is on $\delta^{(0)}_{w_i}$ (moment dependence)

Sum rules for the spectral functions

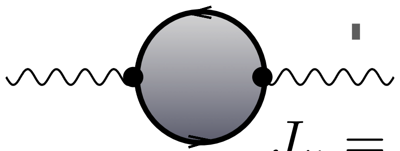
(in tau decays) Braaten, Narison, and Pich, 1992



$$\int_0^{s_0} ds\, \boxed{w(s)}\, \frac{1}{\pi} \mathrm{Im}\tilde{\Pi}(s) = \frac{-1}{2\pi i} \oint_{|z|=s_0} dz\, \boxed{w(z)}\, \tilde{\Pi}(z)$$



experiment (OPAL and ALEPH)



theory

$J_\mu = \bar{u}\gamma_\mu(\gamma_5)q(x)$

- Kinematical moment is a special case:

$$w_\tau(s) = (1 - s/s_0)^2(1 + 2s/s_0) = (1 - x)^2(1 + 2x)$$

$$s_0 = m_\tau^2 \Longrightarrow R_\tau = \frac{\Gamma[\tau \to \text{hadrons}\, \nu_\tau]}{\Gamma[\tau \to e^- \bar{\nu}_e\, \nu_\tau]} = 3.6280 \pm 0.0094$$

but the choice of $w(s)$ is free

- In the literature several weight functions are used

  DB et al ’11, ’12, Davier et al ’08, Maltman and Yavin ’08, ALEPH ’98, ‘05, OPAL ‘99

  - One often employs $w_\tau(x) = (1-x)^2(1+2x)$ (gives $R_\tau = \frac{\Gamma[\tau \to \mathrm{hadrons}\,\nu_\tau]}{\Gamma[\tau \to e^-\bar{\nu}_e\,\nu_\tau]}$)

    and many others:

    $w(x) = 1,\ w(x) = 1 - x^2,\ w(x) = x(1-x)^2,\ w^{(k)}(x) = (1-x)^3 x^k (1+2x),$
    $w^{(n)}(x) = 1 - \frac{n}{n-1}\,x + \frac{n}{n-1}\,x^n \ldots$

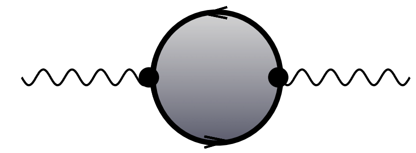
  - Different emphasis on the experimental spectrum. Change the relative contributions on the theory side (pert., OPE, DVs)

  - α_s dependence comes mainly from $\delta^{(0)}_{w_i}$

- Open questions in $\delta^{(0)}_{w_i}$:

  - Renormalization group improvement: what is the best prescription?
    Contour Improved PT vs Fixed Order PT
  - Moment dependence?
  - Are there better moments to determine α_s ?

renormalization group

- Description in terms of the Adler function (derivative of )

$$D^{(1+0)}_{\rm pert}(s) = \frac{N_c}{12\pi^2}\sum_{n=0}^{\infty} a_\mu^n \sum_{k=1}^{n+1} k\, c_{n,k} \left(\log\frac{-s}{\mu^2}\right)^{k-1} \qquad a_\mu = \alpha(\mu)/\pi$$

- only $c_{n,1}$ are independent (known up to $c_{4,1}$). $c_{n,k}$ depend on $c_{n,1}$ and β_m .
- Prescriptions for the RG improvement

FOPT

$\mu = s_0$

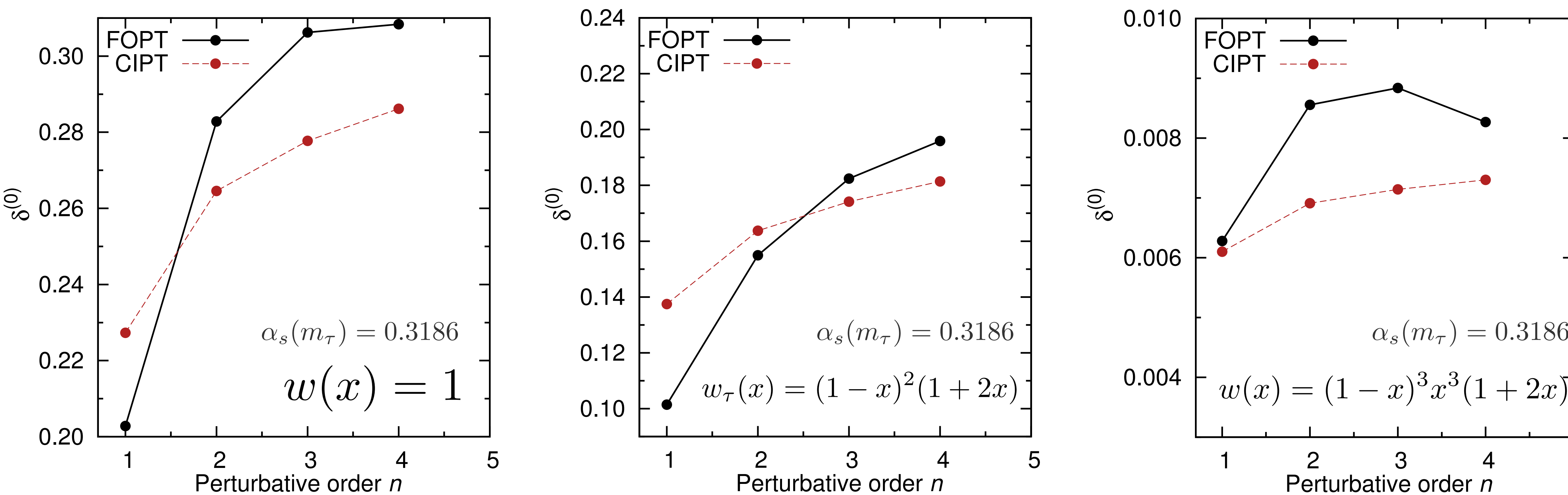
$$\delta^{(0)}_{{\rm FO},w_i} = \sum_{n=1}^{\infty} a(s_0)^n \sum_{k=1}^{n} k\, c_{n,k}\, J^{{\rm FO},w_i}_{k-1} \qquad J^{{\rm FO},w_i}_n \equiv \frac{1}{2\pi i}\oint_{|x|=1}\frac{dx}{x} W_i(x)\log^n(-x)$$

CIPT

$\mu = -s_0 x$

Le Diberder and Pich '92

$$\delta^{(0)}_{{\rm CI},w_i} = \sum_{n=1}^{\infty} c_{n,1}\, J^{{\rm CI},w_i}_n(s_0) \qquad J^{{\rm CI},w_i}_n(s_0) \equiv \frac{1}{2\pi i}\oint_{|x|=1}\frac{dx}{x} W_i(x)\, a^n(-s_0 x)$$



higher orders

$$R \sim \sum_n^{\infty} r_n \alpha_{(s)}^{n+1}$$

divergent but (hopefully) **asymptotic**

Dyson 1952

$R \downarrow$? $\qquad$ $\downarrow$ *in QFT we only know the expansion*

- Define the Borel transformed series

$$B[R](t) \equiv \sum_{n=0}^{\infty} r_n \frac{t^n}{n!}$$

which can be "summed" $\Longrightarrow$ $\tilde{R} \equiv \int_0^{\infty} dt\, \mathrm{e}^{-t/\alpha} B[R](t)$

- Divergent behaviour encoded in the singularities of $B[R](t)$

(review) Beneke 1999

Strategy:

sing. of $B[R](t)$: $\frac{c}{(1+at)^b} + \cdots$ *(renormalons)* $\Rightarrow$ $B[R](t)$ $\Rightarrow$ $\sum_n^{\infty} r_n \alpha_{(s)}^{n+1}$

$$D_{\rm pert}^{(1+0)}(Q^2) = \frac{N_c}{12\pi^2} \sum_{n=0}^{\infty} \left(\frac{c_{n,1}}{\pi^n}\right) \alpha_Q^n$$

- General structure of large-order behavior (believed to be) known

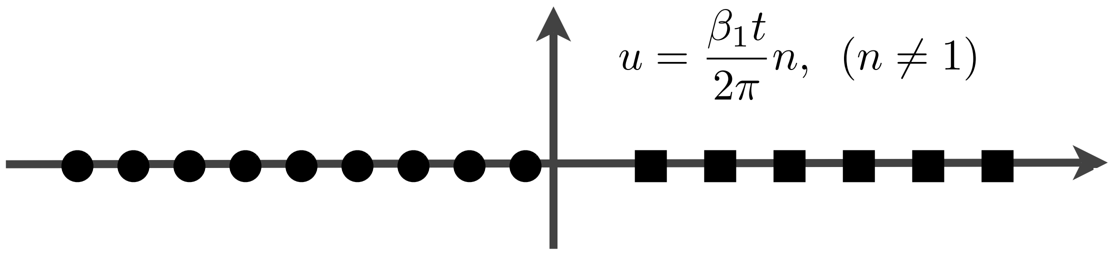
Borel transformed Adler function

(review) Beneke 1999

$$B[\hat{D}](t) \equiv \sum_{n=0}^{\infty} \frac{c_{n,1}}{\pi^n} \frac{t^n}{n!}$$

Borel sum: $$\hat{D}(\alpha) \equiv \int_0^\infty dt\, \mathrm{e}^{-t/\alpha} B[\hat{D}](t)$$

- Singularities in the *t* plane



UV renormalons

- sign alternating
- leading sing. in the Adler function at u =-1
- no-sing alternation in known coeff.: small residue for the leading UV pole

IR renormalons

- fixed sign
- sing. at u = 2, 3, 4... related to dim-4, dim-6, dim-8... contributions
- u = 2 related to the gluon condensate

$$B[D_p] = \frac{c_p}{(p-u)^\gamma}\left[1 + \tilde{b}_1(p-u) + \cdots\right]$$

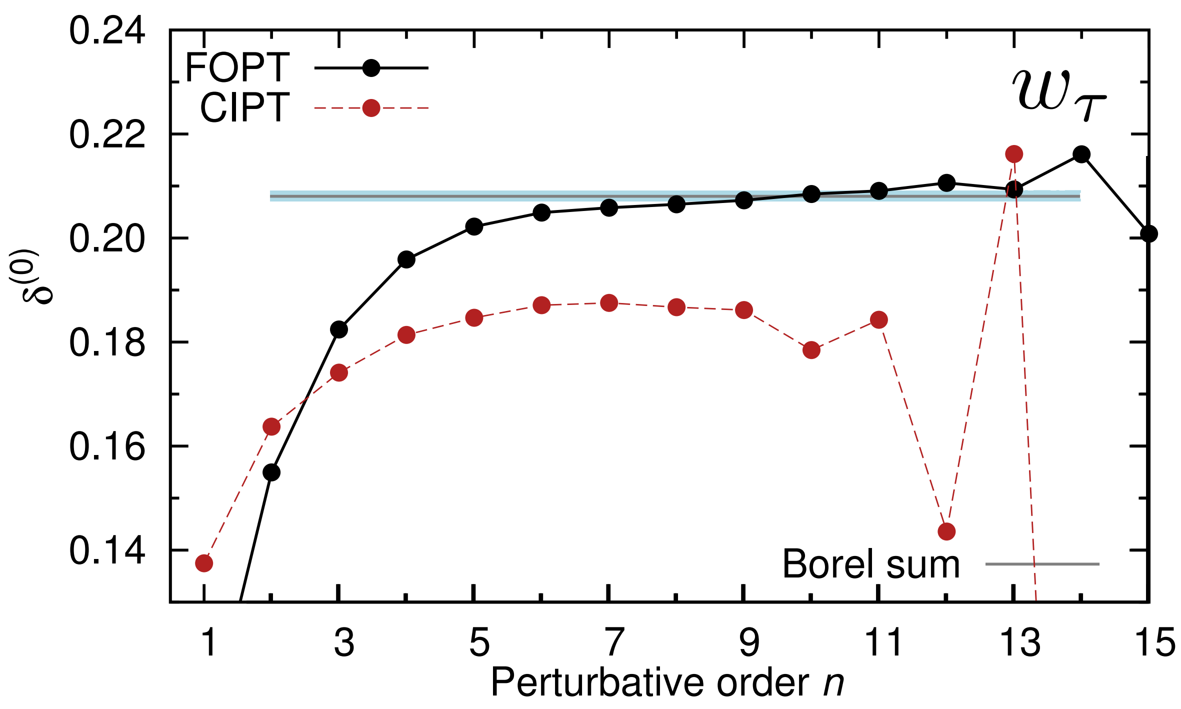
Structure of each singularity in principle calculable (up to c_p)

Reference model (RM)

Beneke & Jamin '08

$$B[\widehat{D}](u) = B[\widehat{D}_1^{\mathrm{UV}}](u) + B[\widehat{D}_2^{\mathrm{IR}}](u) + B[\widehat{D}_3^{\mathrm{IR}}](u) + d_0^{\mathrm{PO}} + d_1^{\mathrm{PO}}\, u$$

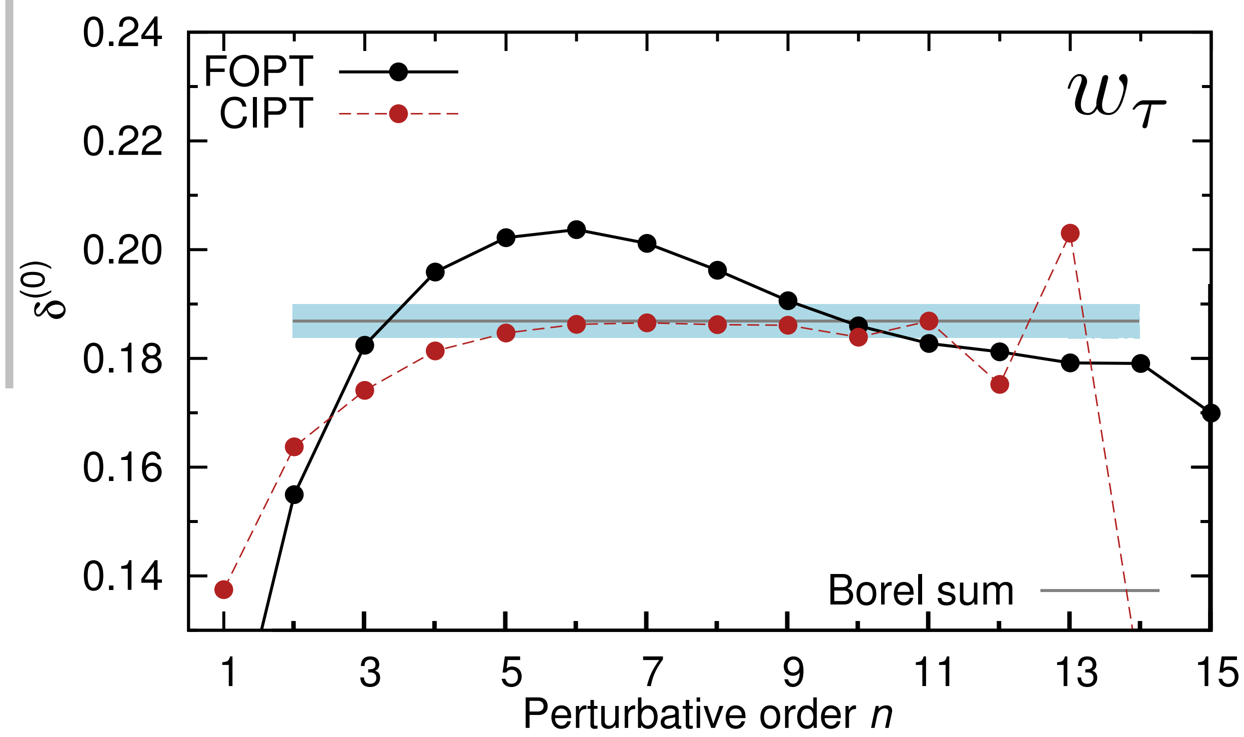
- Model with the leading UV and the first two IR singularities.
- Small polynomial terms to fix $c_{1,1}$ and $c_{2,1}$.
- Favors FOPT (related to the presence of u = 2 sing).



Alternative model (AM)

$$B[\widehat{D}](u) = B[\widehat{D}_1^{\mathrm{UV}}](u) + B[\widehat{D}_3^{\mathrm{IR}}](u) + B[\widehat{D}_4^{\mathrm{IR}}](u) + d_0^{\mathrm{PO}} + d_1^{\mathrm{PO}}\, u$$

- No IR singularity at u = 2 (related to the gluon condensate)
- Favors CIPT.



First conclusions:

- Decision in favor of FOPT or CIPT depends on the higher order coefficients.
- Highly correlated with the first IR renormalon (gluon condensate).

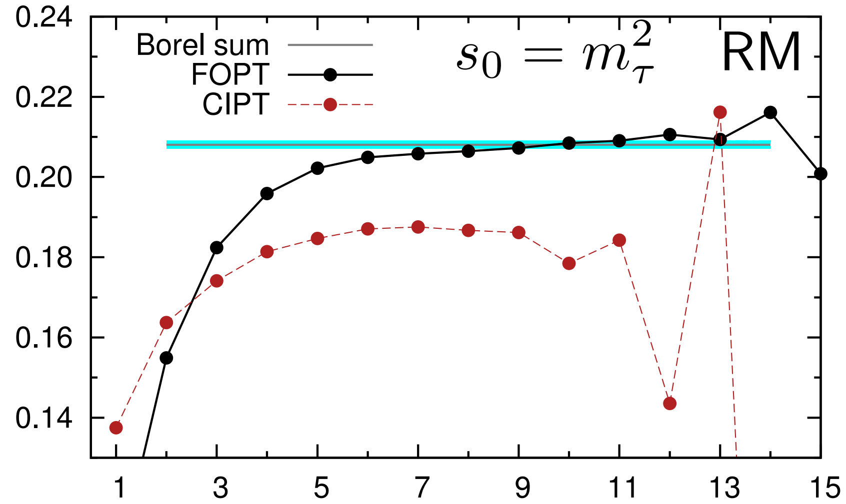
moment analysis

- Moments studied can have their behavior separated in classes

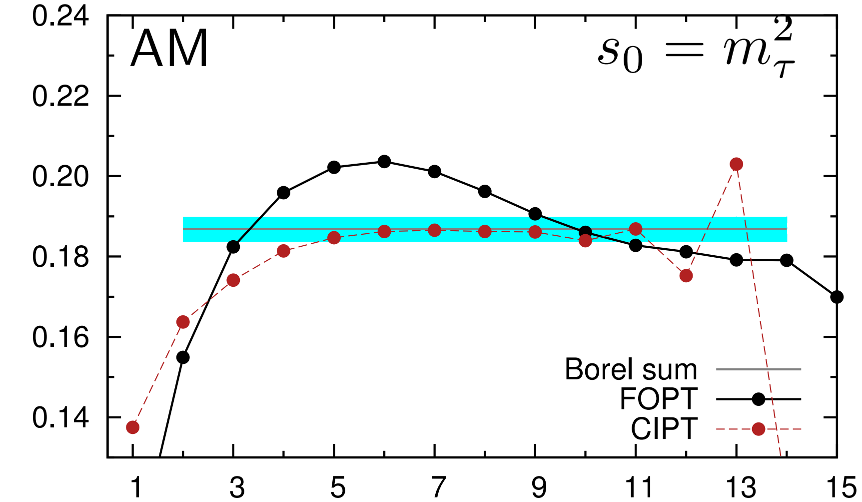
k	$w_k(x)$		
1	1	[DB et al]	Monomials
2	x		
3	x^2		
4	x^3		
5	x^4		
6	$1-x$		Pinched [$w(1)=0$] with a "1"
7	$1-x^2$	[DB et al]	
8	$1-x^3$	[DB et al]	
9	$1-\frac{3x}{2}+\frac{x^3}{2}$	[Maltman & Yavin]	
10	$(1-x)^2$	[Maltman & Yavin]	
11	$(1-x)^3$		
w_τ	$(1-x)^2(1+2x)$	[All recent works]	
13	$(1-x)^3(1+2x)$	[Davier et al, ALEPH, OPAL]	
14	$(1-x)^2x$	[Maltman & Yavin]	Pinched with no "1"
15	$(1-x)^3x(1+2x)$	[Davier et al, ALEPH, OPAL]	
16	$(1-x)^3x^2(1+2x)$	[Davier et al, ALEPH, OPAL]	
17	$(1-x)^3x^3(1+2x)$	[Davier et al, ALEPH, OPAL]	

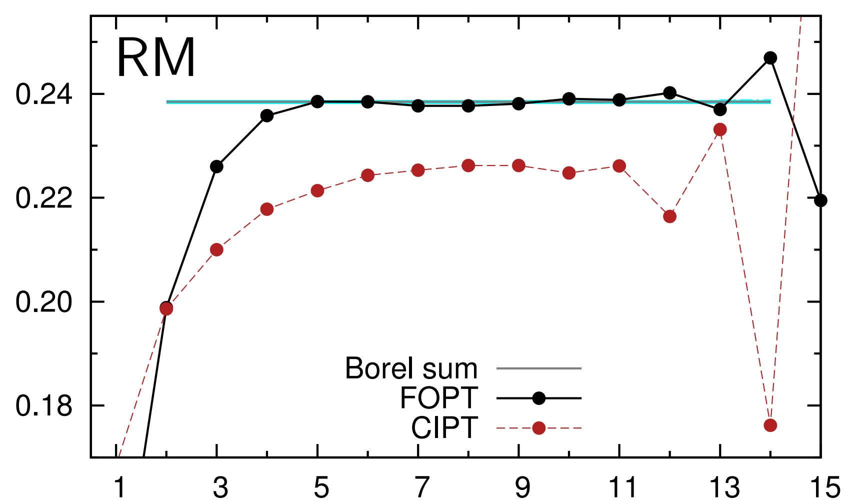
Note: moments with a term "*x*" form a separate class.

- Pinched moments with a “1”
- At least one of the methods approach the Borel result at relatively low orders

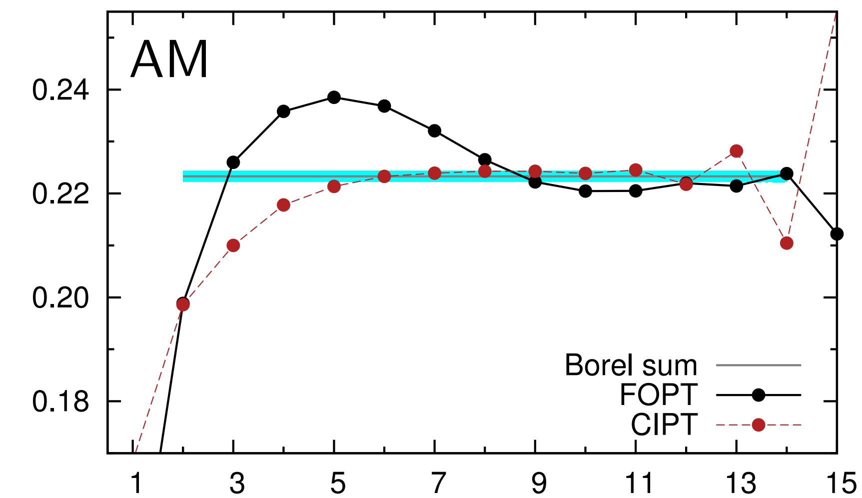


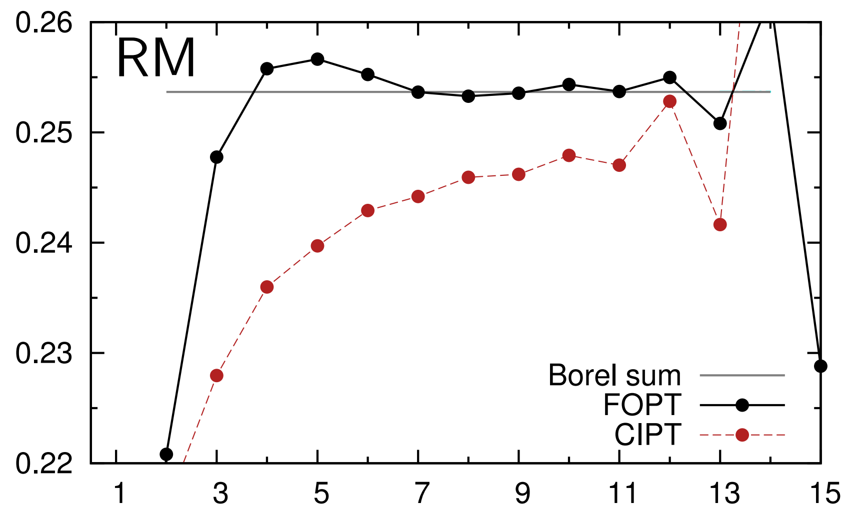
$$w_\tau(x) = (1-x)^2(1+2x)$$



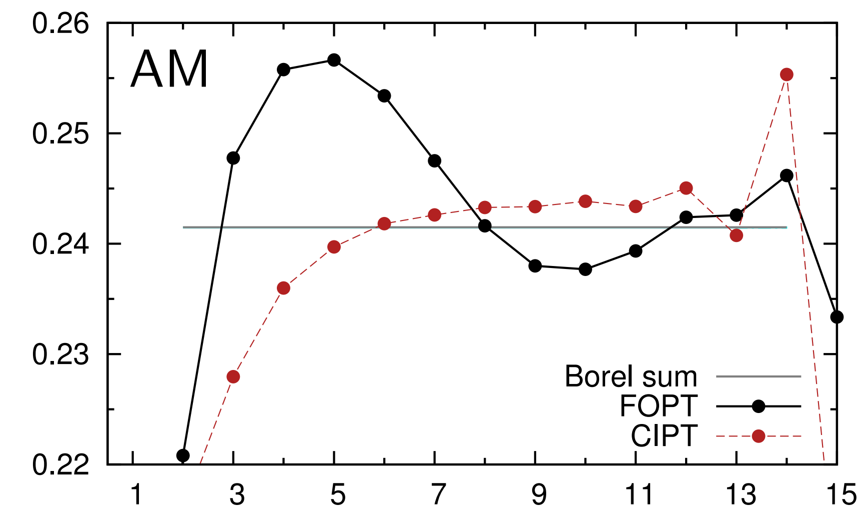


$$w(x) = 1 - x^2$$



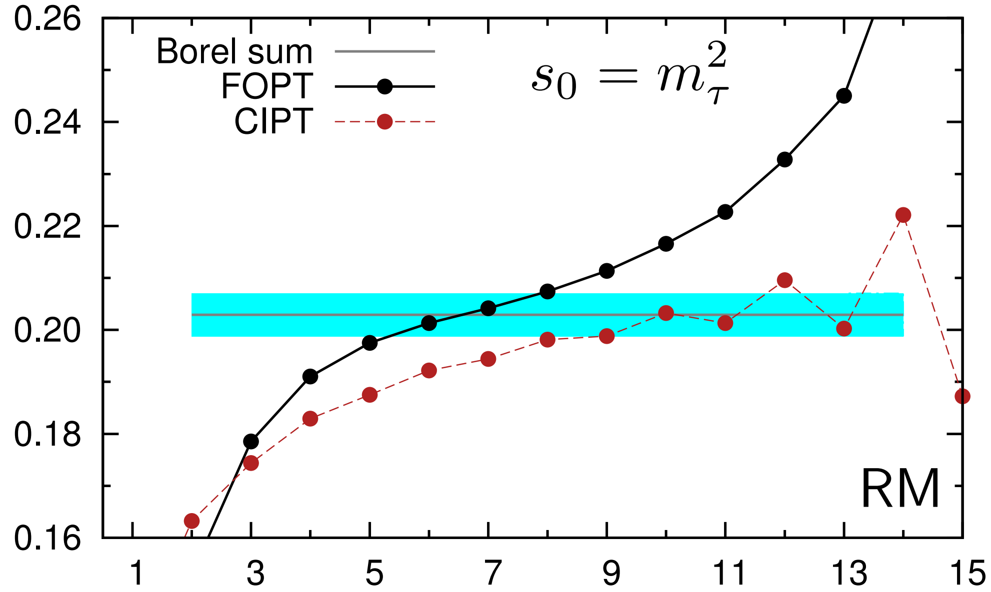


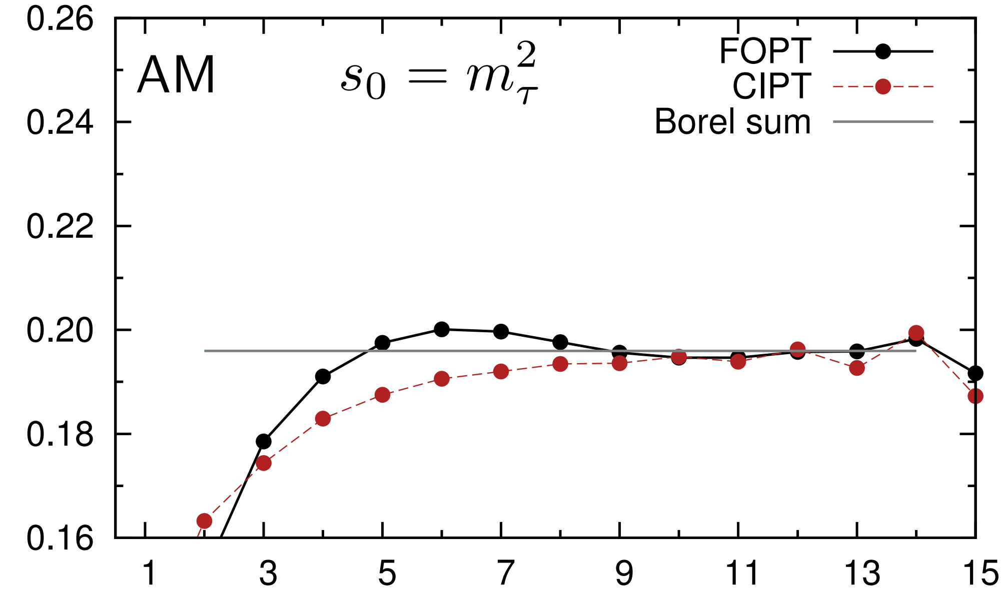
$$w(x) = 1 - x^3$$



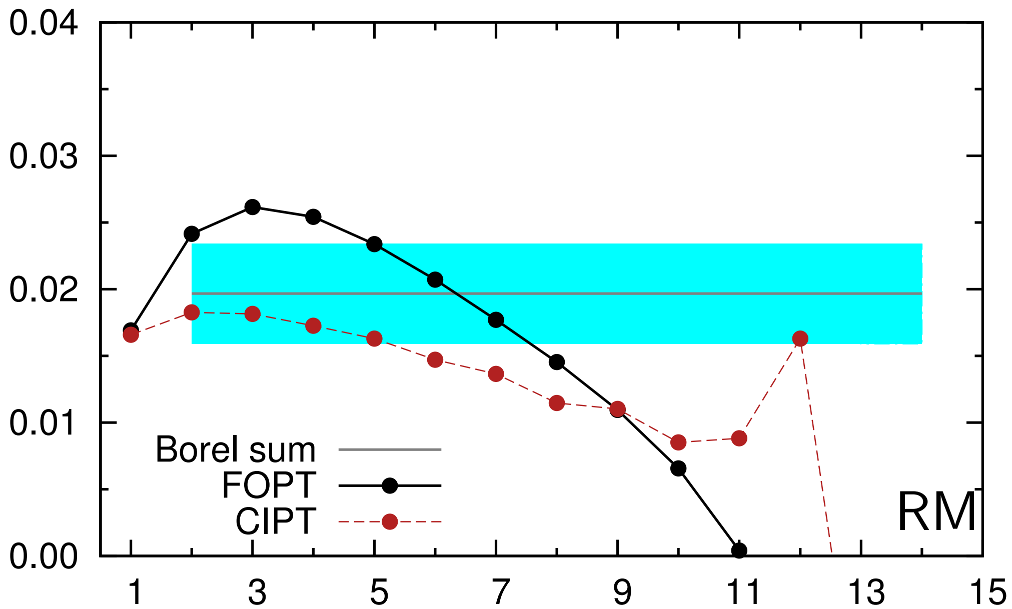
dangerous moments

- Moments with the term x
- Very sensitive to D = 4. Unstable results if the u = 2 singularity is sizable.

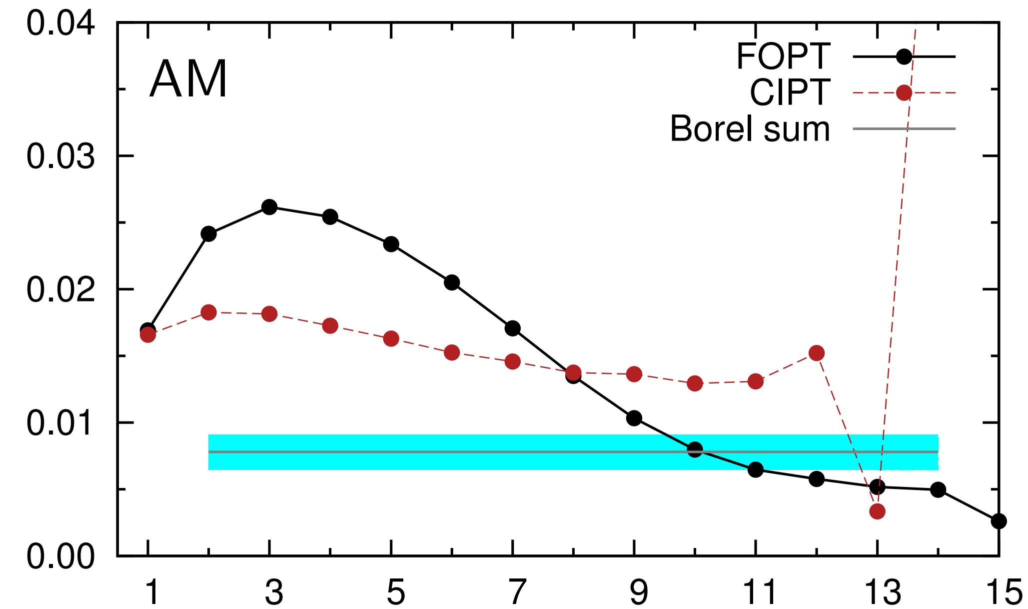




$$w(x) = 1 - x$$

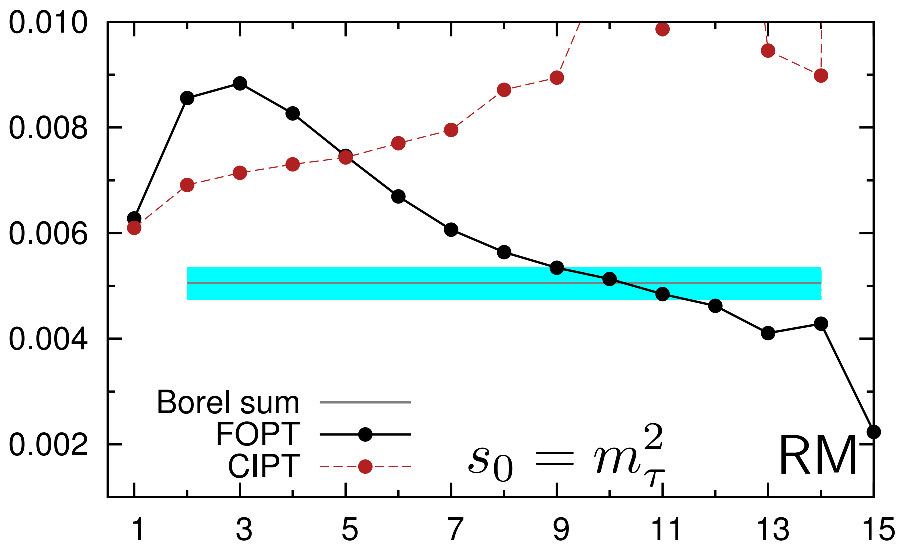


$$w(x) = (1-x)^3 x(1+2x)$$

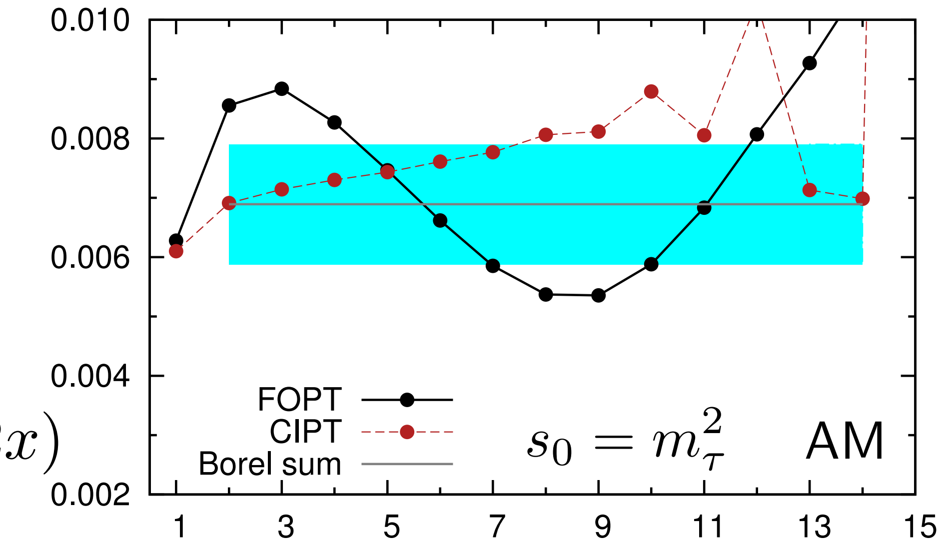


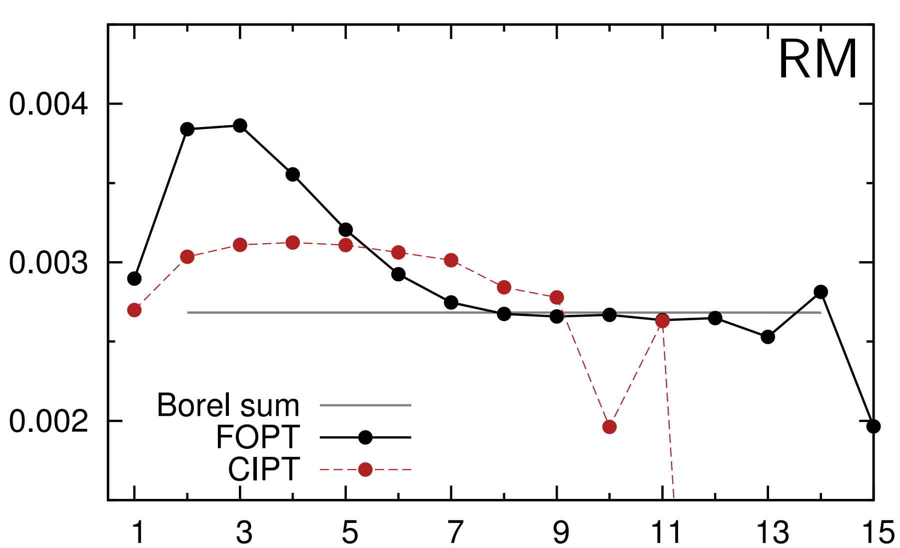
problematic moments

- Pinched moments starting at x^2 (or higher)
- Borel results are never well reproduced at low orders.



$$w(x) = (1-x)^3 x^2 (1+2x)$$





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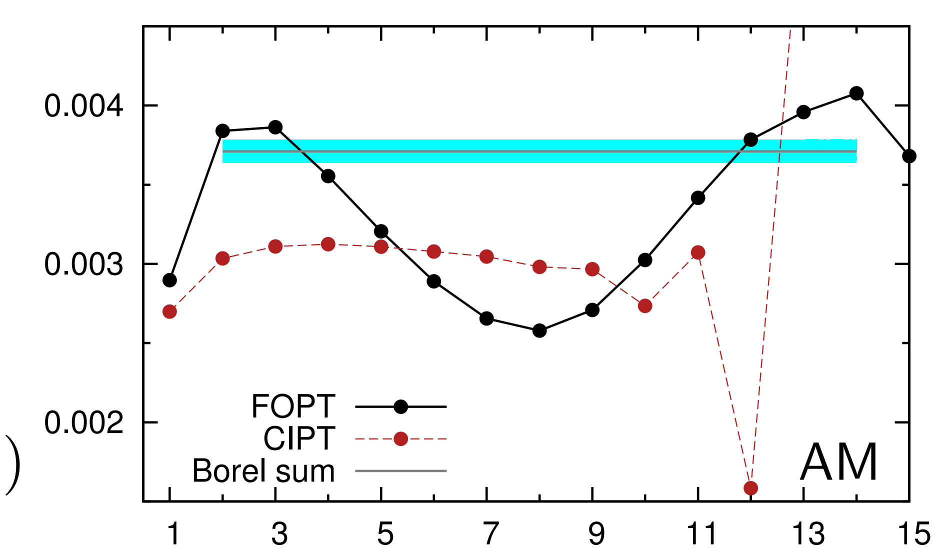


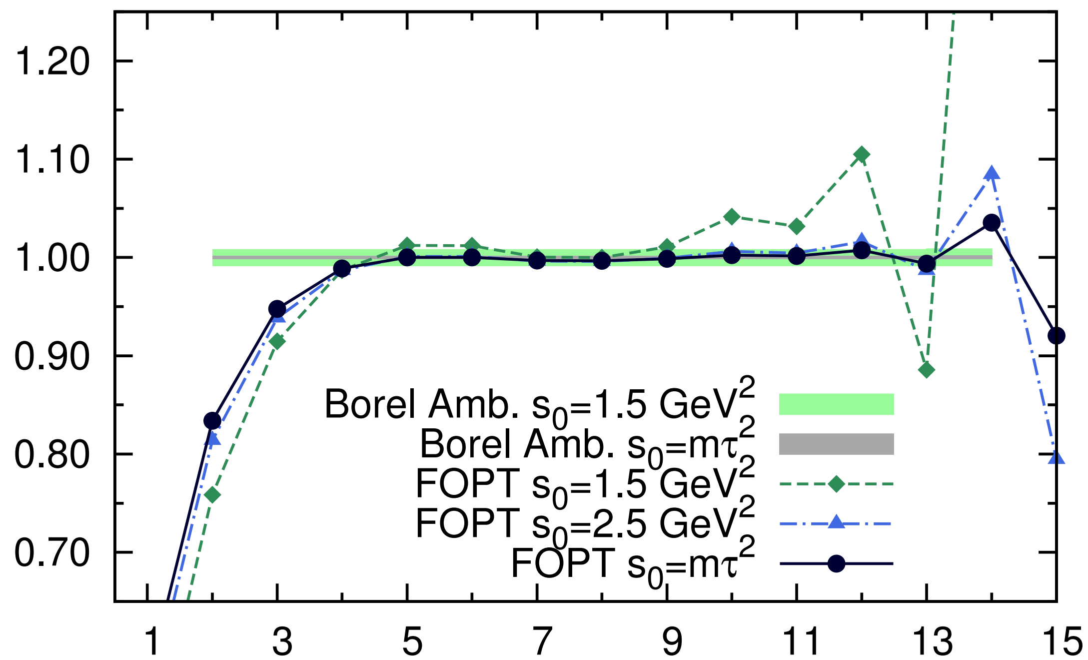
energy dependence

energy dependence: reference model

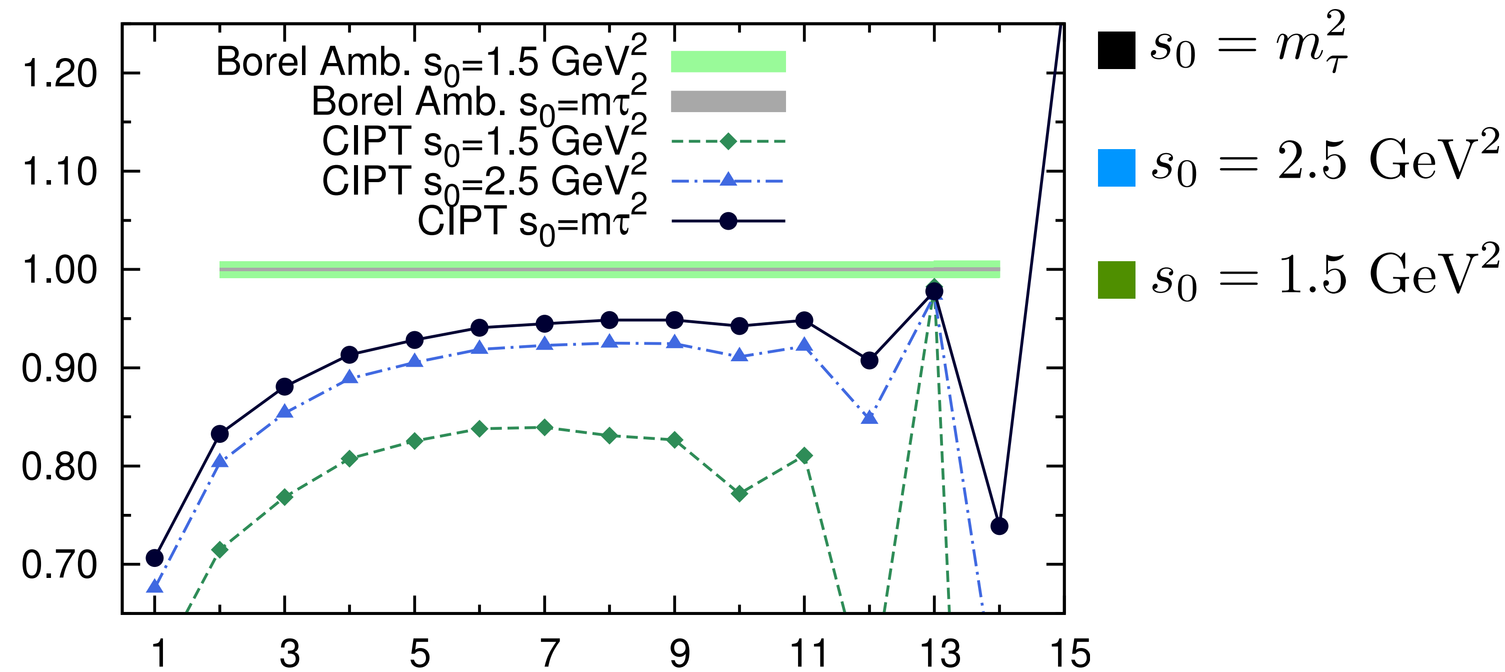
Reference model

- $w(x) = 1 - x^2$

FOPT

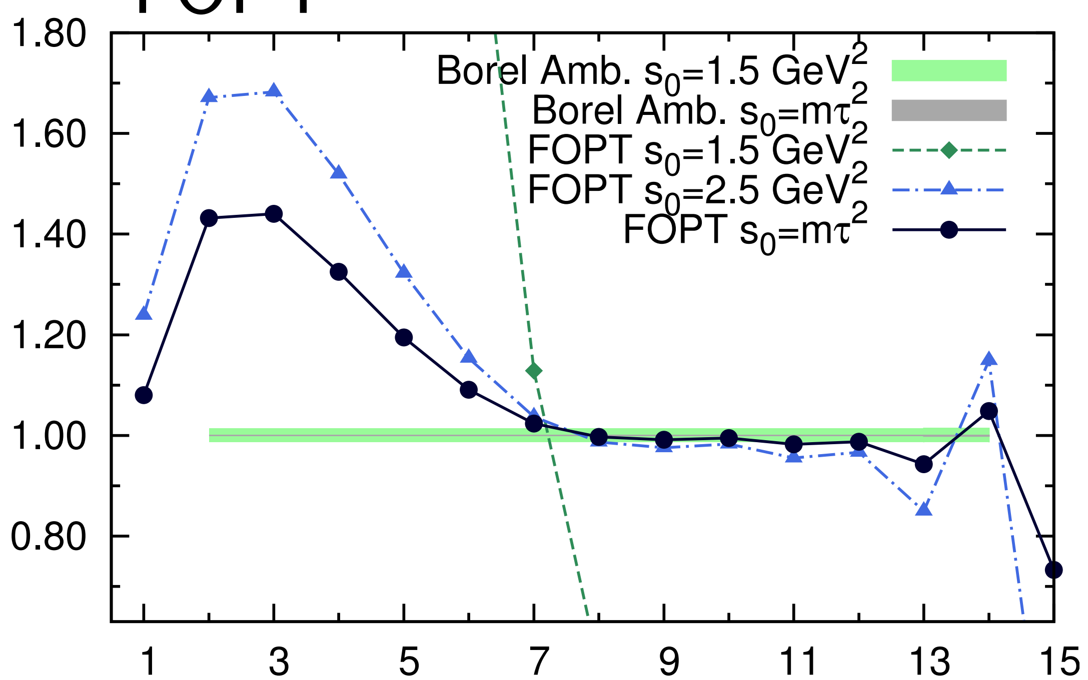


CIPT

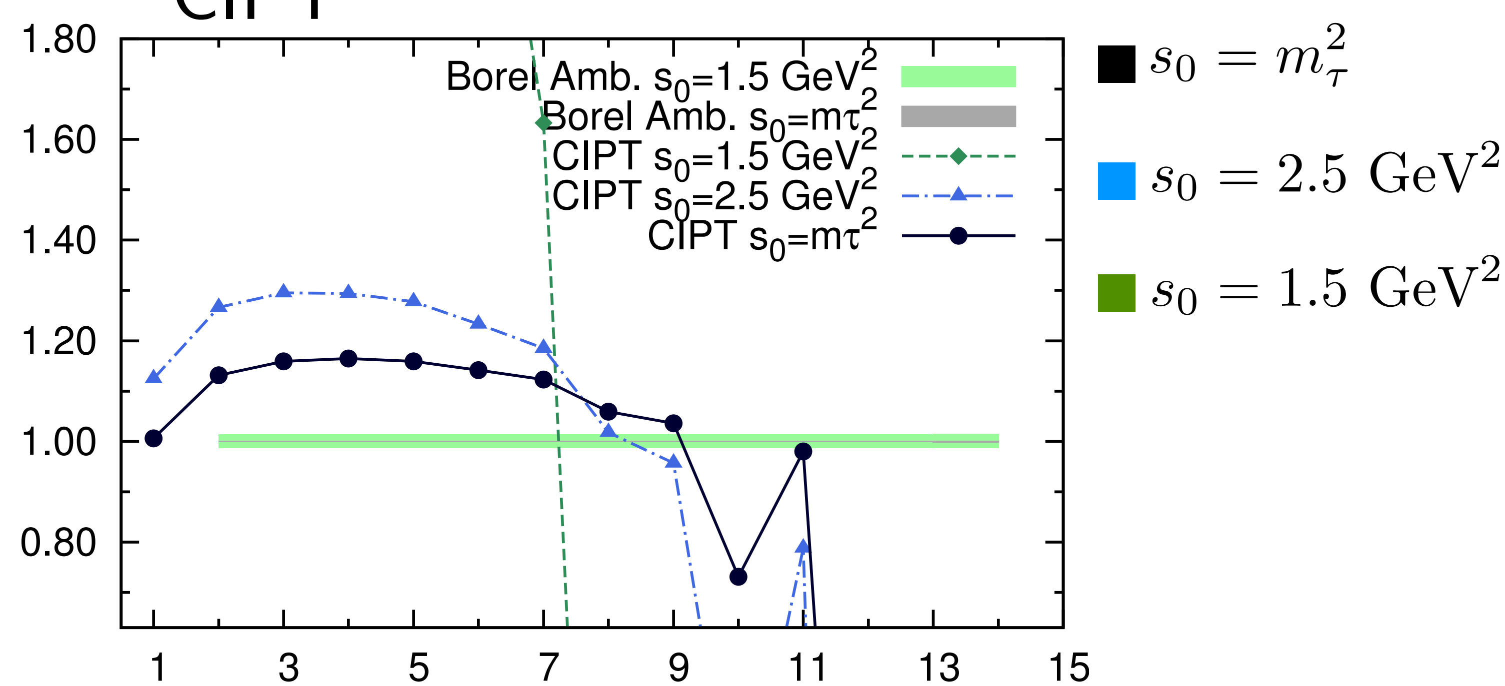


- $w(x) = (1-x)^3 x^3 (1+2x)$

FOPT

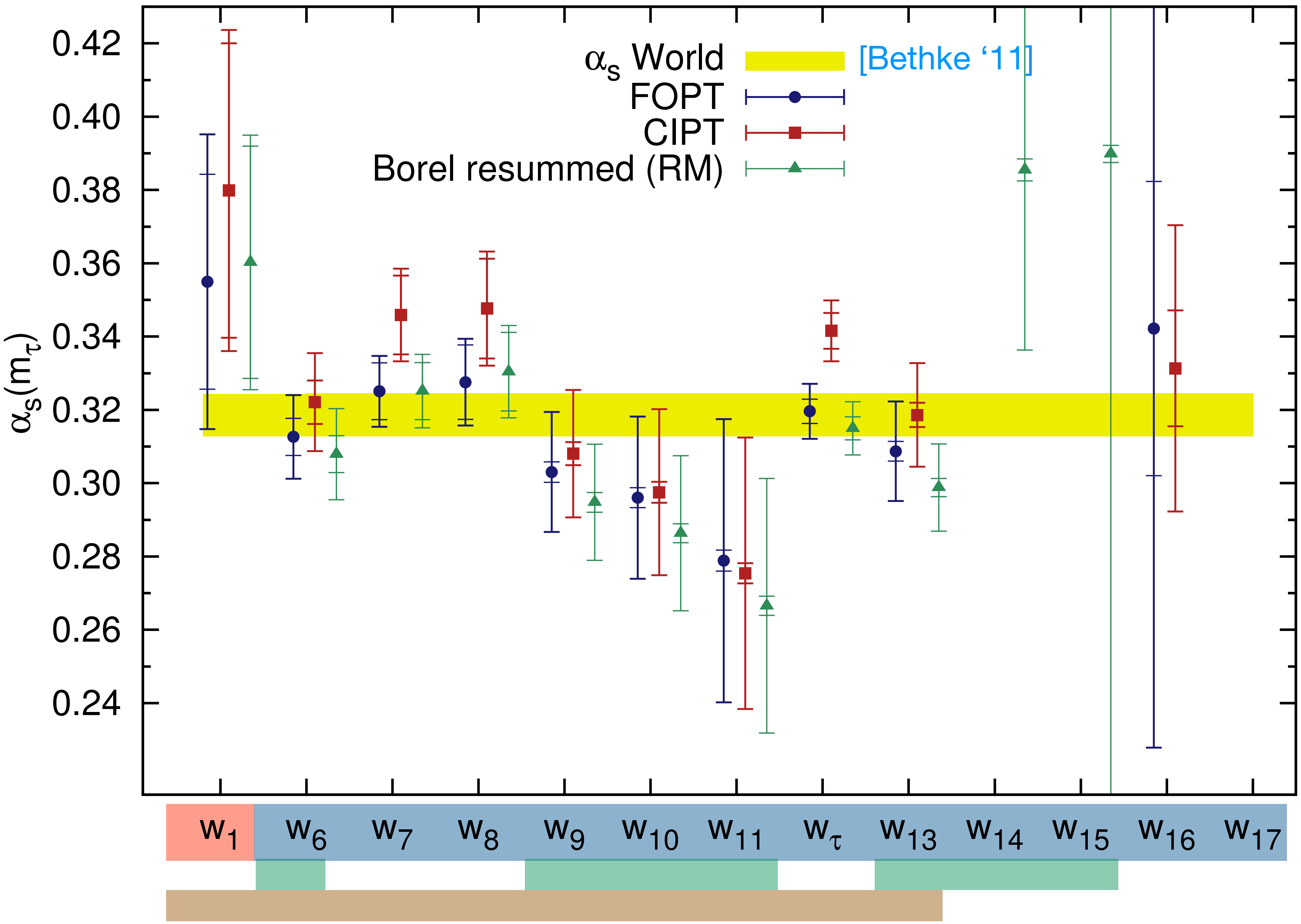


CIPT



consequences for α_s

(exploratory study: power corrections and DVs as external inputs)



k	$w_k(x)$
1	1
2	x
3	x^2
4	x^3
5	x^4
6	$1-x$
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8	$1-x^3$
9	$1-\frac{3x}{2}+\frac{x^3}{2}$
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w_τ	$(1-x)^2(1+2x)$
13	$(1-x)^3(1+2x)$
14	$(1-x)^2x$
15	$(1-x)^3x(1+2x)$
16	$(1-x)^3x^2(1+2x)$
17	$(1-x)^3x^3(1+2x)$

Power Corrections from Beneke & Jamin 2008

unpinched

pinched

contains a term "1"

contains a term "x"

Data: Updated ALEPH [Davier et al (2008)]. Warning: Correlations due to unfolding missing in this data set. Experimental errors potentially underestimated!

conclusions

- Decision in favor of FOPT or CIPT depends on the higher order coefficients.

- Some moments are more suitable for the extraction of α_s.

- The pinched moments with a "1" and without an "x" are ideal:
  - Good convergence of FOPT (RM) or CIPT (AM) at low orders

- Moments composed only by powers of "x" should be avoided:
  - problems in the convergence of **both** FOPT and CIPT,
  - power corrections are too important.

- Some of the recent extractions of α_s employed moments that are not optimal. similar conclusion also in Maltman & Yavin 2008

- Conformal mapping: Promising strategy to deal with the RG improvement of pt. series Abbas, Ananthanarayan, Caprini, and Fischer, PRD 88 034026 (2013)

extra

Reference model

Beneke & Jamin '08

- Separating the contributions in FOPT

$$\delta^{(0)}_{\mathrm{FO},w_i} = \sum_{n=1}^{\infty} \left[c_{n,1}\delta^{\mathrm{tree}}_{w_i} + g_n^{[w_i]} \right] a(s_0)^n \qquad g_n^{[w_i]} = \sum_{k=2}^{n} k\, c_{n,k} J^{\mathrm{FO},w_i}_{k-1}$$

- Result at α_s^n . FOPT sums the first *n* ***rows***. Important cancellations.

w_τ

Fixed Order

α_s^n		$c_{1,1}$	$c_{2,1}$	$c_{3,1}$	$c_{4,1}$	$c_{5,1}$	$c_{6,1}$	$c_{7,1}$	$c_{8,1}$	g_n	$\frac{c_n+g_n}{c_n}$
1		1									1
2	g_2	3.56	+ 1.64							3.56	3.17
3	g_3	8.31	+ 11.7	+ 6.37						20.0	4.14
4	g_4	−20.6	+ 30.5	+ 68.1	+ 49.1					78	2.59
⋮											⋮
6	g_6	−2924	−2858	−2280	2214	5041	3275			−807	0.754
⋮											⋮
8	g_8	14652	−29552	−145846	−502719	−393887	260511	467787	388442	−329054	0.153

- CIPT sums the first *n* ***columns to all orders***. Misses the cancellations.