The anomalous process $\gamma\pi \to \pi\pi$ and its impact on the π^0 transition form factor

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$\gamma\pi \to \pi\pi$ and the π^0 transition form factor

Introduction

- testing the Wess–Zumino–Witten chiral anomaly

Dispersion relations...

- ... for two pions: pion vector form factor
- ... for three pions: $\gamma\pi \to \pi\pi, \ \omega/\phi \to 3\pi$
- linking hadronic decays to transition form factors: $\omega/\phi \to \pi^0\gamma^*$

Towards the π^0 transition form factor

Summary / Outlook

work done in collaboration with M. Hoferichter, S. Leupold,
F. Niecknig, D. Sakkas, S. P. Schneider

Testing the Wess–Zumino–Witten chiral anomaly

- controls low-energy processes of odd intrinsic parity
- π^0 decay $\pi^0 \to \gamma\gamma$: $F_{\pi^0\gamma\gamma} = \frac{e^2}{4\pi^2 F_\pi}$
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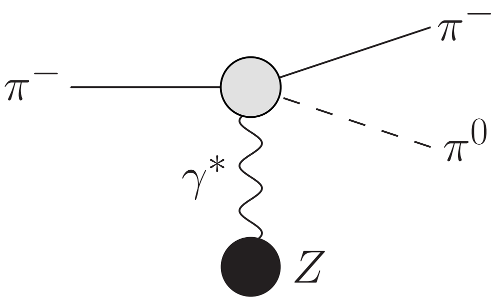
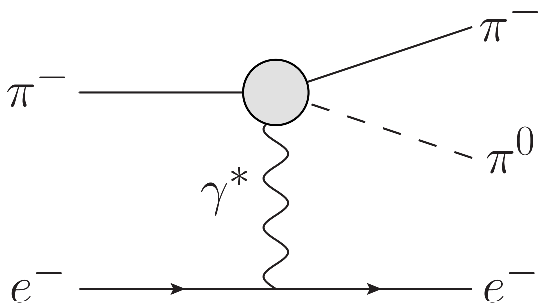
Primakoff reaction



$F_{3\pi} = (10.7 \pm 1.2)$ GeV^{-3}

Serpukhov 1987, Ametller et al. 2001

$\pi^- e^- \to \pi^- e^- \pi^0$



$F_{3\pi} = (9.6 \pm 1.1)$ GeV^{-3}

Giller et al. 2005

$\longrightarrow$ $F_{3\pi}$ tested only at 10% level

Chiral anomaly: Primakoff measurement

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- Primakoff measurement of whole spectrum
  COMPASS, work in progress

- idea: use dispersion relations to exploit all data below 1 GeV for anomaly extraction

- effect of ρ resonance included model-independently via $\pi\pi$ P-wave phase shift

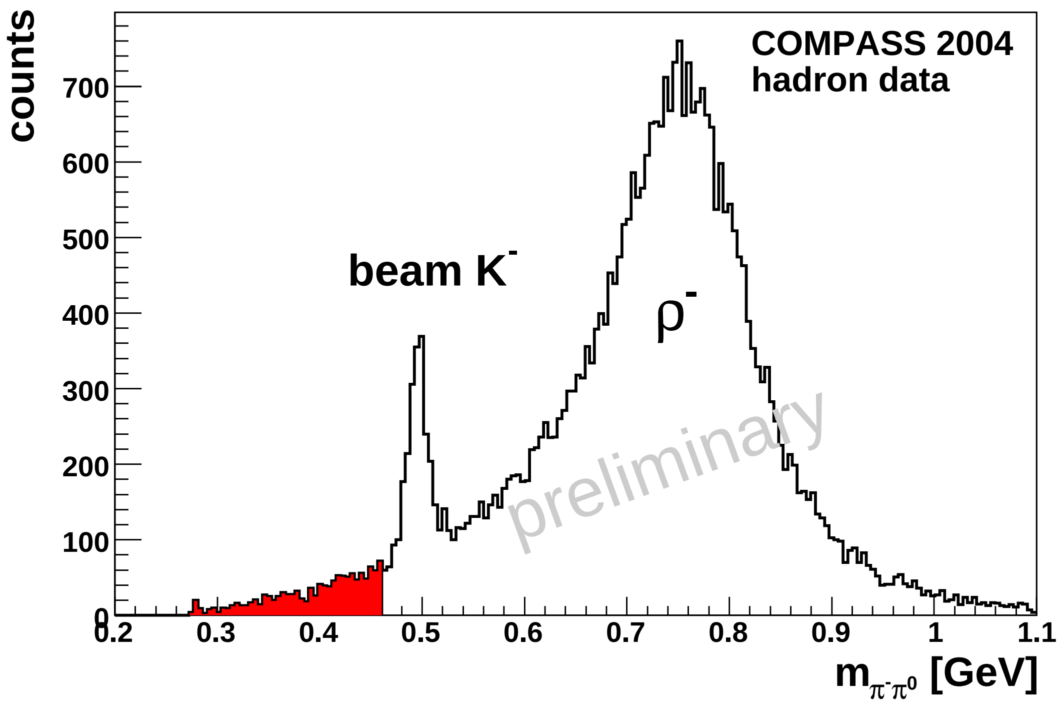
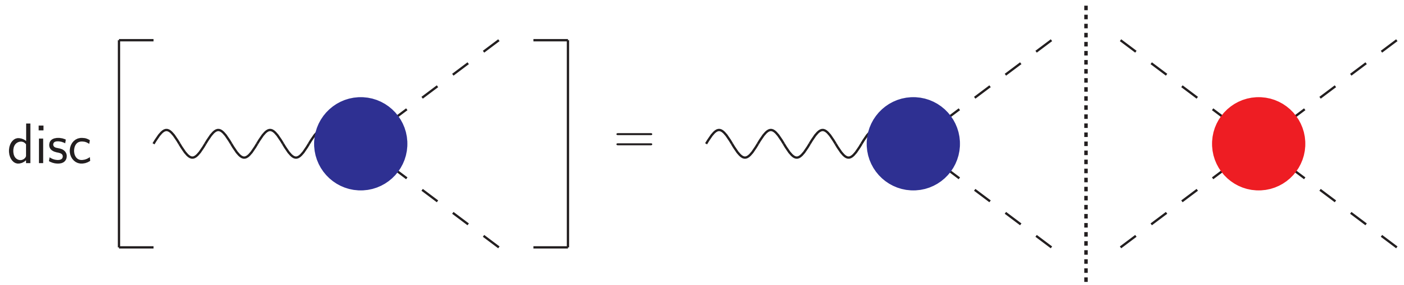


figure courtesy of T. Nagel 2009

Warm-up: pion form factor from dispersion relations

- just two particles in final state: form factor; from unitarity:



$$\frac{1}{2i}\,\text{disc}\,F_I(s) \;=\; \text{Im}\,F_I(s) \;=\; F_I(s)\times\theta(s-4\,M_\pi^2)\times\sin\delta_I(s)\,e^{-i\delta_I(s)}$$

$\longrightarrow$ final-state theorem: phase of $F_I(s)$ is just $\delta_I(s)$ Watson 1954

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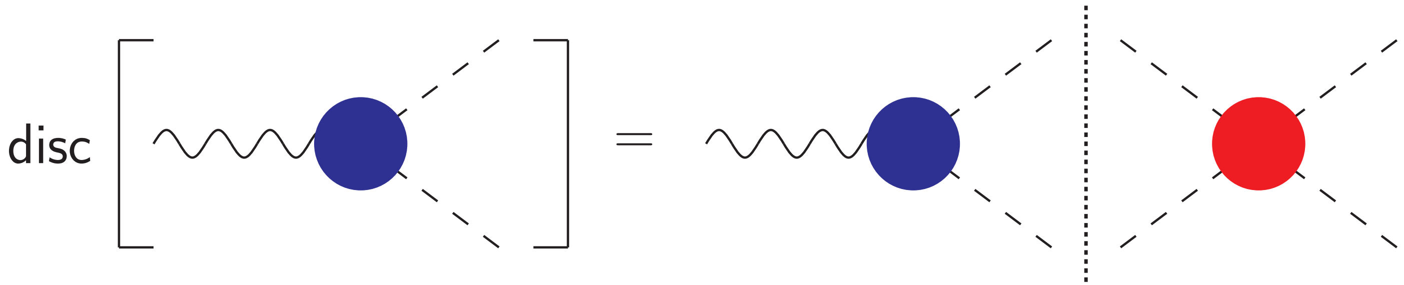

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- solution to this homogeneous integral equation known:

$$F_I(s) = P_I(s)\Omega_I(s)\,, \quad \Omega_I(s) = \exp\left\{\frac{s}{\pi}\int_{4M_\pi^2}^{\infty} ds'\,\frac{\delta_I(s')}{s'(s'-s)}\right\}$$

$P_I(s)$ polynomial, $\Omega_I(s)$ Omnès function Omnès 1958

- constrain polynomial using symmetries / chiral perturbation theory (normalisation/derivatives at $s=0$)

Dispersion relations for 3 pions

- $\gamma\pi \to \pi\pi$ particularly simple system: odd partial waves $\longrightarrow$ P-wave interactions only (neglecting F- and higher)
- decay amplitude decomposed into single-variable functions

$$\mathcal{M}(s,t,u) = i\epsilon_{\mu\nu\alpha\beta} n^\mu p^\nu_{\pi^+} p^\alpha_{\pi^-} p^\beta_{\pi^0}\, \mathcal{F}(s,t,u)$$

$$\mathcal{F}(s,t,u) = \mathcal{F}(s) + \mathcal{F}(t) + \mathcal{F}(u)$$

Unitarity relation for $\mathcal{F}(s)$:

$$\text{disc}\,\mathcal{F}(s) = 2i\{ \underbrace{\mathcal{F}(s)}_{\text{right-hand cut}} + \underbrace{\hat{\mathcal{F}}(s)}_{\text{left-hand cut}} \} \times \theta(s - 4\,M_\pi^2) \times \sin\,\delta_1^1(s)\, e^{-i\delta_1^1(s)}$$

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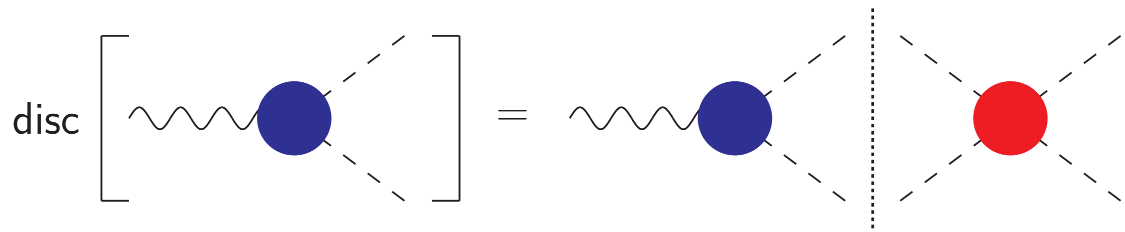
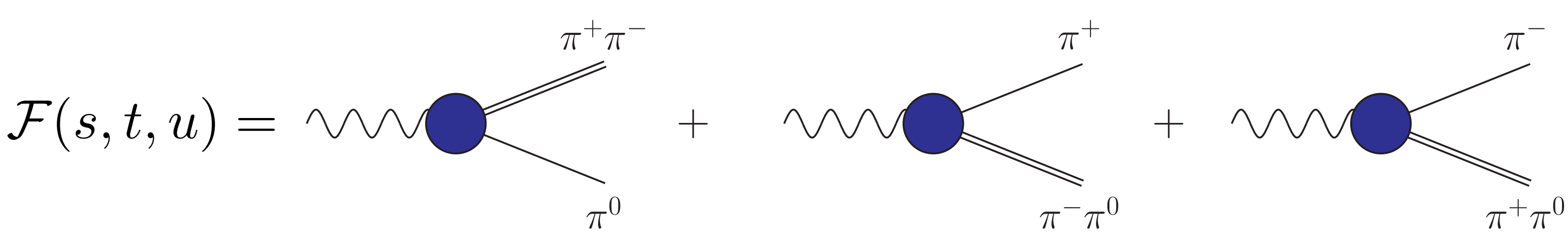


- right-hand cut only $\longrightarrow$ Omnès problem

$$\mathcal{F}(s) = P(s)\,\Omega(s)\,, \qquad \Omega(s) = \exp\left\{\frac{s}{\pi}\int_{4M_\pi^2}^{\infty}\frac{ds'}{s'}\frac{\delta_1^1(s')}{s'-s}\right\}$$

$\longrightarrow$ amplitude given in terms of pion vector form factor



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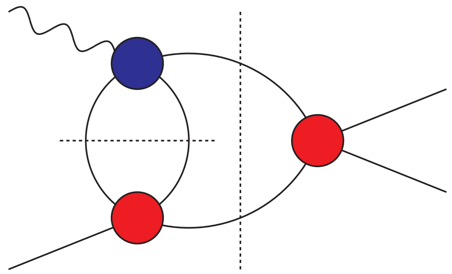

- inhomogeneities $\hat{\mathcal{F}}(s)$: angular averages over the $\mathcal{F}(t)$, $\mathcal{F}(u)$

$$\mathcal{F}(s) = \Omega(s)\left\{ \frac{C_2^{(1)}}{3}\big(1-\dot{\Omega}(0)s\big) + \frac{C_2^{(2)}}{3}s + \frac{s^2}{\pi}\int_{4M_\pi^2}^{\infty} \frac{ds'}{s'^2} \frac{\sin\delta_1^1(s')\hat{\mathcal{F}}(s')}{|\Omega(s')|(s'-s)} \right\}$$

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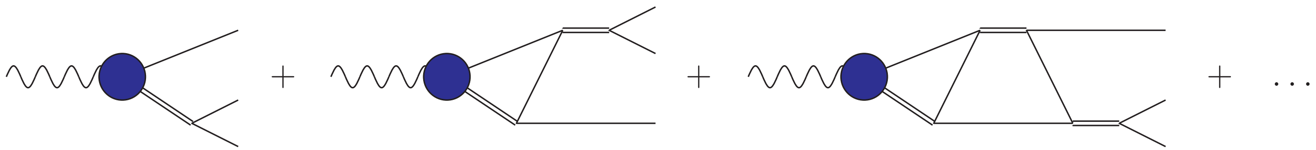
$$\mathcal{F}(s) =$$



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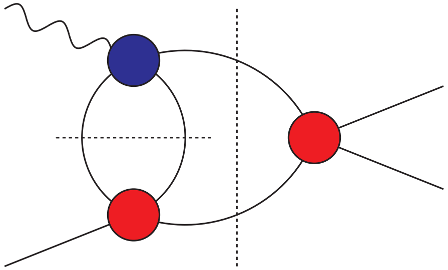

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- admits crossed-channel scattering between s-, t-, and u-channel (left-hand cuts)

Omnès solution for $\gamma\pi \longrightarrow \pi\pi$

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- important observation: $\mathcal{F}(s)$ linear in $C_2^{(i)}$

$$\mathcal{F}(s) = C_2^{(1)}\mathcal{F}^{(1)}(s) + C_2^{(2)}\mathcal{F}^{(2)}(s)$$

$\longrightarrow$ basis functions $\mathcal{F}^{(i)}(s)$ calculated independently of $C_2^{(i)}$

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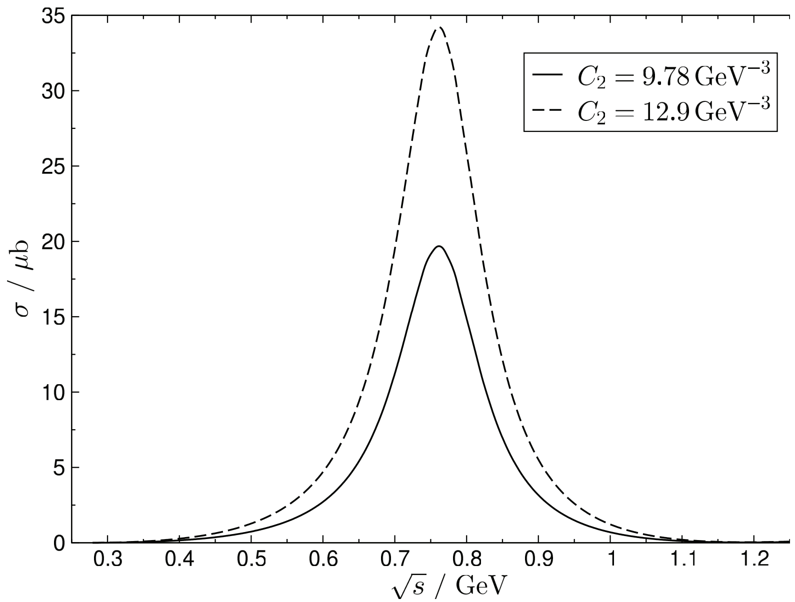
- representation of cross section in terms of two parameters

  $\longrightarrow$ fit to data, extract

$$F_{3\pi} \simeq C_2 = C_2^{(1)} + C_2^{(2)} M_\pi^2$$

$\longrightarrow \sigma \propto (C_2)^2$ also in ρ region



Hoferichter, BK, Sakkas 2012

Extension to decays: $\omega/\phi \to 3\pi$

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- now: fix subtraction constant a to partial width(s) $\omega/\phi \to 3\pi$

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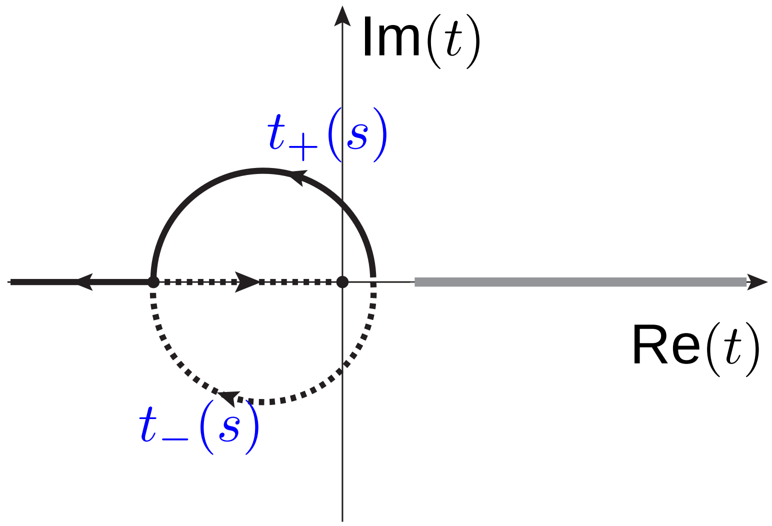
- complication:
  analytic continuation in decay mass M_V required
- $M_V < 3M_\pi$:
  okay



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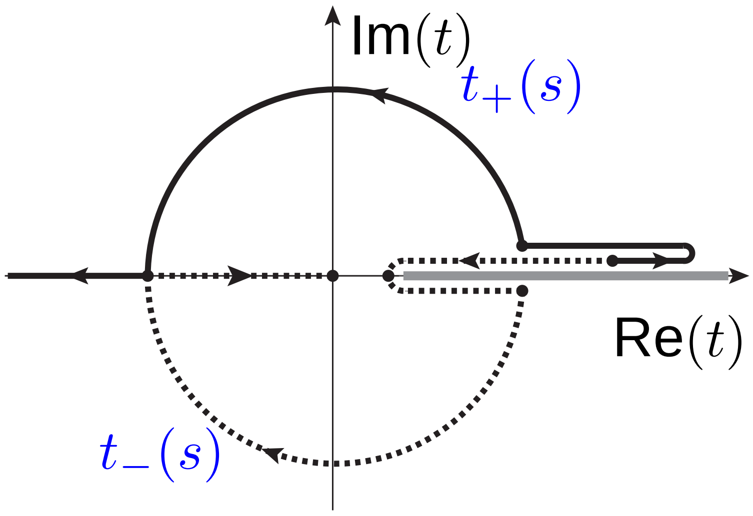

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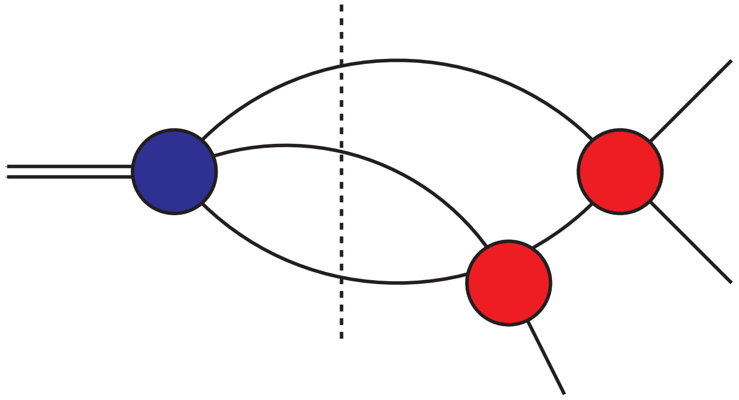

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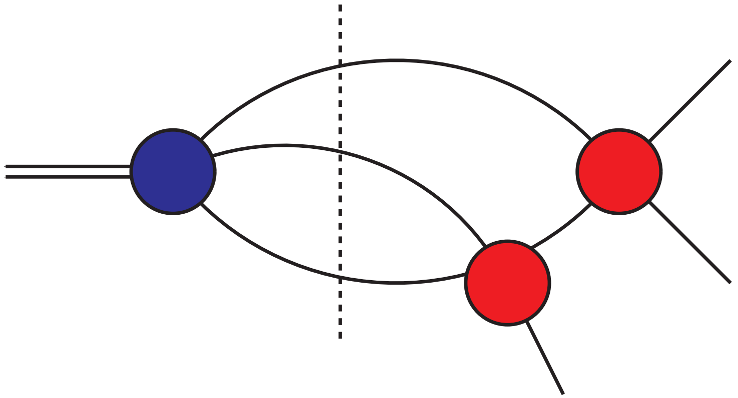

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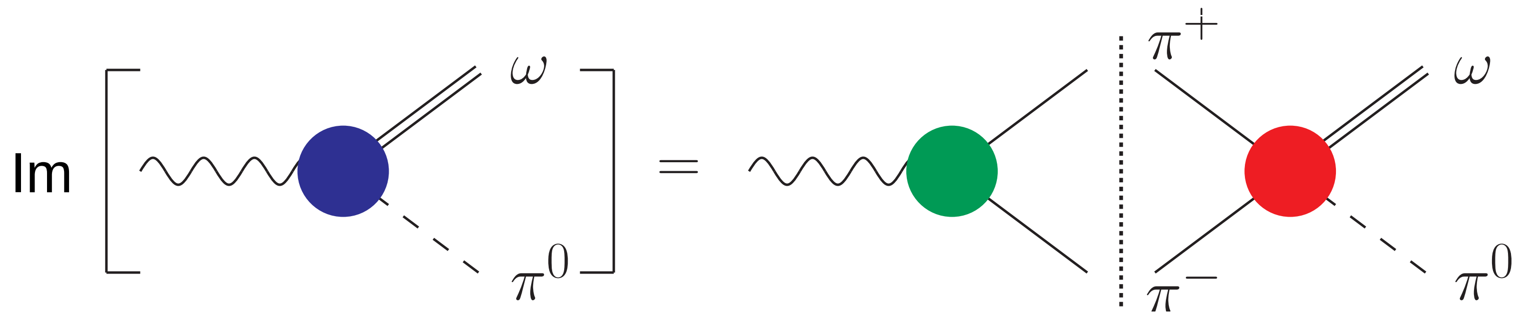
  $\longrightarrow$ generates 3-particle cuts
- fits experimental data on $\phi \to 3\pi$ Niecknig, BK, Schneider 2012
- experimental $\omega \to 3\pi$ Dalitz plots? KLOE, WASA@COSY, CLAS?

Transition form factors $\omega(\phi) \to \pi^0 \ell^+ \ell^-$

- dispersion relations link hadronic to radiative decays:



- ω transition form factor related to

pion vector form factor $\times$ $\omega \to 3\pi$ decay amplitude

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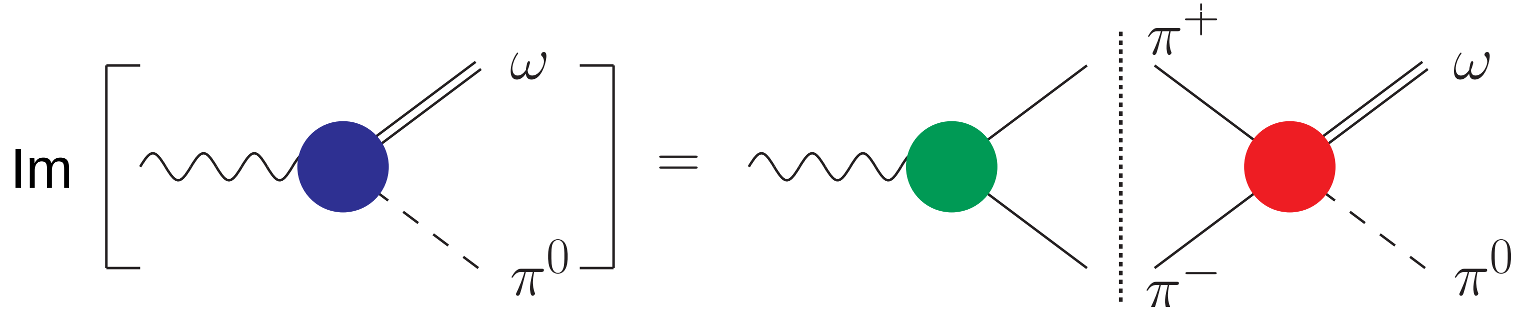

$$f_{\omega\pi^0}(s) = f_{\omega\pi^0}(0) + \frac{s}{12\pi^2}\int_{4M_\pi^2}^{\infty} ds' \frac{q_\pi^3(s')F_\pi^{V*}(s')f_1(s')}{s'^{3/2}(s'-s)}$$ Köpp 1974

- $f_1(s) = f_1^{\omega\to 3\pi}(s) = \mathcal{F}(s) + \hat{\mathcal{F}}(s)$ P-wave projection of $\mathcal{F}(s,t,u)$

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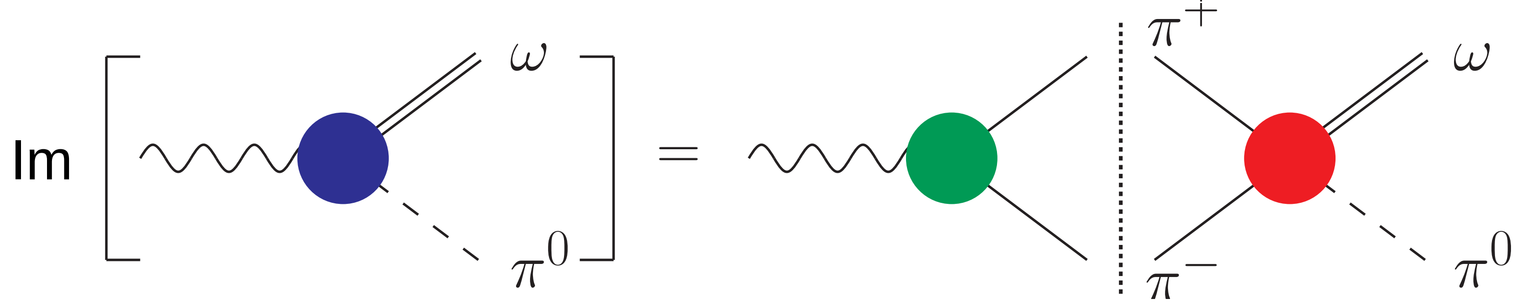

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- $f_1(s) = f_1^{\omega\to3\pi}(s) = \mathcal{F}(s) + \hat{\mathcal{F}}(s)$ P-wave projection of $\mathcal{F}(s,t,u)$

- sum rule for $\omega \to \pi^0\gamma \longrightarrow$ saturated at 90–95%

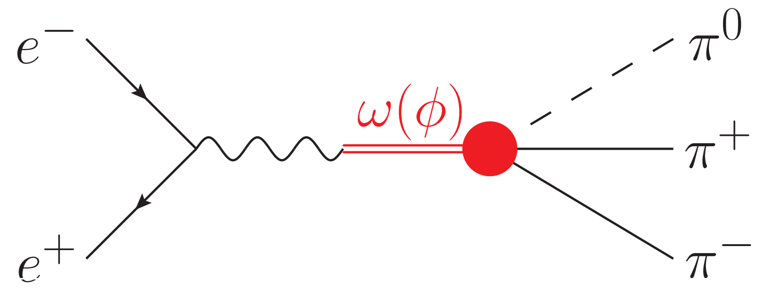
$$f_{\omega\pi^0}(0) = \frac{1}{12\pi^2}\int_{4M_\pi^2}^{\infty} ds' \frac{q_\pi^3(s')}{s'^{3/2}} F_\pi^{V*}(s') f_1(s') \,, \quad \Gamma_{\omega\to\pi^0\gamma} \propto |f_{V\pi^0}(0)|^2$$

Schneider, BK, Niecknig 2012

- comparison to $\omega \to \pi^0\mu^+\mu^-$ mysterious; experiments for $\phi \to \pi^0\ell^+\ell^-$ highly welcome see Sebastian Schneider's poster on Tuesday

One step further: $e^+e^- \to 3\pi$, $e^+e^- \to \pi^0\gamma$

see Sebastian Schneider's poster on Tuesday



- decay amplitude for $\omega/\phi \to 3\pi$: $\mathcal{M}_{\omega/\phi} \propto \mathcal{F}(s) + \mathcal{F}(t) + \mathcal{F}(u)$

$$\mathcal{F}(s) = a_{\omega/\phi}\,\Omega(s)\left\{1 + \frac{s}{\pi}\int_{4M_\pi^2}^{\infty} \frac{ds'}{s'} \frac{\sin\delta_1^1(s')\hat{\mathcal{F}}(s')}{|\Omega(s')|(s'-s)}\right\}$$

$a_{\omega/\phi}$ adjusted to reproduce total width $\omega/\phi \to 3\pi$

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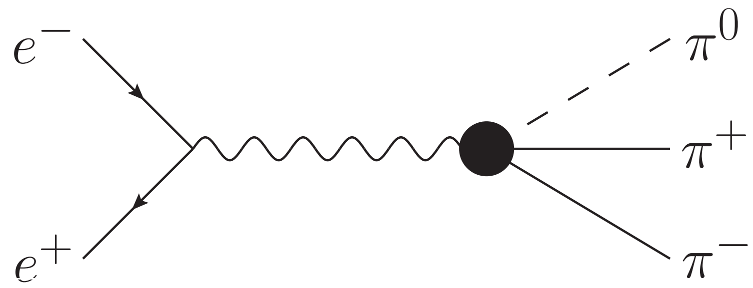

- decay amplitude for $e^+e^- \to 3\pi$: $\mathcal{M}_{e^+e^-} \propto \mathcal{F}(s) + \mathcal{F}(t) + \mathcal{F}(u)$

$$\mathcal{F}(s,q^2) = a_{e^+e^-}(q^2)\,\Omega(s)\left\{1 + \frac{s}{\pi}\int_{4M_\pi^2}^{\infty}\frac{ds'}{s'}\frac{\sin\delta_1^1(s')\hat{\mathcal{F}}(s',q^2)}{|\Omega(s')|(s'-s)}\right\}$$

$a_{e^+e^-}(q^2)$ adjusted to reproduce spectrum $e^+e^- \to 3\pi$

contains 3π resonances $\longrightarrow$ parameterise

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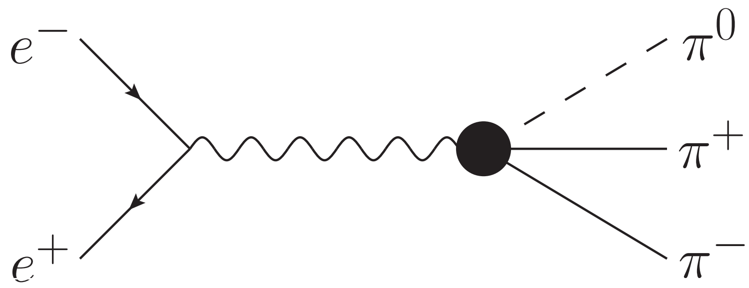

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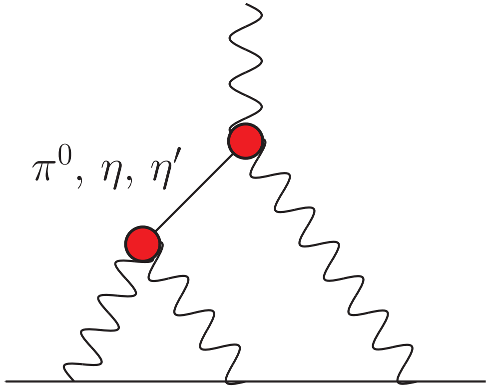
- fit to $e^+e^- \to 3\pi$ data $\longrightarrow$

  prediction for isoscalar $e^+e^- \to \pi^0\gamma$:

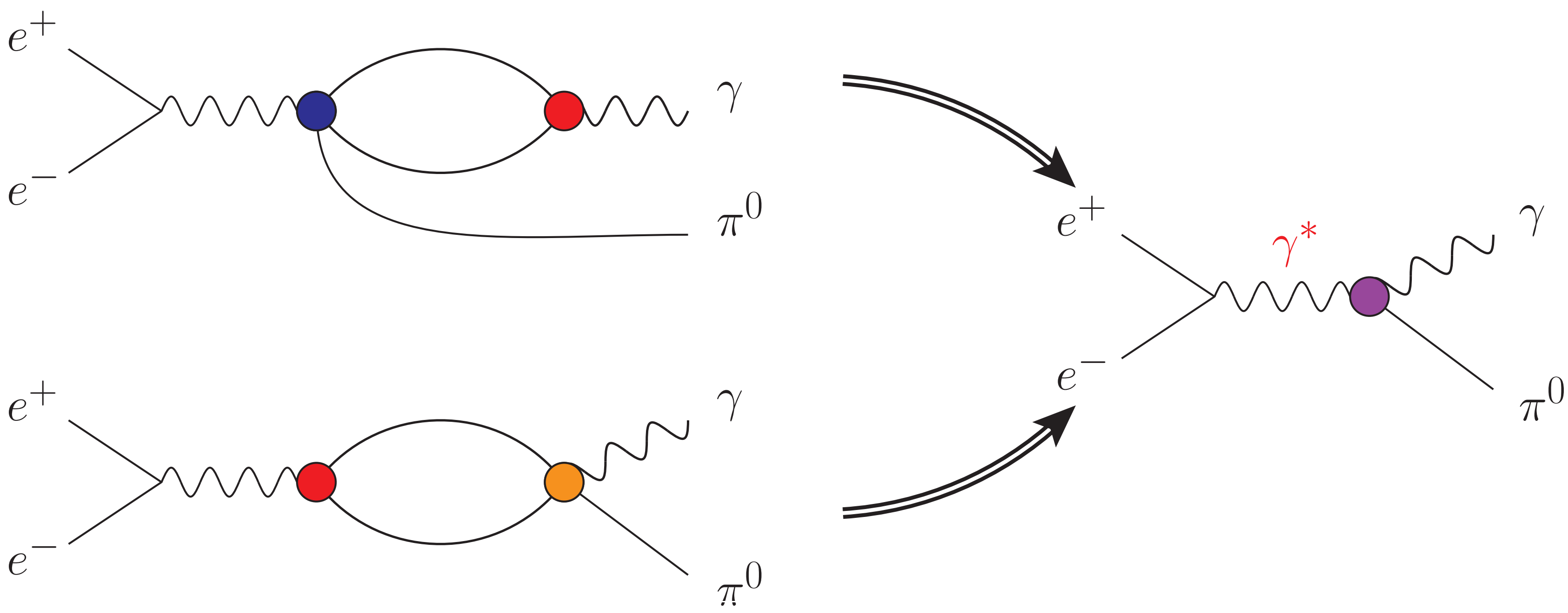
$$F_{\pi\gamma^*\gamma}(q^2,0) = F_{vs}(q^2,0) + F_{vs}(0,q^2)$$

$\longrightarrow$ important for $(g-2)_\mu$



Towards a dispersive analysis of $e^+e^- \to \pi^0\gamma$

see Sebastian Schneider's poster on Tuesday

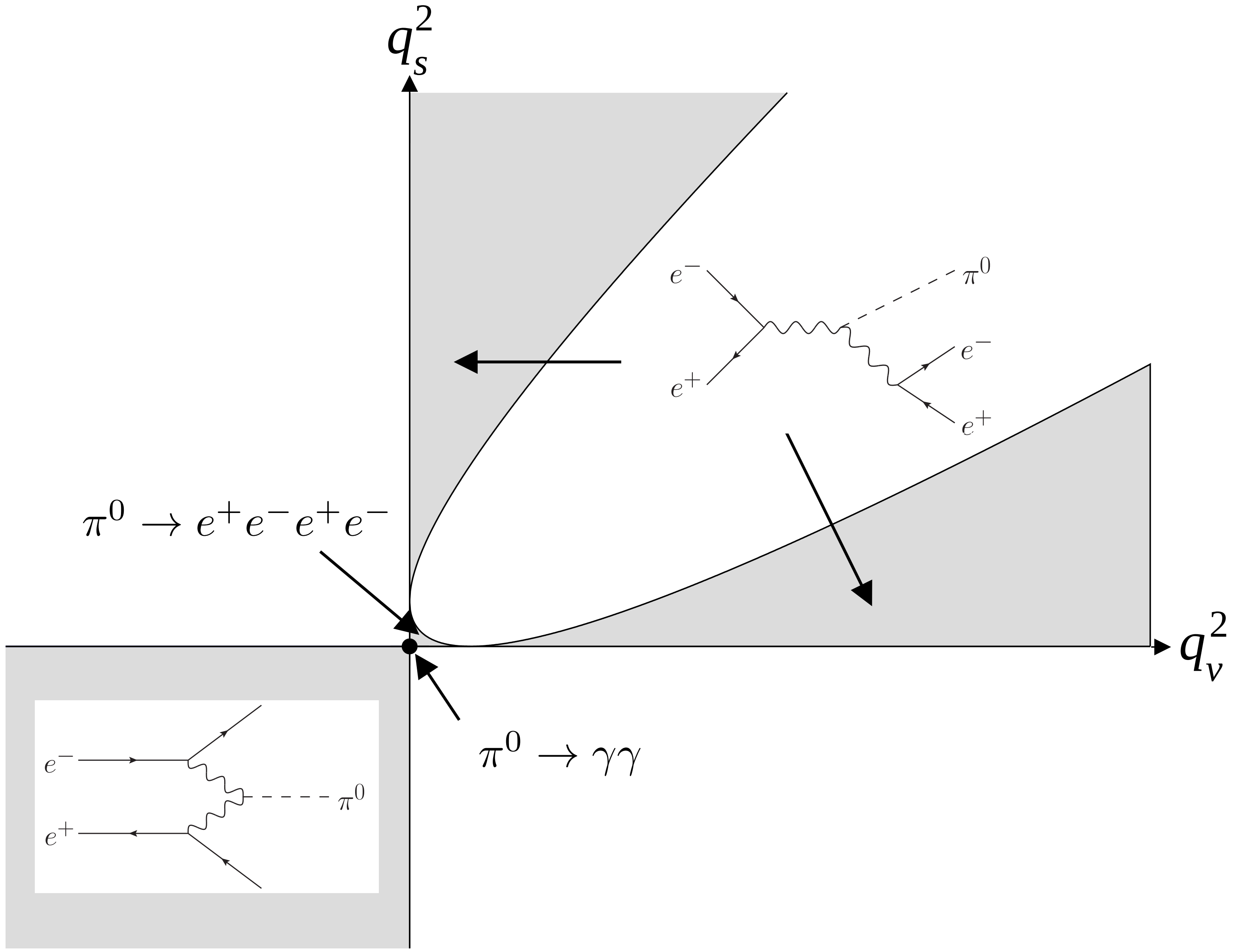


- combine isoscalar and isovector contribution to $e^+e^- \to \pi^0\gamma$

$$F_{\pi\gamma^*\gamma}(q^2,0) = F_{vs}(0,q^2) + F_{vs}(q^2,0)$$

$$= \frac{1}{12\pi^2}\int_{4M_\pi^2}^{\infty} ds' \frac{q_\pi^3(s')}{\sqrt{s'}} \left\{ \frac{f_1^{\gamma^*\to 3\pi}(s',q^2)}{s'} + \frac{f_1^{\gamma\pi\to\pi\pi}(s')}{s'-q^2} \right\} F_\pi^{V*}(s')$$

$\pi^0 \to \gamma^*(q_v^2)\gamma^*(q_s^2)$ transition form factor



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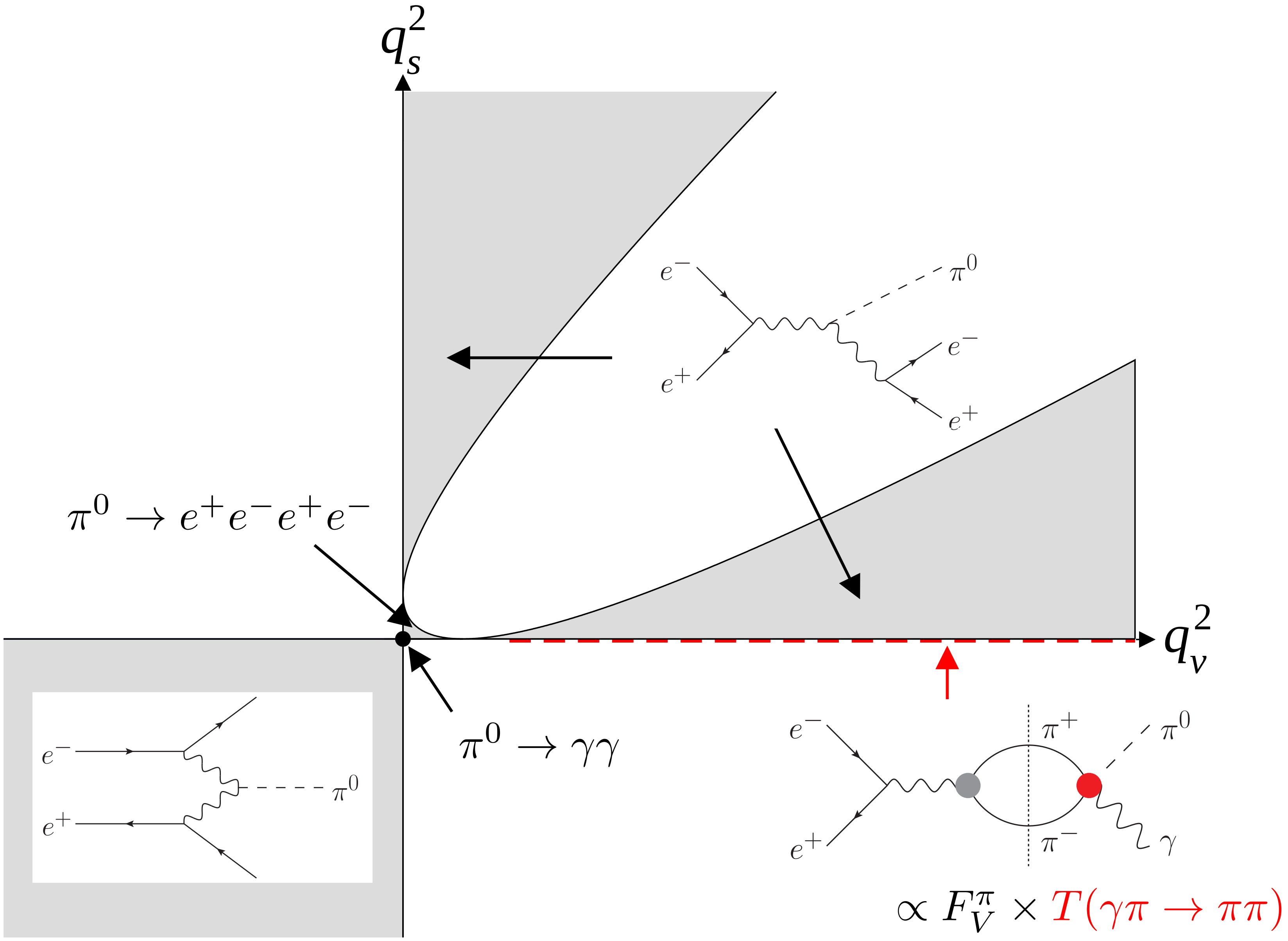

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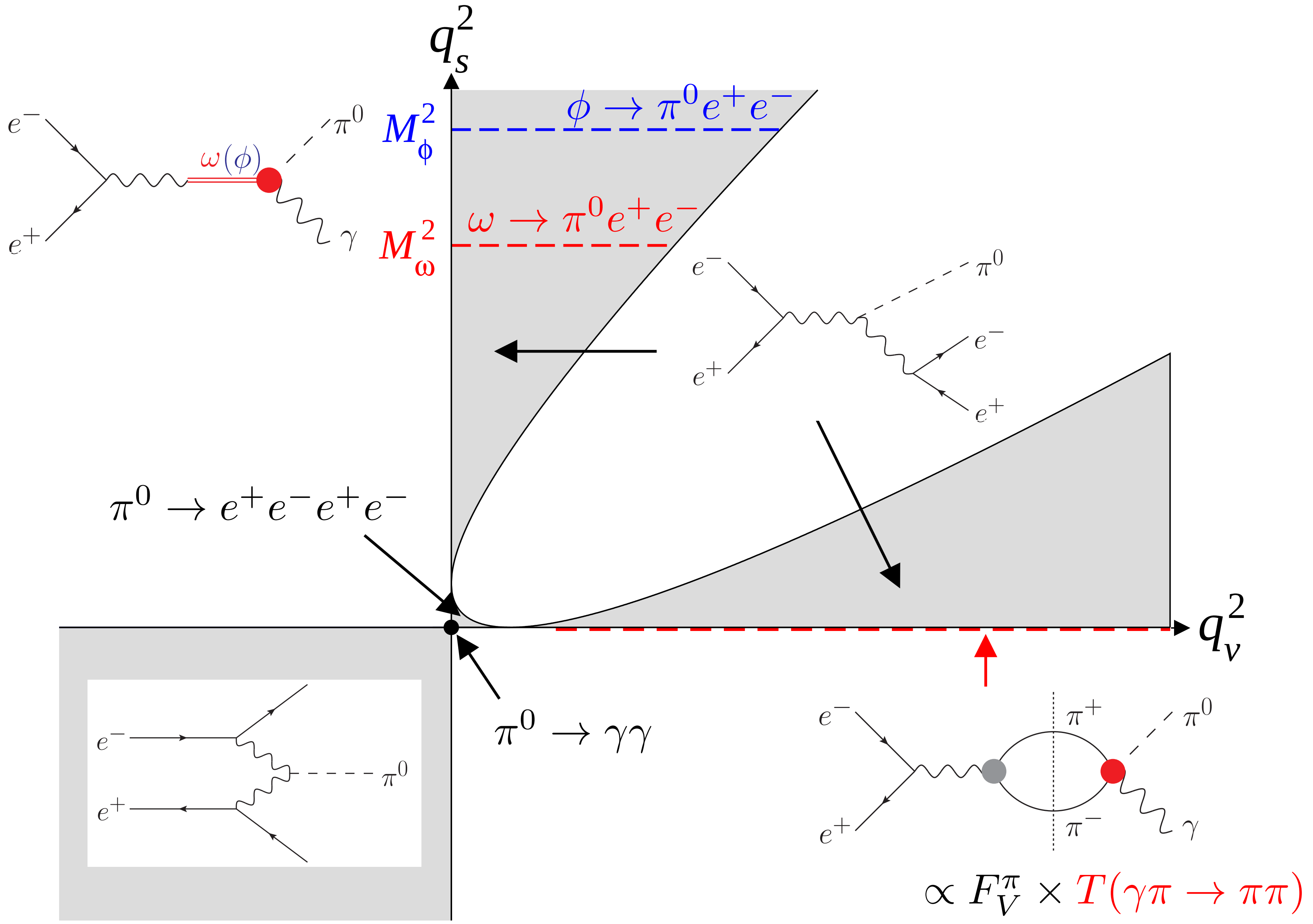


Summary / Outlook

Dispersion relations for $\gamma\pi \to \pi\pi$

- based on unitarity, analyticity, crossing symmetry
- enable improved extraction of $F_{3\pi}$ from data up to 1 GeV

Vector meson decays $\omega/\phi \to 3\pi$, $\omega/\phi \to \pi^0\gamma^*$

- analytic-unitary description of $\phi \to 3\pi$ Dalitz plot
- dispersion relations predict transition form factors therefrom

Towards π^0 transition form factors

- short-term: combine all input for $e^+e^- \to \pi^0\gamma$
  Hoferichter, BK, Leupold, Niecknig, Schneider, in progress
- long-term: full description of doubly-virtual π^0 transition form factor, based on dispersion theory

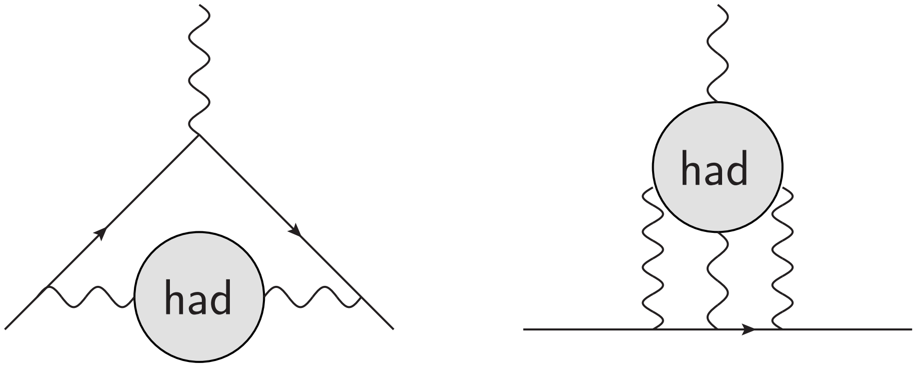
$\longrightarrow$ interrelate as much experimental information as possible to constrain hadron physics in $(g-2)_\mu$

Spares

Meson transition form factors and $(g-2)_\mu$

Czerwinski et al., arXiv:1207.6556 [hep-ph]

- leading and next-to-leading hadronic effects in $(g-2)_\mu$:



$\longrightarrow$ hadronic light-by-light soon dominant uncertainty

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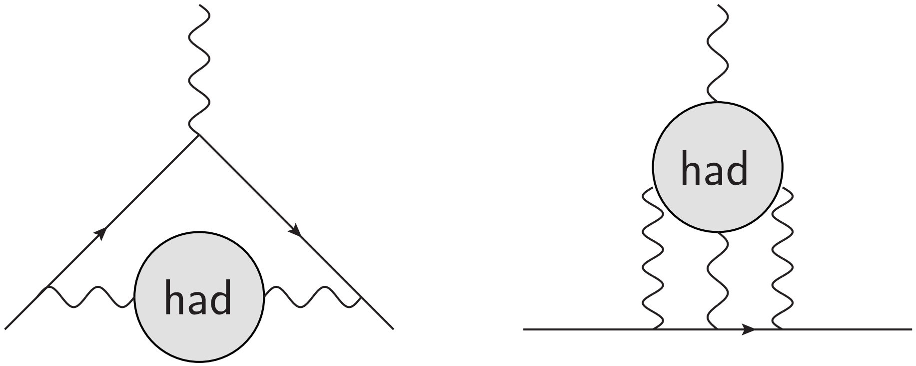
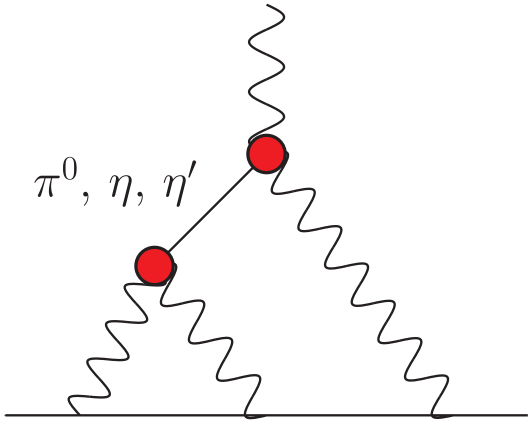


$\longrightarrow$ hadronic light-by-light soon dominant uncertainty

- important contribution: pseudoscalar pole terms
  singly / doubly virtual form factors
  $F_{P\gamma\gamma^*}(q^2,0)$ and $F_{P\gamma^*\gamma^*}(q_1^2,q_2^2)$



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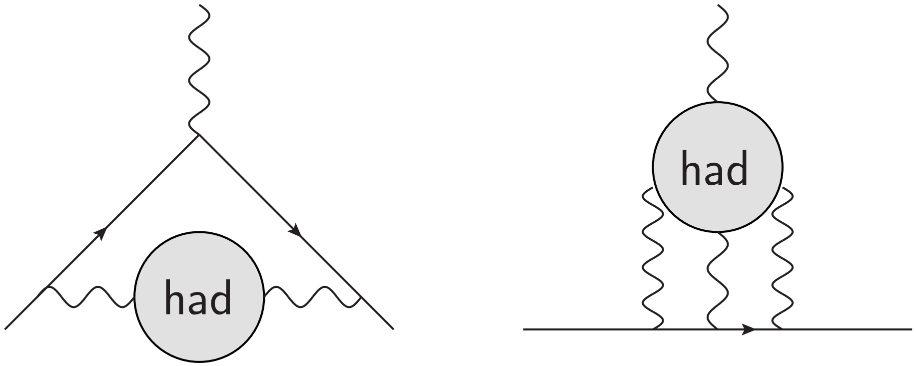

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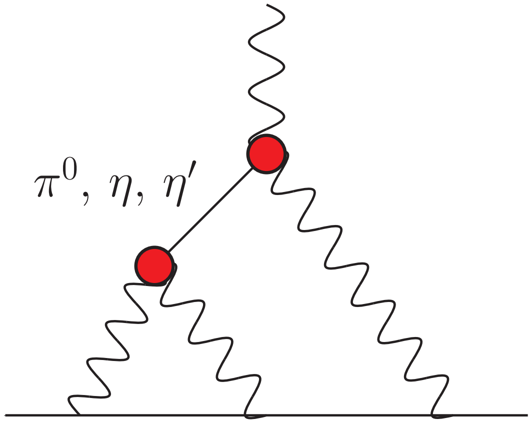
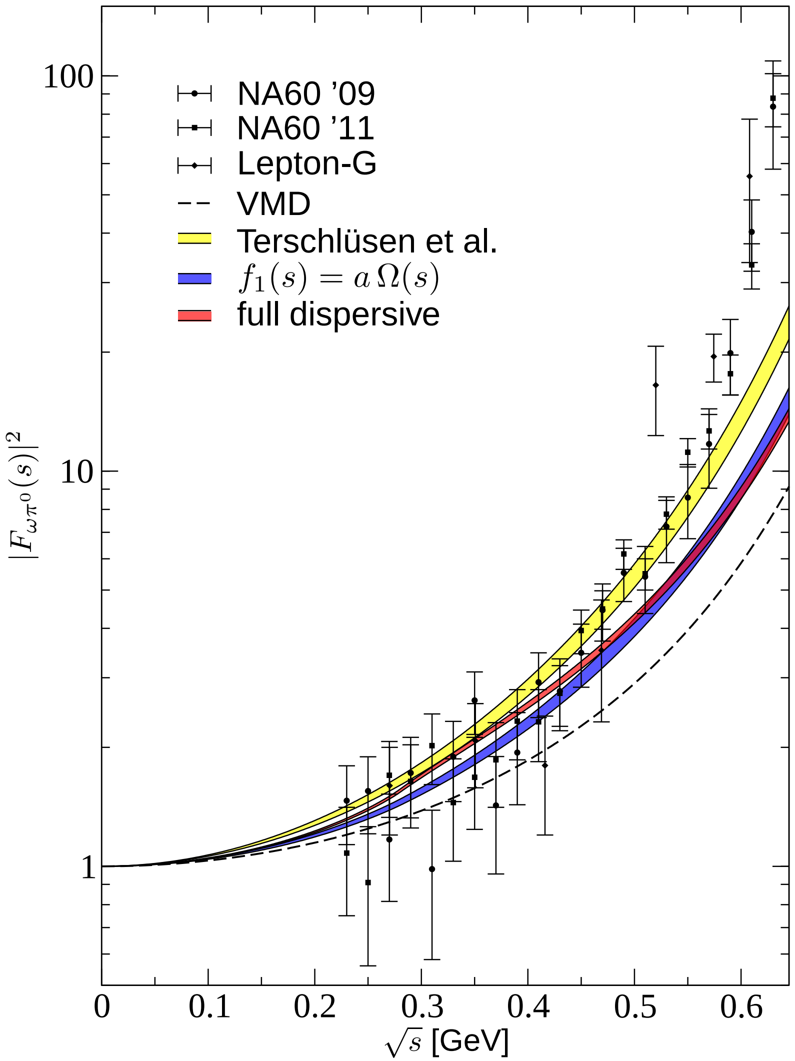
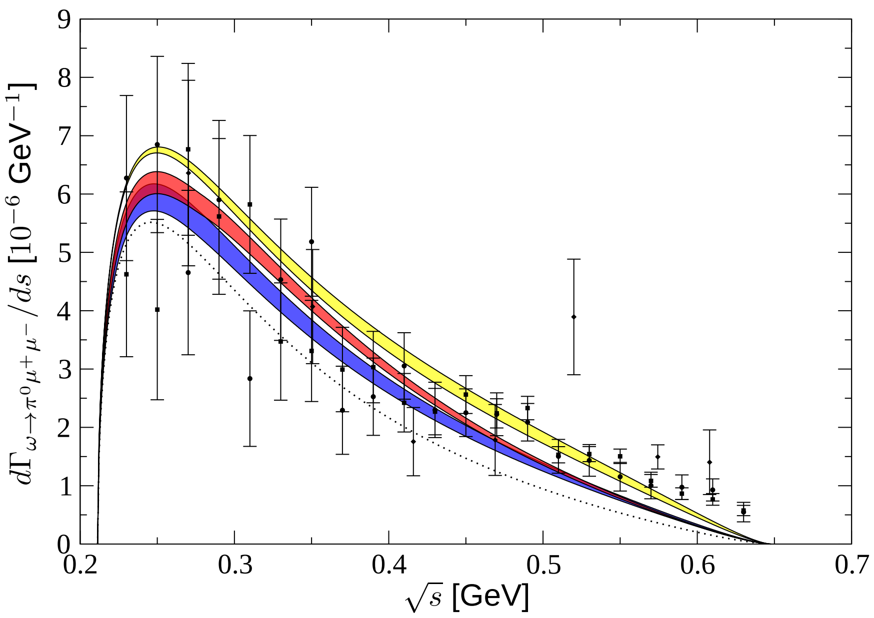
  $F_{P\gamma\gamma^*}(q^2,0)$ and $F_{P\gamma^*\gamma^*}(q_1^2,q_2^2)$
- for specific virtualities: linked to
  vector-meson conversion decays



$\longrightarrow$ e.g. $F_{\pi^0\gamma^*\gamma^*}(q_1^2,M_\omega^2)$ measurable in $\omega\to\pi^0\ell^+\ell^-$ etc.

Numerical results: $\omega \to \pi^0\mu^+\mu^-$

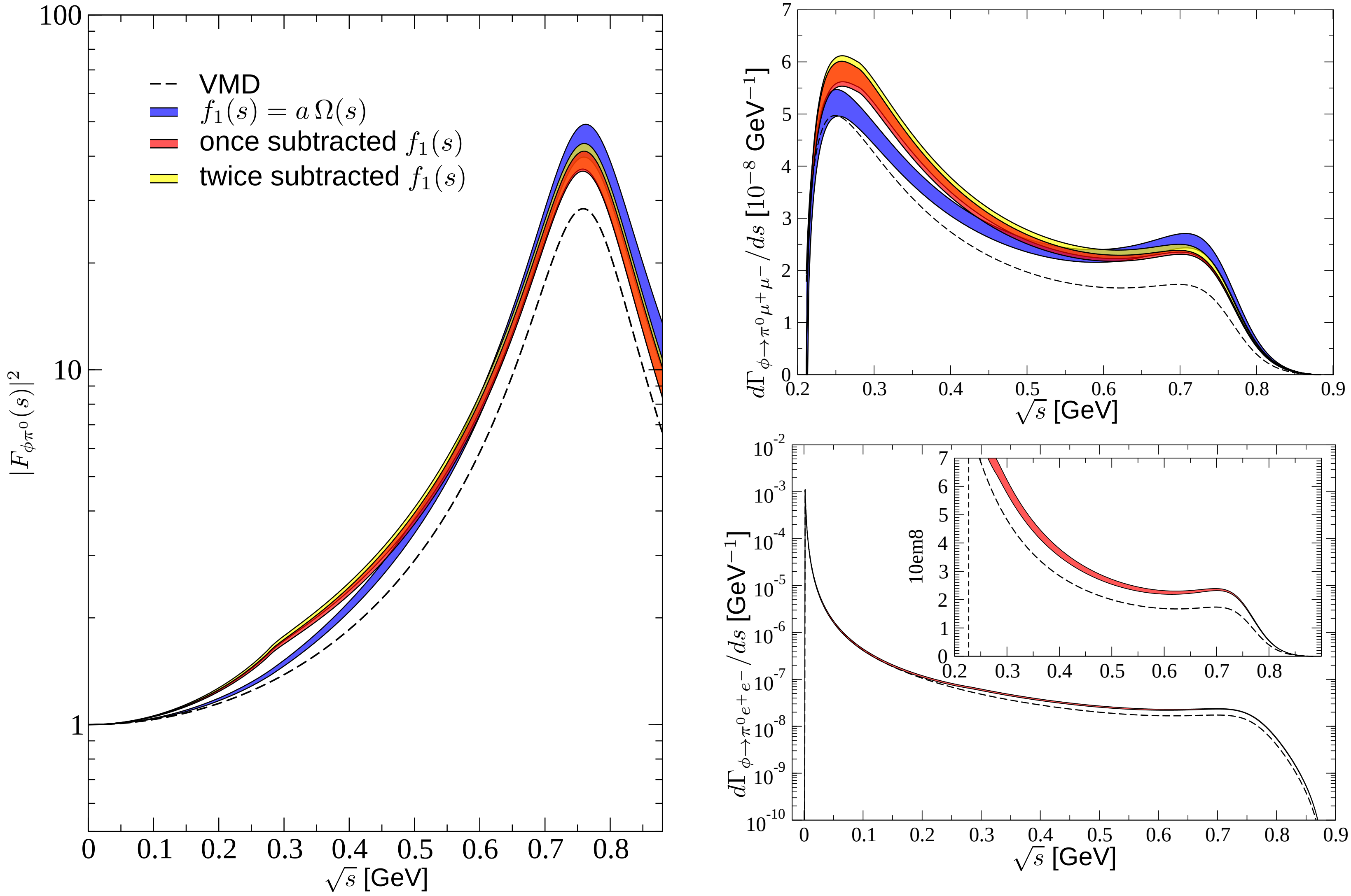




- unable to account for steep rise in data (from heavy-ion collisions) NA60 2009, 2011
- more "exclusive" data?!
- $\omega \to 3\pi$ Dalitz plot?

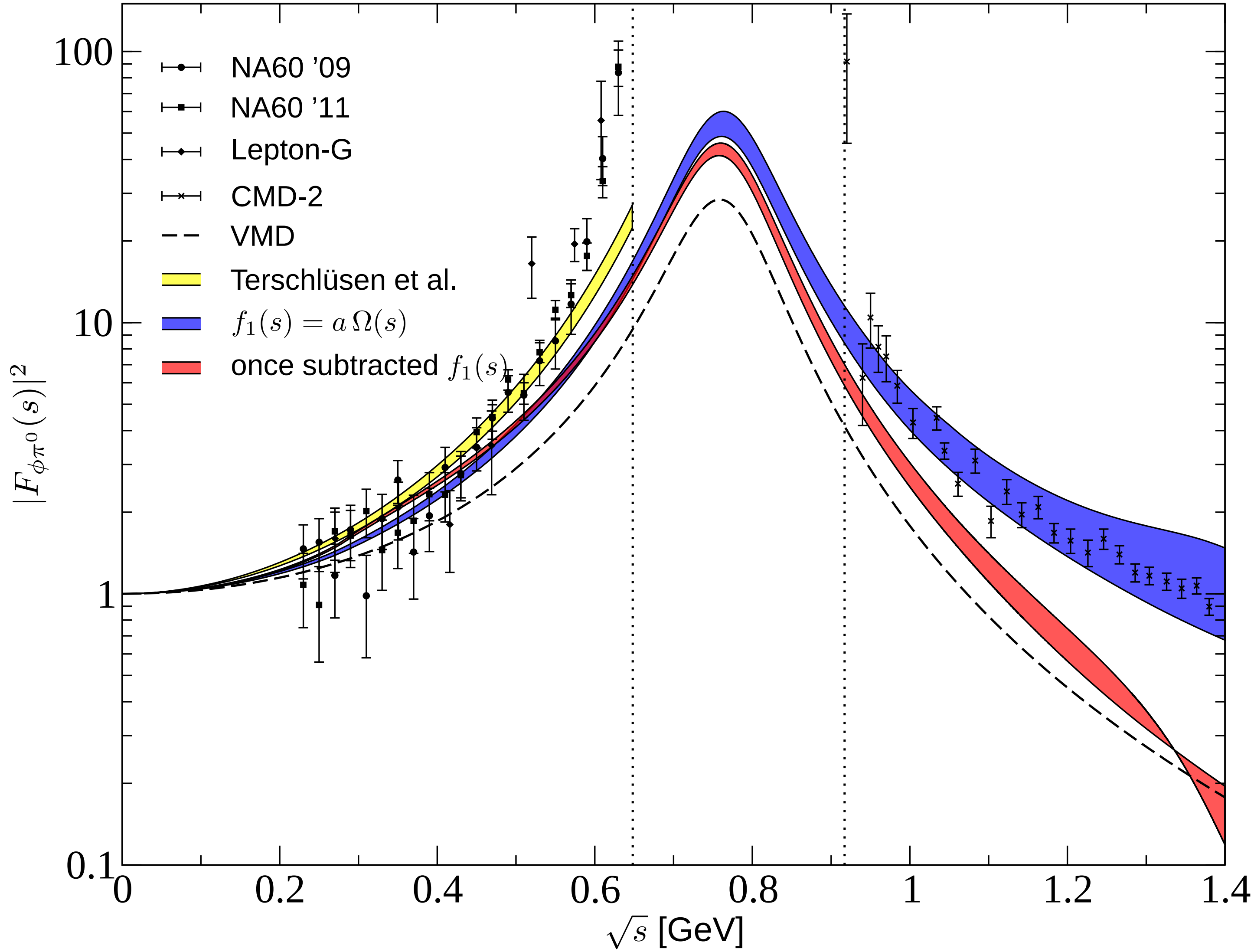
KLOE, WASA-at-COSY, CLAS?

Numerical results: $\phi \to \pi^0 \ell^+ \ell^-$



- measurement would be extremely helpful: ρ in physical region!
- partial-wave amplitude backed up by experiment

Transition form factor beyond the $\pi\omega$ threshold



- full solution above naive VMD, but still too low
- higher intermediate states (4π / $\pi\omega$) more important?

Fit to $e^+e^- \to 3\pi$ data

- parametrisation in terms of dispersively reconstructed $\omega + \phi$ Breit–Wigner propagators $\longrightarrow$ good analytic properties

  Lomon, Pacetti 2012; Moussallam 2013

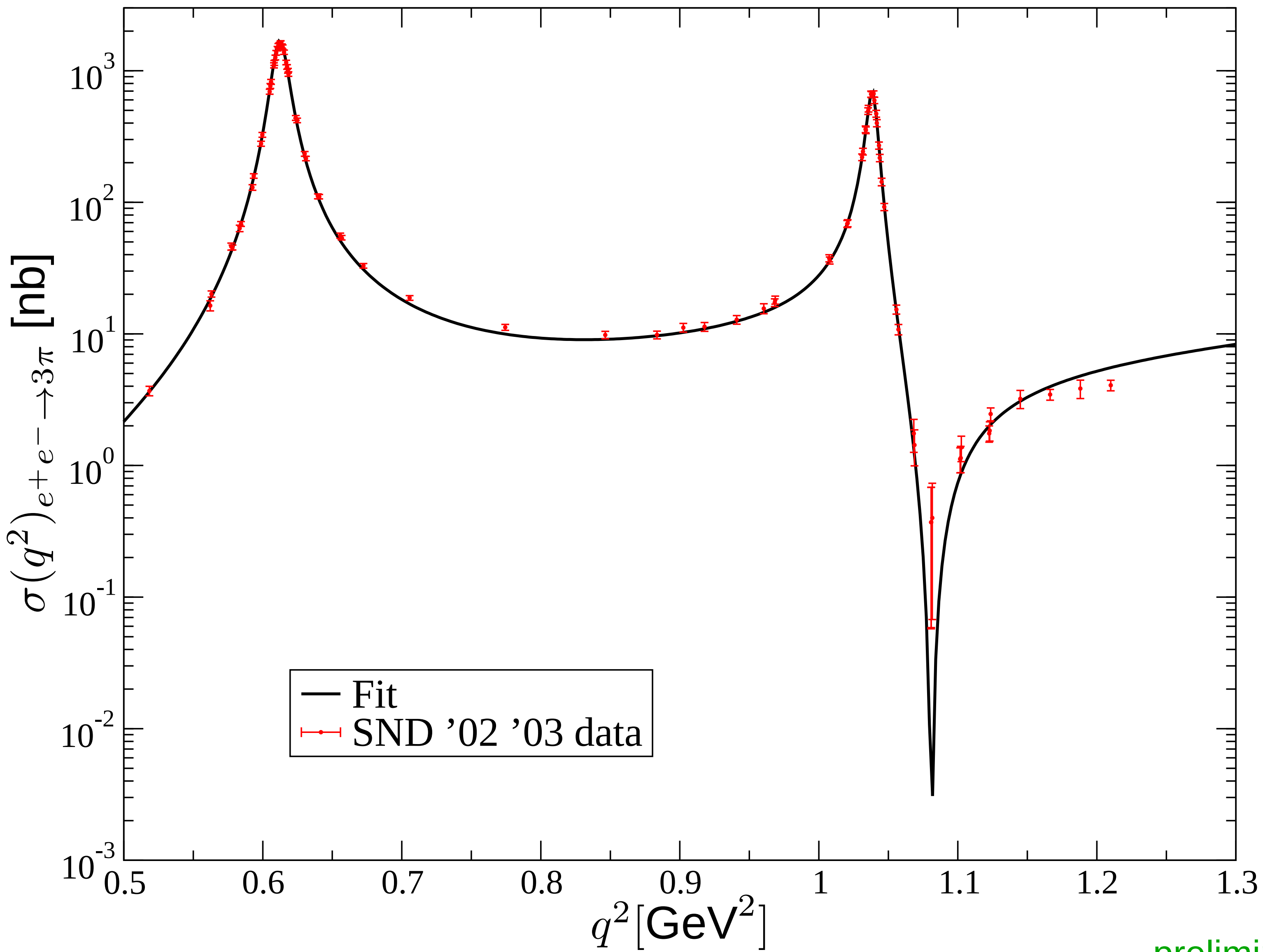
- "standard" Breit–Wigner function with energy-dependent width

$$B(q^2) = \frac{c_\omega}{M_\omega^2 - q^2 - iM_\omega\Gamma_\omega(q^2)} + \frac{c_\phi}{M_\phi^2 - q^2 - iM_\phi\Gamma_\phi(q^2)}$$

$$\tilde{B}(q^2) = \alpha + \beta q^2 + \frac{q^4}{\pi}\int_{9M_\pi^2}^\infty \frac{ds'}{s'^2}\frac{\operatorname{Im} BW(s')}{s' - q^2} = a_{e^+e^-}(q^2)$$

- subtraction constant $\alpha = \tilde{B}(q^2 = 0)$: fixed by $\gamma \to 3\pi$
- c_ω, c_ϕ, β from fit to data

Fit to $e^+e^- \to 3\pi$ data



preliminary

Improved Breit–Wigner resonances

Lomon, Pacetti 2012; Moussallam 2013

- "standard" Breit–Wigner function with energy-dependent width

$$B^\ell(q^2) = \frac{1}{M_{\rm res}^2 - q^2 - iM_{\rm res}\Gamma_{\rm res}^\ell(q^2)}$$

$$\Gamma_{\rm res}^\ell(q^2) = \theta(q^2-4M_\pi^2)\frac{M_{\rm res}}{\sqrt{q^2}}\bigg(\frac{q^2-4M_\pi^2}{M_{\rm res}^2-4M_\pi^2}\bigg)^\ell\,\Gamma_{\rm res}(M_{\rm res}^2)$$

  - ▷ no correct analytic continuation below threshold $q^2 < 4M_\pi^2$
  - ▷ wrong phase behaviour for $\ell \geq 1$:

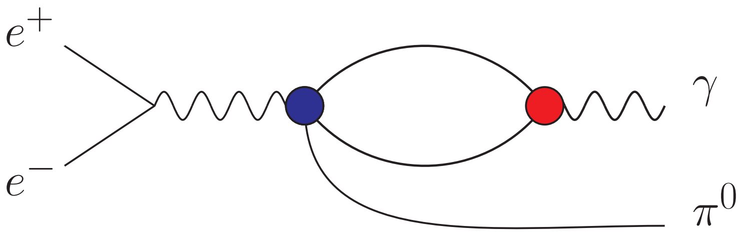
$$\lim_{q^2\to\infty} \arg B^1(q^2) \approx \pi - \arctan\frac{\Gamma_{\rm res}}{M_{\rm res}} \qquad \lim_{q^2\to\infty} \arg B^{\ell\geq 2}(q^2) = \frac{\pi}{2}\ (!)$$

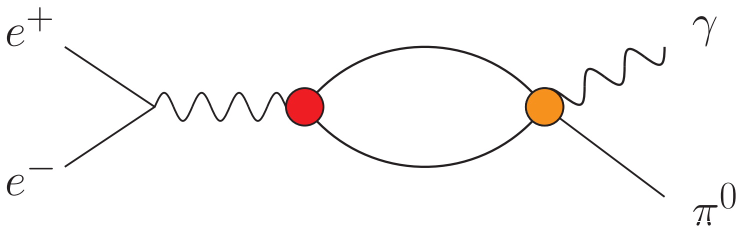
- remedy: reconstruct via dispersion integral

$$\tilde{B}^\ell(q^2) = \frac{1}{\pi}\int_{4M_\pi^2}^\infty \frac{{\rm Im}\,B^\ell(s')ds'}{s'-q^2} \quad\longrightarrow\quad \lim_{s\to\infty}\arg B^\ell(q^2) = \pi$$

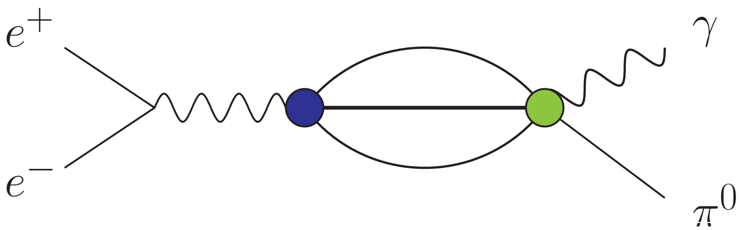
On the approximation for the 3-pion cut

Compare:





$\longrightarrow$ isoscalar contribution looks simplistic; why not instead



$\longrightarrow$ contains amplitude $3\pi \to \gamma\pi$

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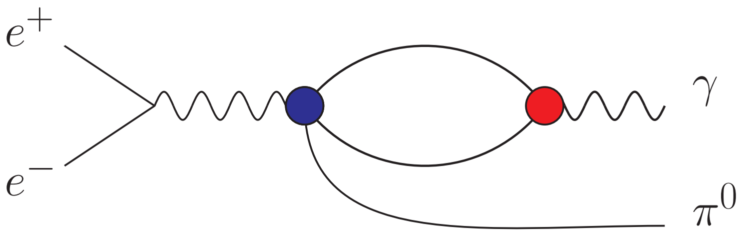

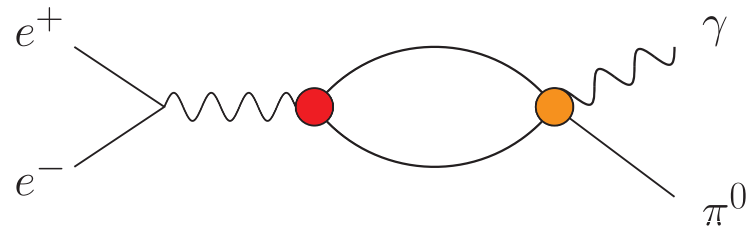

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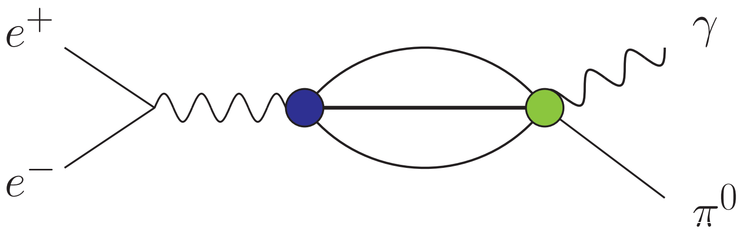

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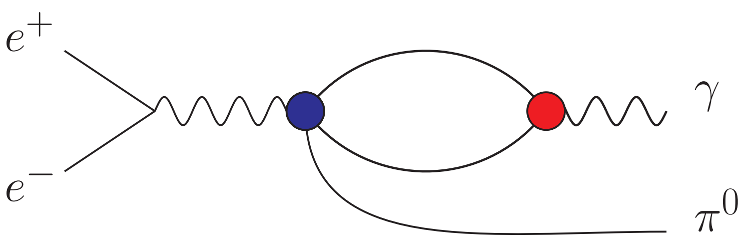
Our approximation:



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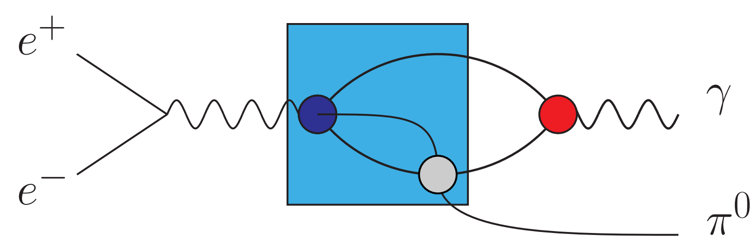

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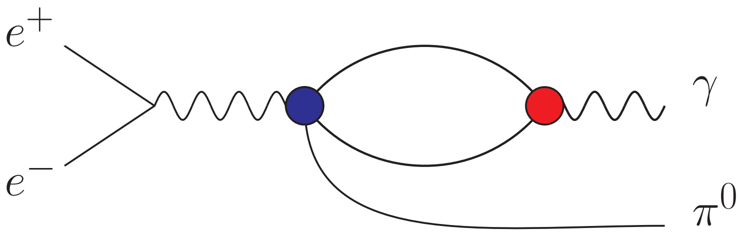

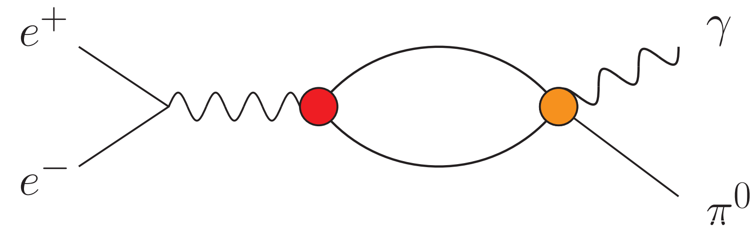

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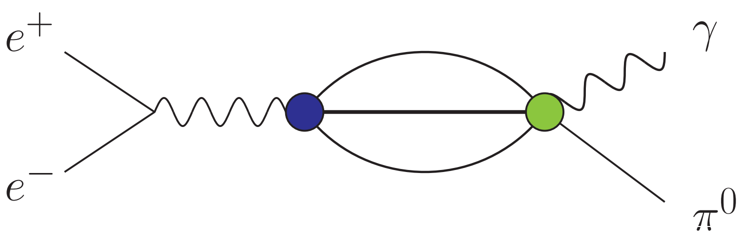

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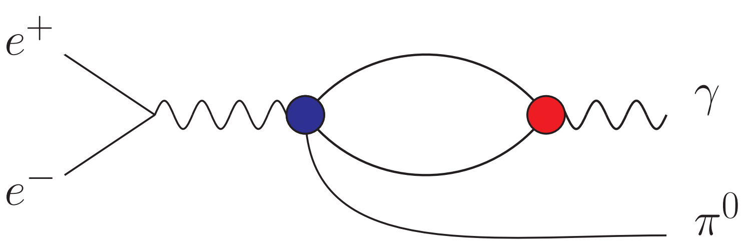

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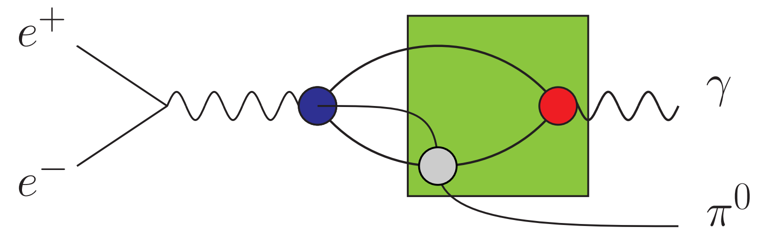

$\longrightarrow$ simplifies left-hand-cut structure in $3\pi \to \gamma\pi$ to pion pole terms