Hadronic Vacuum Polarisation in g-2 and α_{QED}

UNIVERSITY OF LIVERPOOL

Thomas Teubner

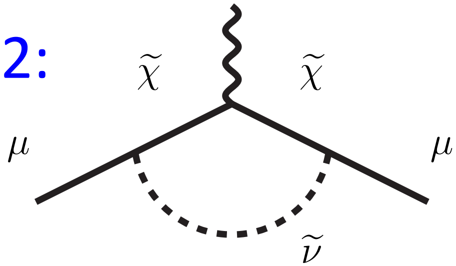
LIVERPOOL THEORETICAL PHYSICS

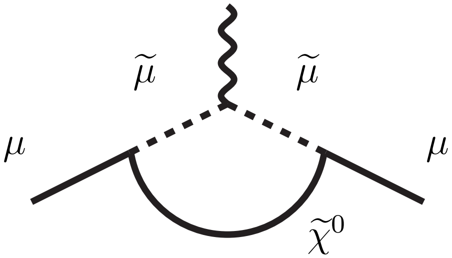
- Introduction/Motivation
- HVP in g-2. Old `puzzles'
- Recent evaluations. New data
- VP-routines. $\Delta\alpha$ at low and high energies
- Future improvements
- Conclusions

Motivation: g-2, a sign for `new' physics?! SUSY?

SUSY could easily explain g-2:

Main 1-loop contributions:





Simplest case:

$$a_\mu^{\rm SUSY} \simeq sgn(\mu)\, 130 \times 10^{-11}\, \tan\beta \left(\frac{100\,{\rm GeV}}{\Lambda_{\rm SUSY}}\right)^2$$

- Needs $\mu>0$, `light' SUSY-scale Λ and/or large tan β to explain 260×10^{-11}

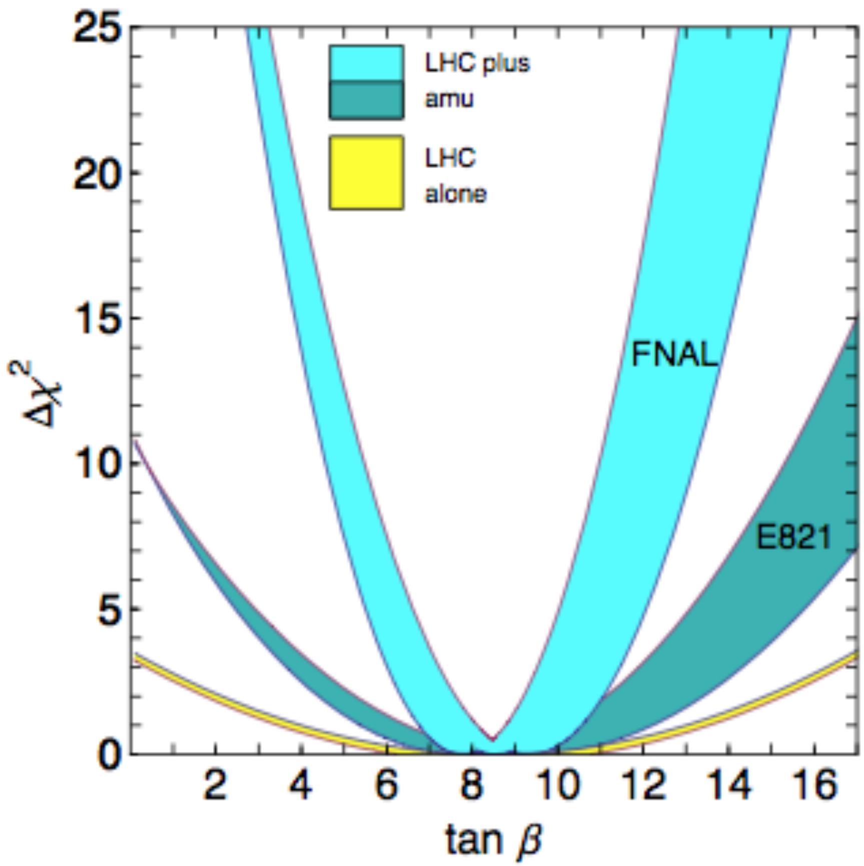
- This is already `excluded' by LHC searches in the simplest SUSY scenarios (like CMSSM); causes large χ^2 in simultaneous SUSY-fits with LHC data and g-2

- However note: SUSY does not have to be minimal (w.r.t. Higgs),
  could have large mass splittings (with lighter sleptons), or
  corrections (to g-2 and Higgs mass) different from simple models,
  or not be there at all

- g-2 constrains params, distinguishes between NP models `degenerate' for LHC

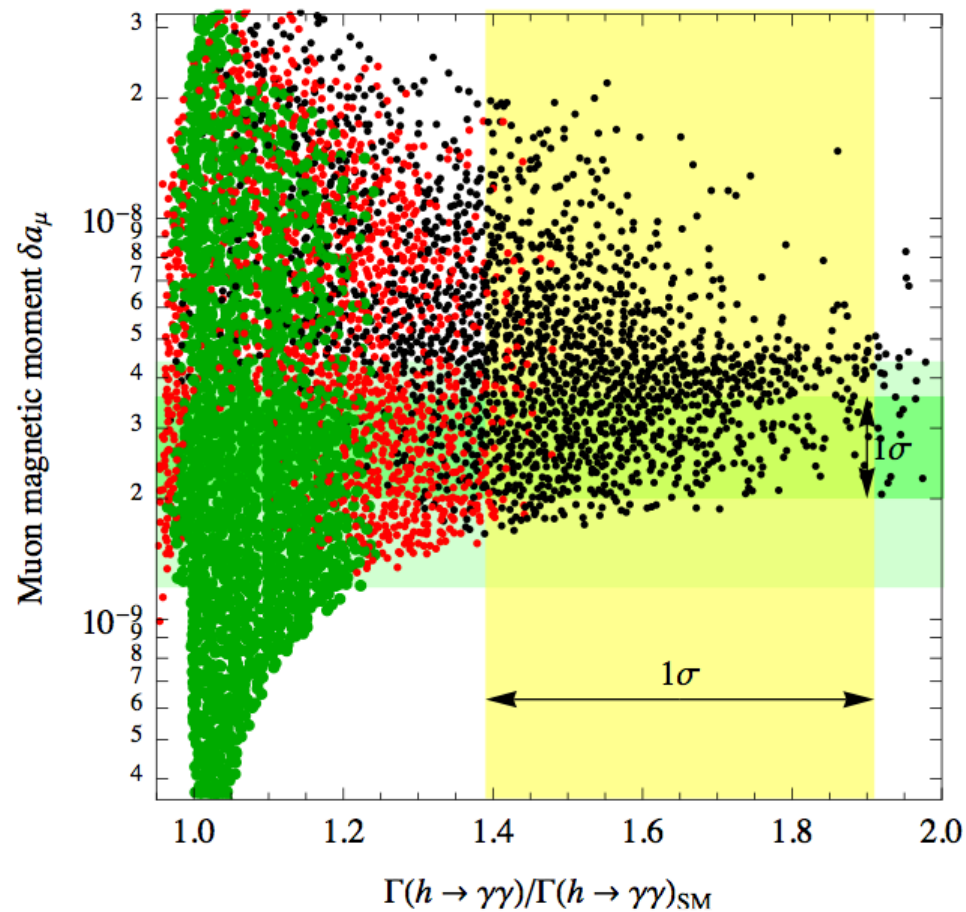
Motivation: Precision g-2 constrains SUSY

LHC with (100 fb^{-1}) can determine tan(β) to 50%, with g-2 to 10%



Miller, de Rafael, Roberts and Stöckinger, Ann. Rev. Nucl. Part. Sci. 62 (2012) 237

g-2 complements LHC data selecting in the vast SUSY (param/model) space



Guidice, Paradisi, Strumia JHEP 1210, 186

g-2: SM prediction

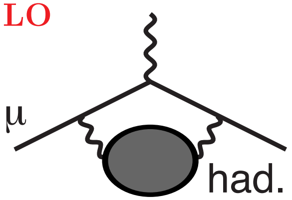
$$a_\mu = a_\mu^{\rm QED} + a_\mu^{\rm EW} + a_\mu^{\rm hadronic} + a_\mu^{\rm NP?}$$

see talks by M Knecht, M Steinhauser

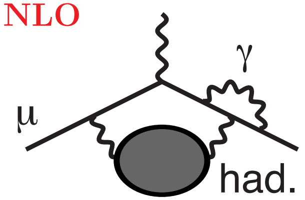
- QED: Kinoshita et al. 2012: 5-loop completed (12672 diagrams) ✓
- EW: 2-loop (and Higgs mass now known, see talk by D Stoeckinger) ✓
- Hadronic: the limiting factor of the SM prediction ✗

$$a_\mu^{\rm had} = a_\mu^{\rm had,VP\ LO} + a_\mu^{\rm had,VP\ NLO} + a_\mu^{\rm had,Light-by-Light}$$

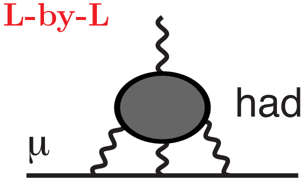
LO



NLO



L-by-L



L-by-L: - so far use of model calculations, form-factor data (KLOE-2) will help improving (see talks in γγ session, talk by A Radzhabov, poster by A Nyffeler)
- for the future: lattice QCD predictions (first results encouraging)
- several groups: USQCD, UKQCD, ETMC, … much increased effort

HVP: - most precise prediction by using e^+e^- hadronic cross section data (this talk)
- alternative: lattice QCD, see talk by M Golterman

g-2: SM prediction

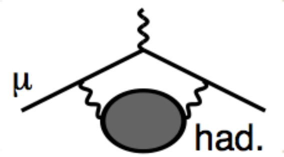
- Several groups have produced hadronic compilations over the years.
- Here: Hagiwara+Liao+Martin+Nomura+T
- No new `global' compilations; more precise data expected for near future
- At present the Hadronic Vacuum Polarisation dominates the SM error

QED contribution	11 658 471.808 (0.015) $\times 10^{-10}$	Kinoshita & Nio, Aoyama et al
EW contribution	15.4 (0.2) $\times 10^{-10}$	Czarnecki et al
Hadronic contribution		
LO hadronic	**694.9 (4.3)** $\times 10^{-10}$	HLMNT11
NLO hadronic	−9.8 (0.1) $\times 10^{-10}$	HLMNT11
light-by-light	10.5 (2.6) $\times 10^{-10}$	Prades, de Rafael & Vainshtein
Theory TOTAL	**11 659 182.8 (4.9)** $\times 10^{-10}$	
Experiment	**11 659 208.9 (6.3)** $\times 10^{-10}$	world avg
Exp − Theory	**26.1 (8.0)** $\times 10^{-10}$	**3.3 σ discrepancy**

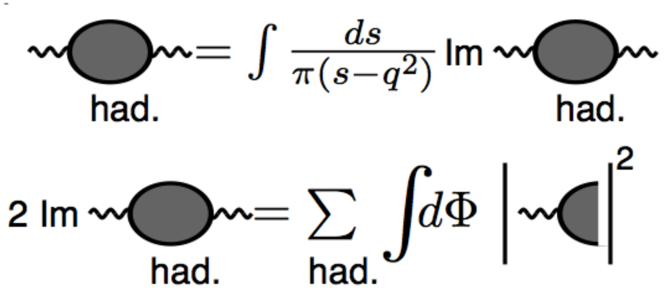
(Numbers taken from HLMNT11, arXiv:1105.3149)

g-2, hadronic VP, essentials

Use of data compilation for HVP:



pQCD not useful. Use the dispersion relation and the optical theorem.



$$a_\mu^{\rm had,LO} = \frac{m_\mu^2}{12\pi^3} \int_{s_{\rm th}}^{\infty} ds\, \frac{1}{s} \hat{K}(s) \sigma_{\rm had}(s)$$

- Weight function $\hat{K}(s)/s = \mathcal{O}(1)/s$

$\Longrightarrow$ Lower energies more important

$\Longrightarrow$ $\pi^+\pi^-$ channel: 73% of total $a_\mu^{\rm had,LO}$

How to get the most precise $\sigma^0_{\rm had}$? e^+e^- data:

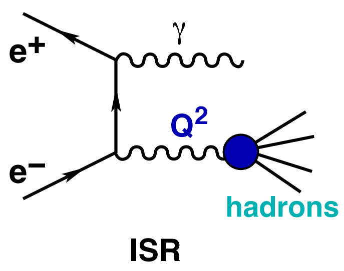
- Low energies: sum ~ 25 exclusive channels
- √s = 1.4 – 2 GeV: sum exclusive channels, use iso-spin relations for missing channels, old inclusive data disfavoured, chance for new incl. measurements?!
- Above ~1.8 GeV: can start to use pQCD (away from flavour thresholds), supplemented by narrow resonances (J/Ψ, Υ)

- Challenge of data combination (locally in √s): from many experiments, in different energy bins, errors from different sources, correlations; sometimes inconsistencies/bias

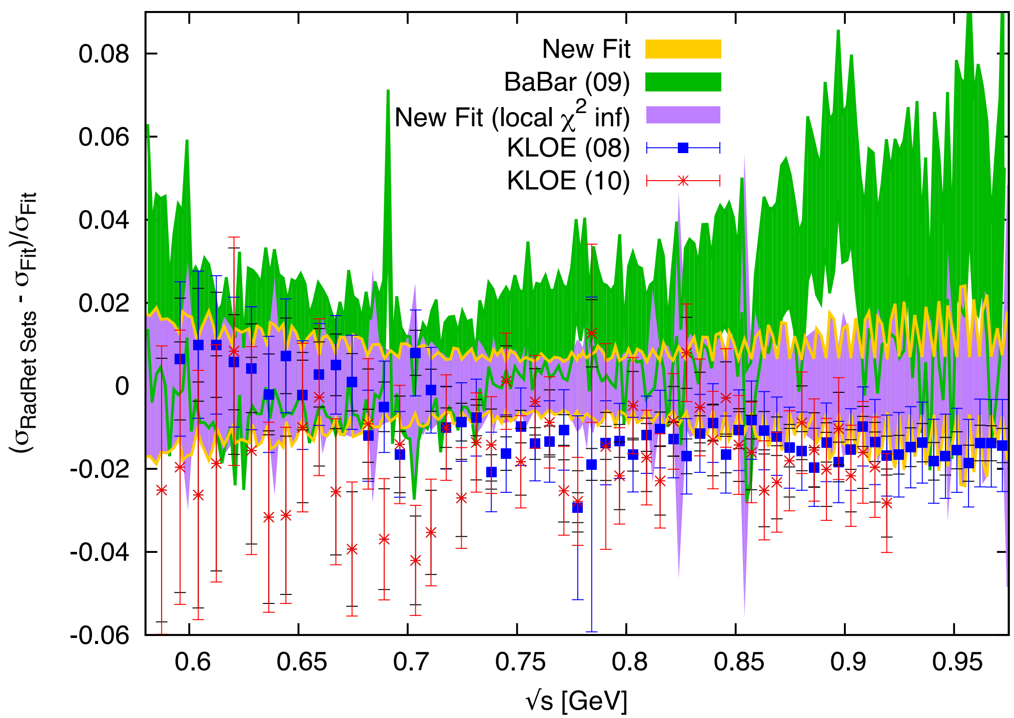
- $\sigma^0_{\rm had}$ means \`bare' σ, but WITH FSR: RadCors [HLMNT: $\delta a_\mu^{\rm had,\, RadCor\, VP+FSR} = 2 \times 10^{-10}$!]
- traditional direct \`scan' (tunable $e^+e^-$ beams) vs \`Radiative Return' [+ τ spectral functions]

Status of data combination in the $\pi^+\pi^-$ channel

Radiative Return $\pi\pi(\gamma)$ data [KLOE 08/10 and BaBar 09] compared to combination of all



Normalised difference of cross sections [HLMNT '11]



$\rightarrow$ Radiative Return (at fixed e^+e^- energy) a powerful method, *complementary to direct energy scan*

$\rightsquigarrow$ Differences in shape and BaBar high at medium and higher energies

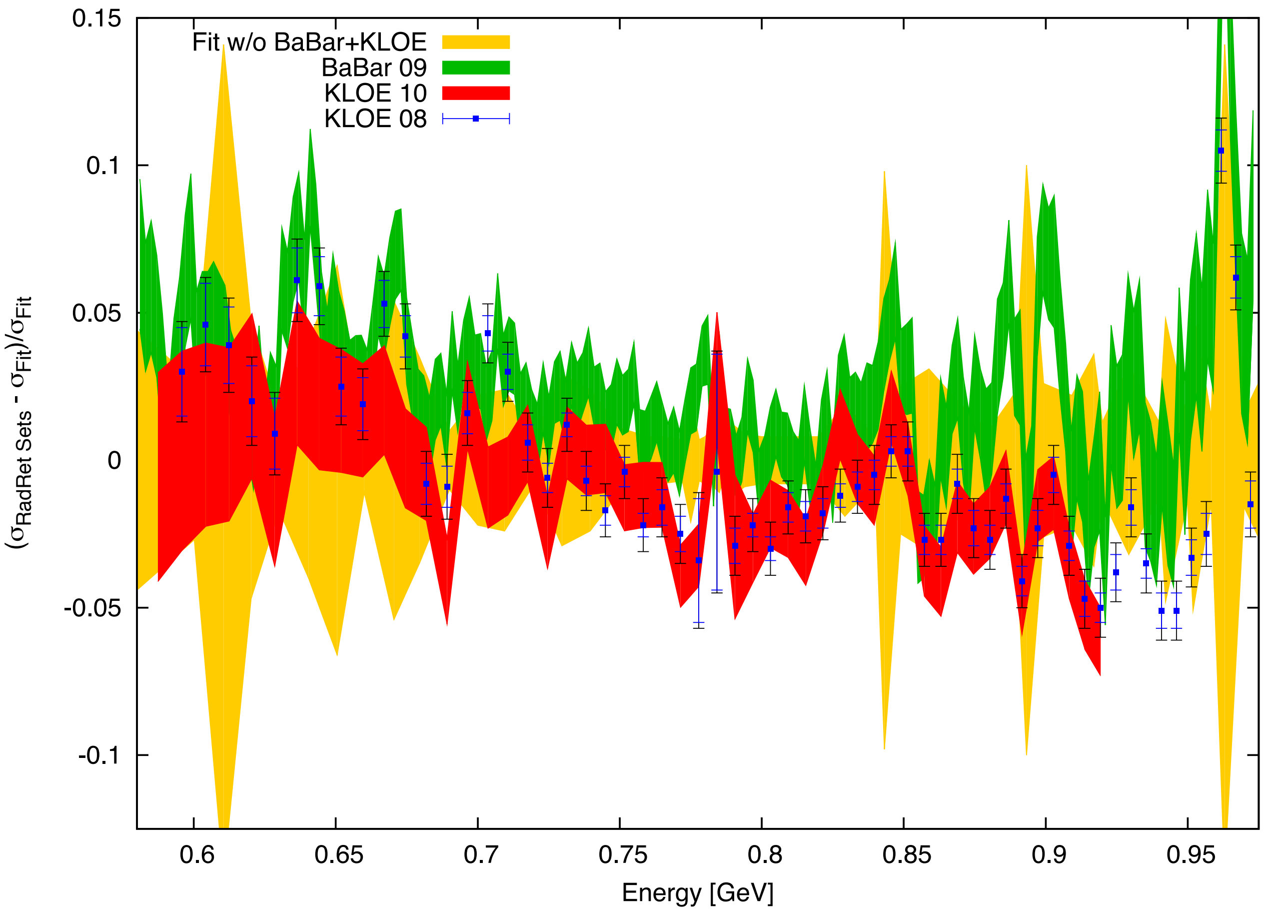
$\rightsquigarrow$ limited gain in accuracy due to 'tension'; pull-up (mainly from BaBar)

- Comb. of all data on same footing, before integration (purple band): still good $\chi^2_{\rm min}/{\rm d.o.f.} \sim 1.5$ of fit
- $a_\mu^{2\pi}(0.32 - 2\,\text{GeV}) = (\mathbf{504.2 \pm 3.0}) \cdot 10^{-10}$, $a_\mu^{2\pi,\text{w/outRad. Ret.}} = (\mathbf{498.7 \pm 3.3}) \cdot 10^{-10}$

  $\rightsquigarrow$ Pull-up of a_μ from Rad. Ret. by $\sim \mathbf{5.5}$; and: comp. w. DHMZ: Their $a_\mu^{2\pi}$ is higher by about $\mathbf{2.1}$ units.
- Clarification/improvement with more, possibly even more precise data (from both scan and ISR) needed!

Status of the data combination in the $\pi^+\pi^-$ channel

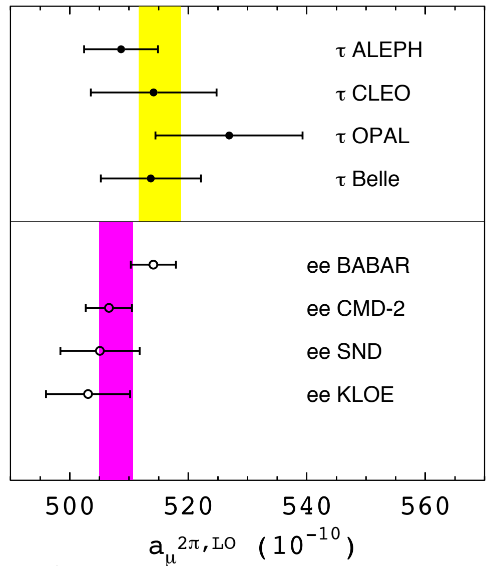
A different way to look at it: **Rad. Ret. data compared to 2π fit w/out them**



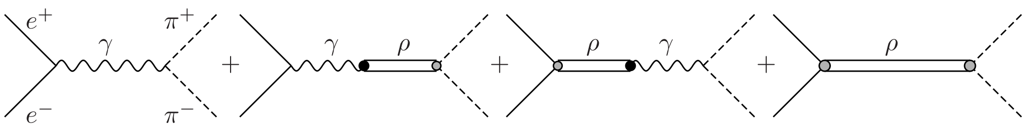
New KLOE12 data will add to this tension, see below and A Palladino

Another `puzzle': Use of tau spectral function data?

- Use CVC (iso-spin symmetry) to connect $\tau^- \to \pi^0 \pi^- \nu_\tau$ spectral functions to $e^+ e^- \to \omega, \rho \to \pi^+ \pi^-$ but have to apply iso-spin corrections
- Early calculations by Alemany, Davier, Hoecker: use of τ data complementing e^+e^- data originally resulted in an improvement w.r.t. use of e^+e^- data alone; discrepancy smaller with tau data; later increased tension between e^+e^- and τ
- Recent compilation by Davier et al (Fig. from PRD86, 032013):



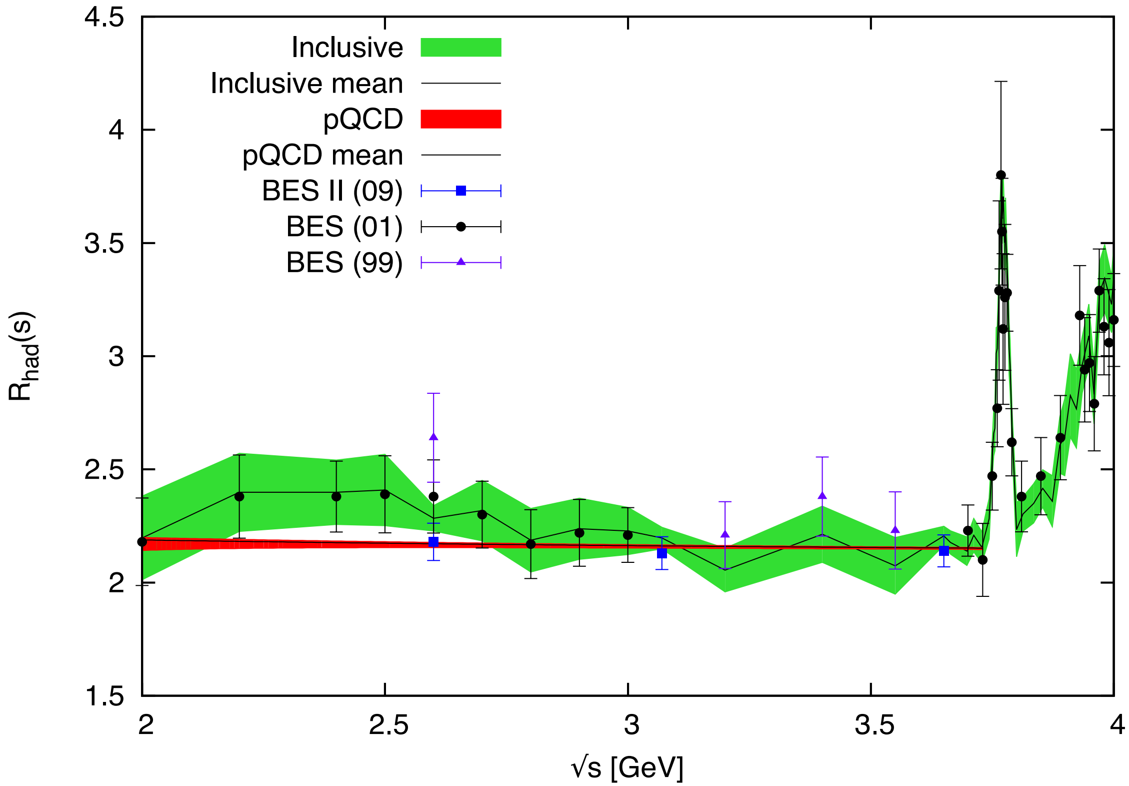
- Jegerlehner+Szafron: crucial role of γ-ρ mixing:



- They found discrepancy gone but τ data improved e^+e^- analysis only marginally, however BaBar $\pi^+\pi^-$ data not used
- Analyses by Benayoun et al: combined fit of e^+e^- and τ based on Hidden Local Symmetry (HLS) (see talk by M Benayoun): no big tension betw. e^+e^- and τ but for BaBar e^+e^-, increased Δa_μ: of ~ 4.5σ
- Davier+Malaescu refute criticism, claim fair agreement betw. BaBar and their τ comp.
- HLMNT: stick to e^+e^- (do not use τ data). With e^+e^- (incl. BaBar) discrepancy of 3-3.5σ

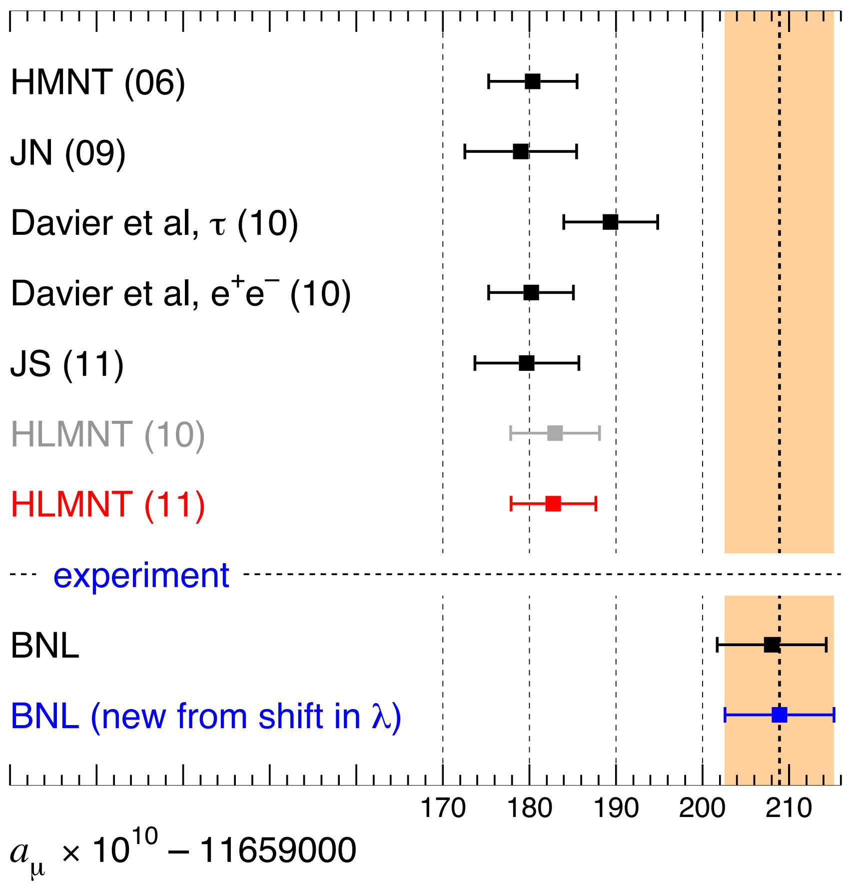
New data from BESIII eagerly awaited... 2011 status:

- Perturbative QCD vs. inclusive data above 2 GeV (below the charm threshold)



- Latest BES data (blue markers) in perfect agreement with perturbative QCD; data slightly higher than pQCD for $\sqrt{s} > 2.6$ GeV
- HLMNT use pQCD for $2.6 < \sqrt{s} < 3.7$ GeV and with (larger) BES errors
  - would have small shift downwards ($\sim -\mathbf{1.4 \cdot 10^{-10}}$ for a_μ) if used from 2 GeV
  - Davier et al. use pQCD from 1.8 GeV

Recent `history' plot.   g-2 HVP numbers



$a_\mu^{HVP, LO}$ (10^{-10}):

- Fair agreement between different e^+e^- analyses, including recent updates:

HLMNT (11): 694.9 ± 3.7 (exp) ± 2.1 (rad)
Jegerlehner (11): 690.8 ± 4.7
Davier et al (11): 692.3 ± 4.2

- The `extremes' (both with τ data):

Davier et al (11): 701.5 ± 4.7
Benayoun et al (12): 681.2 ± 4.5

- New data available now will not shift the mean value strongly, but incrementally improve determination of a_μ^{HVP}

σ_{had}: some recent new data

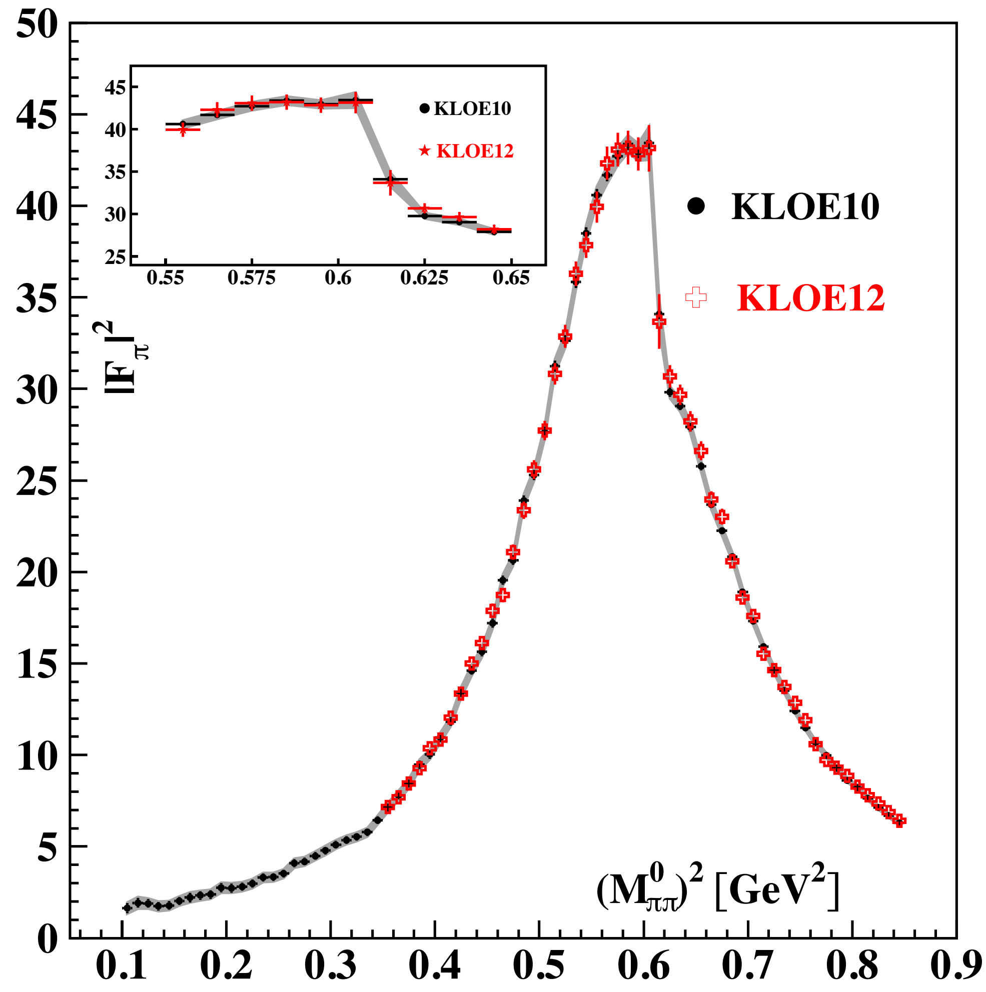
KLOE

$\pi^+\pi^-$ data with $\sigma_{\mu\mu}$ normalisation:

- confirm previous KLOE measurements

- will not decrease tension with BaBar once included in next round of `global' σ_{had} compilations, but increase significance of KLOE (so may lead to slight decrease)

- task for WG RadioMonteCarLow to compare in detail possible reasons for difference, like the MCs and analysis techniques used

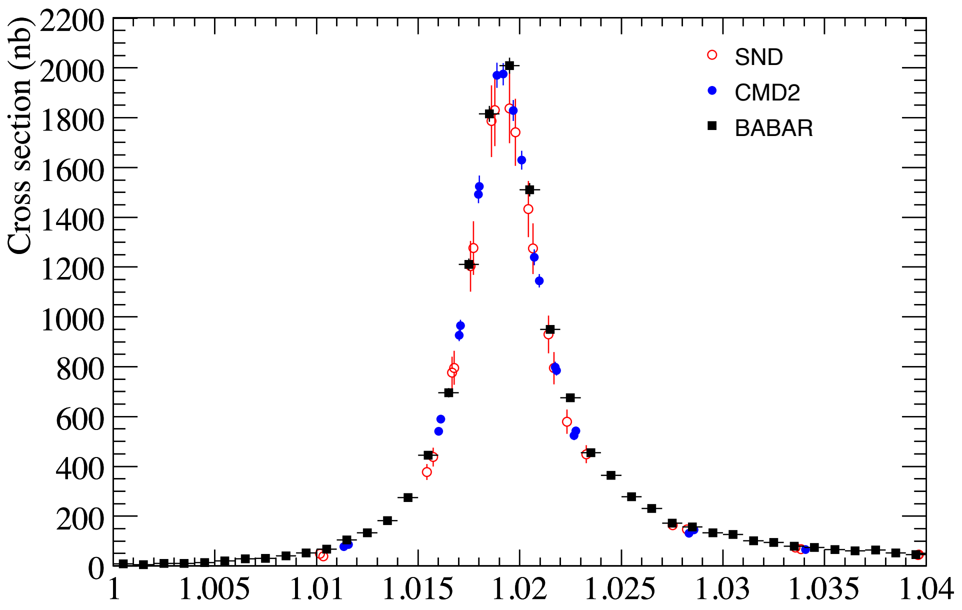


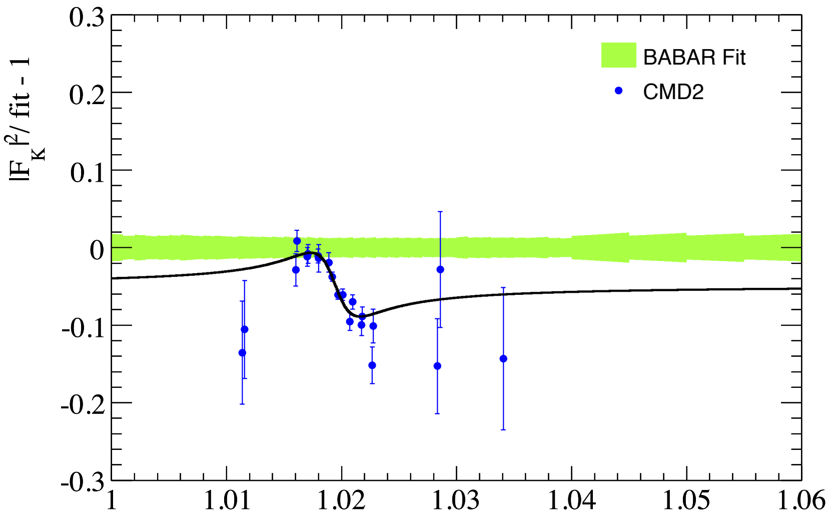
PLB720(2013)336, see talk by A Palladino

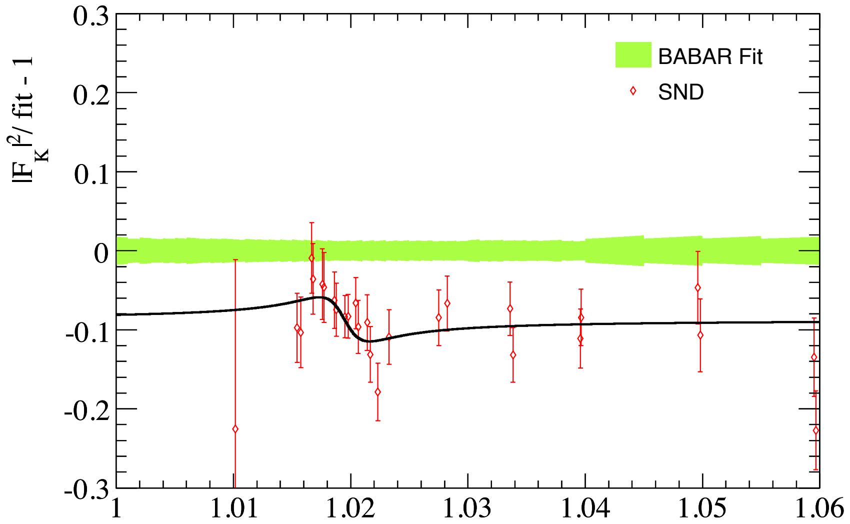
σ_{had}: some recent new data: $K^+K^-(\gamma)$ from BaBar

arXiv:1306.3600, see talk by E. Solodov

- $a_\mu = 22.94 \pm 0.18 \pm 0.22$ up to 1.8 GeV
  vs. $21.63 \pm 0.27 \pm 0.68$ for combined previous data
- significant shift up
- may need to take into account mass shift for best combination
- Comp. plots BaBar vs Novosibirsk:



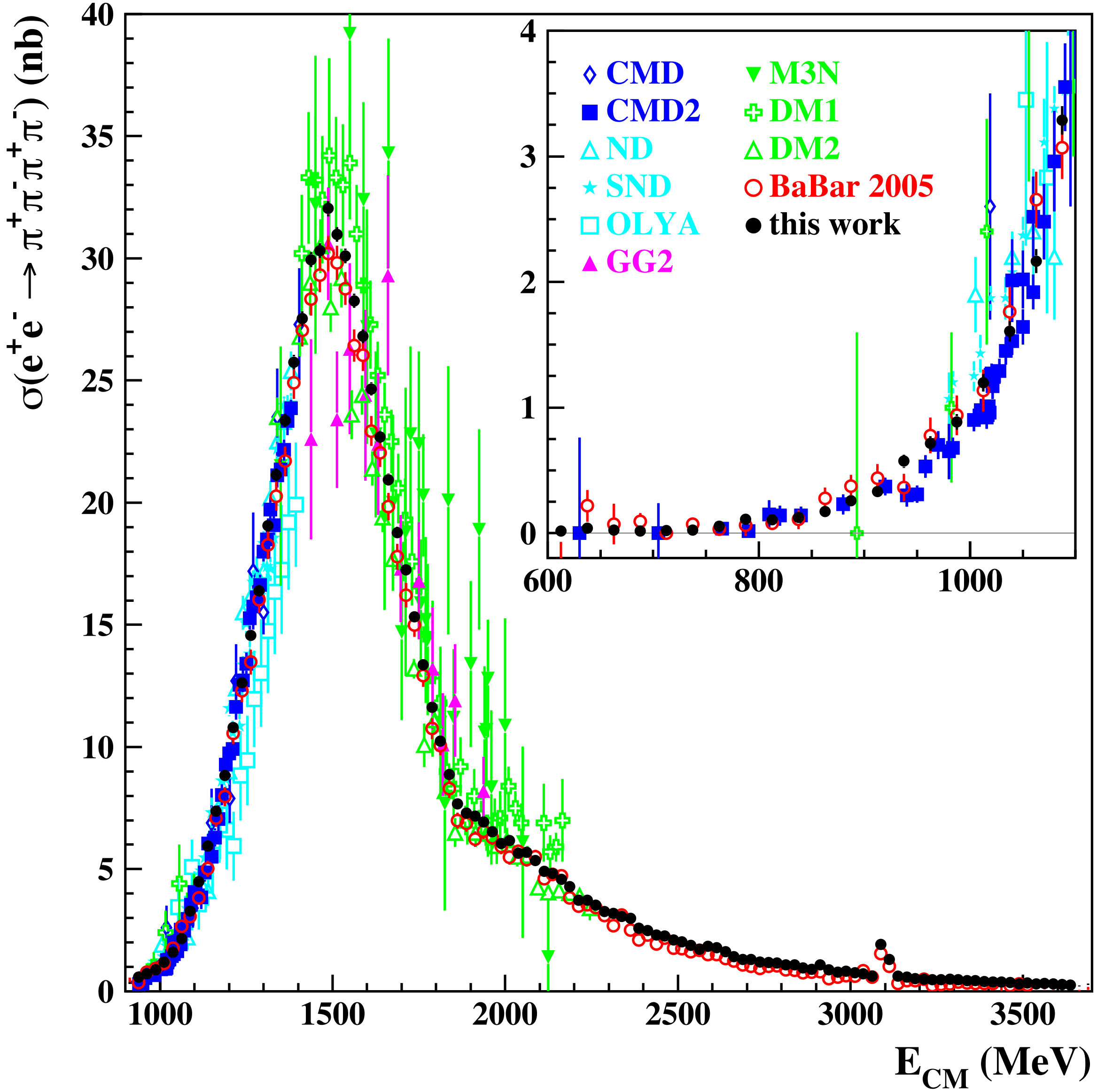




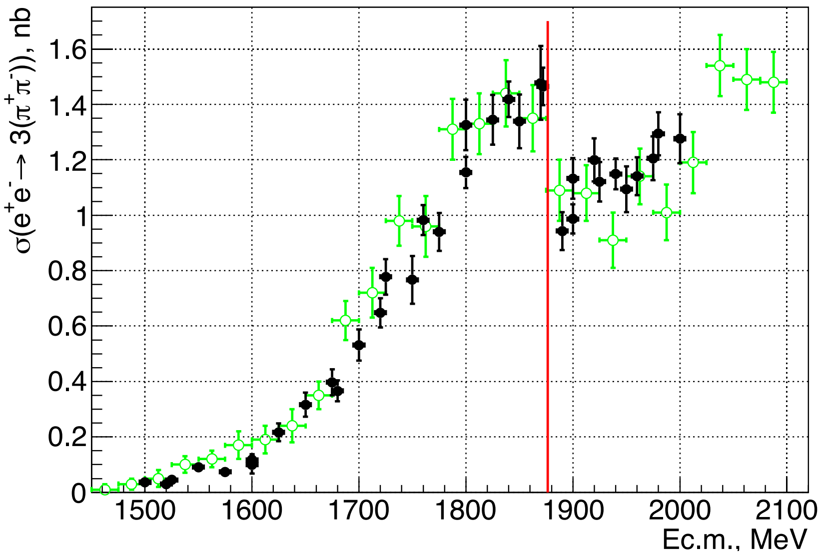
σ_{had}: some recent new data: $2\pi^+2\pi^-(\gamma)$ from BaBar

PRD85(2012)112009,
see talk by E. Solodov

- shift of $+0.3 \times 10^{-10}$ for a_μ
- error down to a third
- combination?

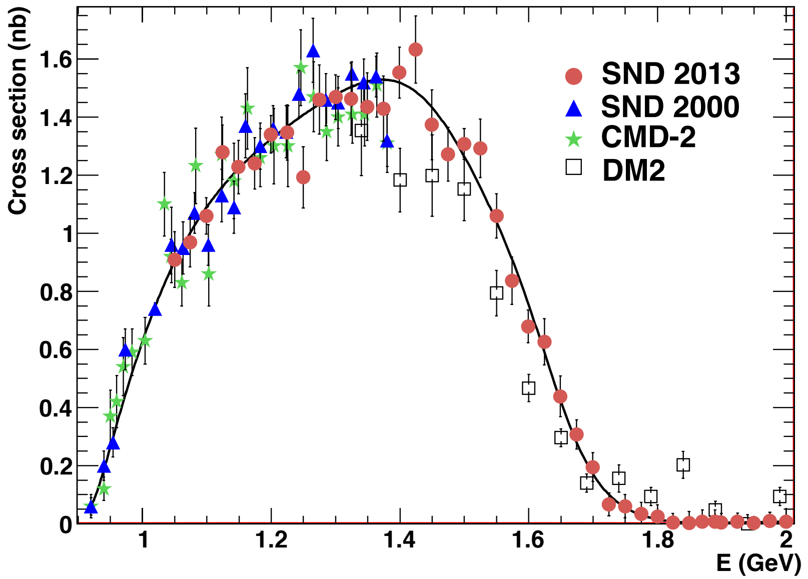


σ_{had}: some recent new data from Novosibirsk



CMD-3 6π charged
PLB723(2013)82

- solid black: CMD-3 open green: BaBar
- full analysis will include $2(\pi^+\pi^-\pi^0)$

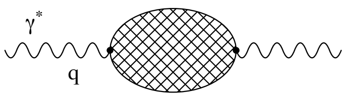


SND $\omega\pi^0$, arXiv:1303.5198

- many more anlayses reported with preliminary results, incl. 3π, 4π(2n)
- looking forward to rich harvest from SND and CMD-3
- see talks by I Logashenko and T Dimova

Hadronic VP: Δα

- Dyson summation of Real part of one-particle irreducible blobs $\mathbf{\Pi}$ into the effective, real running coupling $\boldsymbol{\alpha}_{\mathbf{QED}}$:

$\Pi \;=$ 

Full photon propagator $\sim\; 1 \;+\; \Pi \;+\; \Pi\cdot\Pi \;+\; \Pi\cdot\Pi\cdot\Pi \;+\; \ldots$

$$\rightsquigarrow \qquad \alpha(q^2) \;=\; \frac{\alpha}{1-\mathrm{Re}\Pi(q^2)} \;=\; \alpha\,/\,\left(1-\Delta\alpha_{\mathrm{lep}}(q^2)-\Delta\alpha_{\mathrm{had}}(q^2)\right)$$

- The Real part of the VP, $\mathrm{Re}\Pi$, is obtained from the Imaginary part, which via the *Optical Theorem* is directly related to the cross section, $\mathrm{Im}\Pi \sim \sigma(e^+e^- \to hadrons)$:

$$\Delta\alpha^{(5)}_{\mathrm{had}}(q^2) \;=\; -\frac{q^2}{4\pi^2\alpha}\,\mathrm{P}\int_{m_\pi^2}^{\infty}\frac{\sigma^0_{\mathrm{had}}(s)\,\mathrm{d}s}{s-q^2}\,, \quad \sigma_{\mathrm{had}}(s) = \frac{\sigma^0_{\mathrm{had}}(s)}{|1-\Pi|^2}$$

[$\to$ σ^0 requires 'undressing', e.g. via $\cdot(\alpha/\alpha(s))^2$ $\rightsquigarrow$ iteration needed]

- Observable cross sections σ_{had} contain the $|$full photon propagator$|^2$, i.e. $|$infinite sum$|^2$.

$\to$ To include the subleading Imaginary part, use dressing factor $\frac{1}{|1-\Pi|^2}$.

Hadronic VP: Δα

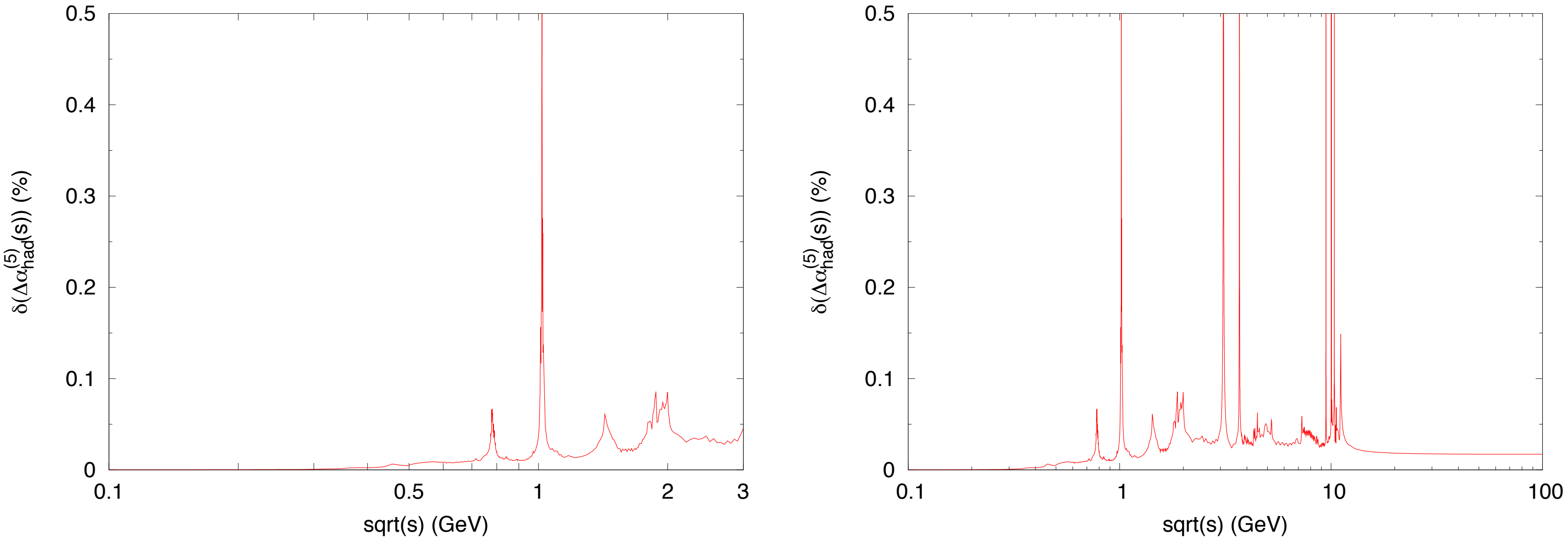
Parametrisations/routines based on `global' data compilations available from a few groups:

- Novosibirsk: http://cmd.inp.nsk.su/~ignatov/vpl/ tabulation with ROOT package
- Davier et al: HVPTools (status of distribution? still in preparation?)
- Fred Jegerlehner's package: http://www-com.physik.hu-berlin.de/~fjeger/software.html
  - set of routines with analytic codes and tabulations
  - uses rhad from Harlander+Steinhauser for Im part
  - regular updates (last 5.4.2012)
- HLMNT routine
  - provided upon request by authors (Daisuke Nomura or TT)
  - standalone Fortran, partly analytic, partly tabulation
  - current version is VP_HLMNT_v2_1 (from 27.1.2012), minor update imminent
  - flag to control if narrow resonances included or not, but Φ and higher ϒ always included through direct data integration

Hadronic VP: Δα

- Typical accuracy $\delta\left(\Delta\alpha^{(5)}_{\rm had}(s)\right)$

Error of VP in the timelike regime at low and higher energies (HLMNT compilation):

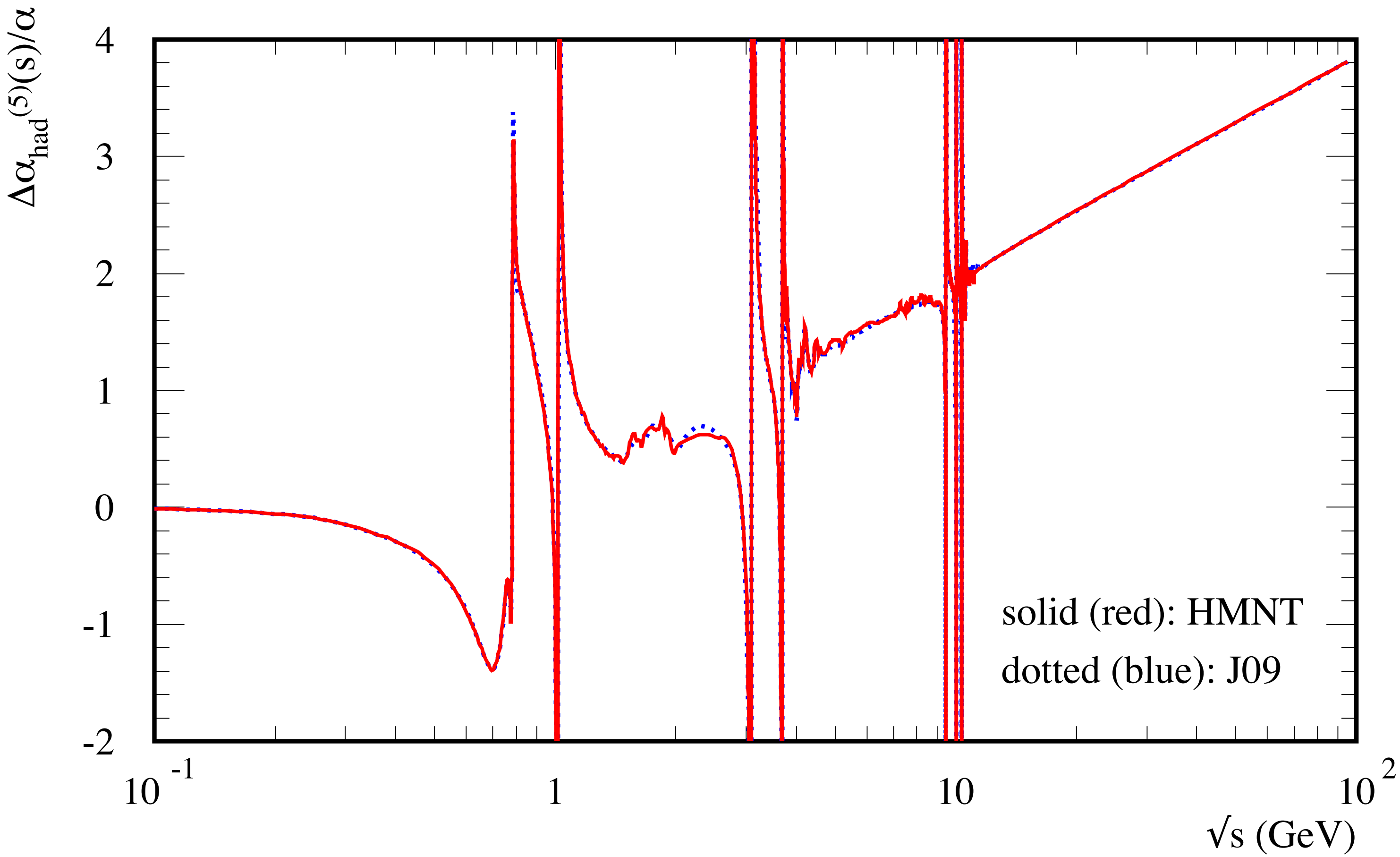


→ Below one per-mille (and typically $\sim 5\cdot10^{-4}$), apart from Narrow Resonances where the bubble summation is not well justified.

Enough in the long term? Need for more work in resonance regions.

Hadronic VP: Δα

- $\Delta\alpha(q^2)$ in the time-like: HLMNT compared to Fred Jegerlehner's new routines



→ with new version big differences (with 2003 version) gone

– smaller differences remain and reflect different choices, smoothing etc.

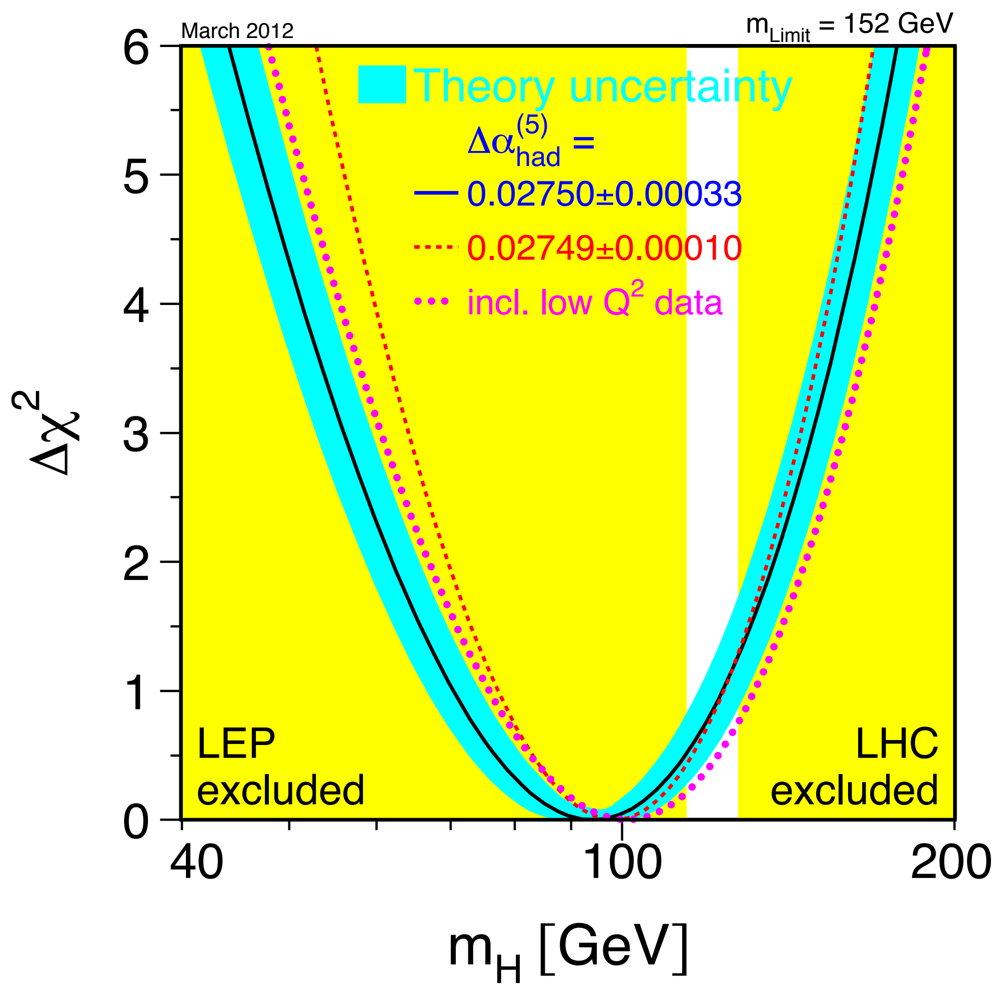
Hadronic VP: Δα

$\Delta\alpha_{had}^{(5)}(M_Z^2)$ (units of 10^{-4})

- DHMZ: 275.7 ± 1.0
- Jegerlehner: 275.10 ± 2.18
  274.98 ± 1.35 (Adler)
- HLMNT: 276.26 ± 1.38

- similar results by all groups now
- limited gain by using more pQCD

LEP EWWG: The last Blueband plot?



Future improvements

Importance of various 'channels'

[Numbers from HLMNT, 'local error infl.', $\cdot 10^{-10}$]

- Errors contributions to a_μ from leading and subleading channels (ordered) up to 2 GeV

Purely from data:

channel	error
$\pi^+\pi^-$	3.09
$\pi^+\pi^-\pi^0\pi^0$	1.26
3π	0.99
$2\pi^+2\pi^-$	0.47
K^+K^-	0.46
$2\pi^+2\pi^-2\pi^0$	0.24
$K_S^0K_L^0$	0.16

'Higher multiplicity' region from 1.4 to 2 GeV with use of isospin relations for some channels: [Use of old inclusive data disfavoured.]

Channel	contr. $\pm$ error
$K\bar{K}2\pi$	3.31 ± 0.58
$\pi^+\pi^-4\pi^0$	0.28 ± 0.28
$\eta\pi^+\pi^-$	0.98 ± 0.24
$K\bar{K}\pi$	2.77 ± 0.15
$2\pi^+2\pi^-\pi^0$	1.20 ± 0.10

- 'Inclusive' region from 2 to ~ 11 GeV: $\mathbf{41.19 \pm 0.82}$

  Can be 'squeezed' by using pQCD (done by DHMZ from 1.8 GeV);

  region from 2 to 2.6 GeV: $\mathbf{15.69 \pm 0.63} \ \rightarrow\ \mathbf{14.49 \pm 0.13}$, only small changes for higher energies.

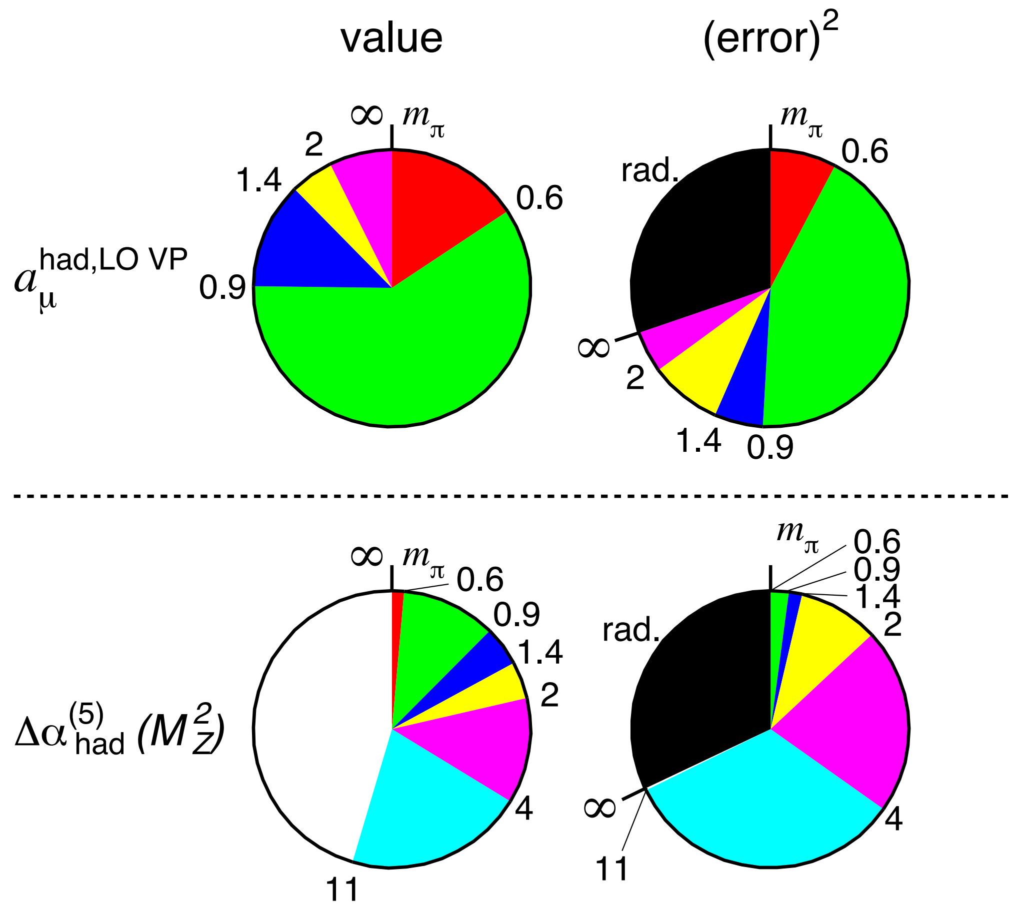
Future improvements

For g-2 the major tasks are

- improve 2π (better data, understand discrepancy between sets!)
- reanalyse/apply radiative corrections (work has started for HLMNT, needs collaboration of Exp and Th)
- higher energies will improve with input from SND, CMD-3 and BESIII

For Δα new data from BESIII and possibly the use of pQCD will squeeze the error.

Pie diagrams from HLMNT 11:



Conclusions

- Low energy precision experiments with leptons strongly test the SM and already exclude/constrain many BSM scenarios
- Only possible with rich experimental programme, including many facilities to measure HVP, efforts to calculate (MC, RadCors, etc)
- Muon g-2 discrepancy consolidated at $> 3\,\sigma$ (and none of the HVP `puzzles' make it go away), but no signs for BSM at the LHC so far
- Will combined efforts (FF from γγ, lattice) help with L-by-L?
- Can we get the required ~ or > factor 2 in HVP? I believe we can.

The race is on. Thank you.