The light-by-light contribution to the muon (g-2) within nonlocal chiral quark model

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Plan

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Motivation

The general form of element of interaction of lepton with external electromagnetic fields is

$$-ie\bar{u}(p')\left\{\gamma_\mu F_1(q^2)+i\sigma_{\mu\nu}\frac{q_\nu}{2m}F_2(q^2)+\gamma_5\sigma_{\mu\nu}\frac{q_\nu}{2m}F_3(q^2)\right\}u(p)e_\mu(q)$$

F_1 is the electric charge distribution $e_l = eF_1(0)$
F_2 corresponds to anomalous magnetic moment (AMM) $a_l = (g_l - 2)/2 = F_2(0)$
F_3 corresponds to anomalous electric dipole moment

Motivation

1. Anomalous magnetic momentum of muon $a_\mu = (g-2)_\mu$ is measured in experiment E821 (BNL) with high precision

$$a_\mu^{\text{exp}} = 11\,659\,208.9(5.4)(3.3) \cdot 10^{-10}$$

Prediction of Standard Model

$$a_\mu^{\text{theory}} = 11\,659\,183.4(0.2)_{\text{EW}}(4.1)_{\text{Had,LO}}(2.6)_{\text{Had,HO}} \cdot 10^{-10}$$

2. Difference between experiment and prediction of Standard Model is 3 standard deviation

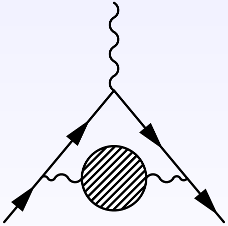
$$a_\mu^{\text{exp}} - a_\mu^{\text{theory}} = 25.5(6.3)(4.9) \cdot 10^{-10}$$

3. The main theoretical error in a_μ^{theory} is due to strong interaction

Motivation. Anomalous magnetic momentum. HVP

4. Contribution of strong interactions can be divided into two parts

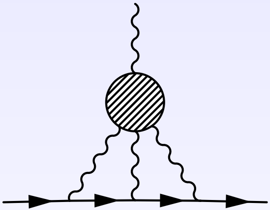
- contribution of hadronic polarization of vacuum (can be extracted from experimental data for process $e^+e^- \to$ in hadrons or hadronic τ-lepton decays)



$$a_\mu^{\text{Had,LO}} = 692.3(4.2) \cdot 10^{-10}$$

Motivation. Anomalous magnetic momentum. LbL

- light-by-light process



LbL scattering amplitude is a complicated object. It is a sum of different diagrams, the quark loop, the meson exchanges, the meson loops and the iterations of these processes. However, there is hierarchy connected to existence of two small parameters: the inverse number of colors $1/N_c$ and the ratio of the characteristic internal momentum to the chiral symmetry parameter $m_\mu/(4\pi f_\pi) \sim 0.1$.

NχQM Lagrangian

The Lagrangian of the nonlocal model has the form

$$\mathcal{L} = \mathcal{L}_{free} + \mathcal{L}_{4q} + \mathcal{L}_{tH}$$
$$\mathcal{L}_{free} = \bar{q}(x)(i\hat{\partial} - m_c)q(x)$$

m_c – current quark mass matrix with diagonal elements $m_c^u = m_c^d$, m_c^s

$$\begin{aligned} \mathcal{L}_{4q} &= \frac{G}{2}[J_S^a(x)J_S^a(x) + J_P^a(x)J_P^a(x)] \\ \mathcal{L}_{tH} &= -\frac{H}{4}T_{abc}[J_S^a(x)J_S^b(x)J_S^c(x) - 3J_P^a(x)J_P^b(x)J_P^c(x)] \end{aligned}$$

Nonlocal quark currents are

$$J_M^a(x) = \int d^4x_1 d^4x_2\, f(x_1)f(x_2)\,\bar{q}(x - x_1)\,\Gamma_M^a q(x + x_2),$$

where $M = S, P$ and $\Gamma_S^a = \lambda^a$, $\Gamma_P = i\gamma^5\lambda^a$, and $f(x)$ is a form factor reflecting the nonlocal properties of the QCD vacuum.

NχQM Lagrangian

The model can be bosonized using the stationary phase approximation which leads to the system of gap equations for the dynamical quark masses $m_{d,i} \quad (i = u, d, s)$

$$m_{d,u} + G S_u + \frac{H}{2} S_u S_s = 0,$$
$$m_{d,s} + G S_s + \frac{H}{2} S_u^2 = 0,$$
$$S_i = -8 N_c \int \frac{d_E^4 k}{(2\pi)^4} \frac{f^2(k^2) m_i(k^2)}{D_i(k^2)},$$

where $m_i(k^2) = m_{c,i} + m_{d,i} f^2(k^2)$, $D_i(k^2) = k^2 + m_i^2(k^2)$ is the dynamical quark propagator obtained by solving the Schwinger-Dyson equation, $f(k^2)$ is the nonlocal form factor in the momentum representation.

NχQM Lagrangian

The vertex functions and the meson masses can be found from the Bethe-Salpeter equation. For the separable interaction the quark-antiquark scattering matrix in pseudoscalar channel becomes

$$\mathbf{T} = \hat{\mathbf{T}}(p^2)\delta^4\left(p_1 + p_2 - (p_3 + p_4)\right)\prod_{i=1}^{4} f(p_i^2),$$

$$\hat{\mathbf{T}}(p^2) = i\gamma_5\lambda_k\left(\frac{1}{-\mathbf{G}^{-1} + \mathbf{\Pi}(p^2)}\right)_{kl} i\gamma_5\lambda_l,$$

where p_i are the momenta of external quark lines, $\mathbf{G}$ and $\mathbf{\Pi}(p^2)$ are the corresponding matrices of the four-quark coupling constants and the polarization operators of pseudoscalar mesons ($p = p_1 + p_2 = p_3 + p_4$). The meson masses can be found from the zeros of determinant $\det(\mathbf{G}^{-1} - \mathbf{\Pi}(-M^2)) = 0$.

NχQM Lagrangian. External fields.

The gauge-invariant interactions with external photon field can be introduced with Schwinger phase factor

$$q(y) \to Q(x,y) = \mathcal{P}\exp\left\{ i \int\limits_x^y dz^\mu V_\mu^a(z) T^a \right\} q(y),$$

apart from kinetic term the additional terms in nonlocal interations are generated

$$J_I(x) = \int d^4x_1 d^4x_2\, f(x_1) f(x_2)\, \bar{Q}(x - x_1, x)\, \Gamma_I\, Q(x, x + x_2)$$

NχQM Lagrangian. External fields.

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The following equations are used for obtaining of nonlocal vertices

$$\frac{\partial}{\partial y^\mu} \int\limits_x^y dz^\nu \; F_\nu(z) = F_\mu(y), \qquad \delta^{(4)}(x - y) \int\limits_x^y dz^\nu \; F_\nu(z) = 0.$$

NχQM Lagrangian. External fields.

Quark-antiquark nonlocal vertex with one photon line

$$\Gamma_\mu(q_1) = -(k+k_1)_\mu m^{(1)}(k,k_1)$$

Here, and below $k_1 = k + q_1$, $k_{ij..k} = k + q_i + q_j + .. + q_k$ and eQ omitted. k is a momentum of incoming quark, and q_i momenta of incoming photons. The first-order finite difference is introduced

$$f^{(1)}(a,b) = \frac{f(a+b) - f(a)}{(a+b)^2 - a^2}$$

Combination with local vertex γ_μ satisfies the Ward identity for dynamical quarks.

NχQM Lagrangian. External fields.

With two lines

$$\begin{aligned}\Gamma_{\mu\nu}(q_1,q_2) &= 2g_{\mu\nu}m^{(1)}(k,k_{12}) \\ &\quad + (k+k_1)_\mu(k_1+k_{12})_\nu m^{(2)}(k,k_1,k_{12}) \\ &\quad + (k+k_2)_\nu(k_2+k_{12})_\mu m^{(2)}(k,k_2,k_{12})\end{aligned}$$

$$f^{(2)}(a,b_1,b_2) = \frac{f^{(1)}(a,b_1) - f^{(1)}(a,b_2)}{(a+b_1)^2 - (a+b_2)^2},$$

NχQM Lagrangian. External fields.

with three lines

$$\Gamma_{\mu\nu\rho}(q_1, q_2, q_3) = .. + 2g_{..}(k + k_b)_{..}m^{(2)}(k, k_b, k_{123})..$$
$$.. + 2g_{..}(k_b + k_{123})_{..}m^{(2)}(k, k_b, k_{123})..$$
$$.. + (k + k_b)_{..}(k_b + k_c)_{..}(k_c + k_{123})_{..}m^{(3)}(k, k_b, k_c, k_{123})..$$

$$f^{(n)}\left(a, \{b_i\}, b_1, b_2\right) = \frac{f^{(n-1)}\left(a, \{b_i\}, b_1\right) - f^{(n-1)}\left(a, \{b_i\}, b_2\right)}{\left(a + b_1\right)^2 - \left(a + b_2\right)^2}.$$

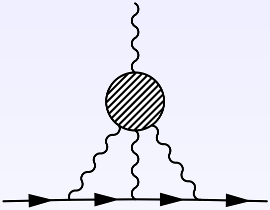
NχQM Lagrangian. External fields.

with four lines

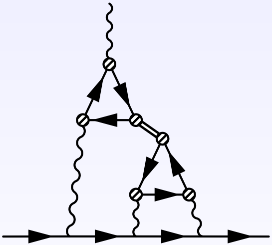
$$\Gamma_{\mu\nu\rho\tau}(q_1, q_2, q_3, q_4) = .. + 4g_{..}g_{..}m^{(2)}(k, .., k_{1234})$$
$$+ 2g_{..}(k + k_b)_{..}(k_b + k_c)_{..}m^{(3)}(k, k_b, k_c, k_{1234})$$
$$+ 2g_{..}(k + k_b)_{..}(k_c + k_{1234})_{..}m^{(3)}(k, k_b, k_c, k_{1234})$$
$$+ 2g_{..}(k_b + k_c)_{..}(k_c + k_{1234})_{..}m^{(3)}(k, k_b, k_c, k_{1234})$$
$$+ (k + k_b)_{..}(k_b + k_c)_{..}(k_c + k_d)_{..}(k_d + k_{1234})_{..}m^{(4)}(k, k_b, k_c, k_d, k_{1234})$$

NχQM Lagrangian. External fields. Gauge simplification

$$
\begin{aligned}
\Gamma_{\mu} q_1^{\mu} =& m_k - m_{k_1} \\
\Gamma^{\mu\nu} q_1^{\mu} q_2^{\nu} =& m_k + m_{k_{12}} - m_{k_1} - m_{k_2} \\
\Gamma^{\mu\nu\rho} q_1^{\mu} q_2^{\nu} q_3^{\rho} =& m_k - m_{k_{123}} - m_{k_1} - m_{k_2} - m_{k_3} \\
& + m_{k_{12}} + m_{k_{13}} + m_{k_{23}} \\
\Gamma^{\mu\nu\rho\tau} q_1^{\mu} q_2^{\nu} q_3^{\rho} q_4^{\tau} =& m_k + m_{k_{1234}} + m_{k_{12}} + m_{k_{13}} \\
& + m_{k_{14}} + m_{k_{34}} + m_{k_{23}} + m_{k_{24}} \\
& - m_{k_1} - m_{k_2} - m_{k_3} - m_{k_4} \\
& - m_{k_{123}} - m_{k_{124}} - m_{k_{134}} - m_{k_{234}}
\end{aligned}
$$



Light-by-light hadronic contribution to the muon AMM. Resonance contribution



Two-photon–pseudoscalar meson. I

Triangular diagram with external pseudoscalar meson and two photon legs with arbitrary virtualities can be written as

$$A\left(\gamma^*_{(q_1,\epsilon_1)}\gamma^*_{(q_2,\epsilon_2)} \to P^*_{(p)}\right) = -ie^2\varepsilon_{\mu\nu\rho\sigma}\epsilon_1^\mu\epsilon_2^\nu q_1^\rho q_2^\sigma \mathrm{F}_{P^*\gamma^*\gamma^*}\left(p^2;q_1^2,q_2^2\right),$$

$$\mathrm{F}_{\pi_0^*\gamma^*\gamma^*}\left(p^2;q_1^2,q_2^2\right) = g_\pi(p^2)F_u\left(p^2;q_1^2,q_2^2\right),$$

$$\mathrm{F}_{\eta^*\gamma^*\gamma^*}\left(p^2;q_1^2,q_2^2\right) = \frac{g_\eta(p^2)}{3\sqrt{3}}\times$$

$$\times\Big[\left(5F_u\left(p^2;q_1^2,q_2^2\right) - 2F_s\left(p^2;q_1^2,q_2^2\right)\right)\cos\theta(p^2) -$$

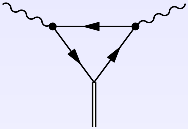
$$-\sqrt{2}\left(5F_u\left(p^2;q_1^2,q_2^2\right) + F_s\left(p^2;q_1^2,q_2^2\right)\right)\sin\theta(p^2)\Big],$$

$$\mathrm{F}_{\eta'^*\gamma^*\gamma^*}\left(p^2;q_1^2,q_2^2\right) = \frac{g_{\eta'}(p^2)}{3\sqrt{3}}\times$$

$$\times\Big[\left(5F_u\left(p^2;q_1^2,q_2^2\right) - 2F_s\left(p^2;q_1^2,q_2^2\right)\right)\sin\theta(p^2) +$$

$$+\sqrt{2}\left(5F_u\left(p^2;q_1^2,q_2^2\right) + F_s\left(p^2;q_1^2,q_2^2\right)\right)\cos\theta(p^2)\Big].$$

Two-photon–pseudoscalar meson. II



$$F_i\left(p^2;q_1^2,q_2^2\right) = 8\int \frac{d_E^4k}{(2\pi)^4}\frac{f(k_1^2)f(k_2^2)}{D_i(k_1^2)D_i(k_2^2)D_i(k^2)}\times$$

$$\times\left[m_i(k^2) - \mathrm{m}_i^{(1)}(k_1,k)J_1 - \mathrm{m}_i^{(1)}(k_2,k)J_2\right],$$

$$J_1 = k^2 + \frac{q_2^2(kq_1)(k_1q_1) - q_1^2(kq_2)(k_1q_2)}{q_1^2q_2^2-(q_1q_2)^2},$$

$$J_2 = k^2 + \frac{q_1^2(kq_2)(k_2q_2) - q_2^2(kq_1)(k_2q_1)}{q_1^2q_2^2-(q_1q_2)^2},$$

where $k_1 = k + q_1,\ k_2 = k - q_2.$

Two-photon–ps meson. III. Special kinematics

1. The kinematics needed for the hadronic exchange LbL

$$F_i\left(q_1^2;q_1^2,0\right)=8\int\frac{d_E^4k}{(2\pi)^4}\frac{f(k_1^2)f(k^2)}{D_i(k_1^2)D_i^2(k^2)}\times$$
$$\times\left[m_i(k^2)-\mathrm{m}_i^{(1)}(k_1,k)\overline{J}_1-m_i'(k^2)\overline{J}_2\right],$$
$$\overline{J}_1\left(k,q_1\right)=(kq_1)+\frac{2}{3}\left[k^2+2\frac{(kq_1)^2}{q_1^2}\right],$$
$$\overline{J}_2=\frac{4}{3}\left[k^2-\frac{(kq_1)^2}{q_1^2}\right],$$

2. Zero momentum kinematics

$$F_i\left(0;0,0\right)=\frac{1}{m_{d,i}}\left[\frac{1}{4\pi^2}-8m_{c,i}\int\frac{d_E^4k}{(2\pi)^4}\frac{m_i(k^2)-2m_i'(k^2)k^2}{D_i^3(k^2)}\right],$$

The first term is due to the axial anomaly, while the second term represents the correction due to explicit breaking of the chiral symmetry by current quark mass.

Two-photon–scalar meson. I

Triangular diagram with external scalar meson and two photon legs with arbitrary virtualities can be written as

$$A\left(\gamma^*_{(q_1,\mu)}\gamma^*_{(q_2,\nu)} \to S^*_{(p)}\right) = e^2 \Delta^{\mu\nu}_{S^*\gamma^*\gamma^*}(q_3, q_1, q_2) =$$

$$= e^2 \left[A_{S^*\gamma^*\gamma^*}\left(p^2; q_1^2, q_2^2\right) T_A^{\mu\nu}(q_1, q_2) \right.$$

$$\left. + B_{S^*\gamma^*\gamma^*}(p^2; q_1^2, q_2^2) T_B^{\mu\nu}(q_1, q_2) \right],$$

$$T_A^{\mu\nu}(q_1, q_2) = \left(g^{\mu\nu}(q_1 \cdot q_2) - q_1^\nu q_2^\mu\right)$$

$$T_B^{\mu\nu}(q_1, q_2) = \left(q_1^2 q_2^\mu - (q_1 \cdot q_2) q_1^\mu\right)\left(q_2^2 q_1^\nu - (q_1 \cdot q_2) q_2^\nu\right),$$

Two-photon–scalar meson. Ia

$$\Delta^{\mu\nu}_{a_0^*\gamma^*\gamma^*}\left(p^2;q_1^2,q_2^2\right) = g_{a_0}(p^2)\delta^{\mu\nu}_u\left(p^2;q_1^2,q_2^2\right),$$

$$\Delta^{\mu\nu}_{\sigma^*\gamma^*\gamma^*}\left(p^2;q_1^2,q_2^2\right) = \frac{g_\sigma(p^2)}{3\sqrt{3}}\times$$

$$\times\Big[\left(5\delta^{\mu\nu}_u\left(p^2;q_1^2,q_2^2\right) - 2\delta^{\mu\nu}_s\left(p^2;q_1^2,q_2^2\right)\right)\cos\theta(p^2) -$$

$$-\sqrt{2}\left(5\delta^{\mu\nu}_u\left(p^2;q_1^2,q_2^2\right) + \delta^{\mu\nu}_s\left(p^2;q_1^2,q_2^2\right)\right)\sin\theta(p^2)\Big],$$

$$\Delta^{\mu\nu}_{f_0'^*\gamma^*\gamma^*}\left(p^2;q_1^2,q_2^2\right) = \frac{g_{f_0}(p^2)}{3\sqrt{3}}\times$$

$$\times\Big[\left(5\delta^{\mu\nu}_u\left(p^2;q_1^2,q_2^2\right) - 2\delta^{\mu\nu}_s\left(p^2;q_1^2,q_2^2\right)\right)\sin\theta(p^2) +$$

$$+\sqrt{2}\left(5\delta^{\mu\nu}_u\left(p^2;q_1^2,q_2^2\right) + \delta^{\mu\nu}_s\left(p^2;q_1^2,q_2^2\right)\right)\cos\theta(p^2)\Big].$$

Light-by-light hadronic contribution to the muon AMM. Resonance contribution

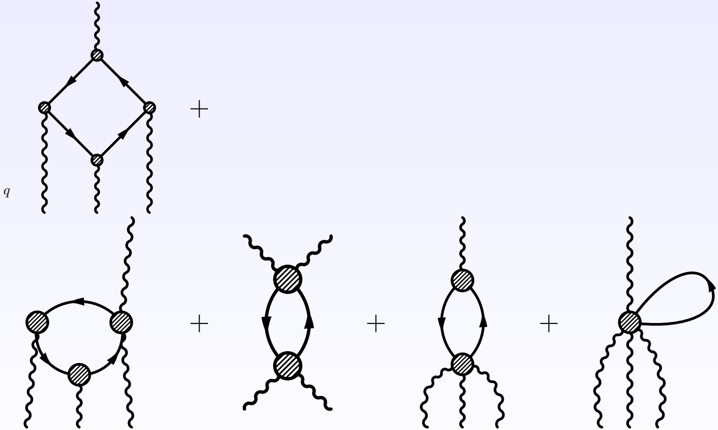
$SU(2)$ estimate for π^0 and σ contribution

$$\begin{aligned} a_\mu^{\mathrm{LbL},\pi^0} &= (5.01 \pm 0.37)\cdot 10^{-10} \\ a_\mu^{\mathrm{LbL},\pi^0+\sigma} &= (5.40 \pm 0.33)\cdot 10^{-10} \end{aligned}$$

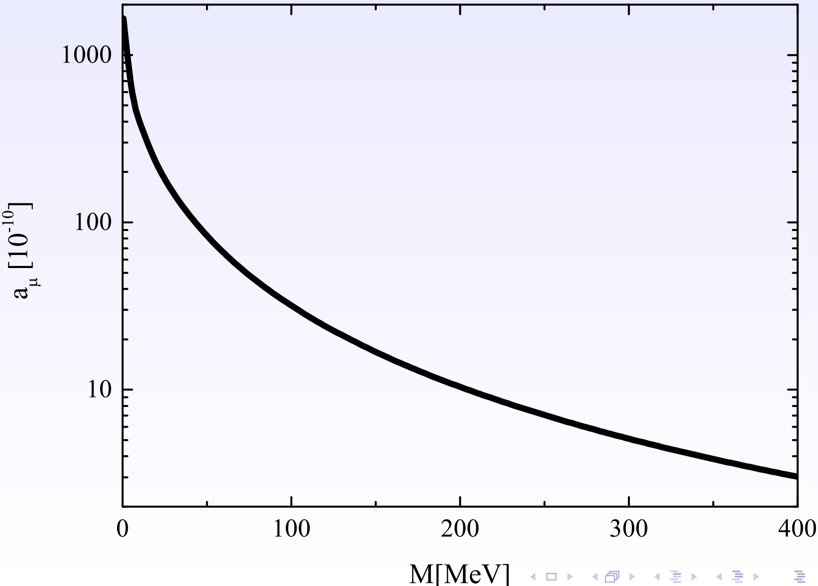
$SU(3)$ estimate for π^0, η, η' and $\sigma, f_0(980), a_0(980)$ contribution

$$\begin{aligned} a_\mu^{\mathrm{LbL,PS}} &= (5.85 \pm 0.87)\cdot 10^{-10} \\ a_\mu^{\mathrm{LbL,PS+S}} &= (6.25 \pm 0.83)\cdot 10^{-10} \end{aligned}$$

Light-by-light hadronic contribution to the muon AMM. Contact zoo



Light-by-light hadronic contribution to the muon AMM. Local contribution.



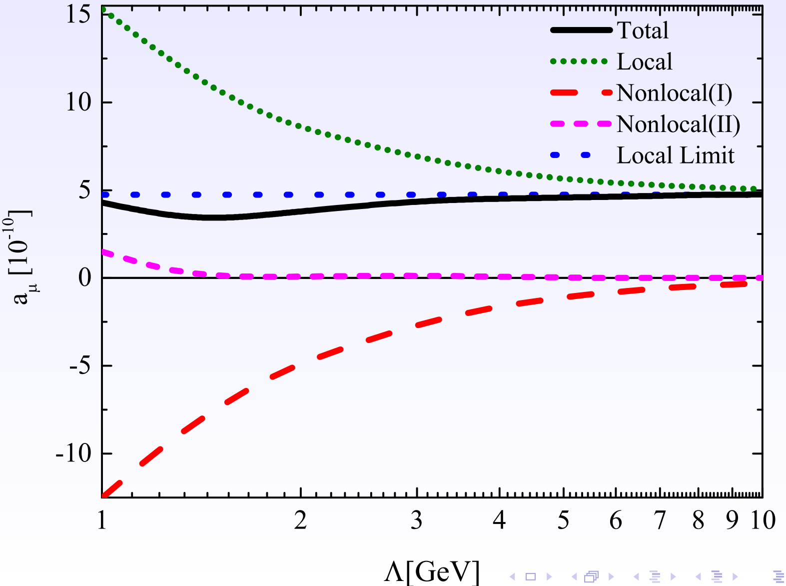
Light-by-light hadronic contribution to the muon AMM. Contact zoo (Preliminary)

Preliminary estimate for contribution of nonstrange contact terms (set G_I)

$$a_\mu^{\mathrm{LbL,local}} = 15.3 \cdot 10^{-10}$$
$$a_\mu^{\mathrm{LbL,nl(I)}} = -12.5 \cdot 10^{-10}$$
$$a_\mu^{\mathrm{LbL,nl(II)}} = 1.5 \cdot 10^{-10}$$
$$a_\mu^{\mathrm{LbL,Total}} = 4.3 \cdot 10^{-10}$$

We found that dynamical momentum dependent mass leads to increasing of contribution of diagram with pure local quark–anti-quark–photon vertices. (This one can expect as $m(k^2 \to \infty) \to m_c$.) However, contribution of diagrams with nonlocal vertices cancels this increase.

Light-by-light hadronic contribution to the muon AMM. Contact zoo (Preliminary)



Conclusions

1. Within the NχQM the pseudoscalar and scalar resonances as well as contact contributions are estimated.
2. The pseudoscalar meson contributions to muon AMM are systematically lower then the results obtained in the other works.
3. The full kinematic dependence of the vertices on the pion virtuality diminishes the result by about 20-30% as compared to the case where this dependence is neglected.
4. The contribution of $\mathrm{PS}+\mathrm{S}$ resonance exchanges is estimated as $a_\mu^{\mathrm{PS+S,LbL}} = (6.25 \pm 0.83) \cdot 10^{-10}$.
5. The contact nonstrange contribution is estimated as $a_\mu^{\mathrm{cont,LbL}} \sim 4.3 \cdot 10^{-10}$.

Conclusions. II

- The total contact and resonance contribution is estimated as $a_\mu^{\mathrm{cont+PS+S,LbL}} \sim 10.55 \cdot 10^{-10}$.

This is in the bounds $a_\mu^{\mathrm{LbL}} = (10.5 \pm 2.6) \cdot 10^{-10}$ quoted in [1]. So, we do not confirm result of recent DSE studies, where it is found $a_\mu^{\mathrm{LbL}} = (18.8 \pm 0.4) \cdot 10^{-10}$.

1. J. Prades, E. de Rafael and A. Vainshtein, (Advanced series on directions in high energy physics. 20) [arXiv:0901.0306 [hep-ph]].
2. T. Goecke, C. S. Fischer and R. Williams, Phys. Rev. D **87**, 034013 (2013).

THANKS !