A short review of New Massive Gravity

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based on

Olaf Hohm, Paul Townsend + E.B.

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Outline

Introduction

Outline

Introduction

3D New Massive Gravity

Outline

Introduction

3D New Massive Gravity

Extension to 4D?

Outline

Introduction

3D New Massive Gravity

Extension to 4D?

Outlook

Outline

Introduction

3D New Massive Gravity

Extension to 4D?

Outlook

Massive Spin-2 by Higher Derivatives

Einstein Gravity is the unique field theory of interacting massless spin-2 particles around a given spacetime background that mediates the gravitational force

Problem: Gravity is perturbative non-renormalizable

$$\mathcal{L} \sim R + a\left(R_{\mu\nu}{}^{ab}\right)^2 + b\,(R_{\mu\nu})^2 + c\,R^2 \;:$$

renormalizable but not unitary

Stelle (1977)

massless spin 2 and massive spin 2 have opposite sign !

Special Case

- In three dimensions there is no (bulk) massless spin 2 !

$\Rightarrow$ "New Massive Gravity"

Hohm, Townsend + E.B. (2009)

Underlying Trick

- higher-derivative theories can be constructed starting from the usual second-order derivative equations and solving for differential subsidiary conditions .....

- in terms of a field with the same index structure

Warming up with Spin 1

- $\mathcal{L} = -\frac{1}{4} F^{\mu\nu} F_{\mu\nu} - \frac{1}{2} m^2 A_\mu A^\mu$: Proca

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Warming up with Spin 1

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- $\left(\Box - m^2\right) A^\nu = 0 \, ,$ $\partial^\mu A_\mu = 0 :$ subsidiary condition

number of propagated modes is $D - 1 = \begin{cases} 3 & \text{for } 4D \\ 2 & \text{for } 3D \end{cases}$

"Boosting up the Derivatives"

- $\partial^\mu \tilde{A}_\mu = 0 \quad \Rightarrow \quad \tilde{A}_\mu = \epsilon_\mu{}^{\nu\rho}\, \partial_\nu A_\rho$

- $\left(\Box - m^2\right) F^\mu = 0\,, \quad F^\mu = \epsilon^{\mu\nu\rho}\partial_\nu A_\rho$: "on-shell equivalence"

- $S = \int d^3x \left(\frac{1}{2}\epsilon^{\mu\nu\rho} F_\mu \partial_\nu F_\rho + \frac{1}{2} m^2\, \epsilon^{\mu\nu\rho} A_\mu \partial_\nu A_\rho\right)$: ghost

Deser, Jackiw (1999)

I will not discuss the parity-odd 3D TME and 3D TMG theories

TMG : Deser, Jackiw, Templeton (1982)

These are based on a factorisation of the 3D Klein-Gordon operator

Now on to spin two !

Outline

Introduction

3D New Massive Gravity

Extension to 4D?

Outlook

3D Einstein-Hilbert Gravity

Deser, Jackiw, 't Hooft (1984)

There are no massless gravitons : "trivial" gravity

Adding higher-derivative terms leads to "massive gravitons"

Free Fierz-Pauli

- $\left(\Box - m^2\right)\tilde{h}_{\mu\nu} = 0\,, \qquad \eta^{\mu\nu}\tilde{h}_{\mu\nu} = 0\,, \quad \partial^\mu \tilde{h}_{\mu\nu} = 0$

- $\mathcal{L}_{\rm FP} = \frac{1}{2}\tilde{h}^{\mu\nu} G^{\rm lin}_{\mu\nu}(\tilde{h}) + \frac{1}{2}m^2\left(\tilde{h}^{\mu\nu}\tilde{h}_{\mu\nu} - \tilde{h}^2\right)\,, \qquad \tilde{h} \equiv \eta^{\mu\nu}\tilde{h}_{\mu\nu}$

no obvious non-linear extension !

number of propagating modes is $\frac{1}{2}D(D+1) - 1 - D = \begin{cases} 5 & \text{for } 4D \\ 2 & \text{for } 3D \end{cases}$

Higher-Derivative Extension in 3D

$$\partial^\mu \tilde{h}_{\mu\nu} = 0 \quad \Rightarrow \quad \tilde{h}_{\mu\nu} = \epsilon_\mu{}^{\alpha\beta} \epsilon_\nu{}^{\gamma\delta} \partial_\alpha \partial_\gamma h_{\beta\delta} \equiv G^{\text{lin}}_{\mu\nu}(h)$$

$$\left(\Box - m^2\right) \; G^{\text{lin}}_{\mu\nu}(h) = 0\,, \qquad R^{\text{lin}}(h) = 0$$

Non-linear generalization : $\quad g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu} \quad \Rightarrow$

$$\mathcal{L} = \sqrt{-g}\left[-R - \frac{1}{2m^2}\left(R^{\mu\nu}R_{\mu\nu} - \frac{3}{8}R^2\right)\right]$$

"New Massive Gravity" : unitary !

Auxiliary Field Formulation

$$\mathcal{L}[g, q] = \sqrt{-g}\left[-R + 2q^{\mu\nu}G_{\mu\nu} - m^2\left(q^{\mu\nu}q_{\mu\nu} - q^2\right)\right]$$

$$q_{\mu\nu} = \frac{1}{m^2}G_{\mu\nu} - \frac{1}{2m^2}g_{\mu\nu}G$$

Mode Analysis

- Take NMG with metric $g_{\mu\nu}$, cosmological constant Λ and coefficient $\sigma = \pm 1$ in front of R

- lower number of derivatives from 4 to 2 by introducing an auxiliary symmetric tensor $q_{\mu\nu}$

- after linearization and diagonalization the two fields describe a massless spin 2 with coefficient $\bar{\sigma} = \sigma - \frac{\Lambda}{2m^2}$ and a massive spin 2 with mass $M^2 = -m^2\bar{\sigma}$

- special cases:
  - 3D NMG — Hohm, Townsend + E.B. (2009)
  - $D \geq 3$ “chiral/critical gravity” for special value of Λ

Li, Song, Strominger (2008); Lü and Pope (2011)

Chiral/Critical Gravity

- a massive graviton disappears but a log mode re-appears

- In general one ends up with a non-unitary theory

- are there unitary truncations ?

Is NMG perturbative renormalizable?

D=4

- $\mathcal{L} \sim +R + R^2$: scalar field coupled to gravity

  unitarity: $\surd$ but renormalizability: X

  propagator $\sim \left(\frac{1}{p^2} + \frac{1}{p^4}\right)_0 + \left(\frac{1}{p^2}\right)_2$

- $\mathcal{L} \sim R + \left(C_{\mu\nu}{}^{ab}\right)^2$: Weyl tensor squared

  propagator $\sim \left(\frac{1}{p^2}\right)_0 + \left(\frac{1}{p^2} + \frac{1}{p^4}\right)_2$

  unitarity: X and renormalizability: X

D=3

How do the NMG propagators behave ?

$$\mathcal{L} \;=\; \sqrt{-g}\left[\sigma R + \frac{a}{m^2}\left(R^{\mu\nu}R_{\mu\nu} - \frac{3}{8}R^2\right) + \frac{b}{m^2}R^2\right] \qquad \sigma = \pm 1$$

$$\text{propagator} \sim \left(\frac{1}{p^2} + \frac{b}{p^4}\right)_0 + \left(\frac{1}{p^2} + \frac{a}{p^4}\right)_2 \quad \Rightarrow \quad ab \neq 0$$

Nishino, Rajpoot (2006)

However, we also need $ab = 0$ $\Rightarrow$

NMG is (most likely) not perturbative renormalizable !

Much work has been dome on different aspects of NMG such as

- solutions
- AdS/(L)CFT
- supersymmetric extensions

Outline

Introduction

3D New Massive Gravity

Extension to 4D?

Outlook

What did we learn?

- two theories can be equivalent at the linearized level (FP and boosted FP) but only one of them allows for a unique non-linear extension i.e. interactions !

- we need massive spin 2 whose massless limit describes 0 d.o.f.

Example :  in 3D

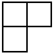
- what about 4D?

Generalized spin-2 FP

standard spin-2 :

$$\text{describes} \begin{cases} 5 \quad \text{d.o.f.} \qquad m \neq 0 \\ 2 \quad \text{d.o.f.} \qquad m = 0 \end{cases}$$

generalized spin-2 :



$$\text{describes} \begin{cases} 5 \quad \text{d.o.f.} \qquad m \neq 0 \\ 0 \quad \text{d.o.f.} \qquad m = 0 \end{cases}$$

Curtright (1980)

Connection-metric Duality

- Use first-order form with independent fields $e_\mu{}^a$ and $\omega_\mu{}^{ab}$
- linearize around Minkowski: $e_\mu{}^a = \delta_\mu{}^a + h_\mu{}^a$
  and add a FP mass term $-m^2(h^{\mu\nu}h_{\nu\mu} - h^2) \quad \rightarrow$

$$\mathcal{L} \sim \text{“}\, h\,\partial\omega + \omega^2 \,\text{”} - m^2(h^{\mu\nu}h_{\nu\mu} - h^2)$$

- solve for ω $\rightarrow$ spin-2 FP in terms of $h_{\mu\nu}$
- solve for $h_{\mu\nu}$ and write $\omega_\mu{}^{ab} = \frac{1}{2}\epsilon^{abcd}\,\tilde{h}_{\mu cd}$ $\rightarrow$ generalized spin-2 FP in terms of $\tilde{h}_{\mu cd}$

Boosting up the Derivatives

- start with generalized spin-2 FP in terms of

and subsidiary conditions

$$\tilde{h}_{\mu\nu,\rho}\,\eta^{\nu\rho} = 0\,, \qquad\qquad \partial^{\rho}\,\tilde{h}_{\rho\mu,\nu} = 0$$

- solve for $\partial^{\rho}\,\tilde{h}_{\rho\mu,\nu} = 0 \to \tilde{h}_{\mu\nu,\rho} = G_{\mu\nu,\rho}(h) \ \to$ "NMG in 4D" :

$$\mathcal{L}_{\rm NMG} \ \sim \ -\tfrac{1}{2}h^{\mu\nu,\rho}G_{\mu\nu,\rho}(h) \ + \ \frac{1}{2m^2}\underbrace{h^{\mu\nu,\rho}\ C_{\mu\nu,\rho}(h)}_{\text{``4 derivatives}}$$

- mode analysis $\ \to$

$$\mathcal{L}_{\rm NMG} \ \sim \ \text{massless spin 2 plus massive spin 2}$$

Interactions ?

- compare to Eddington-Schrödinger theory

$$\mathcal{L}'_{\text{ES}} = \sqrt{-\det g}\,[g^{\mu\nu} R_{\mu\nu}(\Gamma) - 2\Lambda] \quad \Leftrightarrow \quad \mathcal{L}_{\text{ES}} = \sqrt{|\det R_{(\mu\nu)}(\Gamma)|}$$

$$g_{\mu\nu} = \frac{(D-2)}{2\Lambda} R_{(\mu\nu)}(\Gamma)$$

Outline

Introduction

3D New Massive Gravity

Extension to 4D?

Outlook

Open Issues

- relation with massive gravity?
- "trivial gravity" and Chern-Simons formulation ?
- generalization to higher spins ?

NMG is an interesting toy model (like TMG)

to study properties of (quantum) gravity !