
Status of $g - 2$ and $\Delta\alpha$



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- i. What's new in $g - 2$?
- [ii. $\Delta\alpha$: recap (from Frascati)]
- iii. Discussion!

What is new about $g - 2$?

Thankfully everything is said already by previous speakers, so can be short...

→ No new major $g - 2$ compilation to report.

But: a lot of prel. new results and developments on all frontiers.

[See also many talks at TAU2012 on the subject, from where I have stolen some slides]

Hence just some brief remarks:

★ QED: Kinoshita et al. have done it, again:

Now full a_μ^{QED} at order α^5 available!

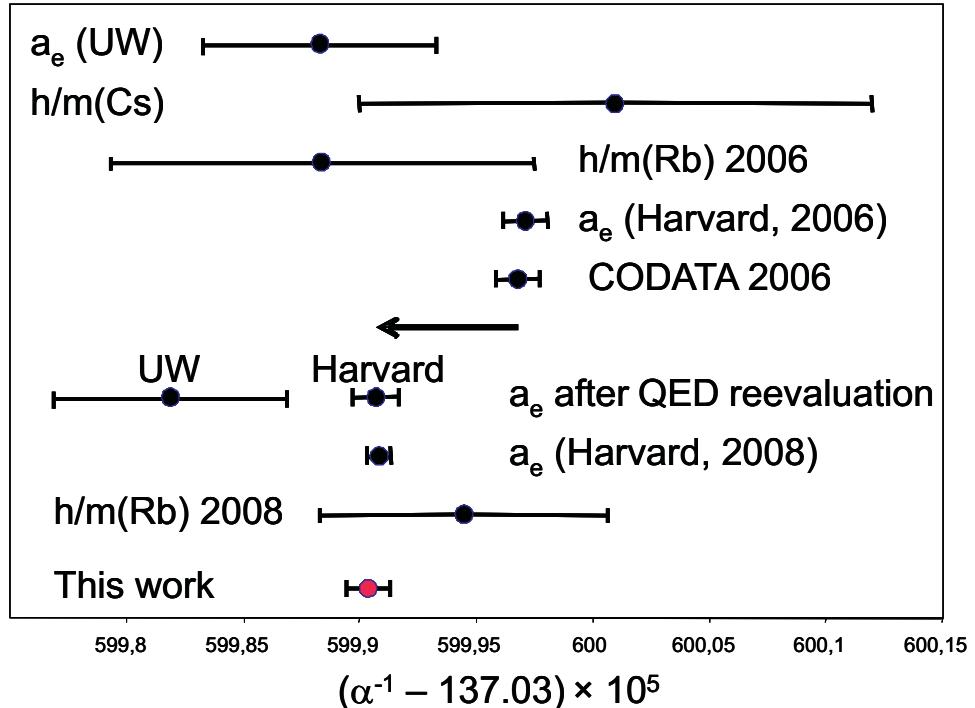
⇝ important for a_e and the determ. of α :

$$\alpha^{-1}(a_e) = 137.035\ 999\ 1736\ (68)(46)(26)(331) \ [0.25\text{ppb}], \text{ against}$$

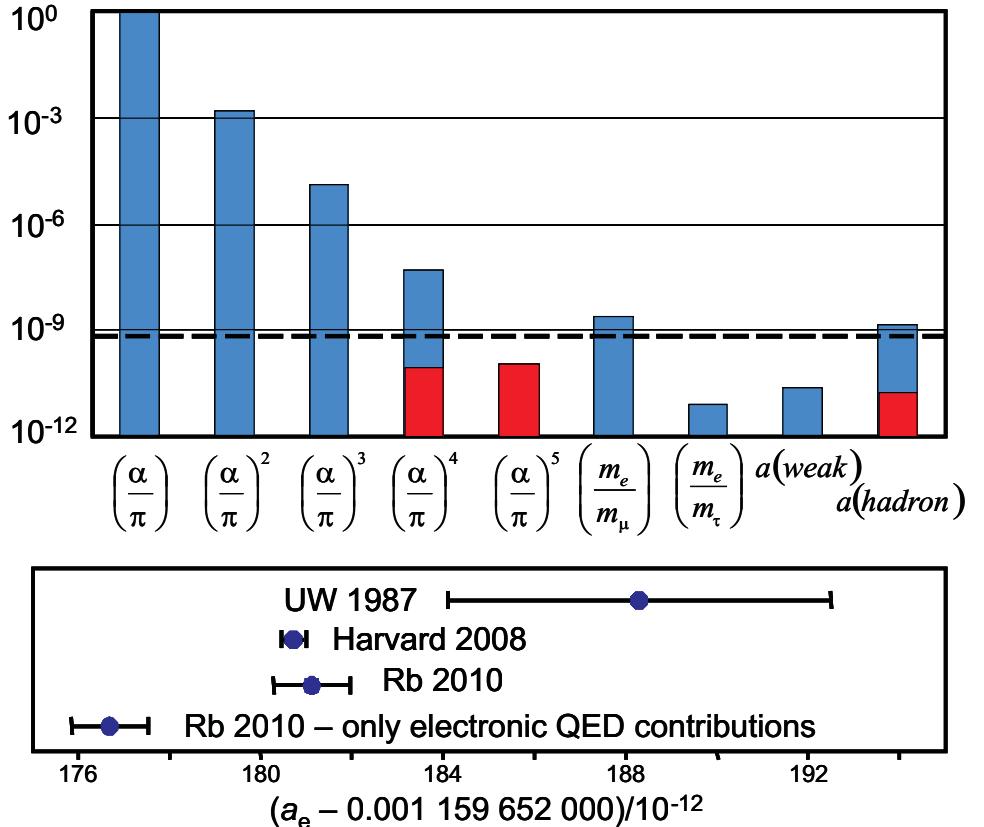
$$\alpha^{-1}(Rb) = 137.035\ 999\ 049\ (90) \ [0.66\text{ppb}], 1.3\sigma \text{ away}$$

⇝ small changes for a_μ : slide from Masashi Hayakawa's talk at Tau2012:

The electron's anomaly a_e



Figures from Bouchendira *et al.*, PRL 106(2011)080801



- Harvard 2008 [Hanneke, Fogwell, Gabrielse, PRL 100(2008)120801]: 15 times better than measurements from 1987, now **0.28 ppt** for $g_e/2 \rightsquigarrow$ best α determination
- but: strongly dependent on a_e^{TH} . Kinoshita *et al.*: **5-loop** ongoing! (12672 diags.)
- Bouchendira *et al.*: very important independent measurement;

h/m_{Rb} from recoil velocity after photon absorption; $\alpha^2 = \frac{2R_\infty}{c} \frac{m_{\text{Rb}}}{m_e} \frac{h}{m_{\text{Rb}}}$

Result

Table: a_μ (QED) at each order $2n$, scaled by 10^{11}

| order $2n$ | using $\alpha(Rb)$ | using $\alpha(a_e)$ |
|------------|----------------------|----------------------|
| 2 | 116 140 973.318 (77) | 116 140 973.213 (30) |
| 4 | 413 217.6291 (90) | 413 217.6284 (89) |
| 6 | 30 141.902 48 (41) | 30 141.902 39 (40) |
| 8 | 381.008 (19) | 381.008 (19) |
| 10 | 5.0938 (70) | 5.0938 (70) |
| sum | 116 584 718.951 (80) | 116 584 718.846 (37) |

The complete calculation of $a_\mu^{(10)}$ eliminates the uncertainty $\sim O(1) \times 10^{-11}$, which has been present unless it is done.

Now, the uncertainties in a_μ (QED) come mostly from

1. uncertainty in the fine structure constant α ($2n = 2$),
2. statistical uncertainty in the Monte Carlo integration of the 8th-order terms.

Recall that $\delta a_\mu(\text{exp}) = 64 \times 10^{-11}$, which may be improved to

$\delta a_\mu(\text{exp}) \sim 2 \times 10^{-11}$ in the future experiments.

$(g - 2)_\mu$: Hadronic contributions

Phase of consolidation

- Hadronic Vacuum Polarisation from exp. $\sigma(e^+e^- \rightarrow \gamma^* \rightarrow \text{hadrons} (+\gamma))$ data

[or from $\tau \rightarrow \nu_\tau + \text{hadrons}$ spectral functions; isospin breaking...]

Use of dispersion integral (based on analyticity and unitarity):

$$a_\mu^{\text{had, VP LO}} = \frac{1}{4\pi^3} \int_{m_\pi^2}^\infty ds \sigma_{\text{had}}^0(s) K(s), \quad \text{with } K(s) = \frac{m_\mu^2}{3s} \cdot (0.4 \dots 1)$$

→ Kernel $K \rightsquigarrow$ weighting towards smallest energies. σ_{had}^0 the **undressed** cross section

→ Similar approach with different kernel functions for NLO VP contributions $a_\mu^{\text{had, VP NLO}}$

- Hadronic Light-by-Light:

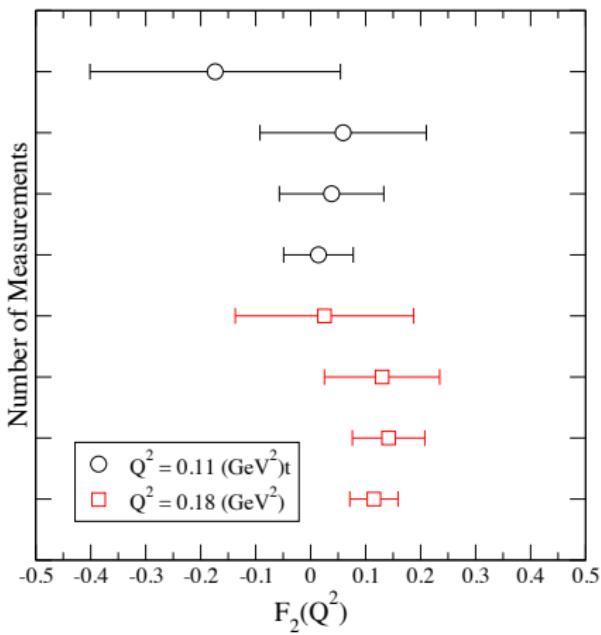
- No dispersion relation. *First Principles* calculations from lattice QCD are underway...

Also first results based on Dyson-Schwinger eqs. by C. Fischer et al. *reliable?*
→ E. de Rafael at Tau2012

- ‘Consensus’ of different recent model calculations. HLMNT numbers below use compilation from J. Prades, E. de Rafael, A. Vainshtein: $a_\mu^{\text{L-by-L}} = (10.5 \pm 2.6) \cdot 10^{-10}$
- Compatible result from F. Jegerlehner, A. Nyffeler: $a_\mu^{\text{L-by-L}} = (11.6 \pm 4.0) \cdot 10^{-10}$

$a_\mu(\text{HLbL})$ in 2+1f lattice QCD+QED (PRELIMINARY)

$F_2(Q^2)$ stable with additional configurations ($20 \rightarrow 40 \rightarrow 80 \rightarrow 160$)



Using AMA, 216 meas/conf

2+1f DW fermions
(RBC/UKQCD)

24^3 lattice size, $a = 0.114$ fm

$Q^2 = 0.11$ and 0.18 GeV 2

$m_\pi \approx 329$ MeV

$m_\mu \approx 190$ MeV

[Blum, Hayakawa, Izubuchi, Lattice 2012]

a_μ (HLbL) more systematic errors

Need to address

- ▶ Finite volume
- ▶ $q^2 \rightarrow 0$ extrap
- ▶ $m_q \rightarrow m_{q, \text{phys}}$
- ▶ $m_\mu \rightarrow m_{\mu, \text{phys}}$
- ▶ excited states/“around the world” effects
- ▶ $a \rightarrow 0$
- ▶ QED renormalization
- ▶ ...

Conclusions

- The results from the elaborated models, which incorporate the known QCD constraints, are well digested by the simple $C\chi$ QM reference model.
- There is interesting work in progress with possible improvements.
- However, at the present time, I see no new result as yet which can modify the *Prades-Vainshtein-de Rafael '10* estimate:

$$a_\mu^{\text{HLbyL}} = (105 \pm 26) \times 10^{-11}.$$

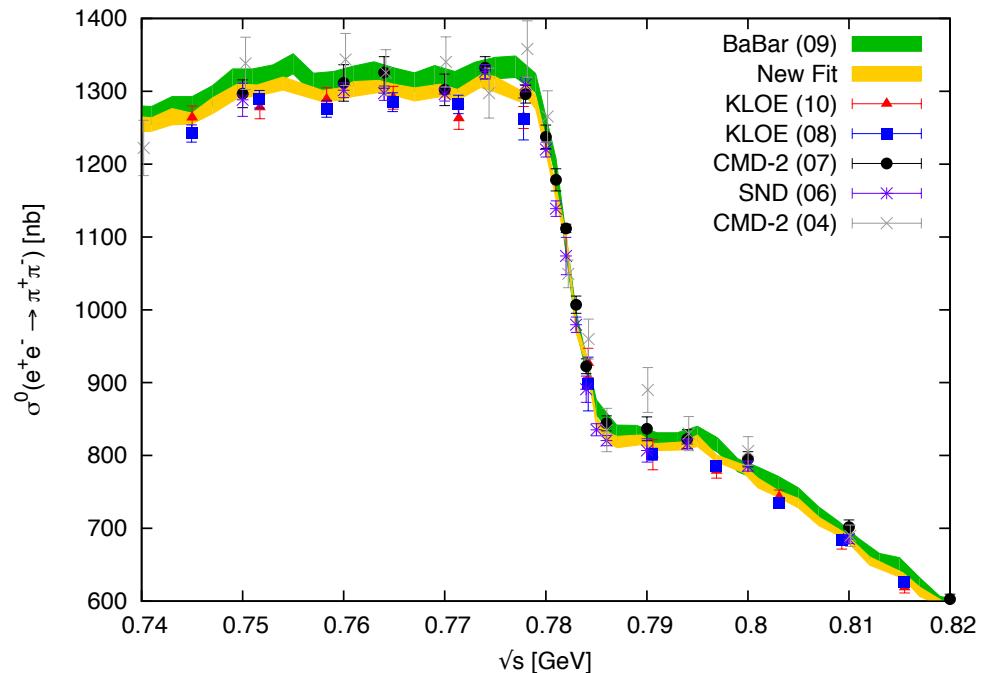
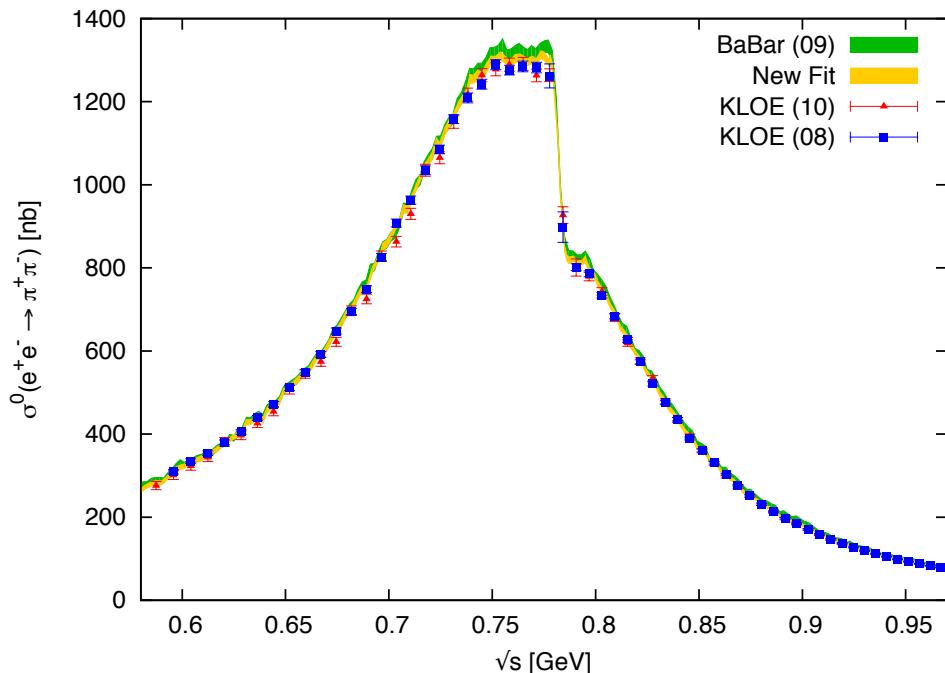
In fact, this estimate is consistent with a recent independent analysis (*Bijnens and Abyaneh august'12*) which gives:

$$a_\mu^{\text{HLbyL}} = (10 \pm 3) \times 10^{-10}.$$

Recent developments in $(g - 2)_\mu$; Hadronic VP contributions

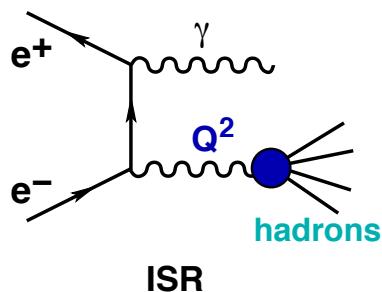
- For low energy $\sigma_{\text{had}}^0(s)$, need to sum ~ 25 exclusive channels [$2\pi, 3\pi, KK, 4\pi, \dots$]
- $\sqrt{s} \sim 1.4 - 2$ GeV: sum exclusive channels and/or use old inclusive data
- above ~ 2 GeV: inclusive data *or* use of perturbative QCD [+narrow resonances]

► The most important 2π channel ($> 70\%$) HLMNT '11 use 879 data points



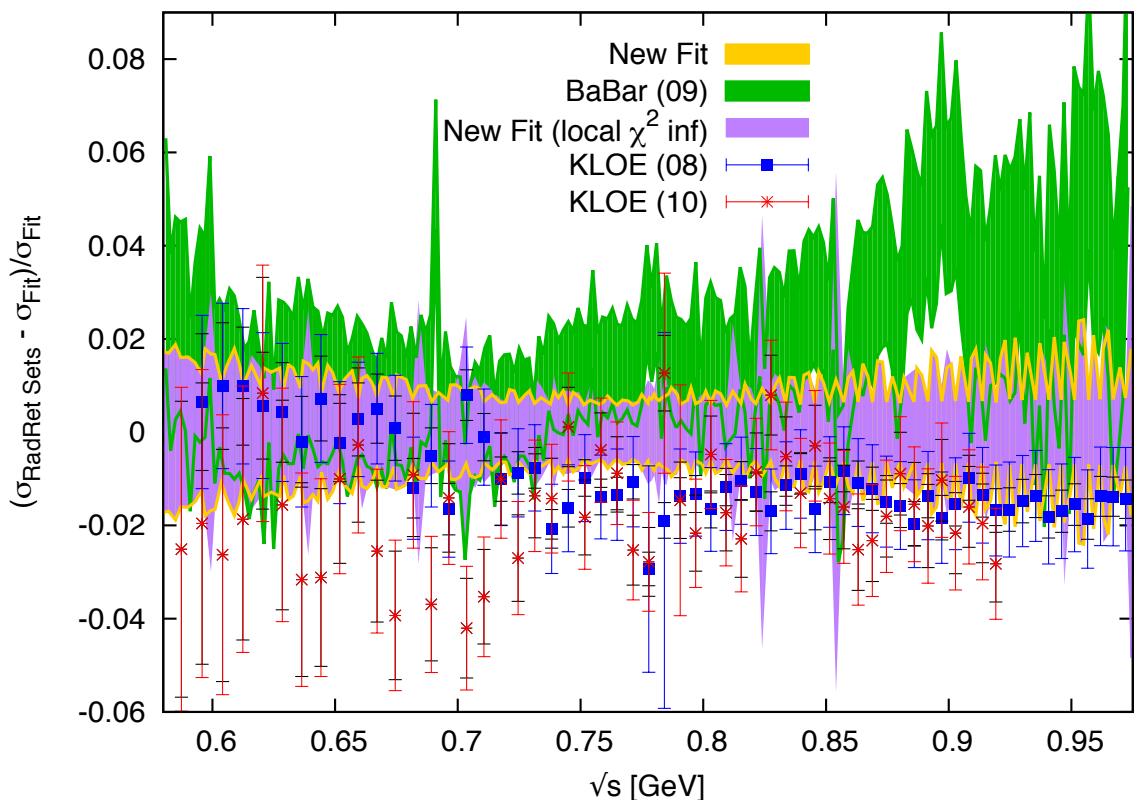
Overall, the data combination incl. 'Direct Scan' and 'Radiative Return' looks fine, but...

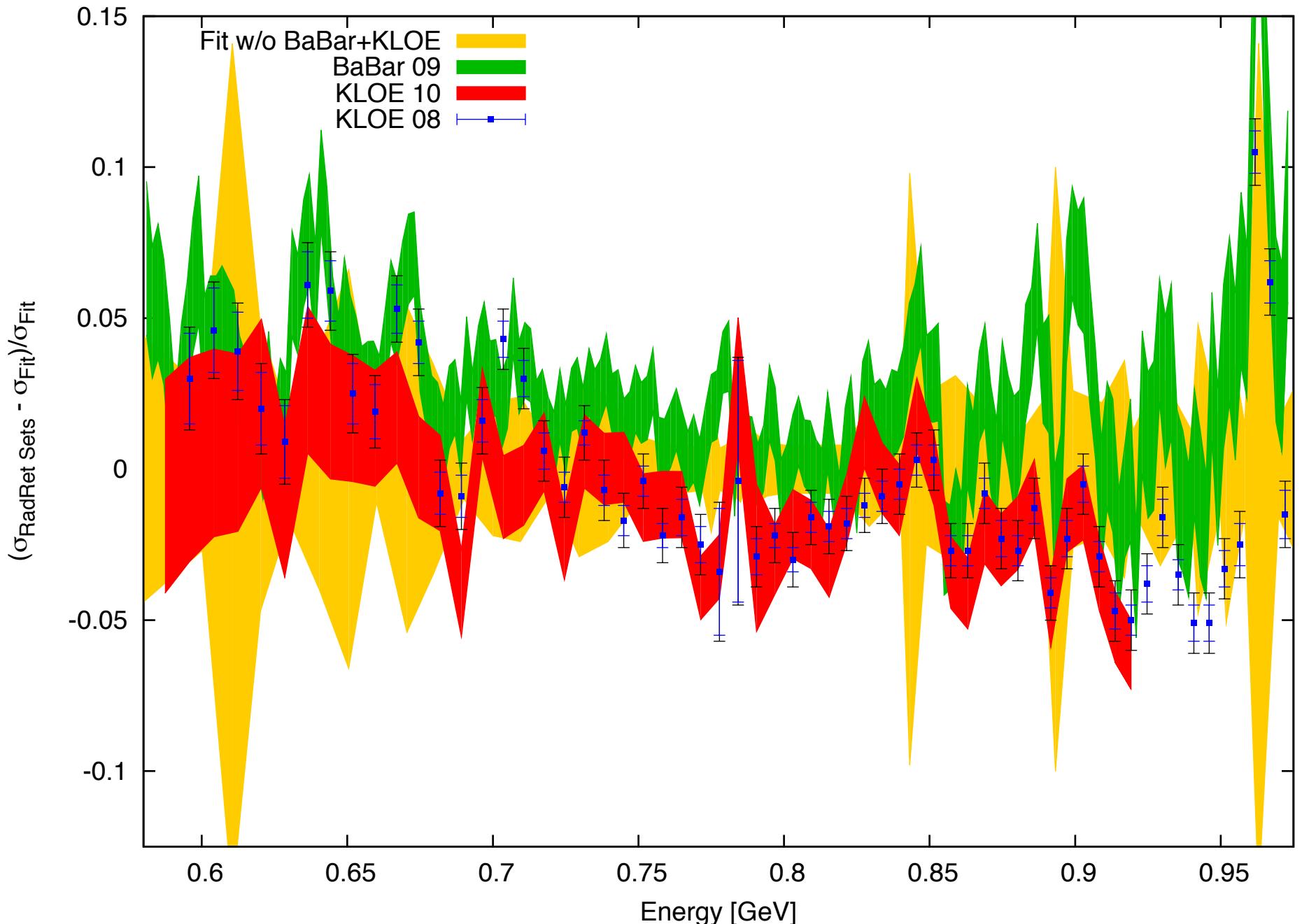
Radiative Return $\pi\pi(\gamma)$ data [KLOE 08/10 and BaBar 09] compared to combination of all



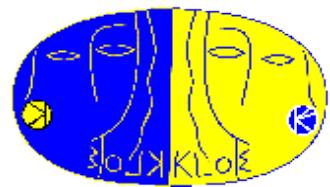
- Radiative Return (at fixed e^+e^- energy) a powerful method, *complementary to direct energy scan*
- ~~ Differences in shape and BaBar high at medium and higher energies
- ~~ limited gain in accuracy due to ‘tension’; pull-up (mainly from BaBar)
- Comb. of all data on same footing, before integration (purple band): still good $\chi^2_{\text{min}}/\text{d.o.f.} \sim 1.5$ of fit]
- $a_\mu^{2\pi}(0.32 - 2 \text{ GeV}) = (504.2 \pm 3.0) \cdot 10^{-10}$, $a_\mu^{2\pi, \text{w/out Rad. Ret.}} = (498.7 \pm 3.3) \cdot 10^{-10}$.
- Clarification/improvement with more, possibly even more precise data (from both scan and ISR) **needed!**

Normalised difference of cross sections [HLMNT '11]

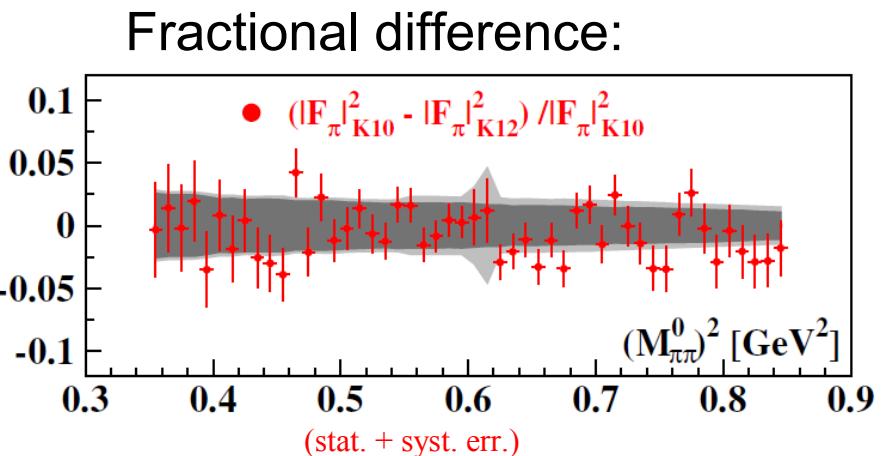
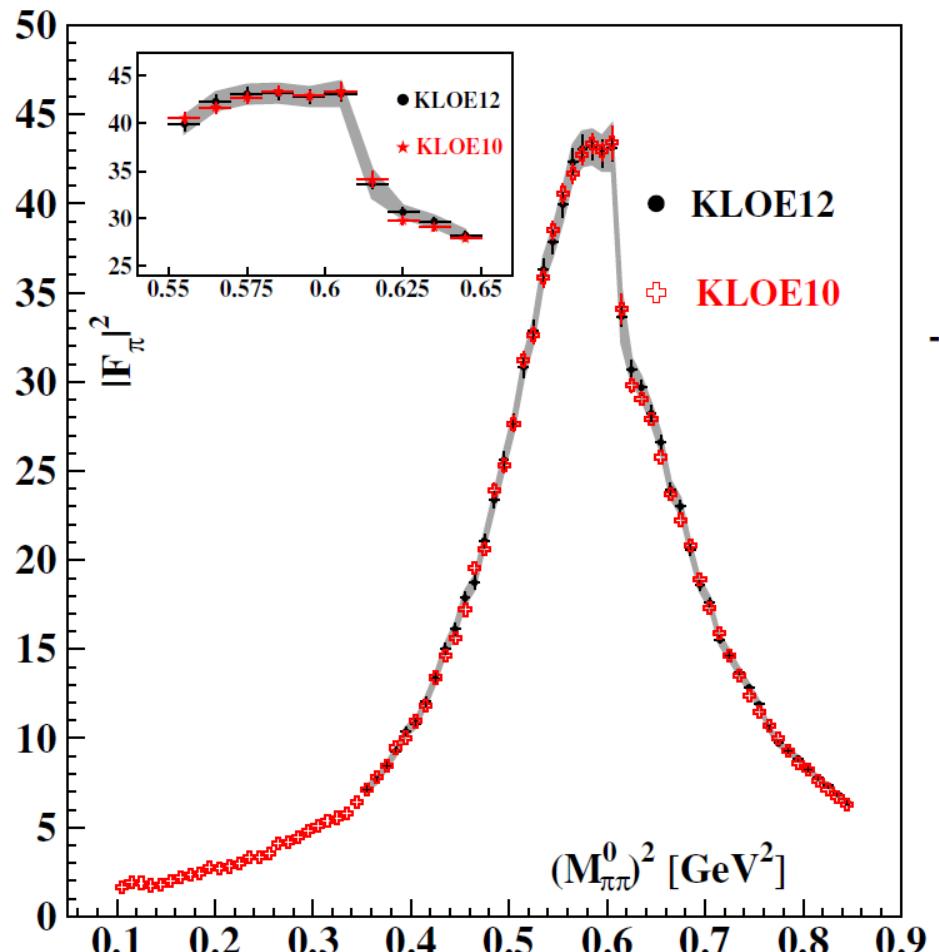




Comparison of results: KLOE12 vs KLOE10



KLOE12 result compared to KLOE10:



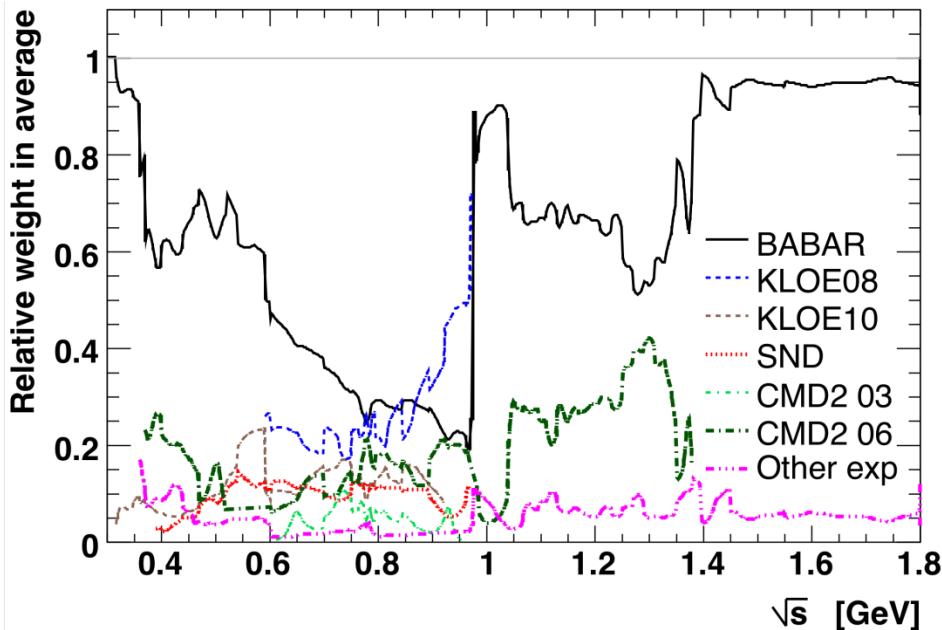
Nice agreement in the whole range
between the two measurements.

| Analysis | $a_\mu^{\pi\pi}(0.35 - 0.85 \text{ GeV}^2) \times 10^{10}$ |
|----------|--|
| KLOE12 | $377.4 \pm 1.1_{\text{stat}} \pm 4.5_{\text{sys+theo}}$ |
| KLOE10 | $376.6 \pm 0.9_{\text{stat}} \pm 3.3_{\text{sys+theo}}$ |

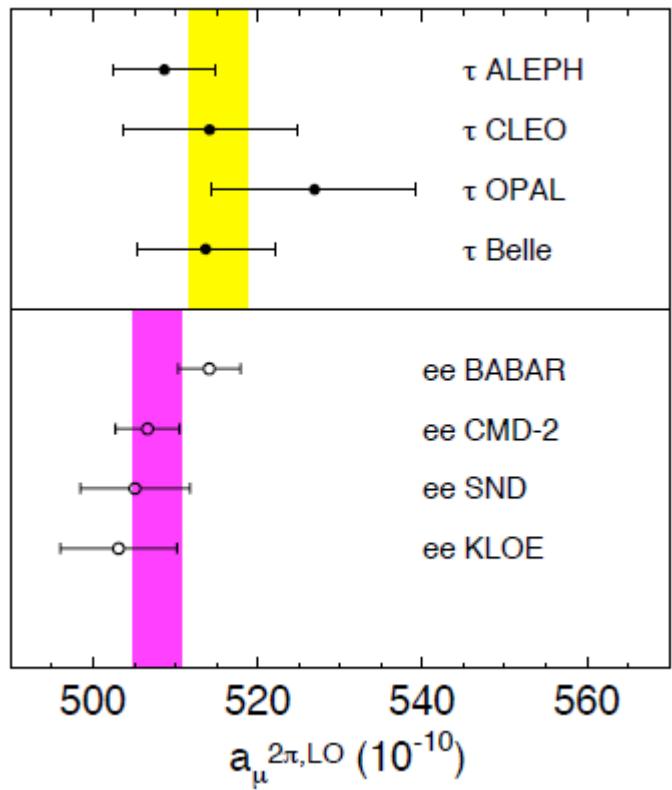
Impact of BABAR data for g-2: $\pi^+\pi^-$

Weights of different experiments in combining their results (DHMZ 2009-2010)

BABAR dominates everywhere, except between 0.8 and 0.93 GeV where KLOE is the most precise



Integral from threshold to 1.8 GeV



BABAR most precise (with CMD-2)
reduces tension between e^+e^- and τ

Impact of BABAR data for g-2: K^+K^-

BABAR preliminary results:

$$a_\mu^{KK, LO} [0.98-1.8] \text{ GeV} = (22.95 \pm 0.14 \text{ (stat)} \pm 0.22 \text{ (syst)}) 10^{-10} \text{ (1.1\%)}$$

DHMZ 2011: update of all results before BABAR:

$$a_\mu^{KK, LO} [0.98-1.8] \text{ GeV} = (21.63 \pm 0.27 \text{ (stat)} \pm 0.68 \text{ (syst)}) 10^{-10} \text{ (3.4\%)}$$

BABAR more precise than previous world average by a factor of 3

Impact of BABAR data for g-2: $2(\pi^+ \pi^-)$

BABAR results:

$$a_\mu^{4\pi, \text{LO}} [0.98-1.8] \text{ GeV} = (13.64 \pm 0.03 \text{ (stat)} \pm 0.36 \text{ (syst)}) 10^{-10} \text{ (2.6%)}$$

DEHZ 2003: all results but BABAR 2007:

$$a_\mu^{4\pi, \text{LO}} [0.98-1.8] \text{ GeV} = (13.95 \pm 0.90 \text{ (exp)} \pm 0.23 \text{ (rad)}) 10^{-10} \text{ (6.7%)}$$

DHMZ 2011: all results but BABAR 2012:

$$a_\mu^{4\pi, \text{LO}} [0.98-1.8] \text{ GeV} = (13.35 \pm 0.10 \text{ (stat)} \pm 0.52 \text{ (syst)}) 10^{-10} \text{ (4.0%)}$$

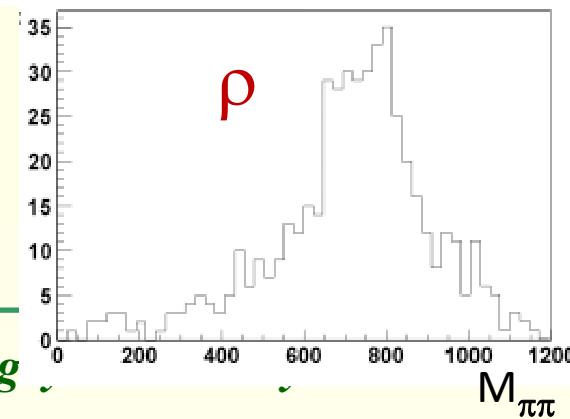
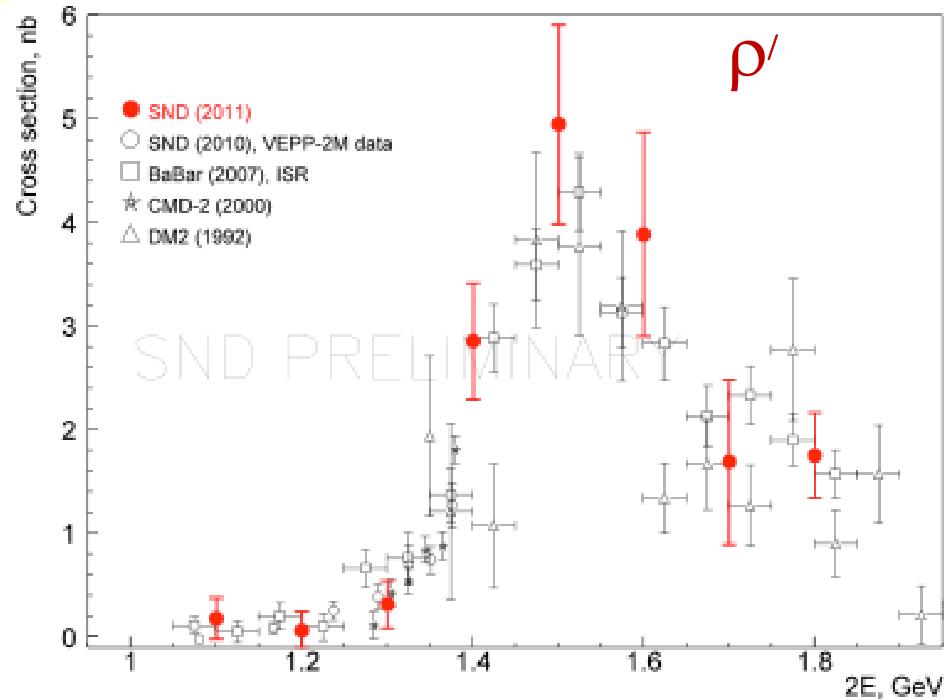
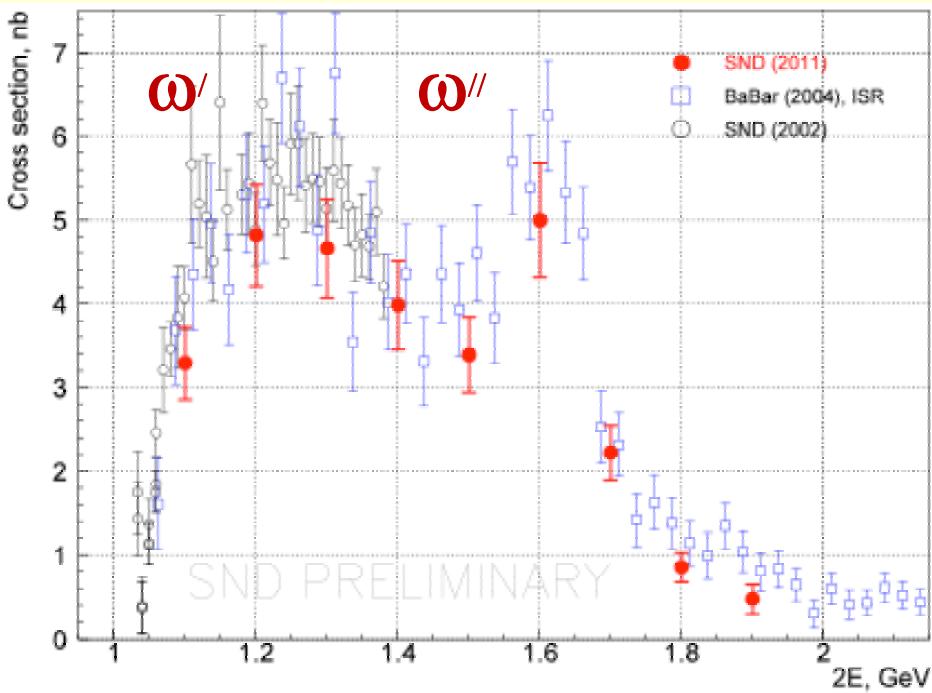
BABAR more precise than previous world average by a factor of 2.6

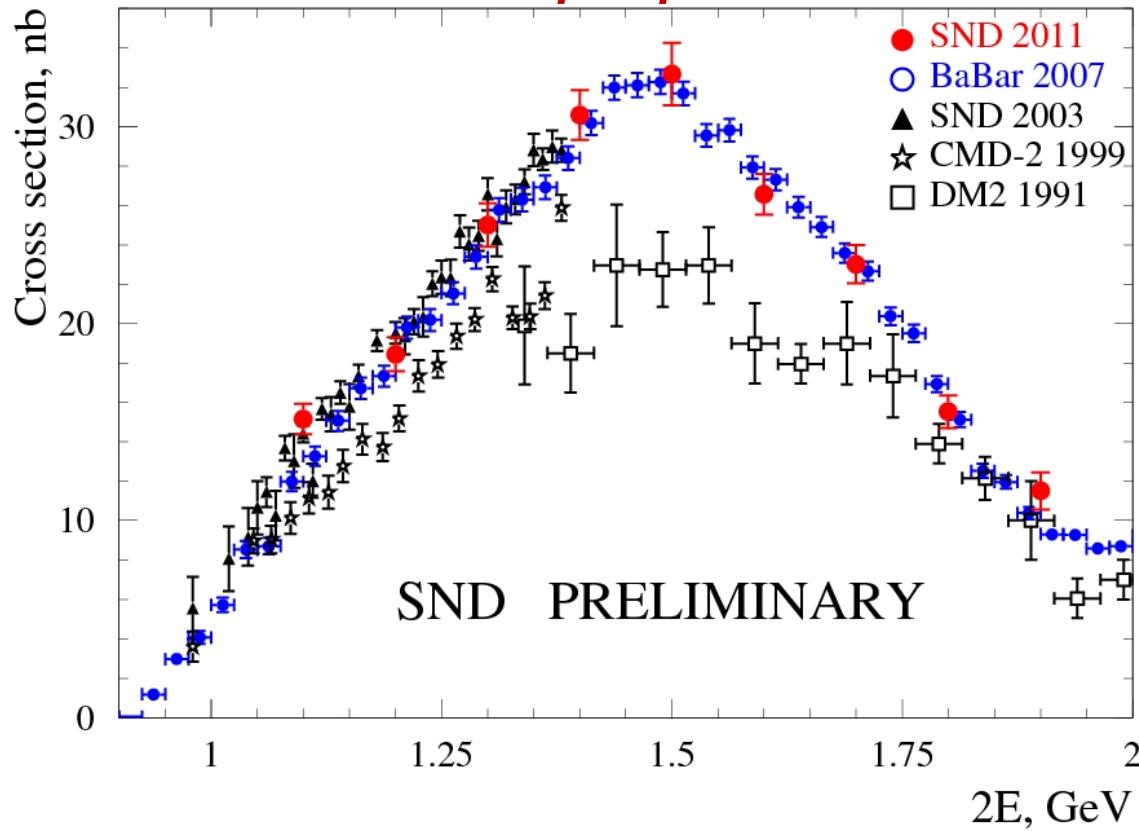


SCAN 2010

$$e^+e^- \rightarrow \pi^0\pi^+\pi^-$$

$$e^+e^- \rightarrow \eta \pi^+\pi^- \rightarrow \eta\rho$$



$e^+e^- \rightarrow \pi^+\pi^-\pi^0\pi^0$
scan 2010 $\rho^/ + \rho^{/\!/}$ 

Only statistical errors are shown

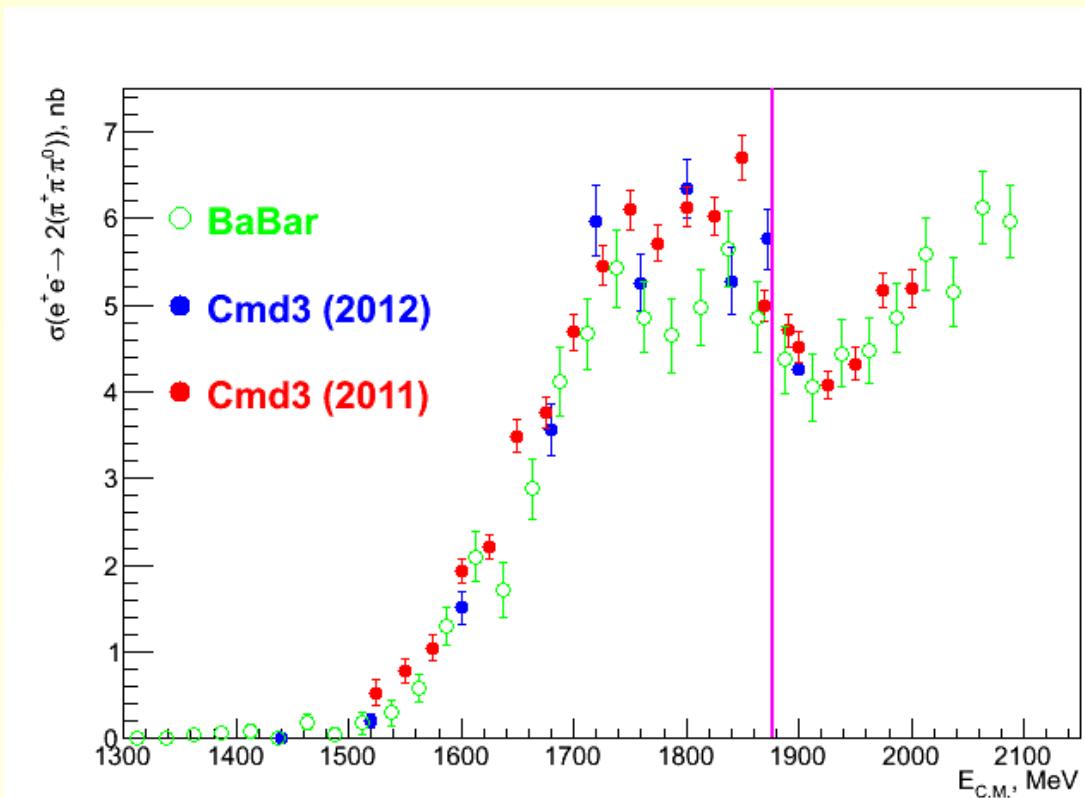
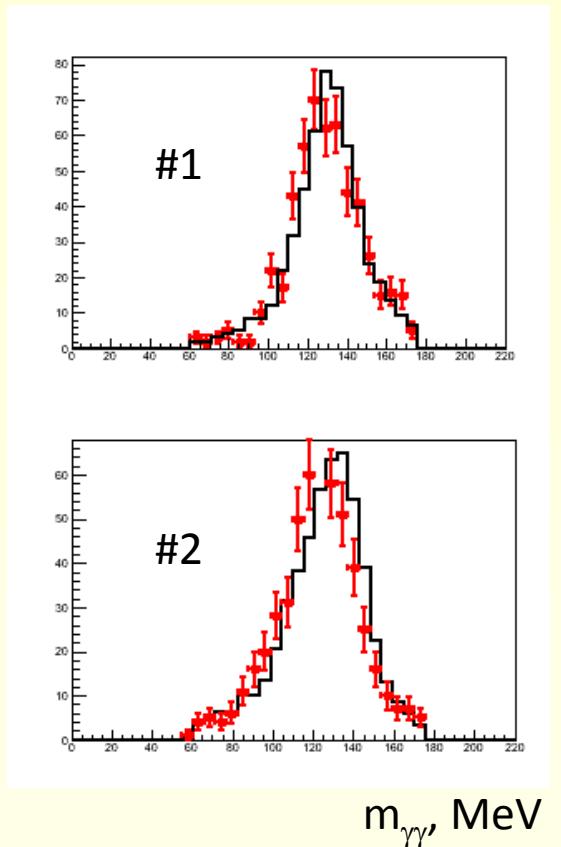
Intermediate states:

- $\omega\pi^0$
- $a_1\pi$
- $\rho^+\rho^-$
- ρf_0
- ρf_2

Preliminary results for the $e^+e^- \rightarrow 2(\pi^+\pi^-\pi^0)$ study



We have relatively clean selection of 2 and 1 π^0 in addition to four charged tr



$\omega\eta, \phi\eta, p4\pi$ intermediate states are seen, systematic errors are under study.

$$2\pi(\gamma)$$

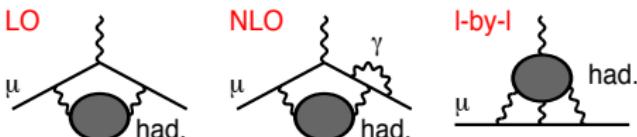
- Will future data fit better together? New KLOE12: NO ?!:(
→ understanding of discrepancy betw. KLOE and BaBar prerequisite to *fully* exploit available data.
- Will systematics of coming analyses be good enough to ‘override’ one of both once enough statistics has been collected and analysed?
- In total big improvements on near horizon
in 2π and many channels relevant for higher energies!

Introduction: Standard Model prediction for muon $g - 2$

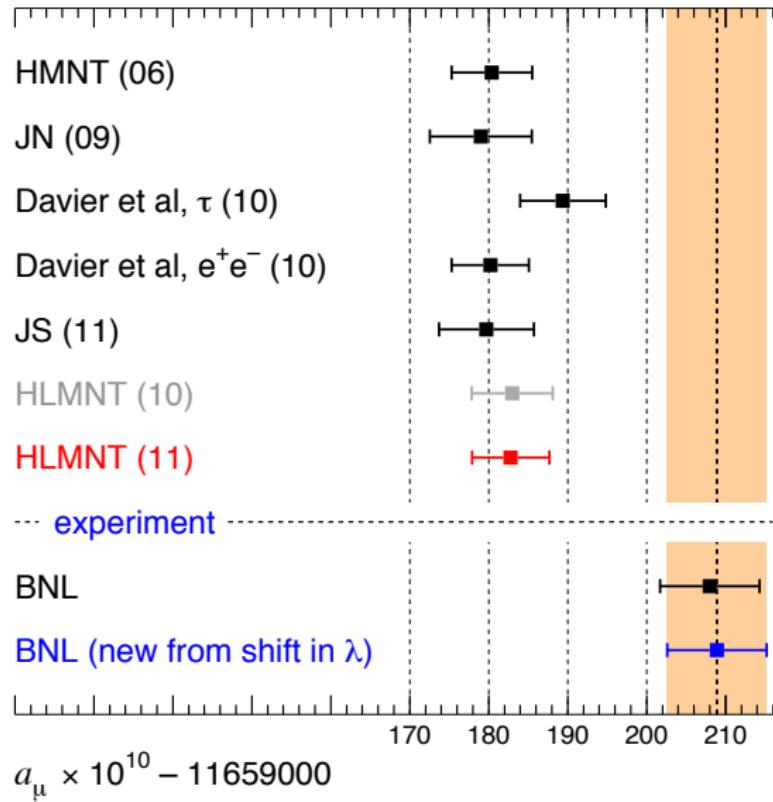
| | | |
|-------------------------------|---------------------------|--|
| QED contribution | 11 658 471.808 (0.015) | Kinoshita & Nio, Aoyama et al |
| EW contribution | 15.4 (0.2) | Czarnecki et al |
| Hadronic contributions | | |
| LO hadronic | 694.9 (4.3) | HLMNT11 |
| NLO hadronic | -9.8 (0.1) | HLMNT11 |
| light-by-light | 10.5 (2.6) | Prades, de Rafael & Vainshtein |
| Theory TOTAL | 11 659 182.8 (4.9) | |
| Experiment | 11 659 208.9 (6.3) | world avg |
| Exp – Theory | 26.1 (8.0) | 3.3 σ discrepancy |

(in units of 10^{-10} . Numbers taken from HLMNT11, arXiv:1105.3149)

n.b.: hadronic contributions:

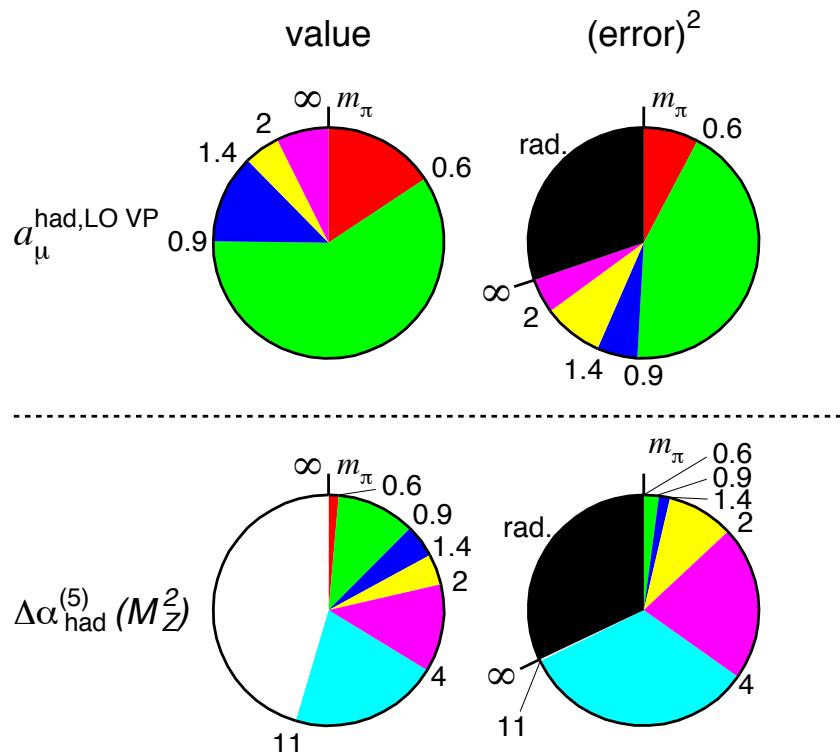


Full SM Result and Comparison with Other Groups



Future improvements: energy regions; experiments

- ▶ New $g - 2$ experiments at Fermilab and J-PARC.
 - ▶ Will a_μ^{SM} match the planned accuracy? ↵ L-by-L may become the limiting factor!
But at present Hadronic VP still contributes the biggest error in a_μ^{SM} .
 - ▶ Contributions from energy regions: Pie diagrams for contr. to a_μ and $\alpha(M_Z)$ and their errors²
- Expected sources for new data:
- More Rad. Ret. in progress at KLOE
 - Great opportunity for KLOE-2, BELLE,
Super $\tau - c$, in a few years SUPER-Bs,
also strong case for DAFNE-HE
 - Big improvement envisaged with
CMD-3 and SND at VEPP2000
 - Higher energies: BES-III at BEPCII in
Beijing is on; KEDR at VEPP-4M



for the discussion:

- WG: what can we contribute?
- Not much VP-WG progress since Novosibirsk/Frascati;
all players under contract for too many (other) things?
- Report for/work on multi-particle final states/critical channels!?!