Light-by-light scattering sum rules

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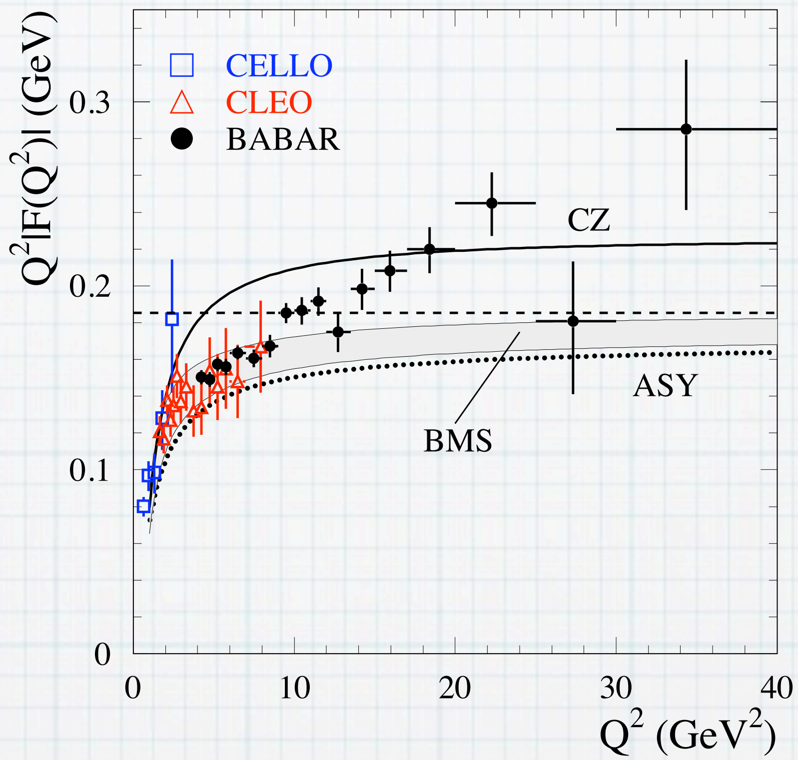
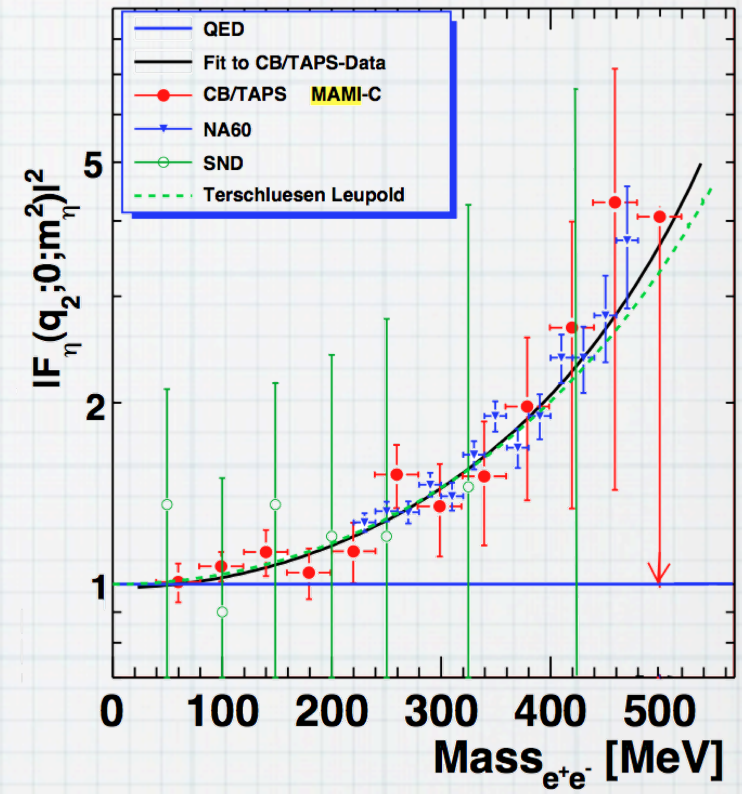
$\gamma\gamma$ physics

transition form-factors of $\gamma\gamma \to M$
for π^0, η, η'

$Q_1^2 = 0$ - one quasi-real photon

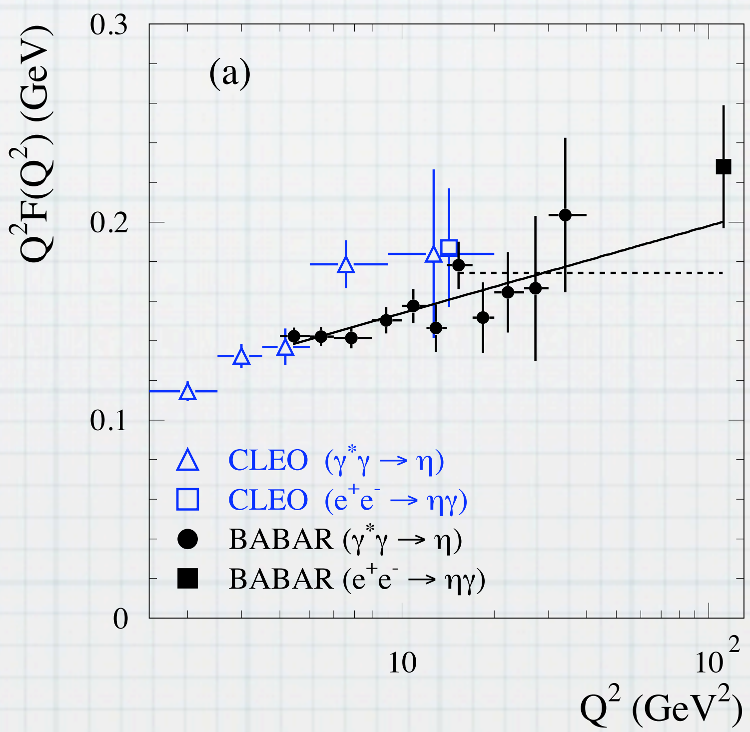
space-like region
1.5 $\mathrm{GeV}^2 < Q_2^2 <$ 40 GeV^2

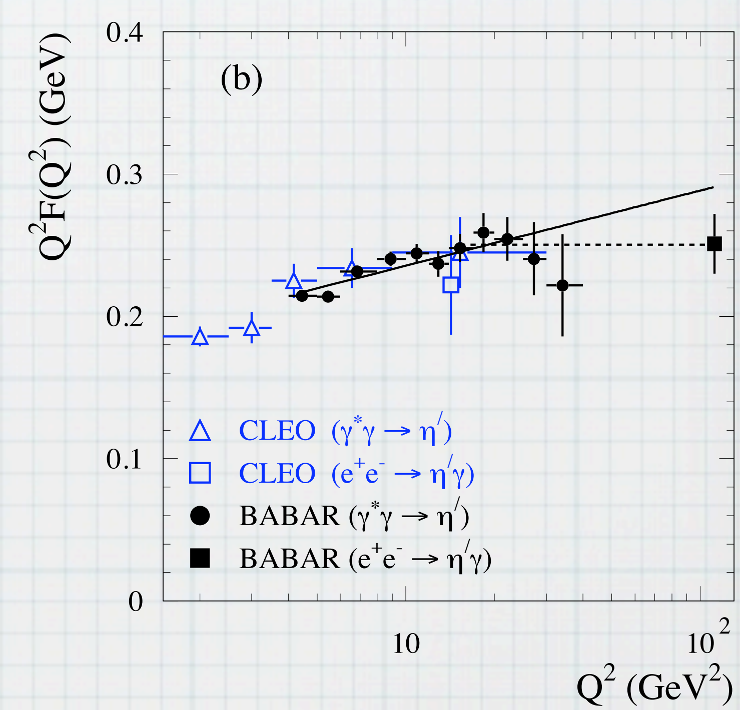




future:

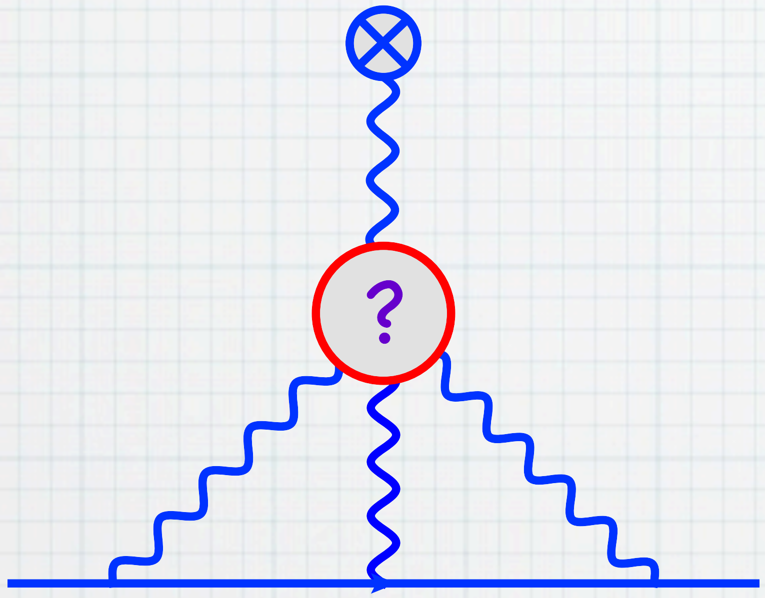
transition form-factors of $\gamma\gamma \to M$
for a2(1320), f2(1270), f2′ (1525)





γγ physics

hadronic contribution
to the $(g-2)_\mu$



loop integrals → full information about $\gamma\gamma \longrightarrow \gamma\gamma$ for all channels in wide energy range!

$\gamma\gamma$ physics

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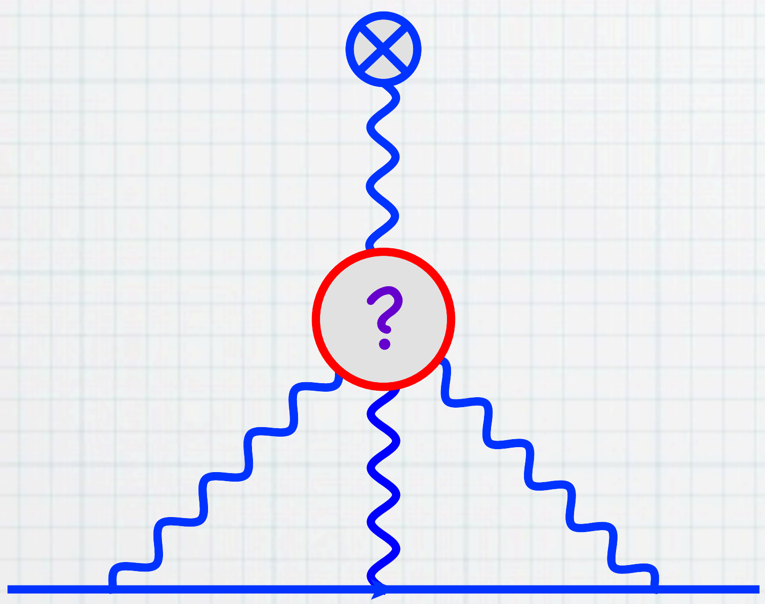
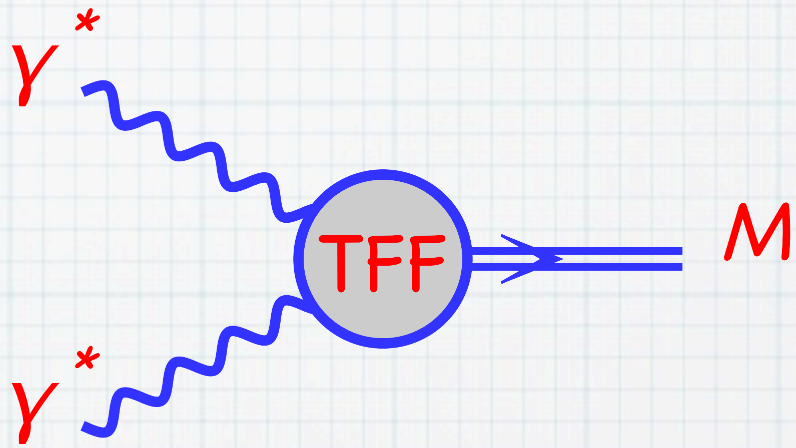


$\gamma^*\gamma^* \to$ meson
Transition Form Factors



$F(Q_1^2, Q_2^2)$

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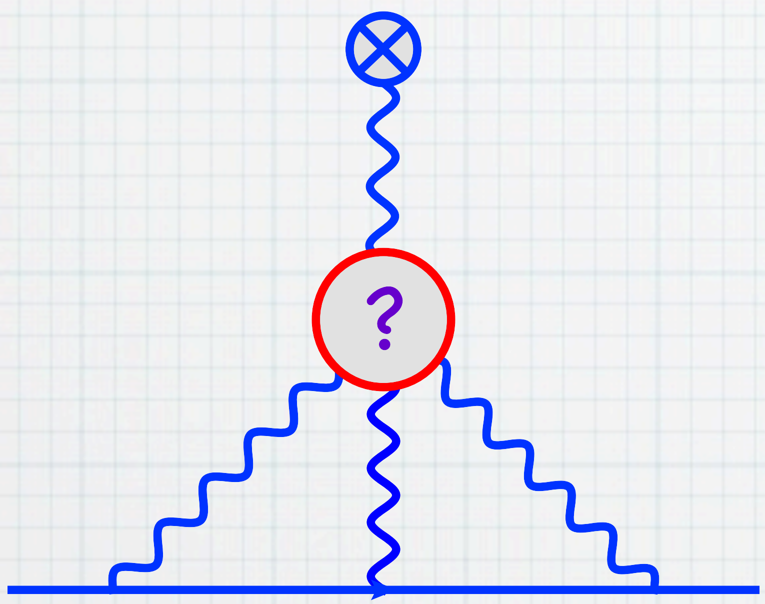

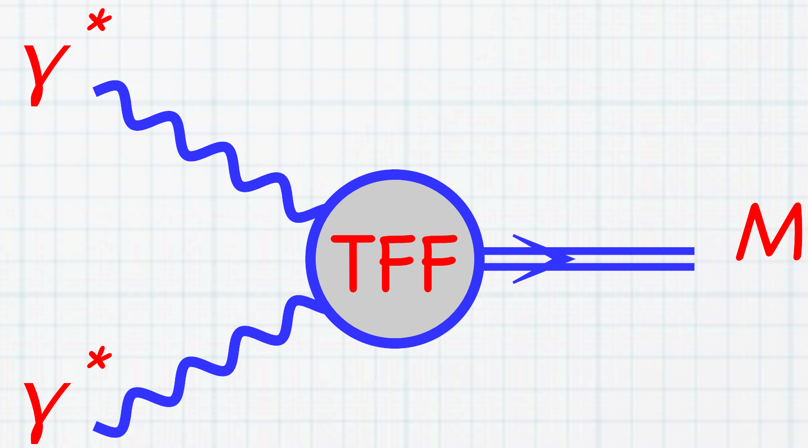

$F(Q_1^2, Q_2^2)$

$\longrightarrow$ low Q_2^2

non-perturbative dynamics of QCD (e.g. η-η' mixing and symmetry breaking mechanisms)

loop integrals $\to$ full information about $\gamma\gamma \longrightarrow \gamma\gamma$ for all channels in wide energy range + a model for extrapolation!

$\gamma\gamma$ physics

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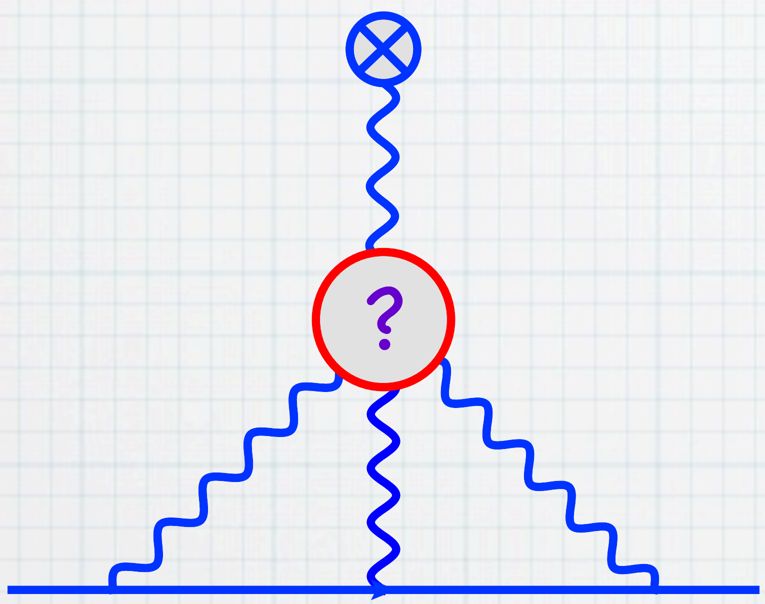

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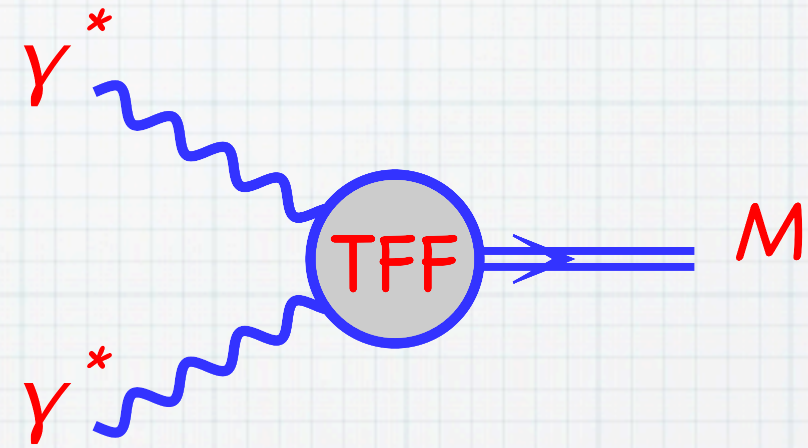


$F(Q_1^2, Q_2^2)$

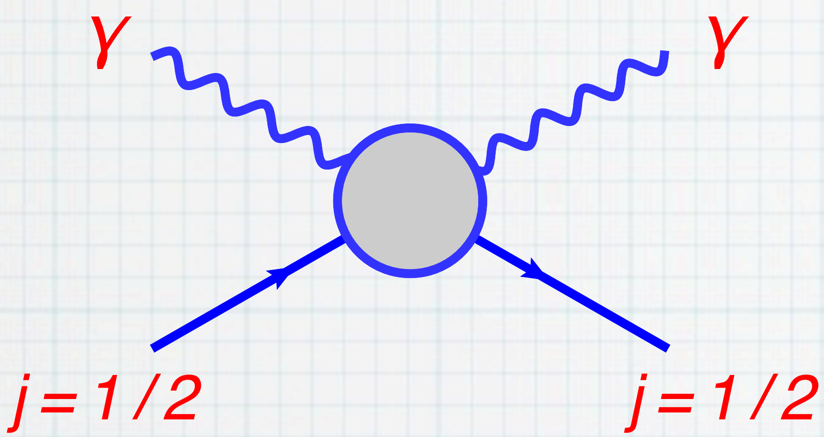
low Q_2^2

non-perturbative dynamics of QCD (e.g. η-η' mixing and symmetry breaking mechanisms)

high Q_2^2

perturbative QCD and quark structure of hadrons (meson distribution amplitudes)

Sum rules



1966

Gerasimov-Drell-Hearn sum rule

$$\frac{e^2}{2M^2}\kappa^2 = \frac{1}{\pi}\int_0^\infty \frac{d\nu}{\nu}\,[\sigma_{3/2}(\nu) - \sigma_{1/2}(\nu)]$$

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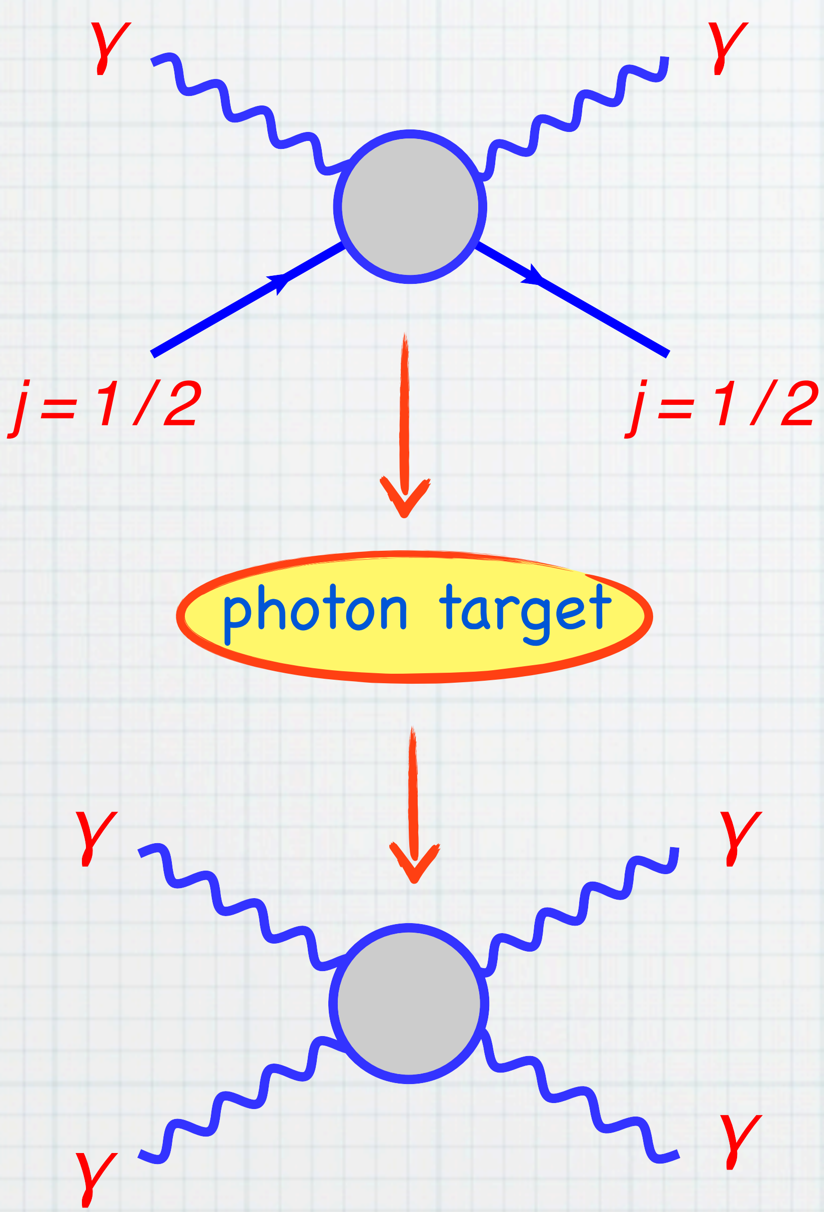

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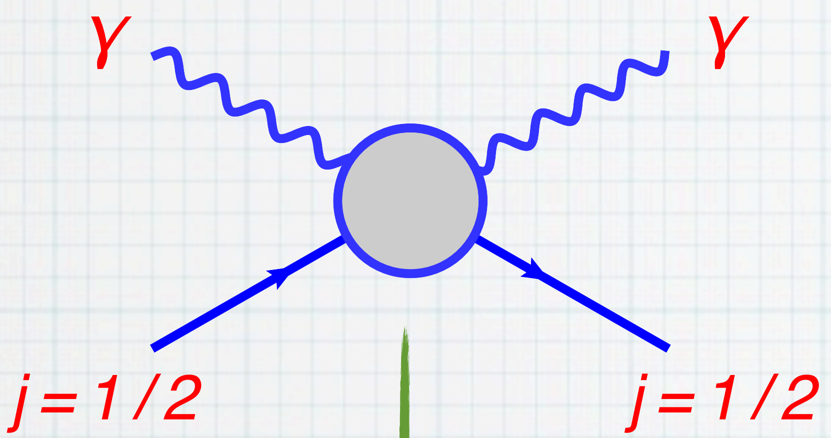

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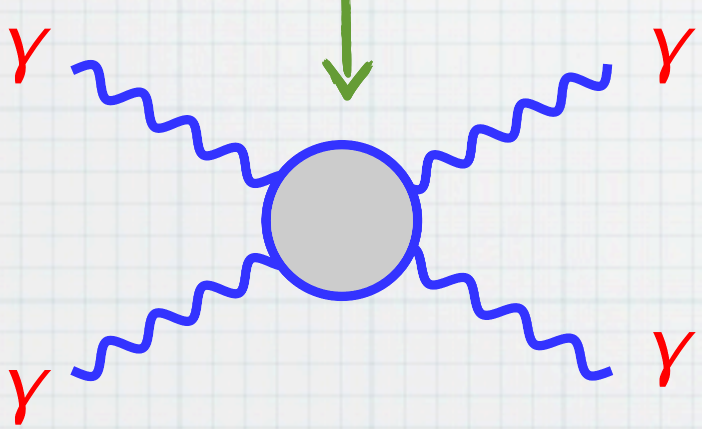
photon target

no anomalous moments!



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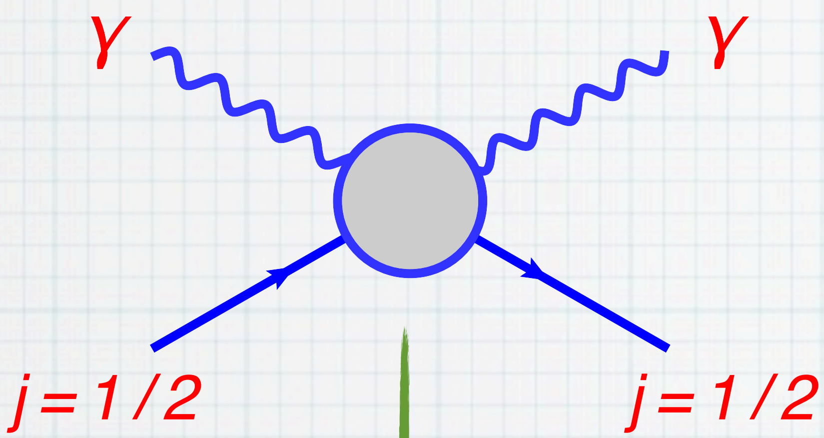

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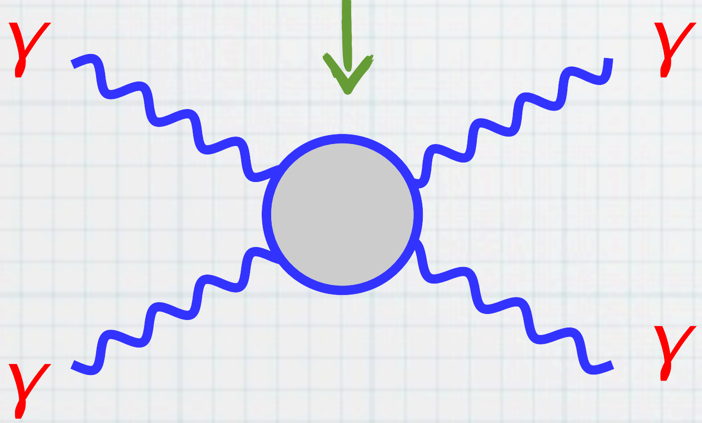

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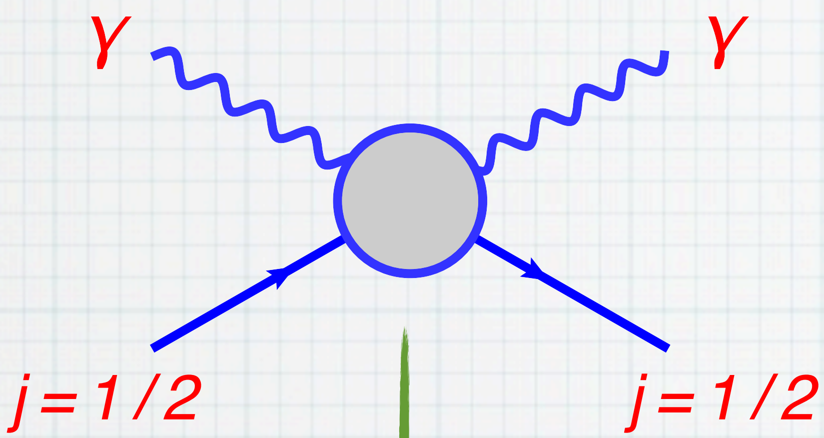

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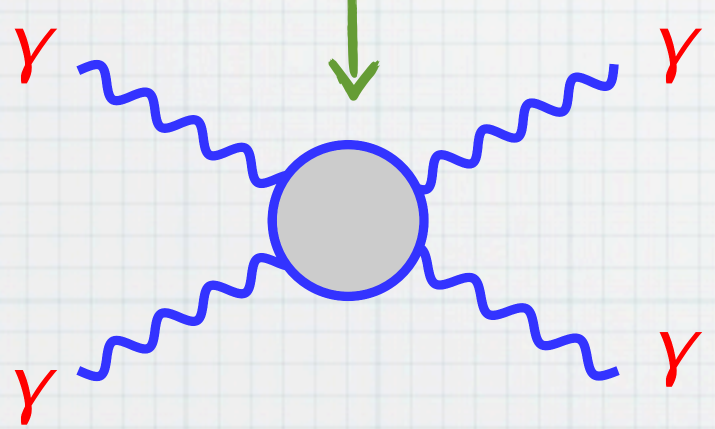

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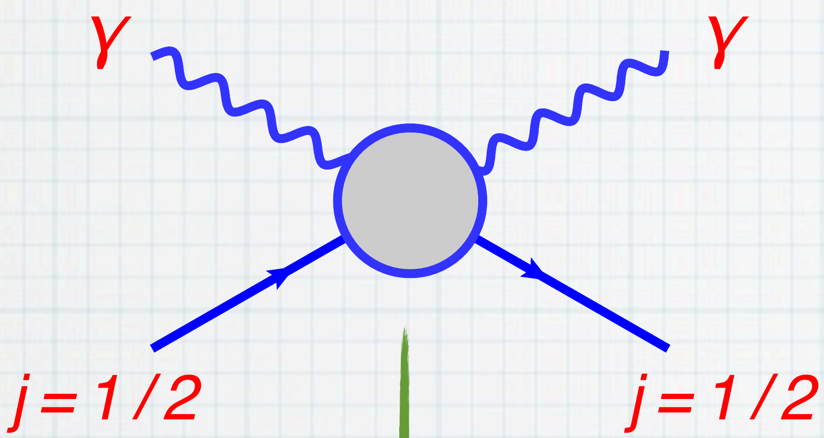

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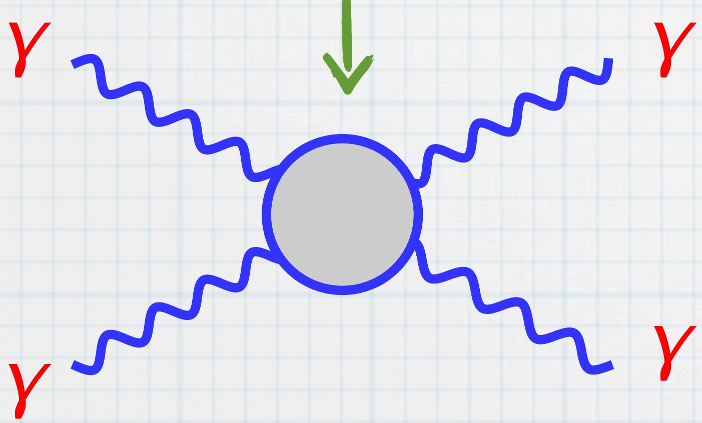

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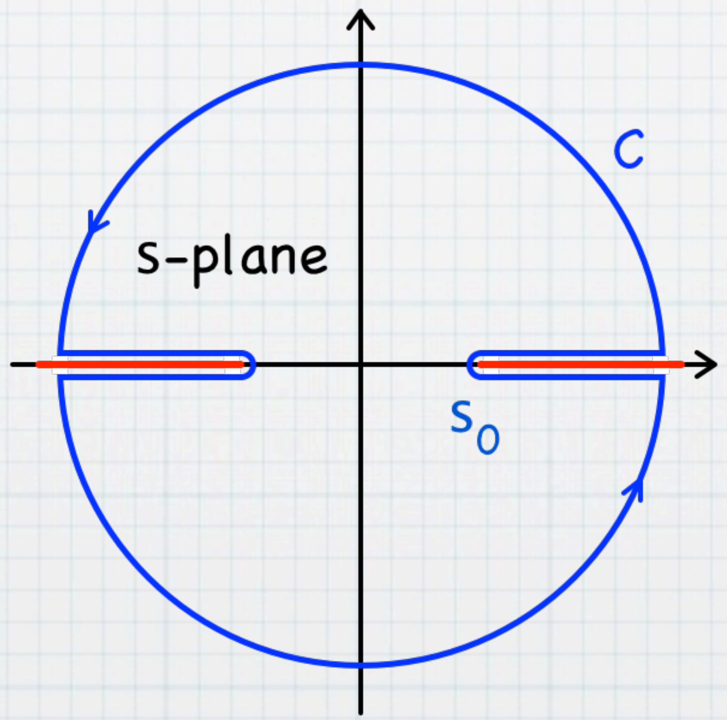
- hadronic physics: lead to constraints on $\gamma\gamma^*$ transition FFs of $q\bar{q}$ and more general meson states
- field theory: provide a model consistency check, give insight into non-perturbative properties of fields dynamics

Outline

- $\gamma\gamma$ sum rules: basic principles
- pair production in QED
- photoproduction of mesons
- conclusions & outlook

V. Pascalutsa, V.P., M. Vanderhaeghen : Phys. Rev. D 85, 116001 (2012)

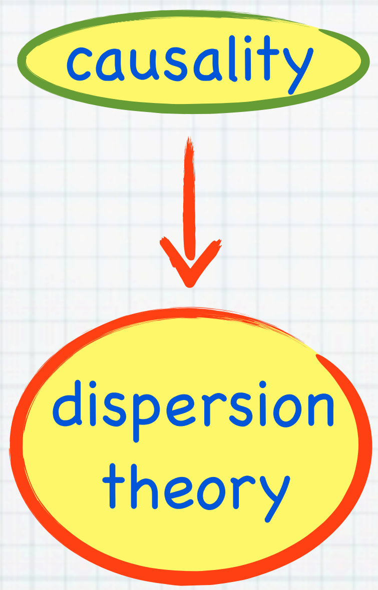
Sum rules : derivation



Sum rules

causality

Sum rules



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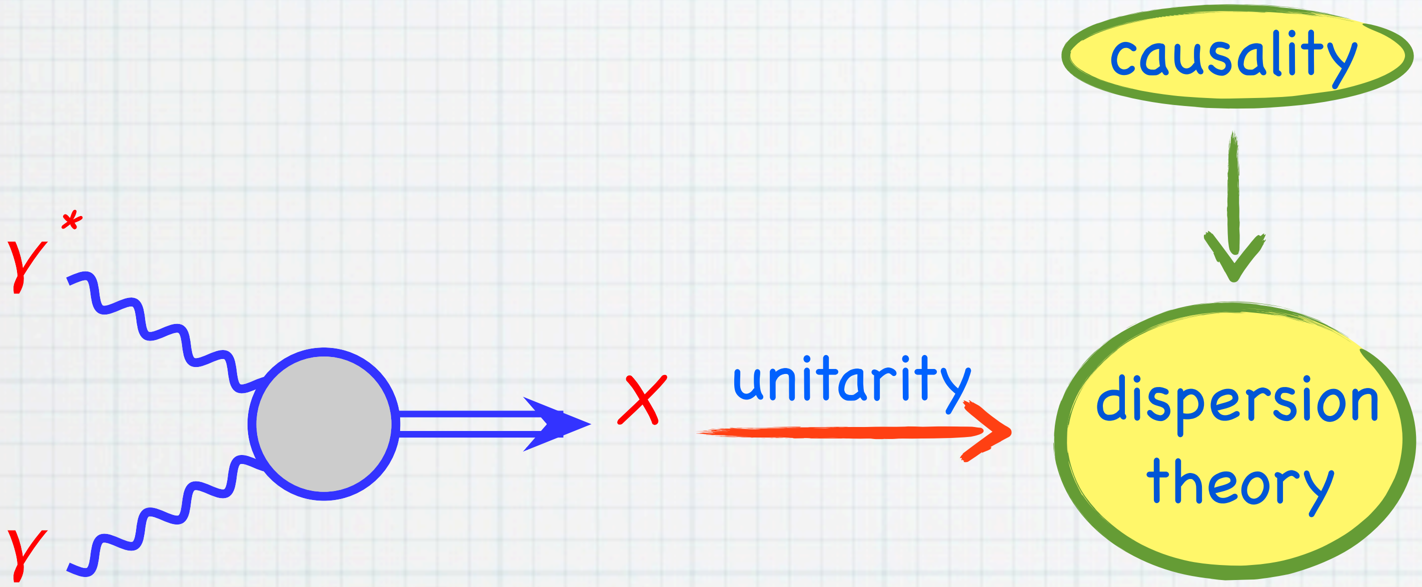
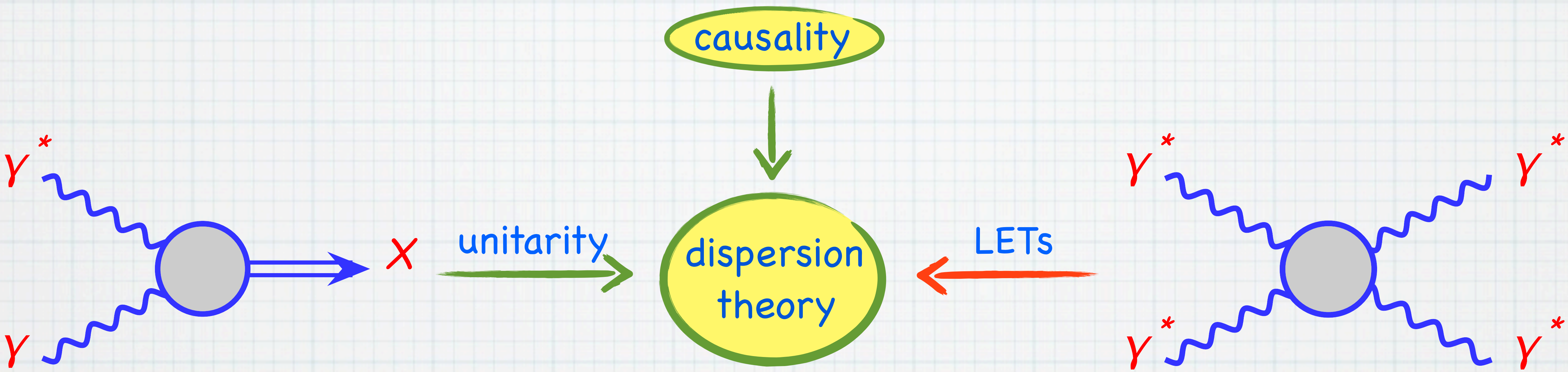


inelastic LbL scattering –
photon-photon fusion into
leptons and hadrons

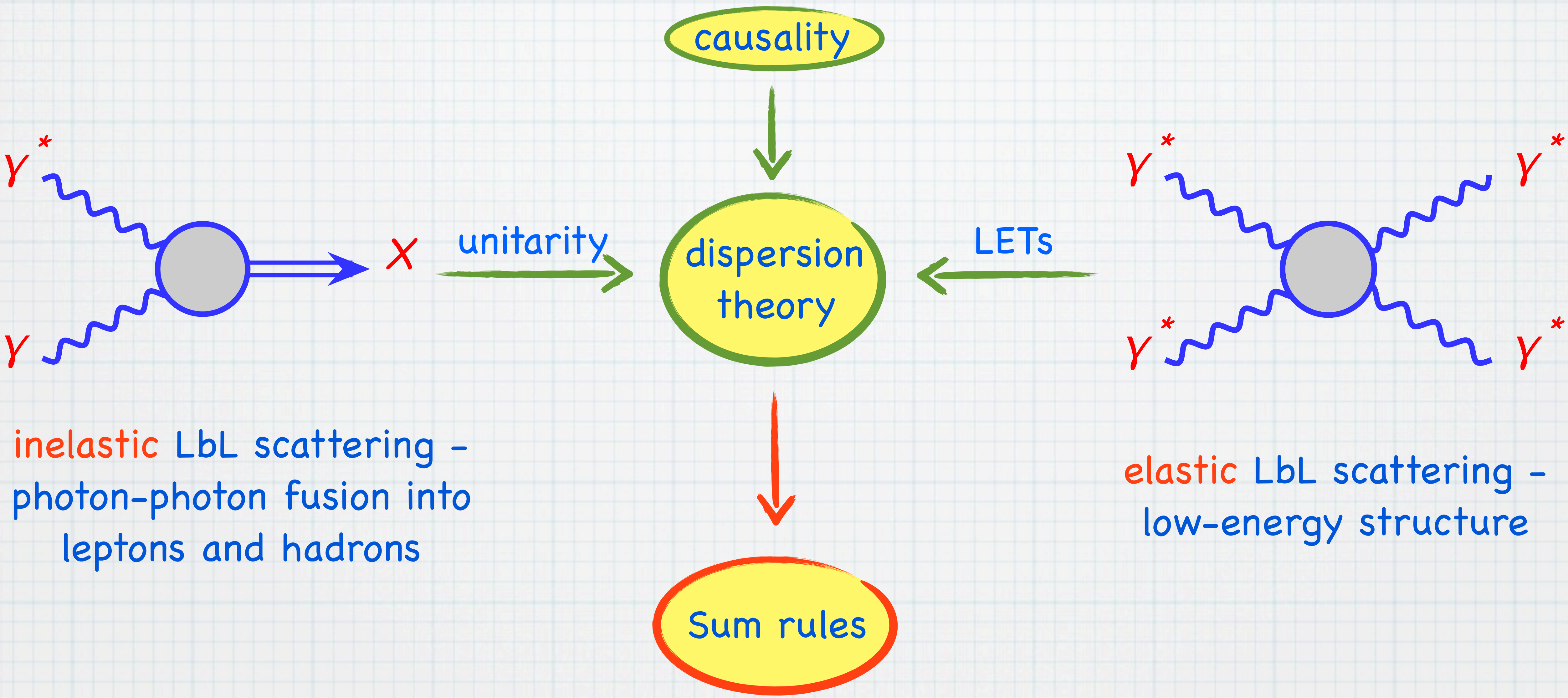
Sum rules



inelastic LbL scattering – photon-photon fusion into leptons and hadrons

elastic LbL scattering – low-energy structure

Sum rules



causality
γ^*
γ
X
unitarity
dispersion theory
LETs
γ^*
γ^*
γ^*
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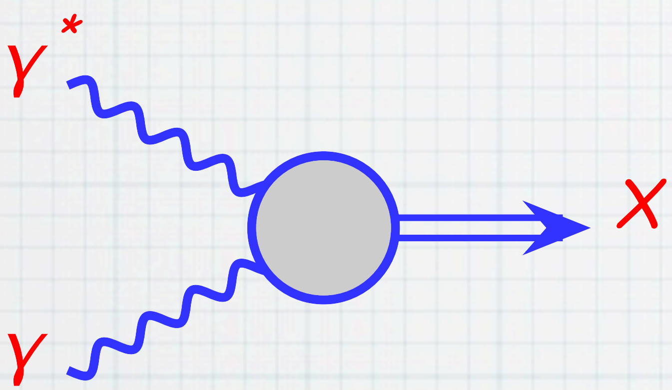

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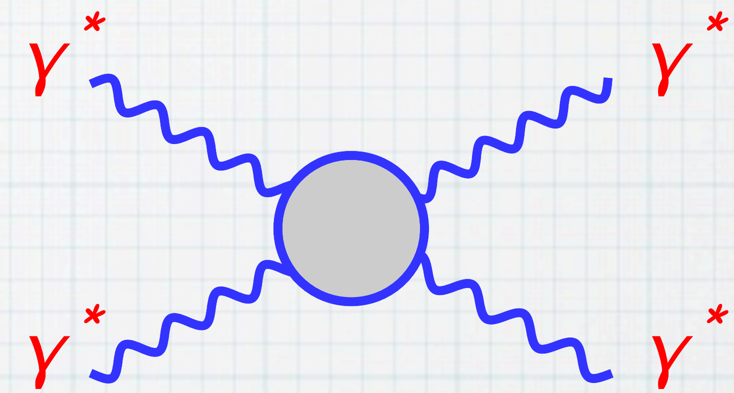


inelastic LbL scattering – photon-photon fusion into leptons and hadrons

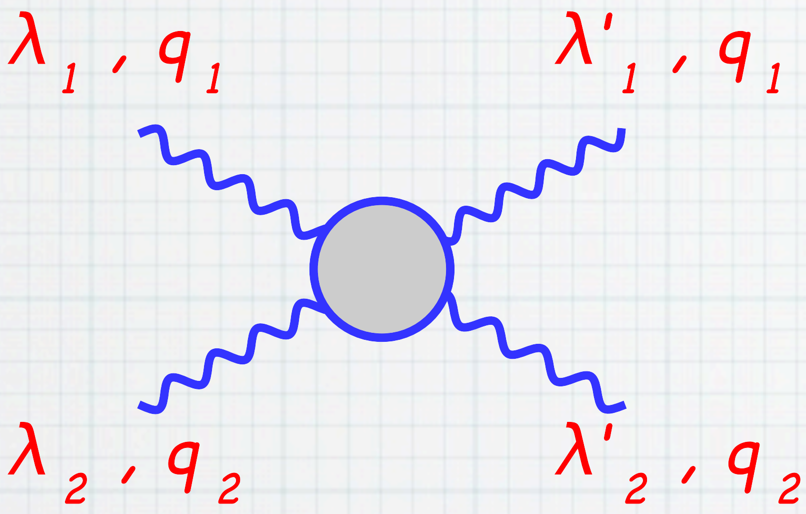
elastic LbL scattering – low-energy structure

Sum rules

$$\int_{s_0}^{\infty} \frac{ds}{s}\,[\sigma_2(s) - \sigma_0(s)] = 0$$

$$c_1 \pm c_2 = \frac{1}{8\pi}\int_{s_0}^{\infty} \frac{ds}{s^2}\,[\sigma_{\parallel}(s) \pm \sigma_{\perp}(s)]$$

$\gamma^*\gamma^* \longrightarrow \gamma^*\gamma^*$ forward scattering



Mandelstam variables: $s = (q_1 + q_2)^2 \quad t = (q_1 - q_3)^2 = 0$

$$u = (q_1 - q_2)^2 = -s$$

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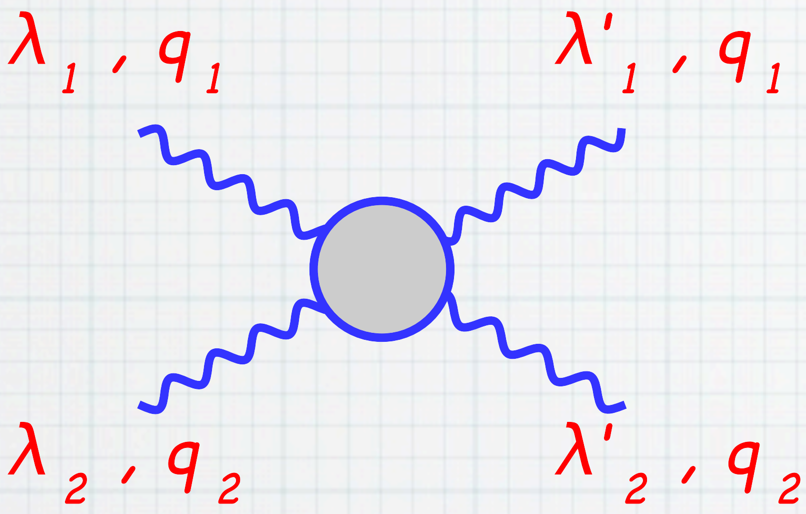

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$\lambda'_{1,2} = \pm 1 \longrightarrow$ 8 helicity amplitudes: $\boxed{M_{\lambda'_1\lambda'_2\lambda_1\lambda_2}(s)}$

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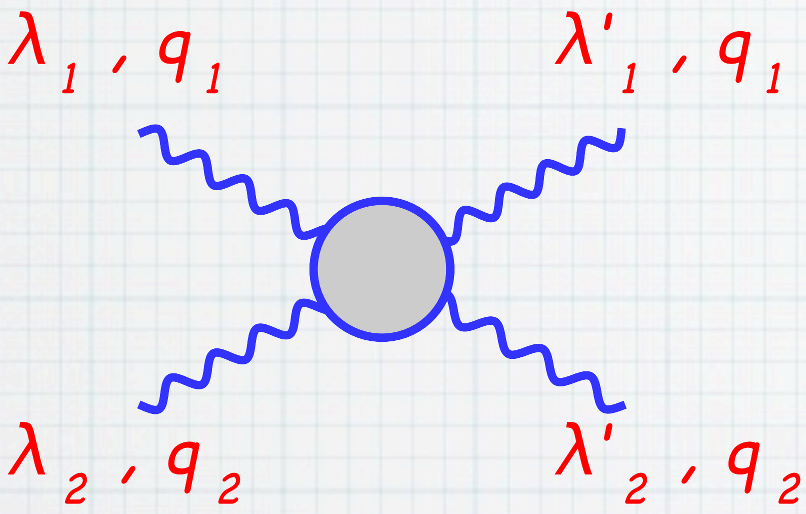

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space & time symmetries:

$T: \quad M_{\lambda'_1\lambda'_2,\lambda_1\lambda_2} = M_{\lambda_1\lambda_2,\lambda'_1\lambda'_2}$

$P: \quad M_{\lambda'_1\lambda'_2,\lambda_1\lambda_2} = M_{-\lambda'_1-\lambda'_2,-\lambda_1-\lambda_2}$

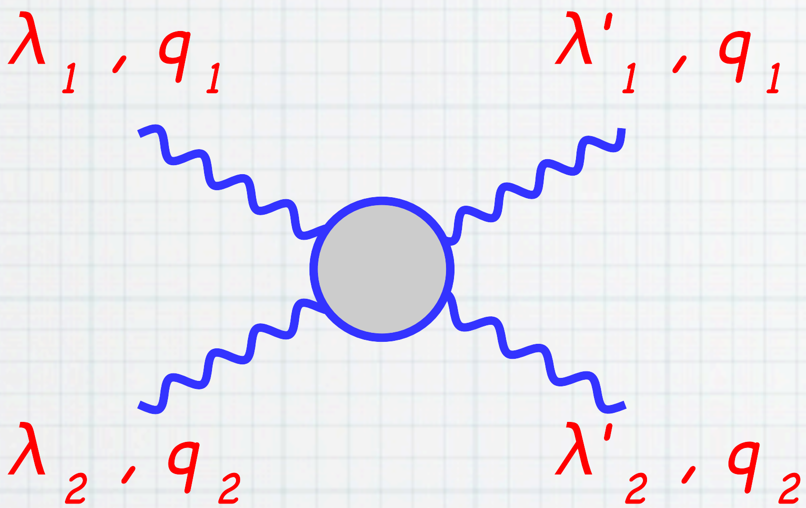
crossing symmetry:

$M_{\lambda'_1\lambda'_2,\lambda_1\lambda_2}(\nu, Q_1^2, Q_2^2) =$

$= M_{\lambda'_1-\lambda'_2,\lambda_1-\lambda_2}(-\nu, Q_1^2, Q_2^2)$

3 independent amplitudes:

$\gamma^*\gamma^* \longrightarrow \gamma^*\gamma^*$ forward scattering



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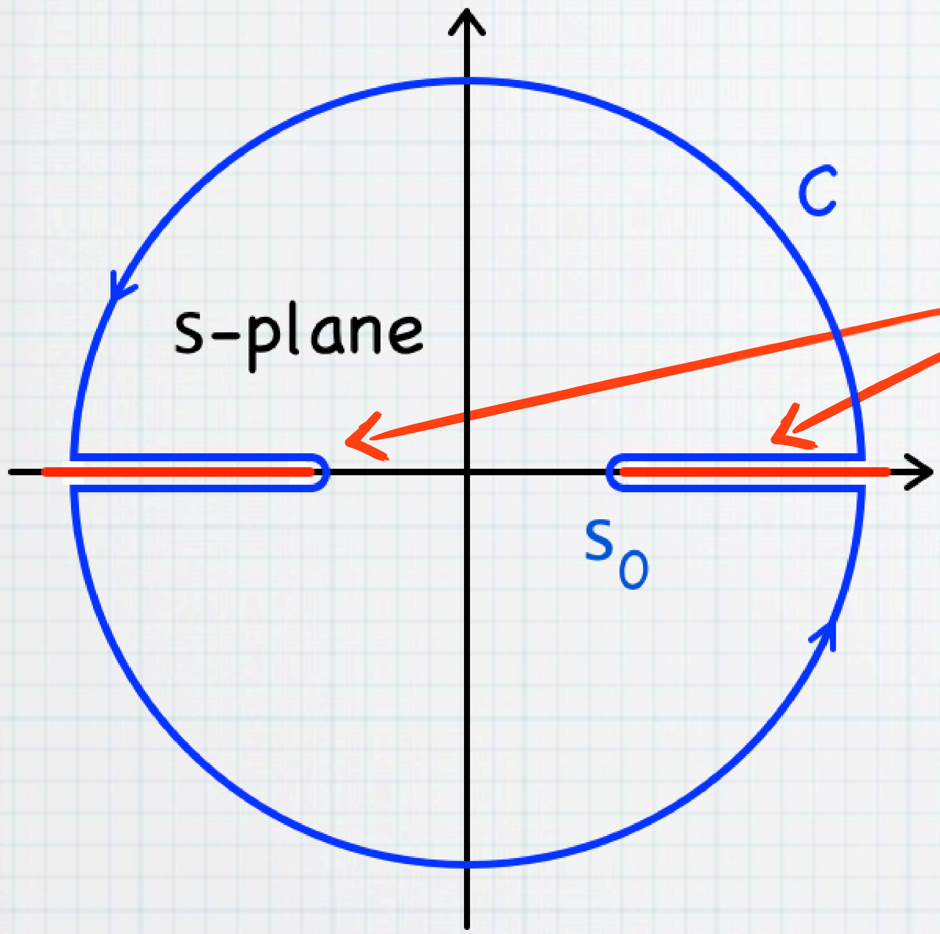
3 independent amplitudes:

even: $\boxed{f^{(+)}(s) = M_{++++}(s) + M_{+-+-}(s)}$ $\quad \boxed{g(s) = M_{++--}(s)}$

odd: $\boxed{f^{(-)}(s) = M_{++++}(s) - M_{+-+-}(s)}$

Dispersion relations



dispersion relations

$$\mathrm{Re}\left\{\begin{array}{c} f^{(+)}(s) \\ g(s) \end{array}\right\} = \frac{2}{\pi}\int_0^\infty \frac{\mathrm{d}s'\, s'}{s'^2 - s^2}\,\mathrm{Im}\left\{\begin{array}{c} f^{(+)}(s') \\ g(s') \end{array}\right\}$$

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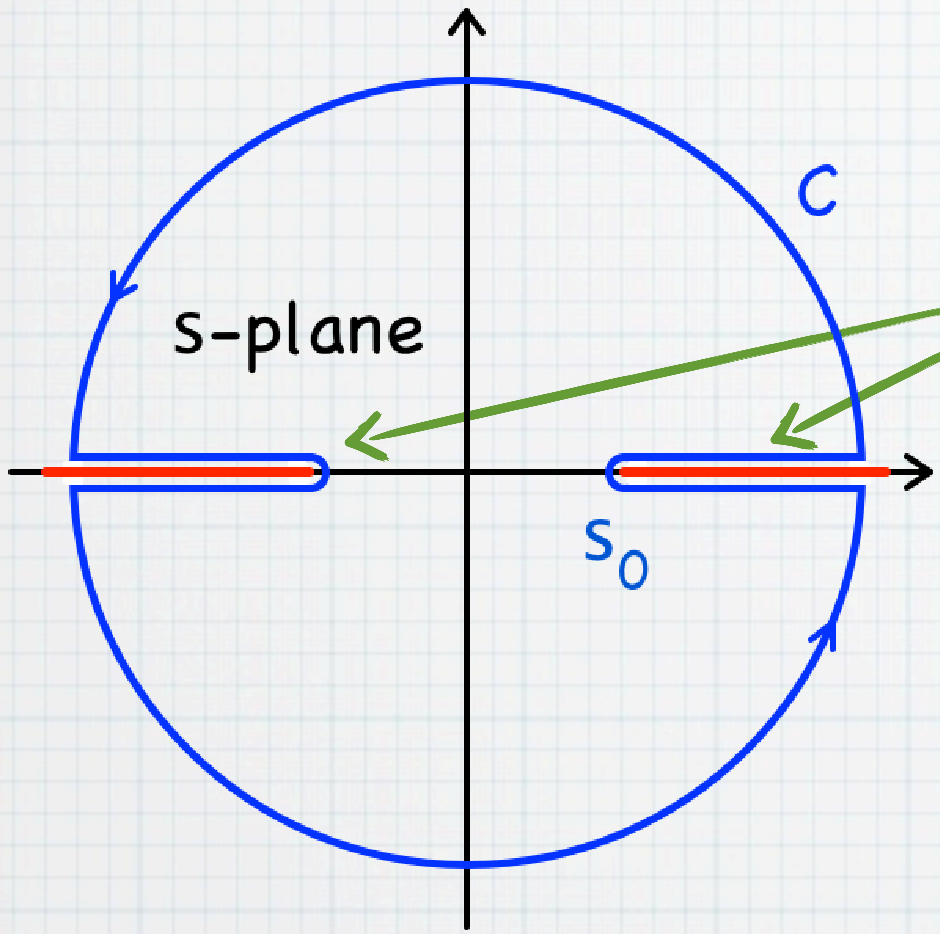

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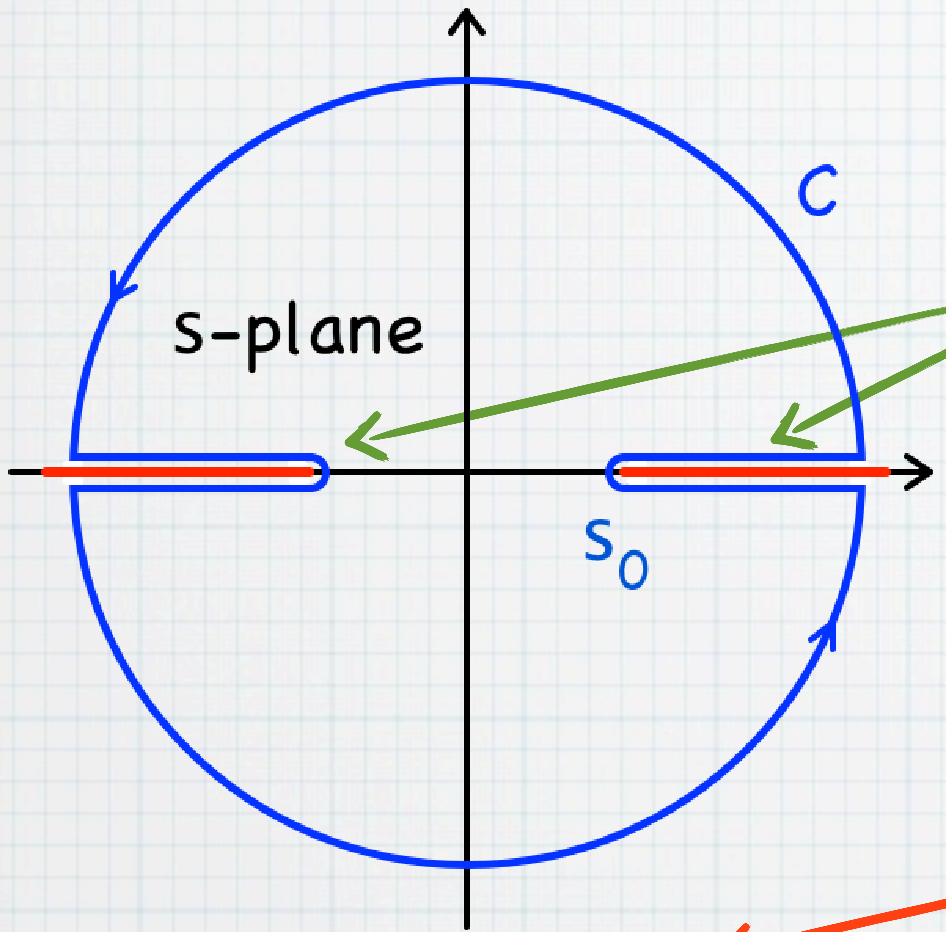

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Observables

Euler-Heisenberg Lagrangian

$$\mathcal{L}^{(8)} = c_1(F_{\mu\nu}F^{\mu\nu})^2 + c_2(F_{\mu\nu}\tilde{F}^{\mu\nu})^2$$

$$f^{(+)}(s) = 16s(c_1 + c_2) + \mathcal{O}(s^4)$$

$$g(s) = 16s(c_1 - c_2) + \mathcal{O}(s^4)$$

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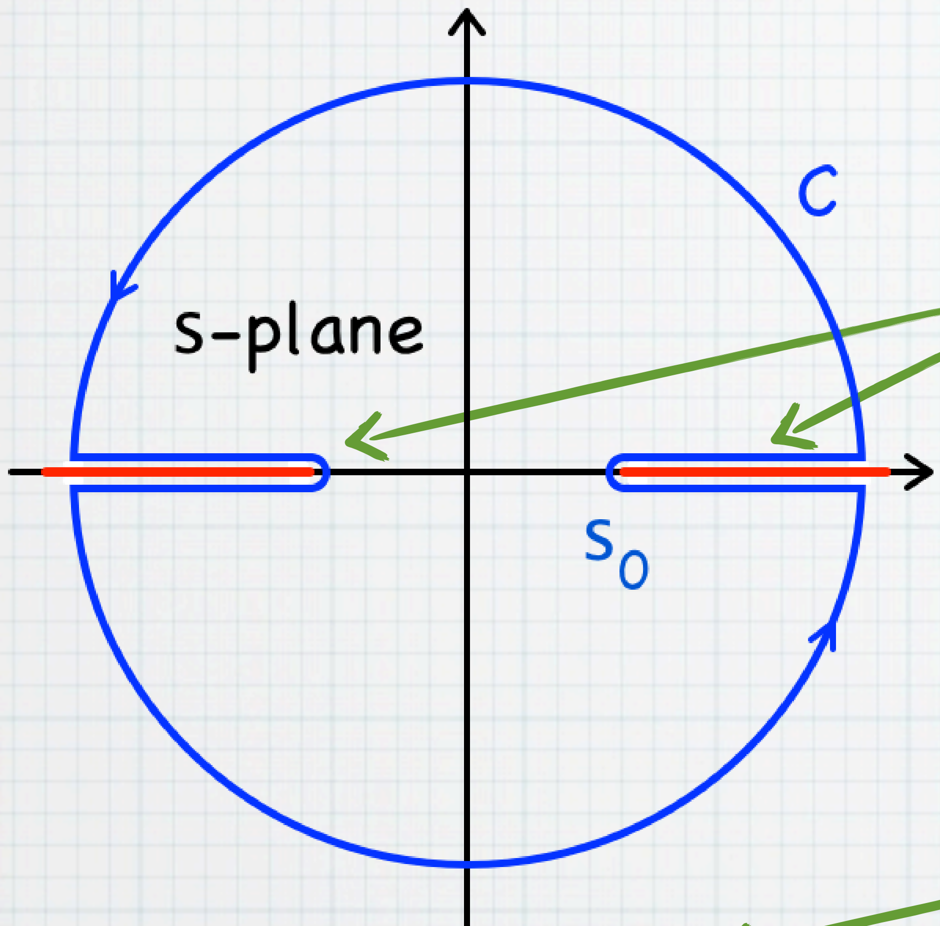
Dispersion relations



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optical theorem

$$\mathrm{Im}\, f^{(-)}(s) = -\frac{s}{8}\,[\,\Delta\sigma(s)\,]$$

$$\mathrm{Im}\, f^{(+)}(s) = -\frac{s}{8}\,[\,\sigma_{tot}(s)\,]$$

$$\mathrm{Im}\, g(s) = -\frac{s}{8}\,[\,\sigma_{||}(s) - \sigma_{\perp}(s)\,]$$

Sum rules

expand the left-hand side and right-hand side of in powers of s and match them at each order

$$\boxed{0 = \int_0^\infty ds\, \frac{\Delta\sigma(s)}{s}}$$

$$\boxed{c_1 \pm c_2 = \frac{1}{8\pi} \int_0^\infty ds\, \frac{\sigma_{||}(s) \pm \sigma_\perp(s)}{s^2}}$$

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virtual photons:

- 8 response functions: $\sigma_{TT}, \tau_{TT}, \tau_{TT}^a, \sigma_{TL}, \sigma_{LT}, \sigma_{LL}, \tau_{TL}, \tau_{TL}^a$
- low-energy expansion up to the order of $\mathcal{O}(s^3)$: 6 new structure constants

enter

- the sum rules can be define only for the case of one quasi-real photon

Sum rules

3 superconvergent relations:

helicity difference sum rule

$$0 = \int_{s_0}^{\infty} ds \frac{1}{(s+Q_1^2)} [\sigma_0 - \sigma_2]_{Q_2^2=0}$$

sum rules involving longitudinal photons

$$0 = \int_{s_0}^{\infty} ds \frac{1}{(s+Q_1^2)^2} \left[\sigma_{\parallel} + \sigma_{LT} + \frac{(s+Q_1^2)}{Q_1 Q_2} \tau_{TL}^a\right]_{Q_2^2=0}$$

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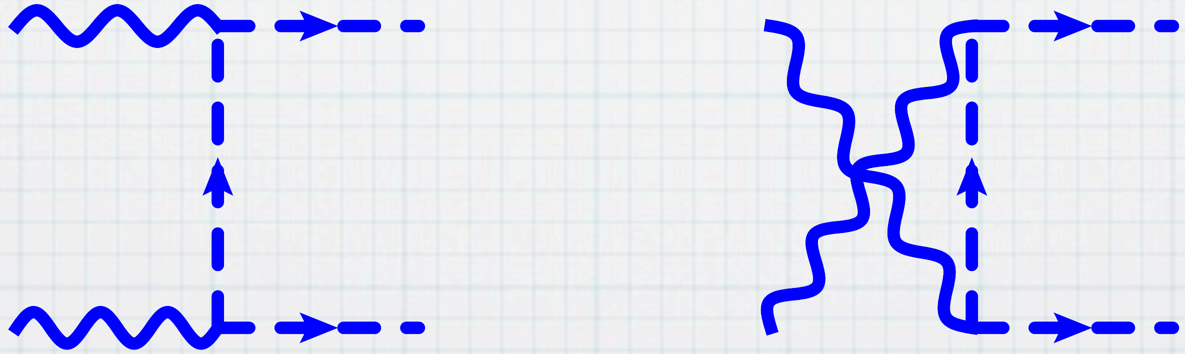
SRs involving LbL low-energy constants:

$$c_1 \pm c_2 = \frac{1}{8\pi} \int_{s_0}^{\infty} \frac{ds}{s^2} \left[\sigma_{\parallel}(s) \pm \sigma_{\perp}(s)\right]$$

...

Digression: production of a pair



Pair production: spinor QED

QED of point-like spin-1/2 particle:

$$\mathfrak{L} = \bar{\psi} i \gamma^\mu \partial_\mu \psi - m \bar{\psi} \psi - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} - e \bar{\psi} \gamma^\mu \psi A_\mu$$

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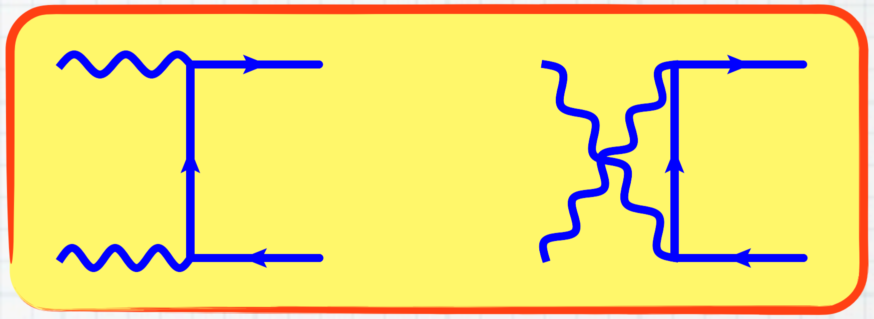
LO



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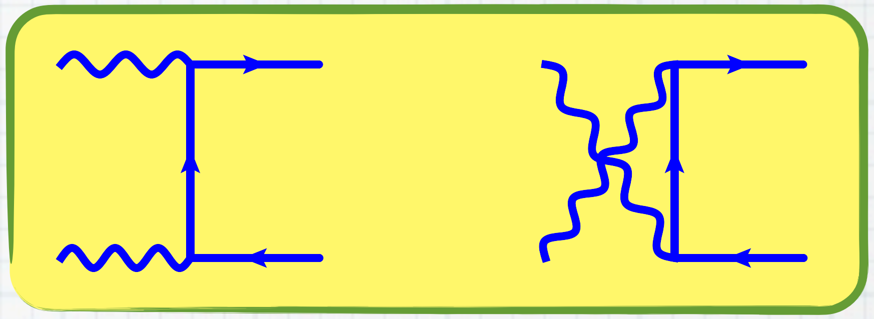

γγ→X response functions:

$$[\sigma_\parallel + \sigma_\perp]_{Q_2^2=0} = \alpha^2 8\pi \frac{s^2}{(s+Q_1^2)^3} \left\{ \beta \left[-\left(1 - \frac{Q_1^2}{s}\right)^2 - \frac{4m^2}{s} \right] + 2\left(1 + \frac{4m^2}{s} - \frac{8m^4}{s^2} + \frac{Q_1^4}{s^2}\right) L \right\},$$

$$[\sigma_\parallel - \sigma_\perp]_{Q_2^2=0} = -\alpha^2 8\pi \frac{s^2}{(s+Q_1^2)^3} \left\{ \beta \left(\frac{2m^2}{s} + \frac{Q_1^4}{s^2} \right) + \frac{8m^2}{s} \left(\frac{m^2}{s} + \frac{Q_1^2}{s} \right) L \right\},$$

$$[\sigma_0 - \sigma_2]_{Q_2^2=0} = \alpha^2 8\pi \frac{s}{(s+Q_1^2)^2} \left\{ \beta \left(3 - \frac{Q_1^2}{s} \right) - 2\left(1 - \frac{Q_1^2}{s} \right) L \right\},$$

$$\left[\frac{1}{Q_1^2} \sigma_{LT} \right]_{Q_2^2=0} = \alpha^2 16\pi \frac{s}{(s+Q_1^2)^3} \left\{ \beta - \frac{4m^2}{s} L \right\},$$

$$\left[\frac{1}{Q_1 Q_2} \tau_{TL} \right]_{Q_2^2=0} = 0,$$

$$\left[\frac{1}{Q_1 Q_2} \tau_{TL}^a \right]_{Q_2^2=0} = \alpha^2 16\pi \frac{s^2}{(s+Q_1^2)^4} \left\{ -\beta \left(1 - \frac{2Q_1^2}{s} \right) + \left(-\frac{2Q_1^2}{s} + \frac{4m^2}{s} \right) L \right\},$$

$$L = \ln\left(\frac{\sqrt{s}}{2m} \left[1 + \sqrt{1 - \frac{4m^2}{s}} \right] \right)$$

$$\beta = \sqrt{1 - \frac{4m^2}{s}}$$

The sum rules

$$0 = \int_{s_0}^{\infty} ds \frac{1}{(s+Q_1^2)} \left[\sigma_2 - \sigma_0\right]_{Q_2^2=0}$$

$$0 = \int_{s_0}^{\infty} ds \frac{1}{(s+Q_1^2)^2} \left[\sigma_\parallel + \sigma_{LT} + \frac{(s+Q_1^2)}{Q_1 Q_2} \tau_{TL}^a\right]_{Q_2^2=0}$$

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$Q_2^2 = 0$

Q_1^2 - arbitrary

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$Q_2^2 = 0$

Q_1^2 - arbitrary

hold exactly at tree-level!

The sum rules

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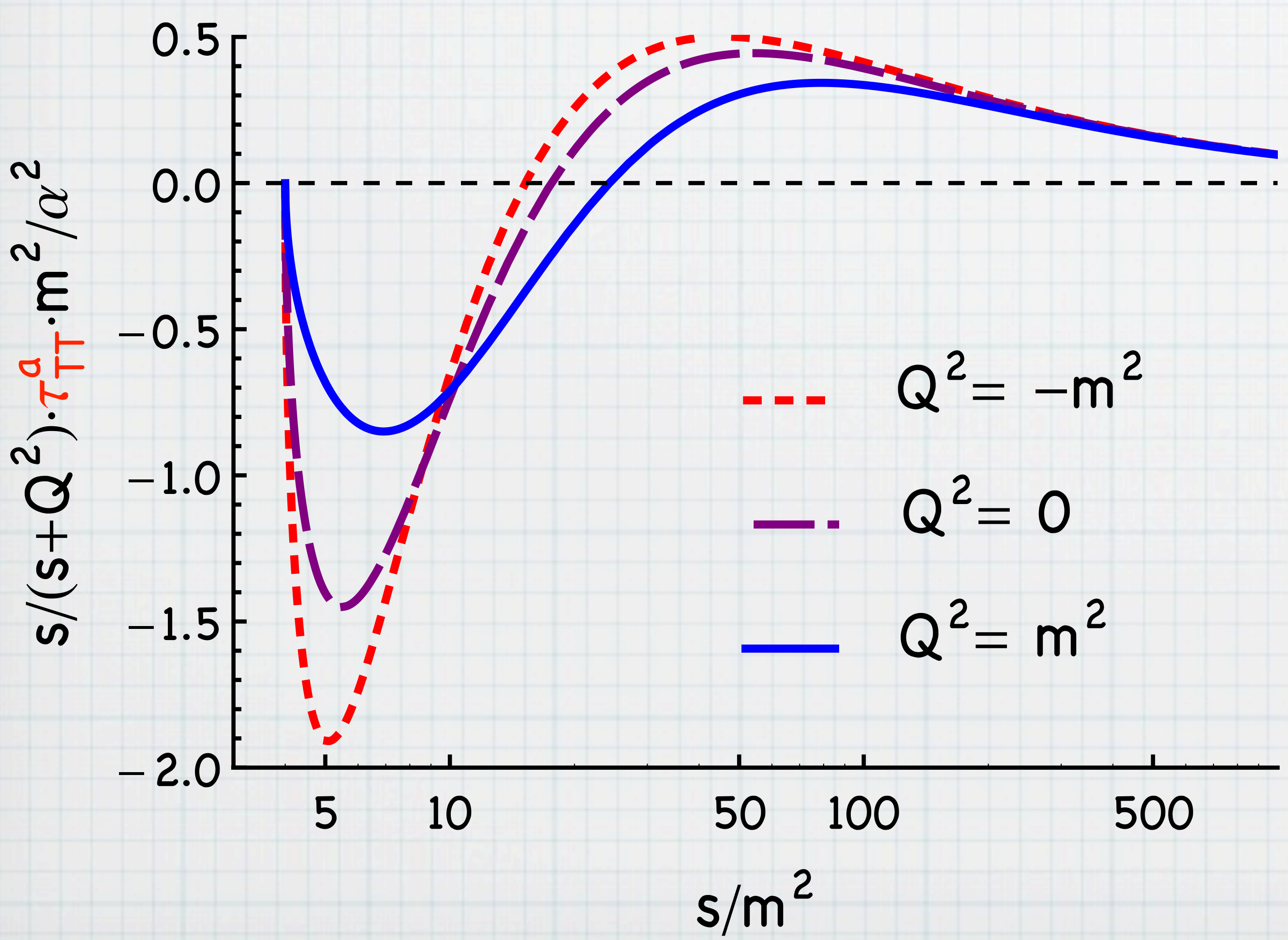

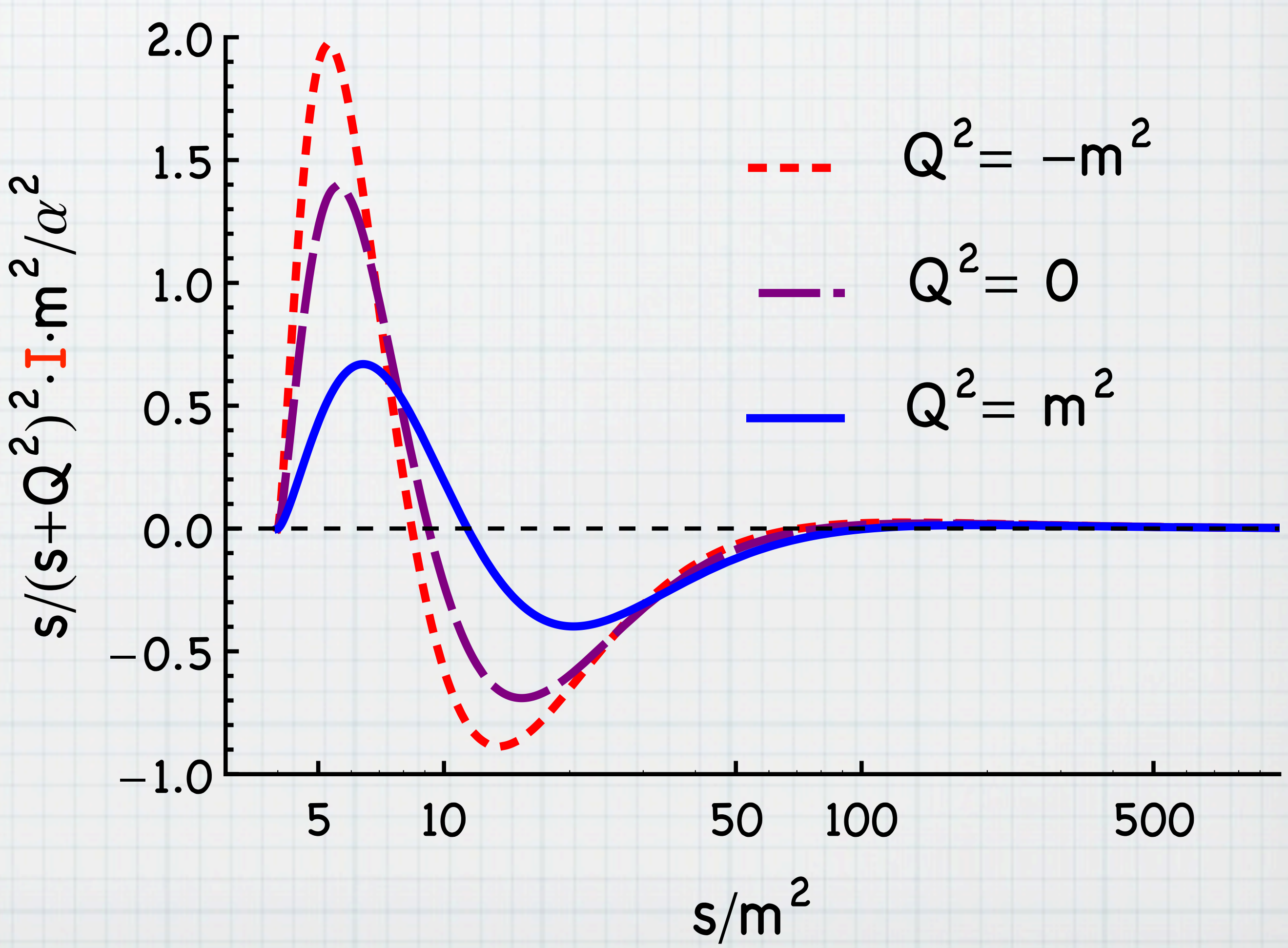

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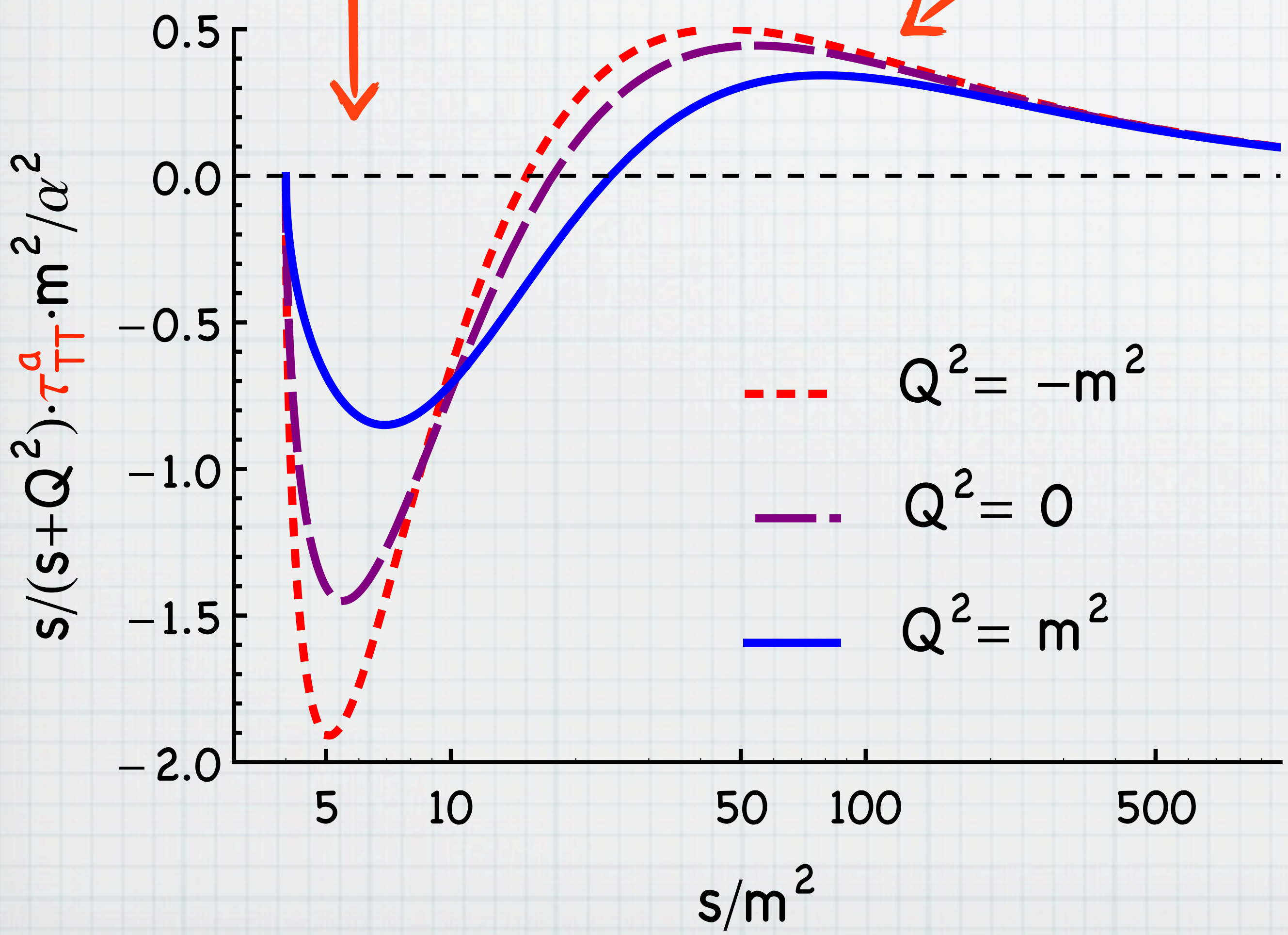
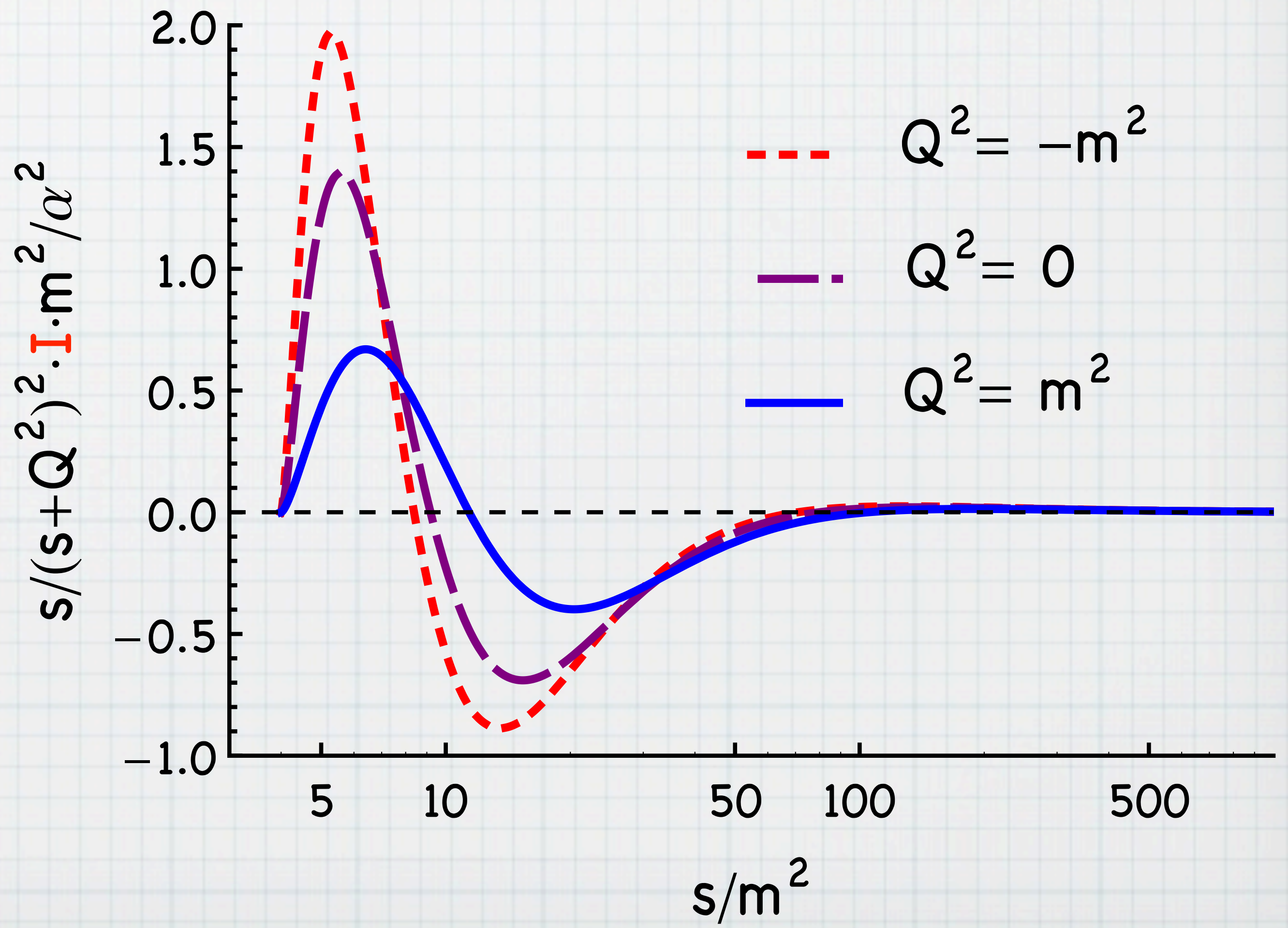
σ_0 dominates at lower energies

σ_2 dominates at higher energies





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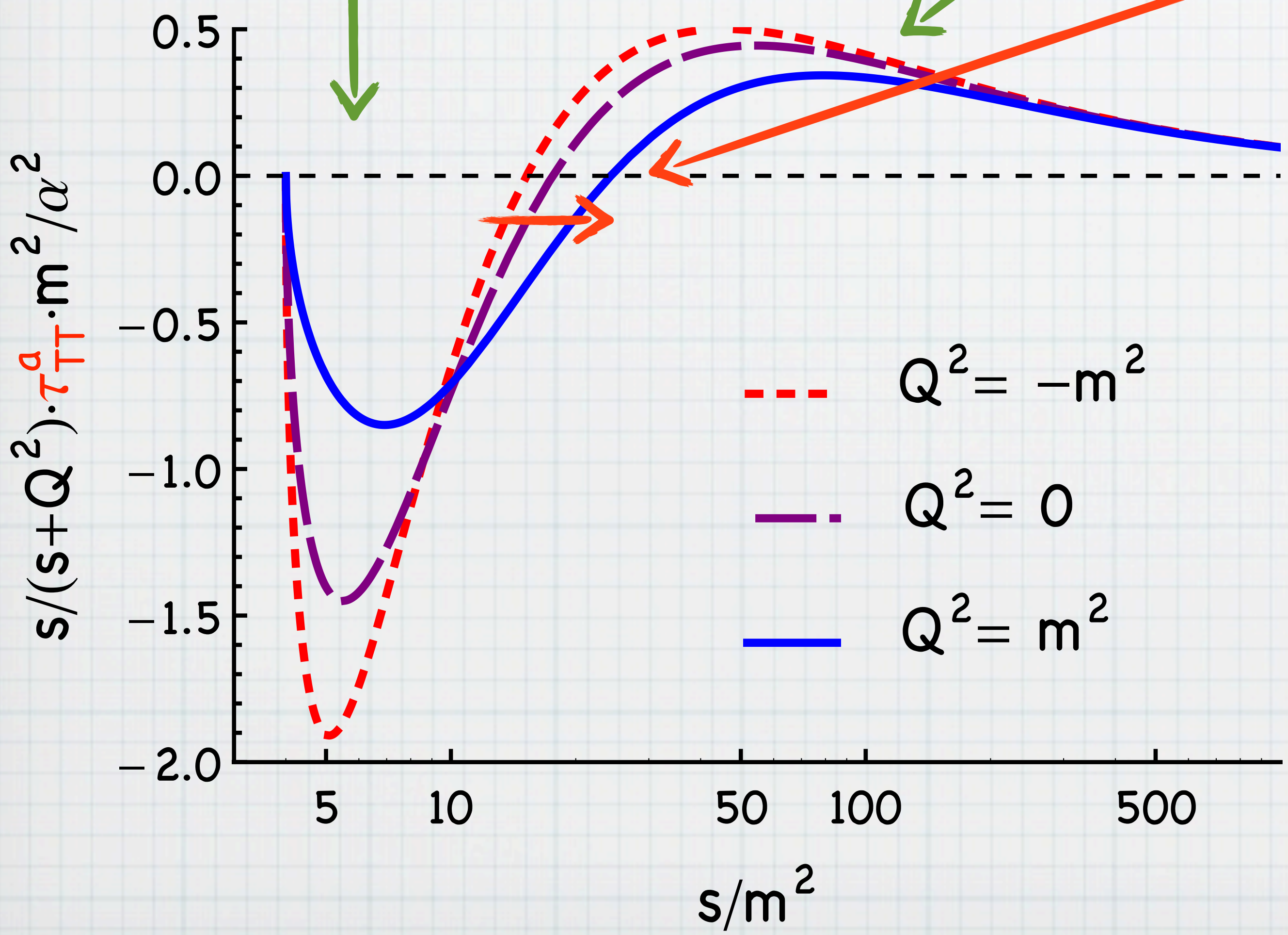
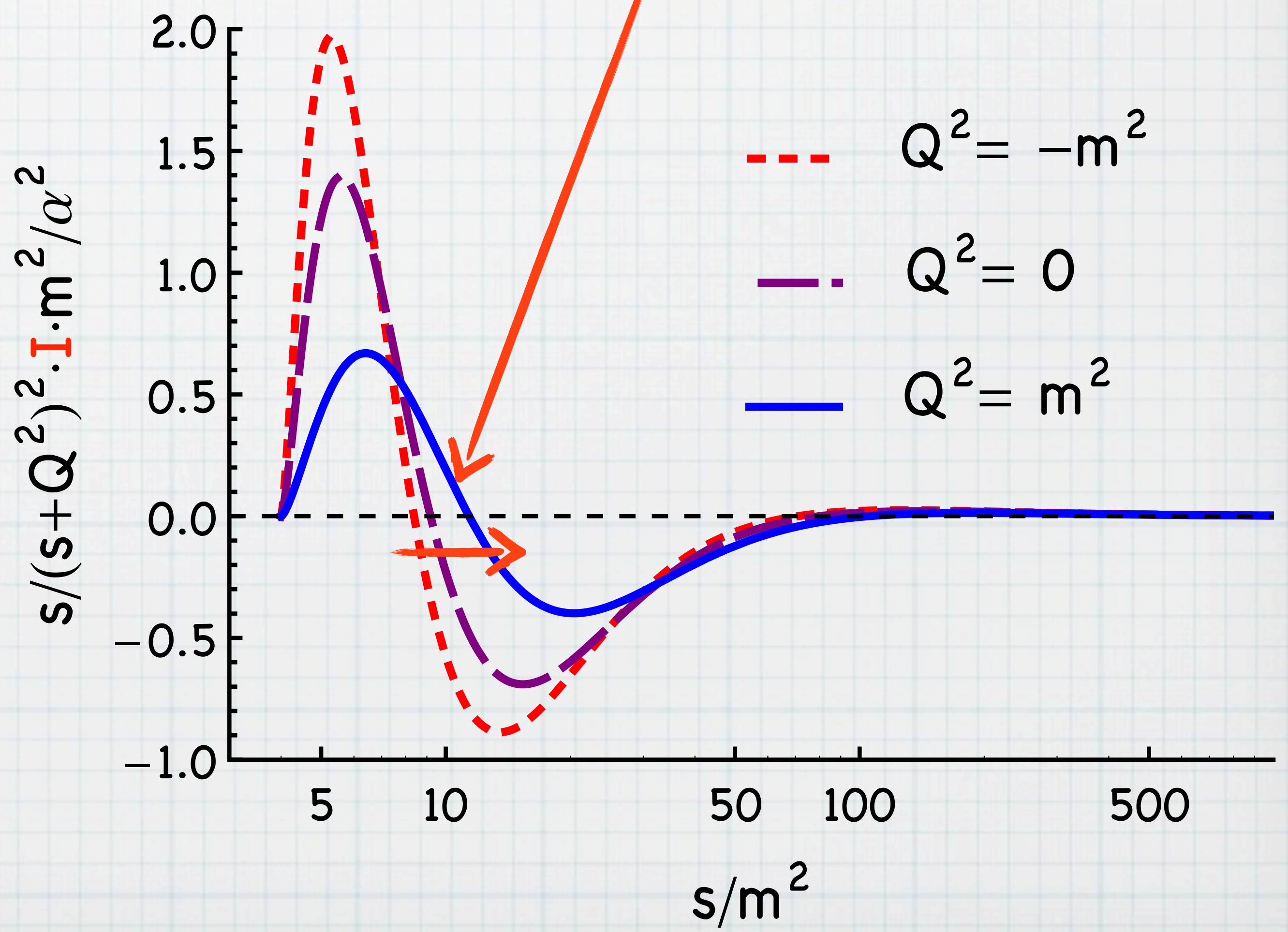
σ_0 dominates at lower energies

σ_2 dominates at higher energies

at larger Q^2 higher energy contributions are required





Tree level: spinor QED

linearly polarized cross sections:

$$[\sigma_\parallel + \sigma_\perp]_{Q_2^2=0} = \alpha^2 8\pi \frac{s^2}{(s+Q_1^2)^3} \left\{ \beta \left[-\left(1-\frac{Q_1^2}{s}\right)^2 - \frac{4m^2}{s} \right] + 2\left(1+\frac{4m^2}{s} - \frac{8m^4}{s^2} + \frac{Q_1^4}{s^2}\right) L \right\},$$

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$$c_1 \pm c_2 = \frac{1}{8\pi} \int_{s_0}^{\infty} \frac{ds}{s^2} \left[\sigma_\parallel(s) \pm \sigma_\perp(s) \right]$$

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↓

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↓

LbL low-energy constants

$$c_1 = \frac{\alpha^2}{m^4} \frac{1}{90} \qquad c_2 = \frac{\alpha^2}{m^4} \frac{7}{360}$$

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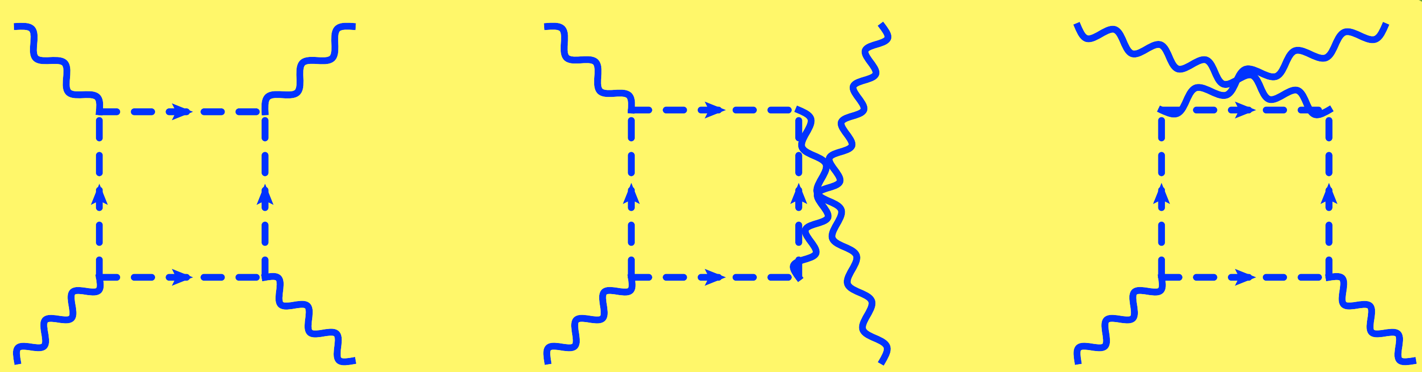
explicit one-loop calculation



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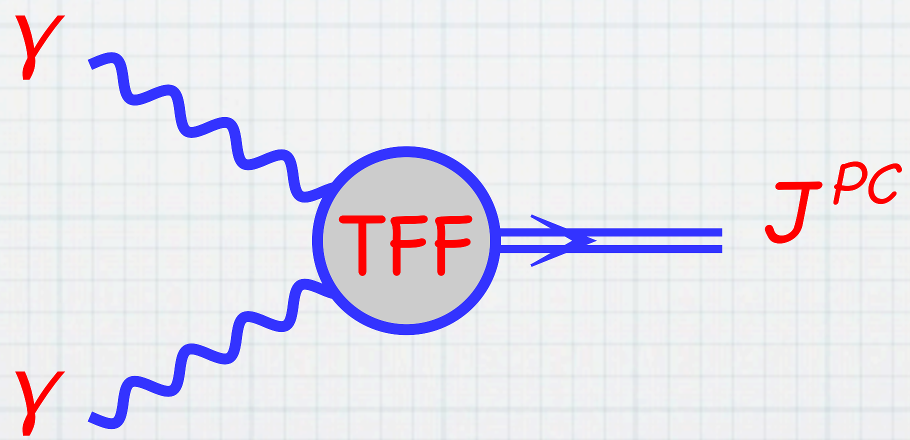
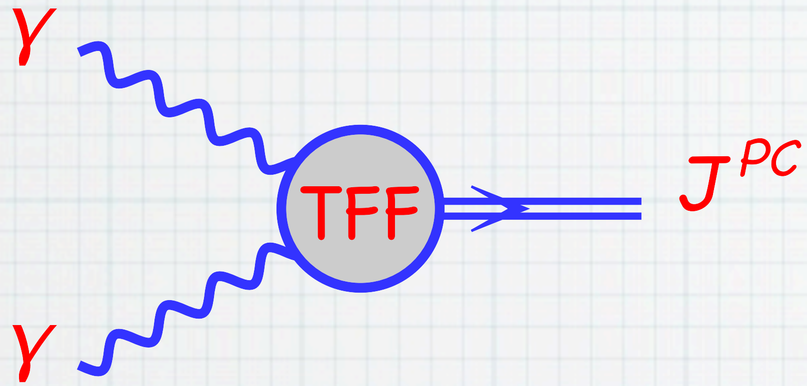
one-loop result is defined by tree-level amplitudes

Applications: meson production



Meson production in γγ collision



- two-photon state: produced meson has C=+1

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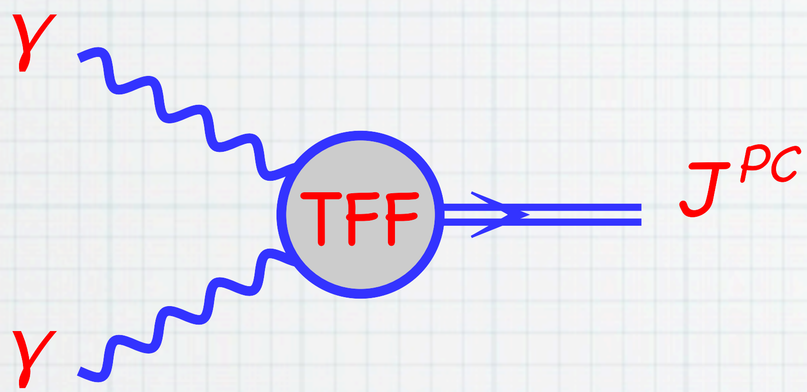

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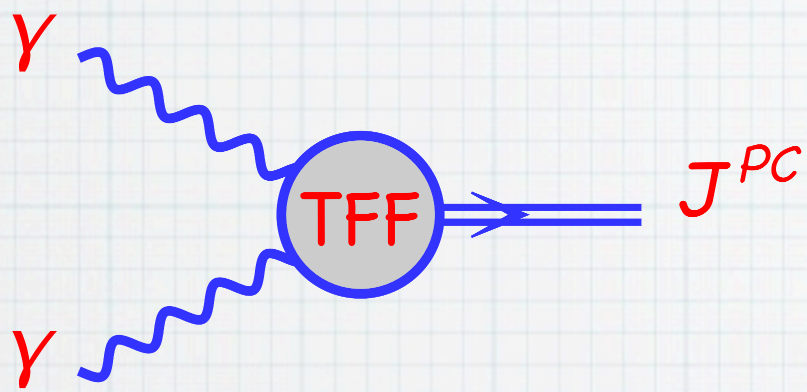
- when both photons are real ~~J=1~~ final state is forbidden

(Landau-Yang theorem);

Meson production in $\gamma\gamma$ collision



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the main contribution comes from J=0: 0^{-+} (pseudoscalar)

and 0^{++} (scalar)

and J=2: 2^{++} (tensor)

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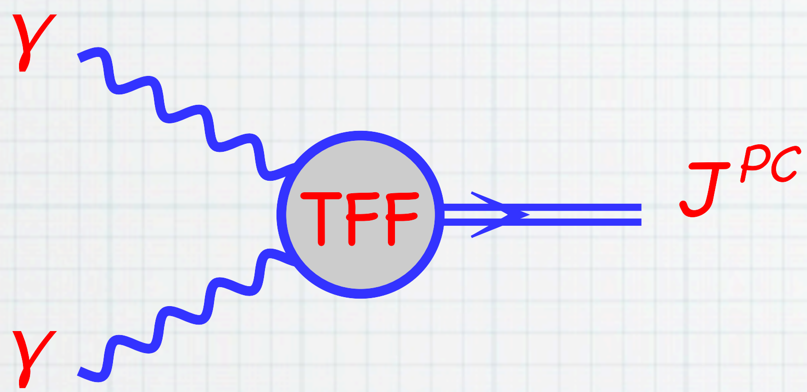

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isoscalar and isovector mesons, $c\bar{c}$ states

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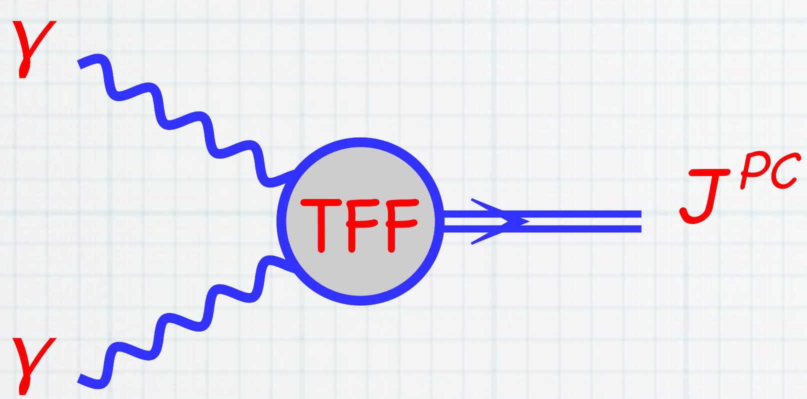

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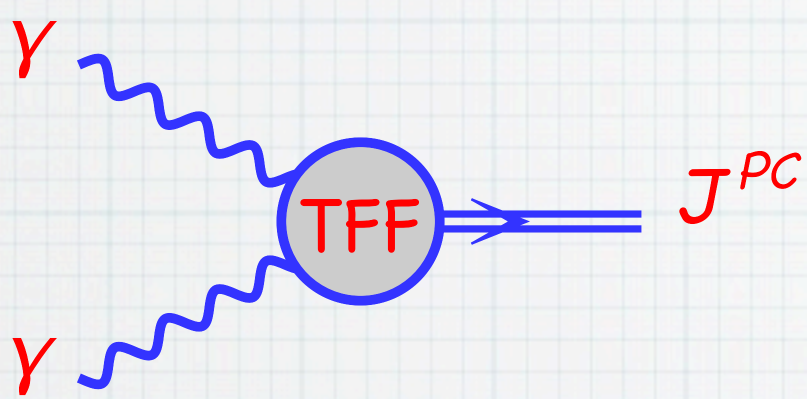
- input for the absorptive part of the SRs: γγ-hadrons response functions, can be expressed in terms of γγ→M transition form factors

Meson production in γγ collision



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$$\boxed{\sigma_\Lambda^{\gamma\gamma\to M}(s) \approx (2J+1)16\pi^2 \frac{\Gamma_{\gamma\gamma}}{m_M}\delta(s-m_M^2)}$$

meson contribution to the cross-section in the narrow-resonance approximation

$$\boxed{\Gamma_{\gamma\gamma}(\mathcal{P}) = \frac{\pi\alpha^2}{4} m^3 \, |F_{\mathcal{M}\gamma^*\gamma^*}(0,0)|^2}$$

two-photons decay rate for the meson

Meson production in $\gamma\gamma$ collision: I=0

the SRs applied to the
I=0 channel

η, η', f_0, f_0', f_2, f_2' ...

Meson production in γγ collision: I=0

the SRs applied to the I=0 channel

$$0 = \int_{s_0}^{\infty} ds \frac{1}{(s+Q_1^2)} [\sigma_2 - \sigma_0]_{Q_2^2=0}$$

$$c_1 \pm c_2 = \frac{1}{8\pi} \int_{s_0}^{\infty} \frac{ds}{s^2} \sigma_\parallel(s) \pm \sigma_\perp(s)$$

		m_M [MeV]	$\Gamma_{\gamma\gamma}$ [keV]	$\int \frac{ds}{s}(\sigma_2-\sigma_0)$ [nb]	c_1 [10^{-4}GeV^{-4}]	c_2 [10^{-4}GeV^{-4}]
0^{-+}	η	547.853 ± 0.024	0.510 ± 0.026	-191 ± 10	0	0.65 ± 0.03
	η'	957.78 ± 0.06	4.29 ± 0.14	-300 ± 10	0	0.33 ± 0.01
0^{++}	$f_0(980)$	980 ± 10	0.29 ± 0.07	-19 ± 5	0.020 ± 0.005	0
	$f_0'(1370)$	$1200 - 1500$	3.8 ± 1.5	-91 ± 36	0.049 ± 0.019	0
2^{++}	$f_2(1270)$	1275.1 ± 1.2	3.03 ± 0.35	449 ± 52	0.141 ± 0.016	0.141 ± 0.016
	$f_2'(1525)$	1525 ± 5	0.081 ± 0.009	7 ± 1	0.002 ± 0.000	0.002 ± 0.000
	$f_2(1565)$	1562 ± 13	0.70 ± 0.14	56 ± 11	0.012 ± 0.002	0.012 ± 0.002
	Sum			-89 ± 66	0.22 ± 0.03	1.14 ± 0.04

Meson production in $\gamma\gamma$ collision: I=0

the SRs applied to the I=0 channel

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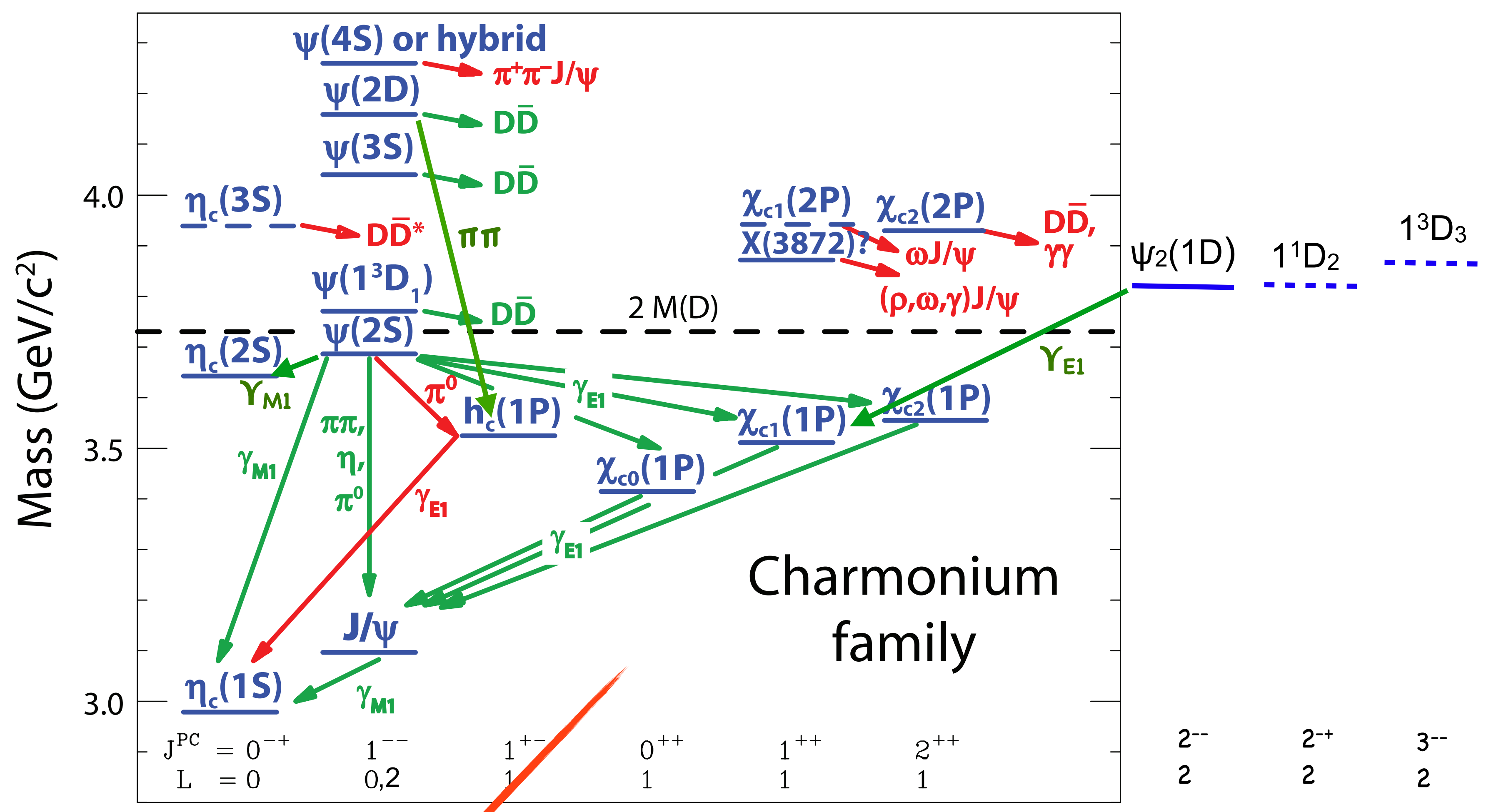
	Sum			-89 ± 66	0.22 ± 0.03	1.14 ± 0.04

- helicity difference SR: the contribution of η, η' is entirely compensated by $f_2(1270)$, $f_2(1565)$ and $f_2'(1525)$
- dominant contribution to low-energy LbL scattering constant c_2 comes from η, η' and $f_2(1270)$

Charmonium states



lower energies:

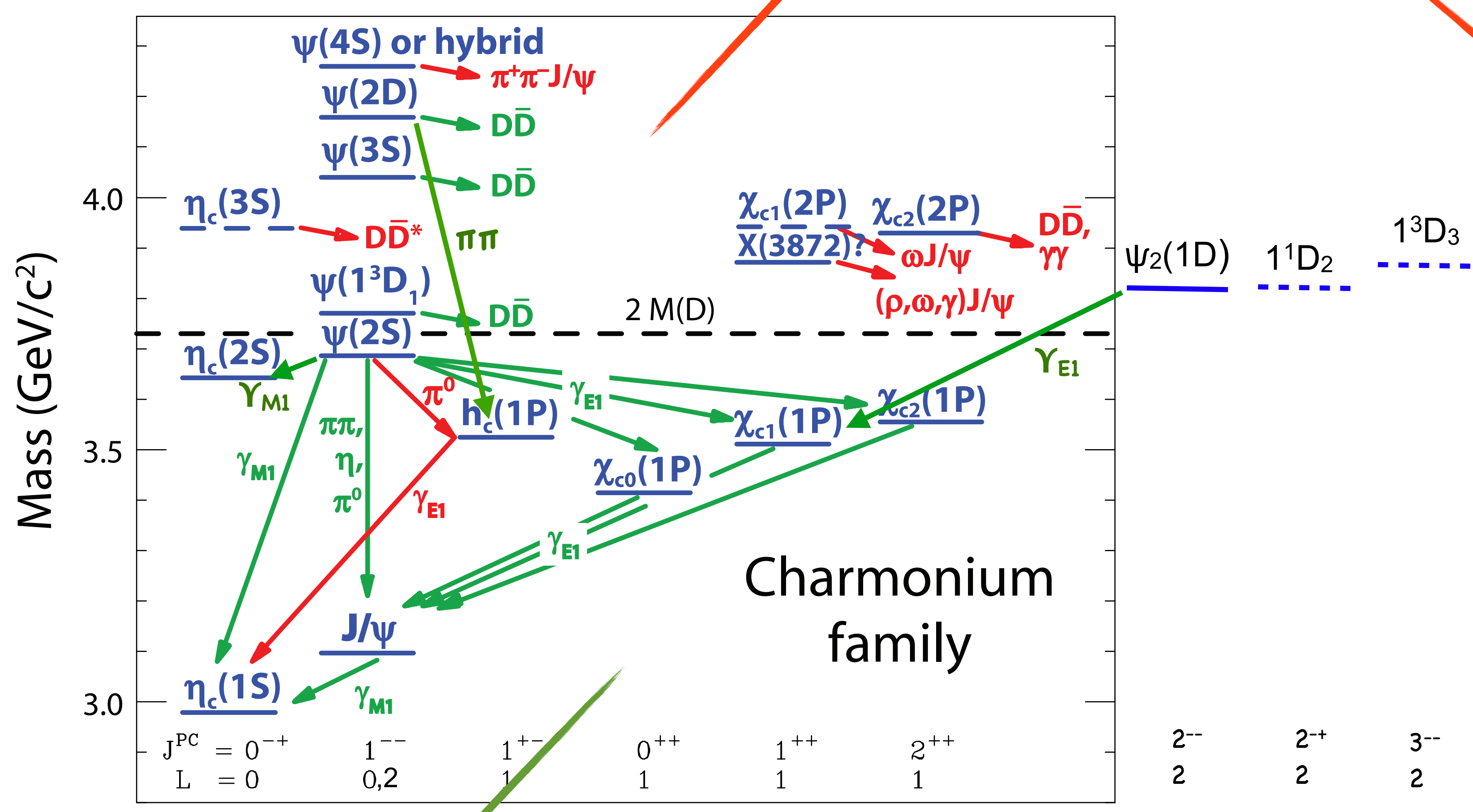
- well understood narrow $c\bar{c}$ states
- only 2 remain to be observed

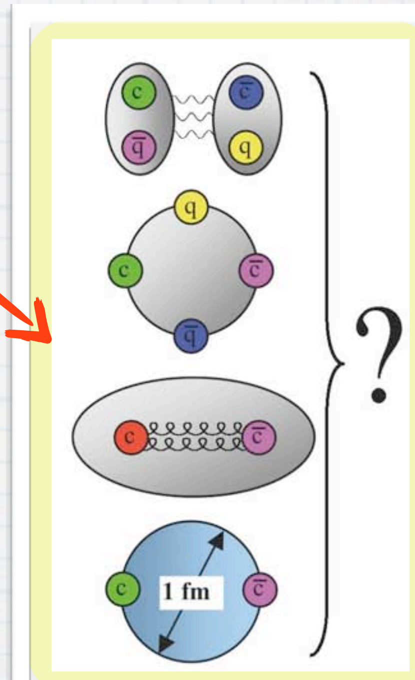
Charmonium states

above $D\bar{D}$ threshold:

- plethora of new states
- ? nature: molecules, tetra-quarks, hybrids,...

charmonium spectrum





lower energies:

- well understood narrow $c\bar{c}$ states
- only 2 remain to be observed

Meson production in γγ collision: $c\bar{c}$ mesons

the SRs evaluated for $c\bar{c}$ states

$$0 = \int_{s_0}^{\infty} ds \frac{1}{(s+Q_1^2)} [\sigma_2 - \sigma_0]_{Q_2^2=0}$$

$$c_1 \pm c_2 = \frac{1}{8\pi} \int_{s_0}^{\infty} \frac{ds}{s^2} \sigma_\parallel(s) \pm \sigma_\perp(s)$$

		m_M [MeV]	$\Gamma_{\gamma\gamma}$ [keV]	$\int \frac{ds}{s} (\sigma_2 - \sigma_0)$ [nb]	c_1 $[10^{-7}\mathrm{GeV}^{-4}]$	c_2 $[10^{-7}\mathrm{GeV}^{-4}]$
0^{-+}	$\eta_c(1S)$	2980.3 ± 1.2	6.7 ± 0.9	-15.6 ± 2.1	0	1.79 ± 0.24
0^{++}	$\chi_{c0}(1P)$	3414.75 ± 0.31	2.32 ± 0.13	-3.6 ± 0.2	0.31 ± 0.02	0
2^{++}	$\chi_{c2}(1P)$	3556.2 ± 0.09	0.50 ± 0.06	3.4 ± 0.4	0.14 ± 0.02	0.14 ± 0.02
	Sum resonances			-15.8 ± 2.1	0.49 ± 0.03	1.97 ± 0.24
	duality estimate continuum ($\sqrt{s} \geq 2m_D$)			15.1		
	resonances + continuum			-0.7 ± 2.1		

Meson production in $\gamma\gamma$ collision: $c\bar{c}$ mesons

the SRs evaluated for
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0^{-+}	$\eta_c(1S)$	2980.3 ± 1.2	6.7 ± 0.9	-15.6 ± 2.1	0	1.79 ± 0.24
0^{++}	$\chi_{c0}(1P)$	3414.75 ± 0.31	2.32 ± 0.13	-3.6 ± 0.2	0.31 ± 0.02	0
2^{++}	$\chi_{c2}(1P)$	3556.2 ± 0.09	0.50 ± 0.06	3.4 ± 0.4	0.14 ± 0.02	0.14 ± 0.02
	Sum resonances			-15.8 ± 2.1	0.49 ± 0.03	1.97 ± 0.24
	duality estimate continuum ($\sqrt{s} \geq 2m_D$)			15.1		
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Meson production in γγ collision: $c\bar{c}$ mesons

the SRs evaluated for $c\bar{c}$ states

$$0 = \int_{s_0}^{\infty} ds \frac{1}{(s+Q_1^2)} [\sigma_2 - \sigma_0]_{Q_2^2=0}$$

$$c_1 \pm c_2 = \frac{1}{8\pi} \int_{s_0}^{\infty} \frac{ds}{s^2} \sigma_\parallel(s) \pm \sigma_\perp(s)$$

		m_M [MeV]	$\Gamma_{\gamma\gamma}$ [keV]	$\int \frac{ds}{s} (\sigma_2 - \sigma_0)$ [nb]	c_1 $[10^{-7}\mathrm{GeV}^{-4}]$	c_2 $[10^{-7}\mathrm{GeV}^{-4}]$
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quark-hadron duality: replace the integral of the cross section for the $\gamma\gamma \to X$ process (X - hadronic final state containing charm quarks) by the corresponding integral of the helicity-difference cross section for perturbative $\gamma\gamma \to c\bar{c}$ process

$$I_{cont} \equiv \int_{s_D}^{\infty} ds \frac{1}{s} [\sigma_2 - \sigma_0](\gamma\gamma \to X) \approx \int_{s_D}^{\infty} ds \frac{1}{s} [\sigma_2 - \sigma_0](\gamma\gamma \to c\bar{c})$$

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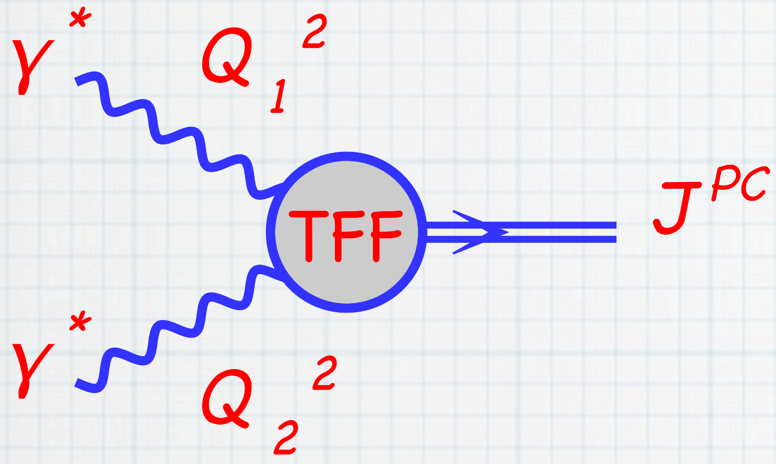
interplay between production of $c\bar{c}$ states and charmed mesons

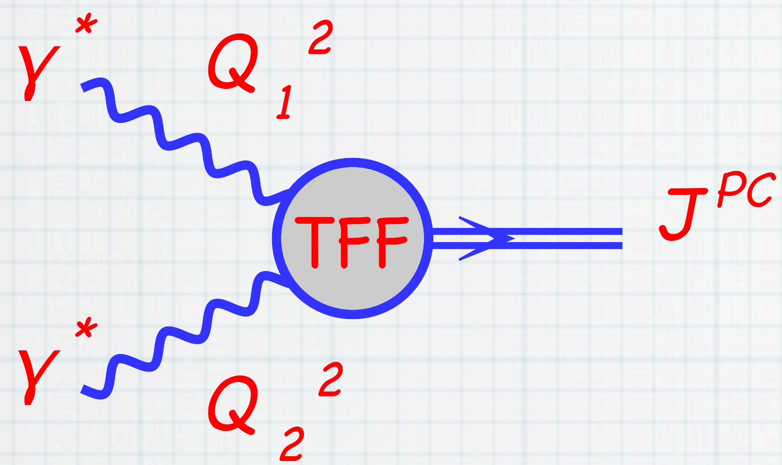
Meson production in $\gamma^*\gamma^*$ collision



one photon is virtual Q_1^2, second photon is real or quasi-real $Q_2^2 \simeq 0$:

axial-vector mesons 1^{++} are also allowed if one of the photons is virtual $\gamma^*\gamma^* \rightarrow f_1(1285)$ / $f_1(1420)$ measured L3 Coll.

Meson production in $\gamma^*\gamma^*$ collision



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sum rules involving longitudinally polarized cross-sections: cancelation mechanism between scalar, axial-vector and tensor mesons

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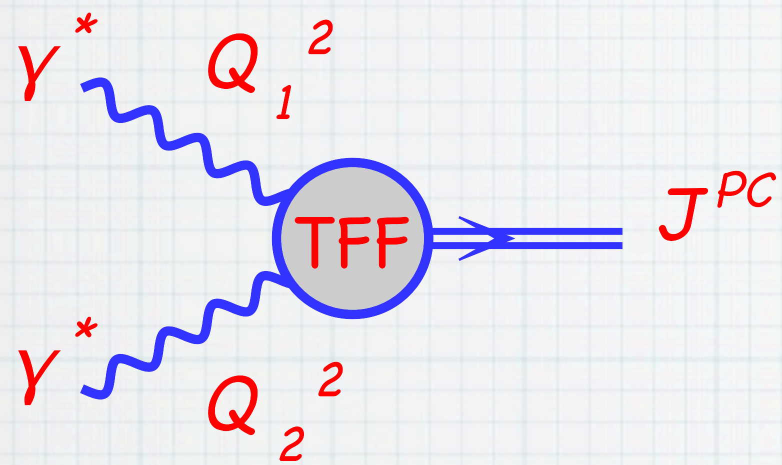

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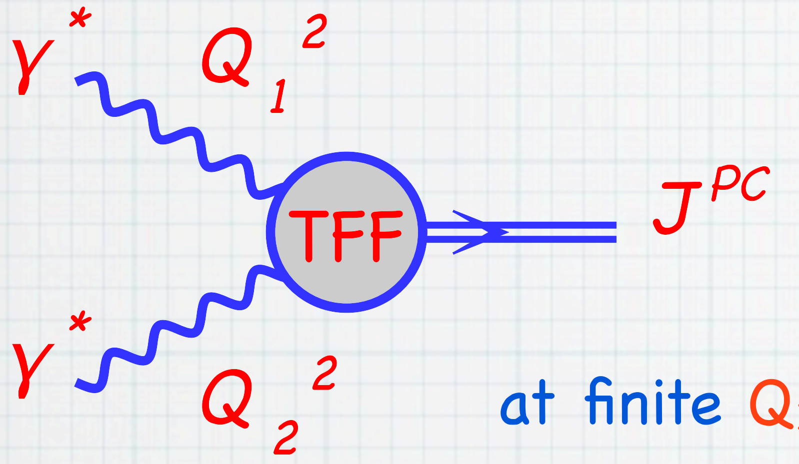
sum rules involving longitudinally polarized cross-sections: cancelation mechanism between scalar, axial-vector and tensor mesons

	m_M [MeV]	$\Gamma_{\gamma\gamma}$ [keV]	$\int \frac{ds}{s^2} \sigma_{\parallel}(s)$ [nb / GeV^2]	$\int ds \left[\frac{1}{s} \frac{\tau_{TL}^a}{Q_1 Q_2} \right]_{Q_i^2=0}$ [nb / GeV^2]	$\int ds \left[\frac{1}{s^2} \sigma_{\parallel} + \frac{1}{s} \frac{\tau_{TL}^a}{Q_1 Q_2} \right]_{Q_i^2=0}$ [nb / GeV^2]
$f_1(1285)$	1281.8 ± 0.6	3.5 ± 0.8	0	-93 ± 21	-93 ± 21
$f_1(1420)$	1426.4 ± 0.9	3.2 ± 0.9	0	-50 ± 14	-50 ± 14
$f_0(980)$	980 ± 10	0.29 ± 0.07	20 ± 5	0	20 ± 5
$f_0'(1370)$	$1200 - 1500$	3.8 ± 1.5	48 ± 19	0	48 ± 19
$f_2(1270)$	1275.1 ± 1.2	3.03 ± 0.35	138 ± 16	$\gtrsim 0$	138 ± 16
$f_2'(1525)$	1525 ± 5	0.081 ± 0.009	1.5 ± 0.2	$\gtrsim 0$	1.5 ± 0.2
$f_2(1565)$	1562 ± 13	0.70 ± 0.14	12 ± 2	$\gtrsim 0$	12 ± 2
Sum					76 ± 36

uncertainty: higher mass states or non-resonant contributions with axial-vector quantum numbers

Meson production in $\gamma^*\gamma^*$ collision



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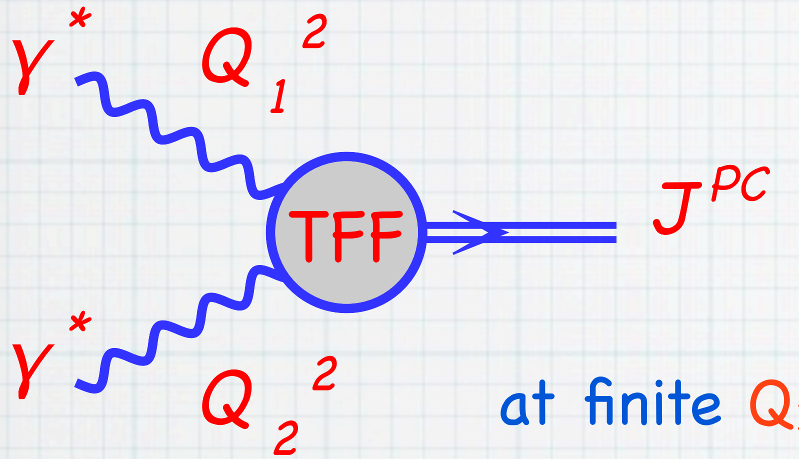
at finite Q_1^2 the SRs imply information on meson transition form-factors: estimate for the $f_2(1270)$ tensor FF in terms of the η and η' FFs and for the $a_2(1320)$ tensor FF in terms of the π^0 FF.

Meson production in $\gamma^*\gamma^*$ collision



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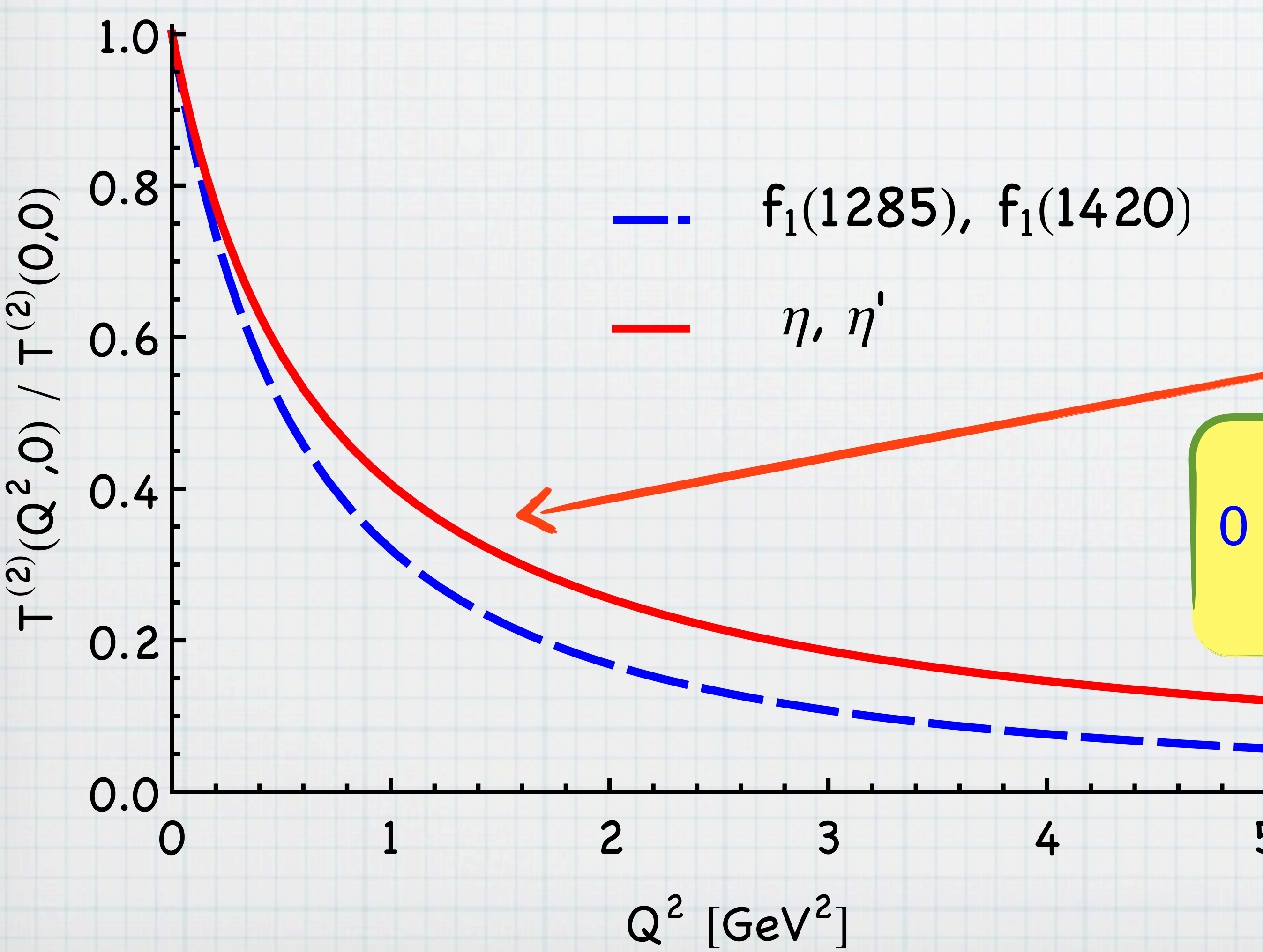

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Thank You
for attention!