Neutrinos versus the flavour puzzle

Belen Gavela

Neutrino Telescopes 2013, Venezia

(Alonso, Dhen, Hambye, B.G.)

(Alonso, D.Hernandez, Merlo, Rigolin, B.G.)

Beyond Standard Model because

1) **Experimental evidence** for new **particle** physics:

*** **Neutrino masses**

*** **Dark matter**

** **Matter-antimatter asymmetry**

2) **Uneasiness with SM fine-tunings**

Beyond Standard Model because

1) **Experimental evidence** for new **particle** physics:

***** Neutrino masses**

***** Dark matter**

**** Matter-antimatter asymmetry**

2) **Uneasiness with SM fine-tunings,** i.e. electroweak:

***** Hierarchy problem**

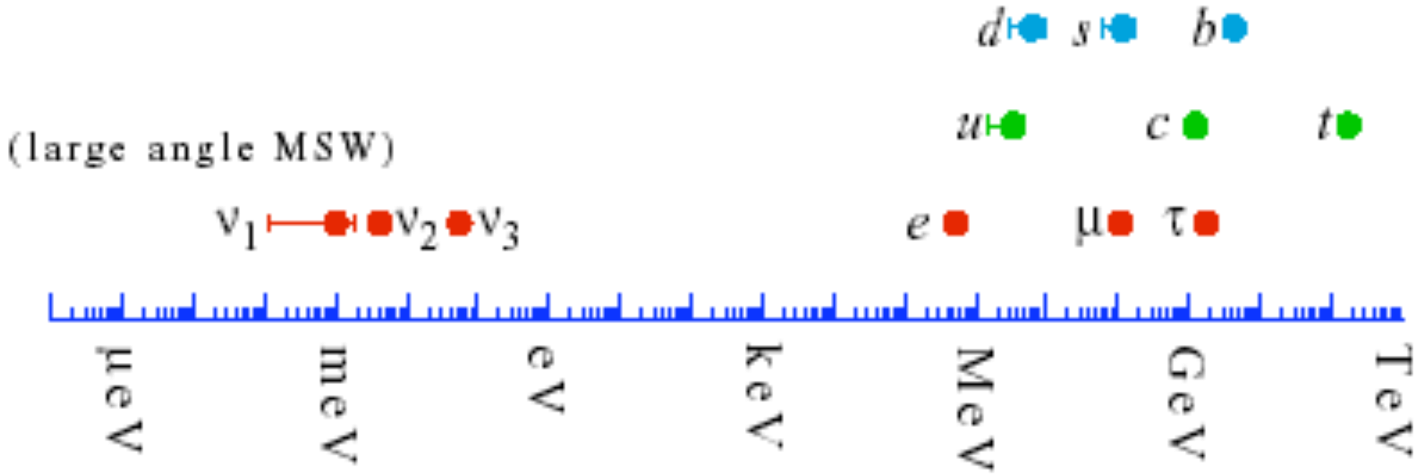
***** Flavour puzzle**

FLAVOUR is the real issue in BSM electroweak

* The understanding of the physics behind is stalled since decades

* Precious data for the puzzle e.g.: B’s, neutrinos

Neutrino light on flavour ?



Neutrinos lighter because Majorana?

Leptons

$$V_{PMNS} = \begin{pmatrix} 0.8 & 0.5 & \sim 9^o \\ -0.4 & 0.5 & -0.7 \\ -0.4 & 0.5 & +0.7 \end{pmatrix}$$

Quarks

$$V_{CKM} = \begin{pmatrix} \sim 1 & \lambda & \lambda^3 \\ \lambda & \sim 1 & \lambda^2 \\ \lambda^3 & \lambda^2 & \sim 1 \end{pmatrix} \quad \lambda \sim 0.2$$

Why so different?

Neutrino are optimal windows into the exotic -dark- sectors

* Can mix with new neutral fermions, heavy or light

* Interactions not obscured by strong and e.m. ones

Dark portals

Only three singlet combinations in SM with d < 4:

$H^{+} H$ Scalar

$B_{\mu\nu}$ Vector

$\bar{L} H$ Fermionic

Any hidden sector, singlet under SM, can couple to the dark portals

Dark portals

Only three singlet combinations in SM with d < 4:

$H^{+} H \mathbf{S}$	Scalar
$B_{\mu\nu} \mathbf{V}_{\mu\nu}$	Vector
$\bar{L} H \mathbf{\Psi}$	Fermionic

Any hidden sector, singlet under SM, can couple to the dark portals

Dark portals

Only three singlet combinations in SM with d < 4:

$H^{+} H \mathbf{S^2}$ Scalar

$B_{\mu\nu} \mathbf{V_{\mu\nu}}$ Vector

$\bar{L} \; H \; \mathbf{\Psi}$ Fermionic

Any hidden sector, singlet under SM, can couple to the dark portals

Dark portals

Only three singlet combinations in SM with d < 4:

$H^+ H \mathbf{S}^2$ Scalar

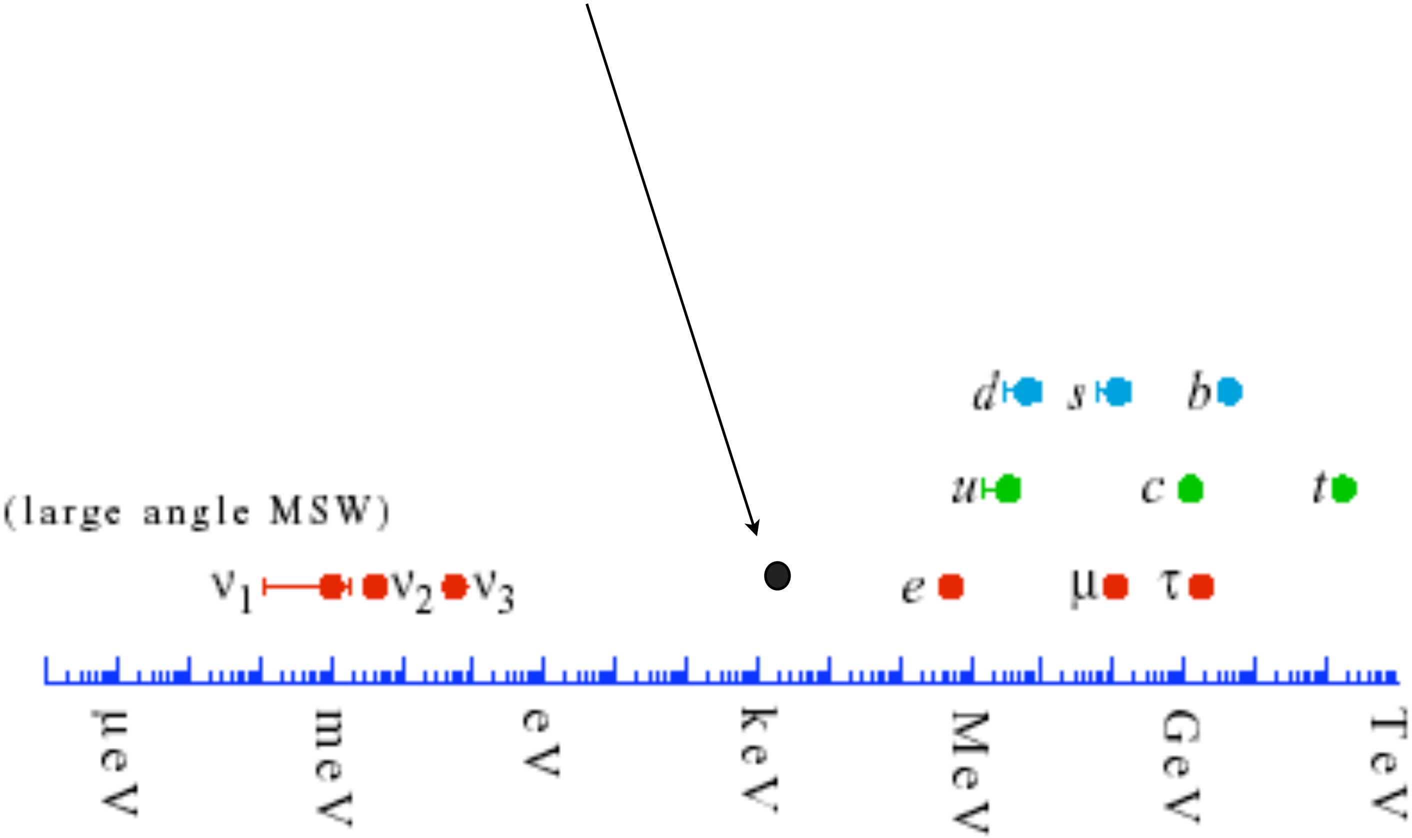
$B_{\mu\nu} \mathbf{V}_{\mu\nu}$ Vector

$\bar{L} \; H \; \mathbf{\Psi}$ Fermionic

Yukawa coupling

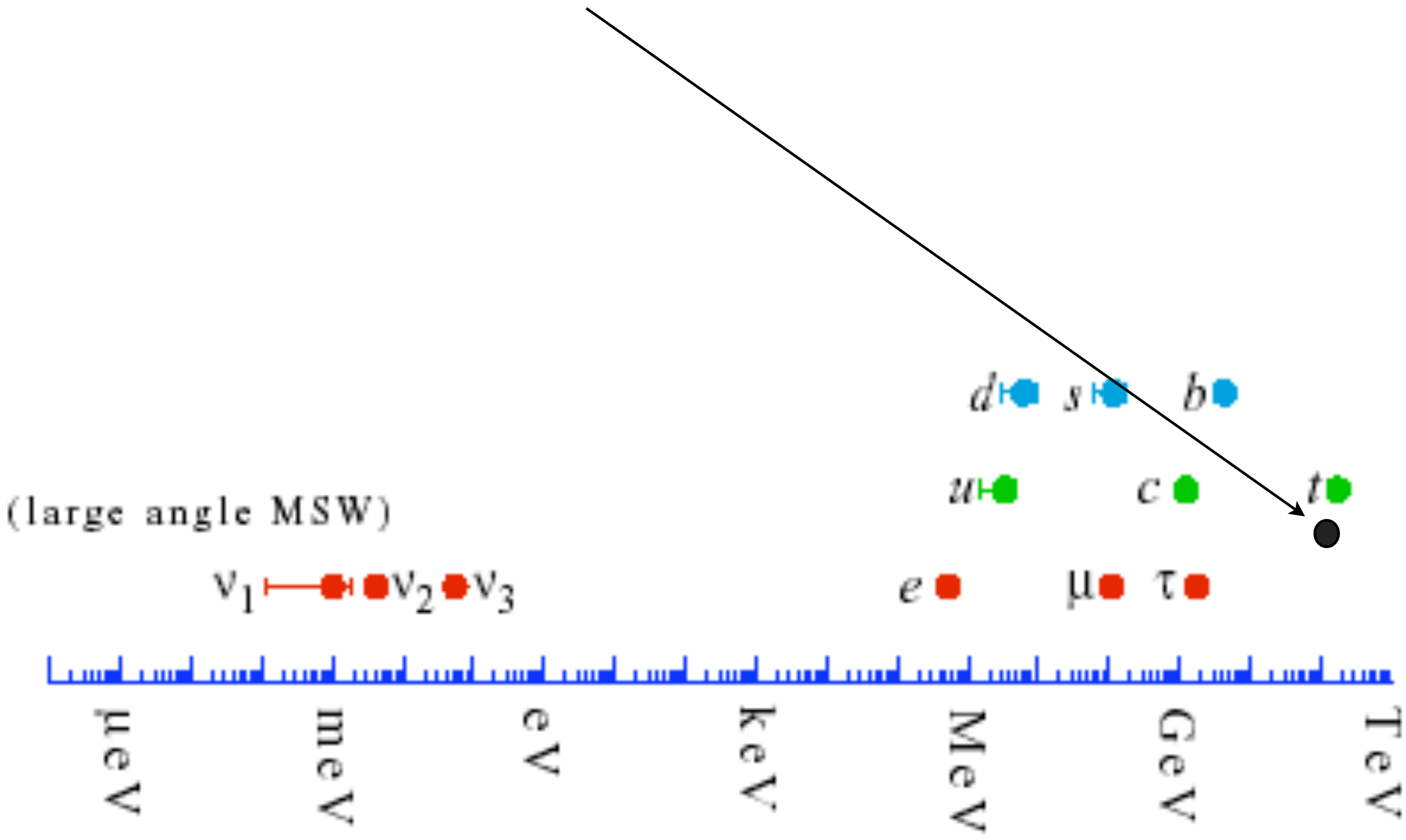
fermion singlets $\mathbf{\Psi}$ = “right-handed” neutrino

DARK FLAVOURS ?

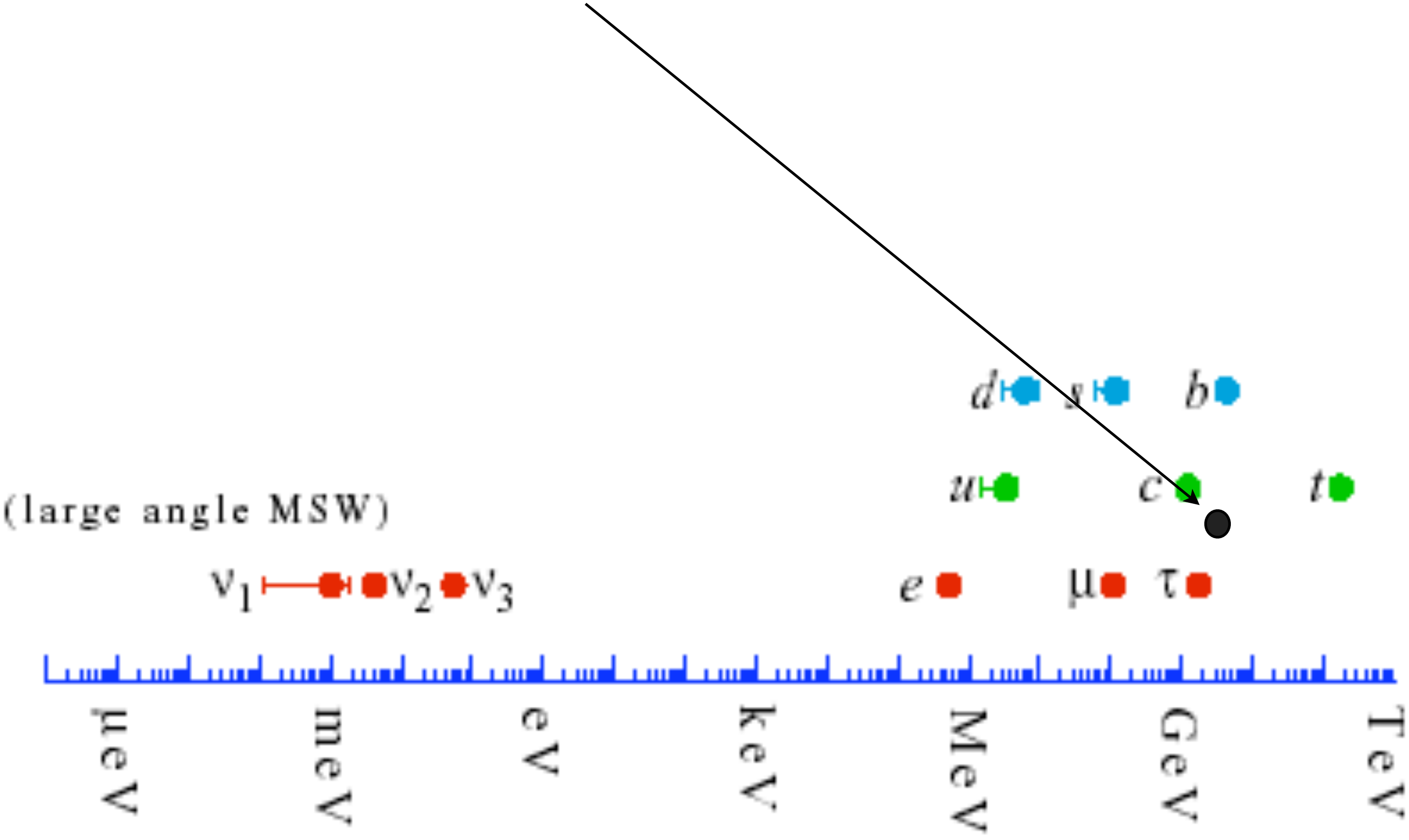


.... **they can be fermions**

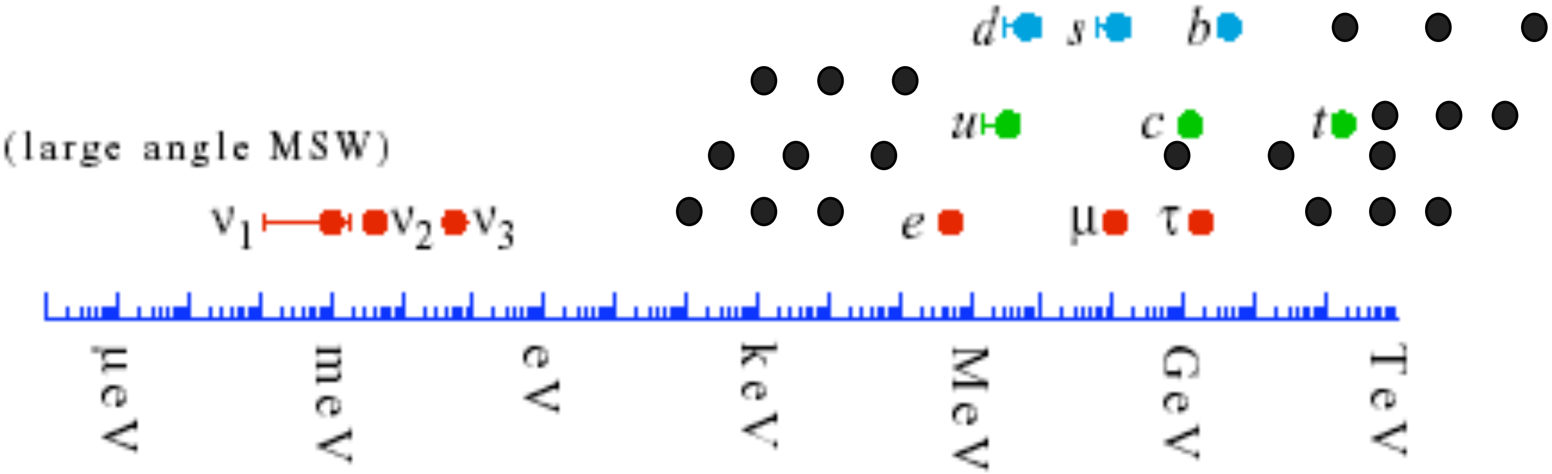
DARK FLAVOURS ?



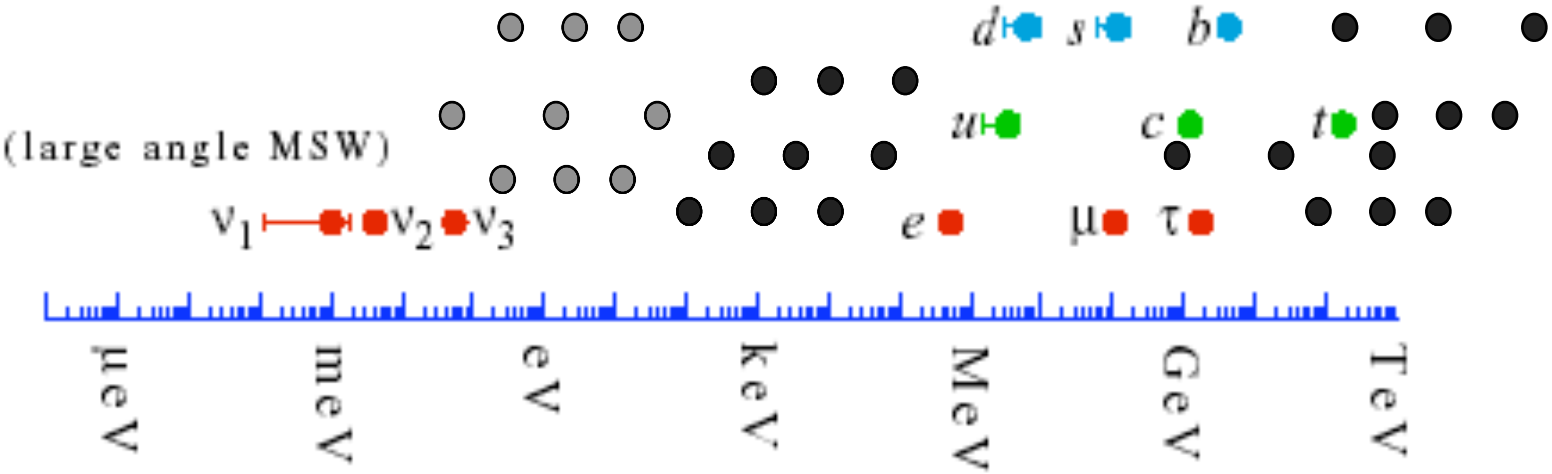
DARK FLAVOURS ?



DARK FLAVOURS ?



DARK FLAVOURS ?



Analysis of SM-**DM** with higer-dimensional ops. (d>= 4) starting:

- with and withour flavour associated to DM:

$$\frac{1}{\Lambda_{DM}^2} \overline{Q}_\alpha \gamma_\mu Q_\beta \quad \overline{\psi}_{DM\gamma} \gamma^\mu \psi_{DM\delta}$$

Lepton Flavour violation (LFV) windows:

*** Neutrino oscillations** <--neutral LFV

*** μ-->e γ**

*** μ-->e e e**

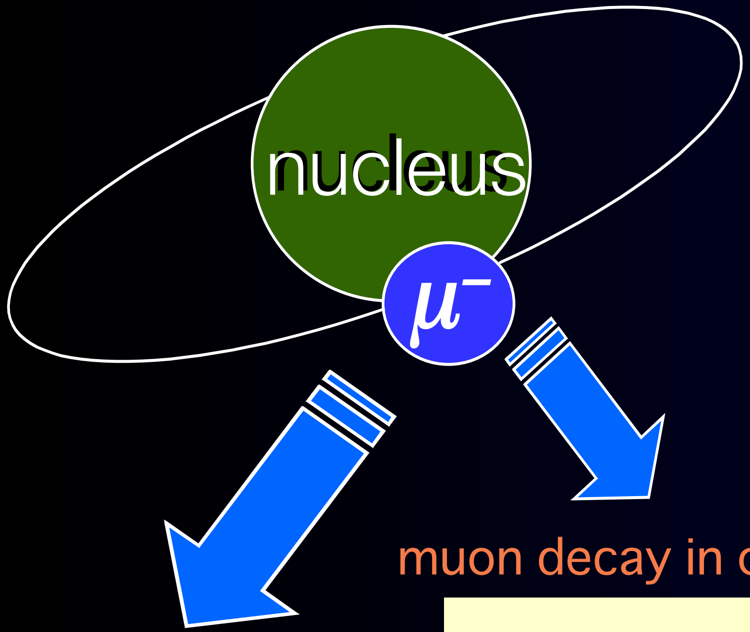
charged LFV

*** A fantastic experimental window being opened on lepton-flavour :**

μ-e conversion in nuclei

What is Muon to Electron Conversion?

1s state in a muonic atom



muon decay in orbit

$$\mu^- \rightarrow e^- \nu_\mu \overline{\nu}_e$$

nuclear muon capture

$$\mu^- + (A,Z) \rightarrow \nu_\mu + (A,Z-1)$$

Neutrino-less muon nuclear capture

$$\mu^- + (A,Z) \rightarrow e^- + (A,Z)$$

Event Signature :
a single mono-energetic electron of 100 MeV

Backgrounds:

(1) physics backgrounds
   ex. muon decay in orbit (DIO)

(2) beam-related backgrounds
   ex. radiative pion capture, muon decay in flight,

(3) cosmic rays, false tracking

courtesy of Yoshi Kuno

Consider together

μ-->e conversion

μ-->e γ

μ-->e e e

now ↓ **expected** ↓

μ-->e conversion

$$R^{Ti}_{\mu\to e} < 4.3\times 10^{-12} \text{ ---------------> } \lesssim 10^{-18}$$

$$R^{Au}_{\mu\to e} < 7\times 10^{-13}$$

$$R^{Pb}_{\mu\to e} < 4.6\times 10^{-11}$$

$$R^{Al}_{\mu\to e} \lesssim 10^{-16}$$

μ-->e γ

$$Br(\mu\to e\gamma) < 2.4\cdot 10^{-12} \text{ -----------------> } < 5\,10^{-14}$$

μ-->e e e

$$Br(\mu\to eee) < 10^{-12} \text{ ---------------> } < 10^{-16}$$

now **expected**

μ-->e conversion

$R^{Ti}_{\mu \to e} < 4.3 \times 10^{-12}$ ---------------> $\lesssim 10^{-18}$

$R^{Au}_{\mu \to e} < 7 \times 10^{-13}$

$R^{Pb}_{\mu \to e} < 4.6 \times 10^{-11}$

$R^{Al}_{\mu \to e} \lesssim 10^{-16}$

μ-->e γ

$Br(\mu \to e\gamma) <$ **5.7 10^{-13}** -----------------> $< 5\, 10^{-14}$

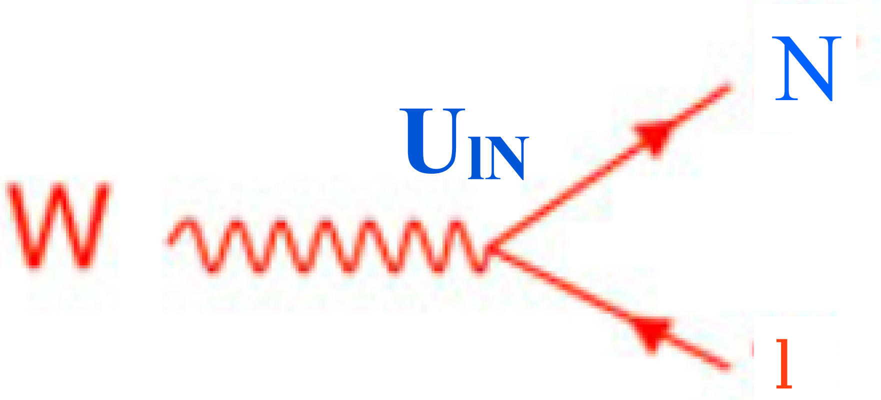
MEG

La Thuile 2013

μ-->e e e

$Br(\mu \to eee) < 10^{-12}$ ---------------> $< 10^{-16}$

Assume that singlet fermion(s) N exists in nature

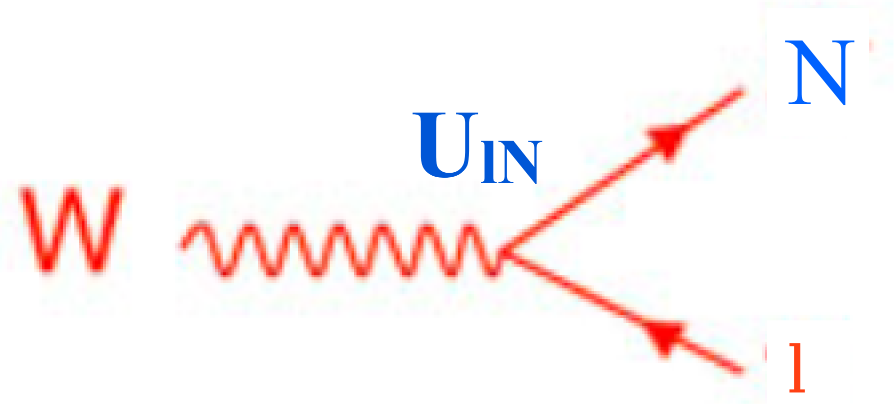


?

What are the limits on their mass m_N and mixings U_{lN} ?

Can we observe them?

Assume that singlet fermion(s) N exists in nature



?

What are the limits on their mass **m_N** and mixings **U_{lN}** ?

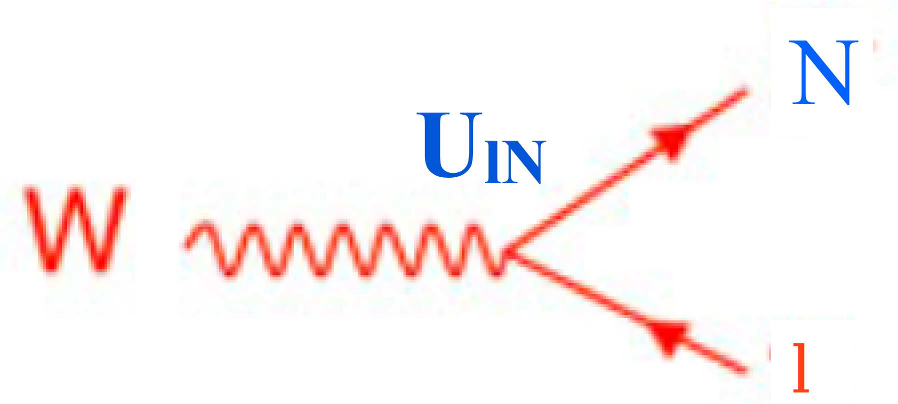
Can we observe them?

The paradigm model: Seesaw type-I N_R

$$\mathcal{L} = \mathcal{L}_{SM} + i\overline{N_R}\partial\!\!\!/ N_R - \left[\overline{N_R}Y_N\tilde{\phi}^\dagger \ell_L + \frac{1}{2}\overline{N_R}MN_R{}^c + h.c.\right]$$

$$U_{lN} \sim Y\ v / M$$

Assume that singlet fermion(s) N exists in nature



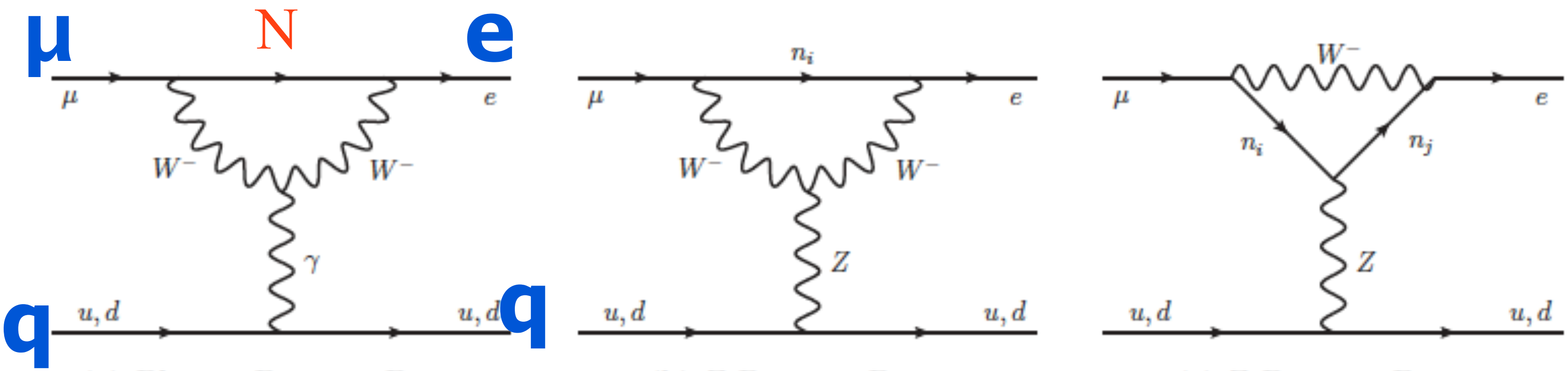
?

What are the limits on their mass m_N and mixings U_{lN} ?

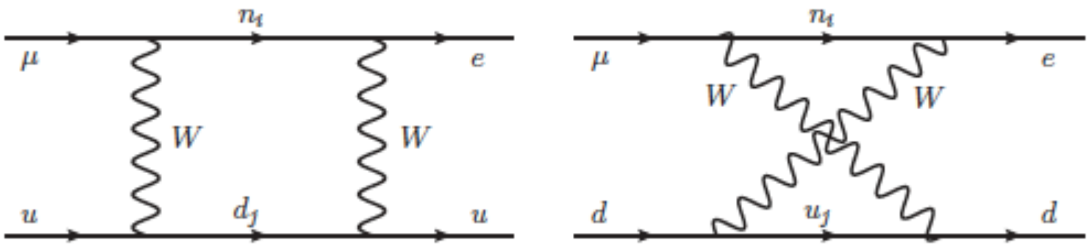
Can we observe them?

μ-->e conversion

mass eigenstates $n_i = \nu_1$, ν_2, ν_3, $N_1 \ldots N_k$



(a) Photon Penguin Diagram (b) Z Penguin Diagram (c) Z Penguin Diagram



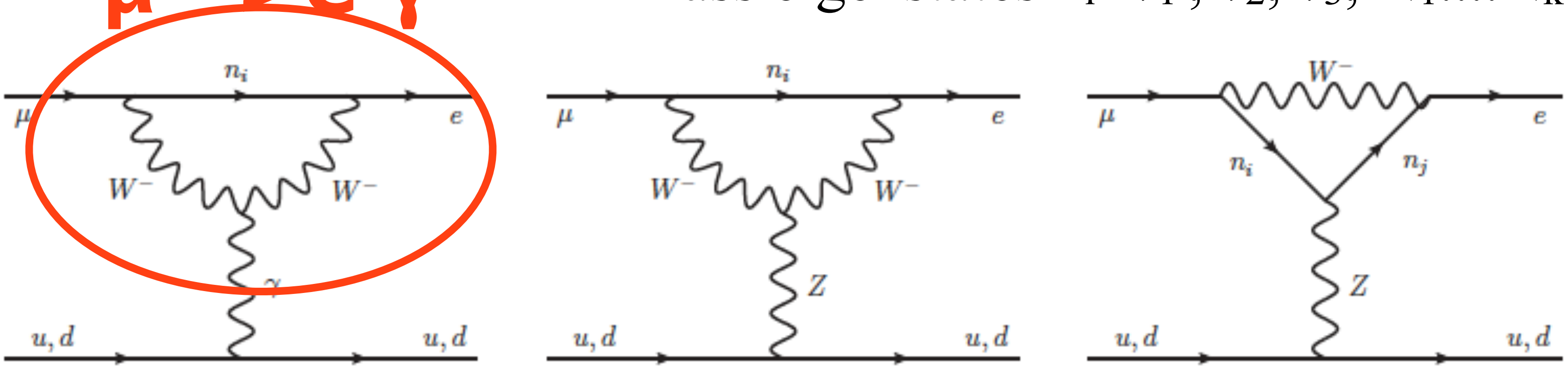
(d) Box Diagram (e) Box Diagram

Figure 1: The five classes of diagrams contributing to μ to e conversion in the type-I seesaw model.

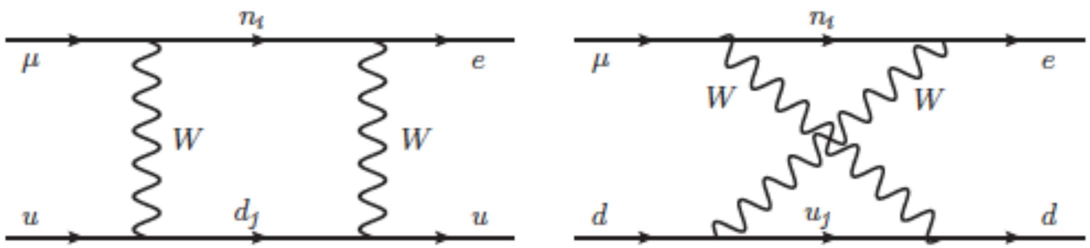
μ-->e conversion

μ-->e γ

mass eigenstates $n_i = \nu_1, \nu_2, \nu_3, N_1 \ldots N_k$



(a) Photon Penguin Diagram (b) Z Penguin Diagram (c) Z Penguin Diagram



(d) Box Diagram (e) Box Diagram

Figure 1: The five classes of diagrams contributing to μ to e conversion in the type-I seesaw model.

They share just one form factor ("dipole")

μ-->e conversion

μ-->e ee

mass eigenstates $n_i = \nu_1, \nu_2, \nu_3, N_1 \ldots N_k$

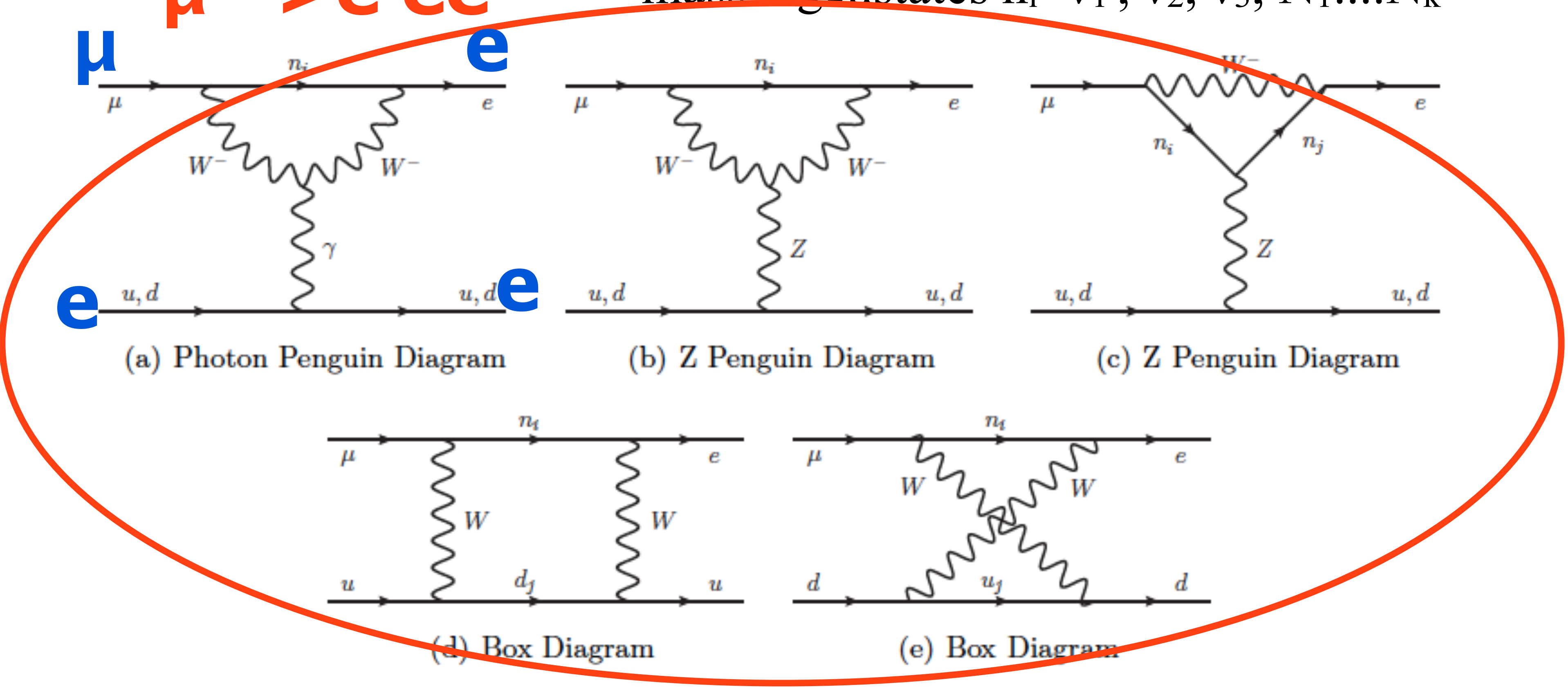


Figure 1: The five classes of diagrams contributing to μ to e conversion in the type-I seesaw model.

Share all form factors, in different combinations

μ-->e conversion

Many people before us computed it for singlet fermions:

De Gouvea
Mohapatra
Riazzudin+Marshak+Mohapatra 91,
Chang+Ng 94,
Ioannisian+Pilaftsis00,
Grimus + Lavoura
Pilaftsis and Underwood05,
Deppish+Kosmas+Valle06,
Ilakovac+Pilaftsis09,
Deppish+Pilaftsis11,
Dinh+Ibarra+Molinaro+Petcov12,
Aristizabal Sierra+Degee+Kamenik12

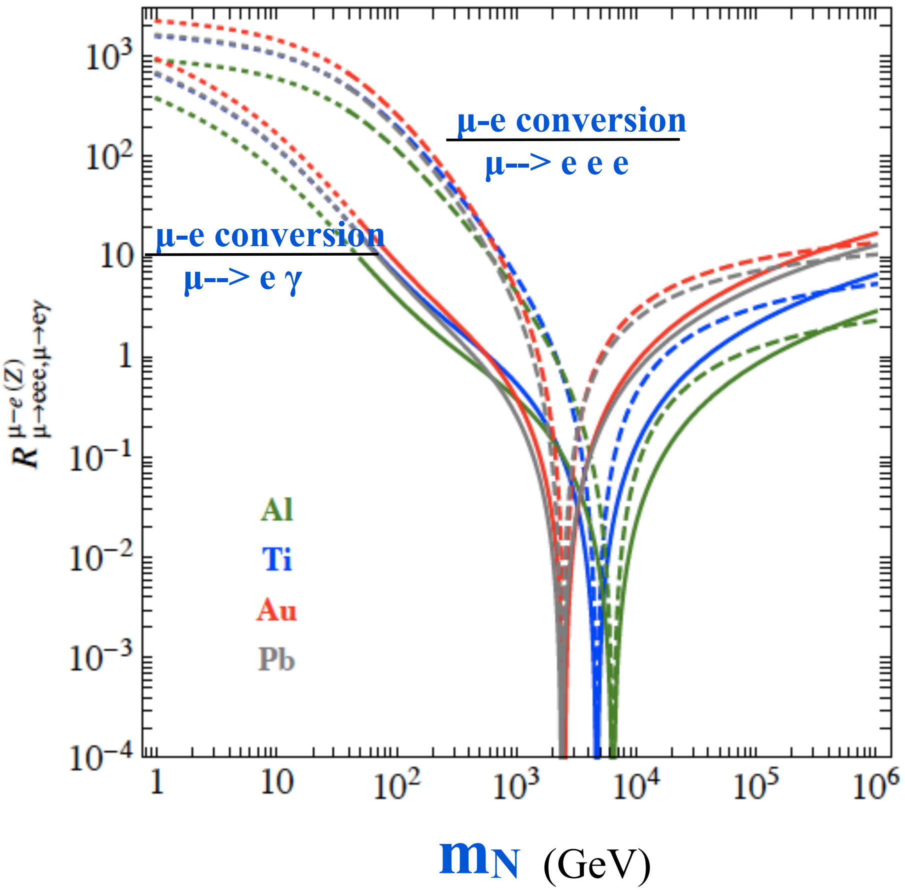
We agree for logarithmic dependence

Not two among those papers completely agree with each other, or they are not complete

typical applications assumed masses over 100 GeV or TeV

* **we computed all contributions (logarithmic and constant)**

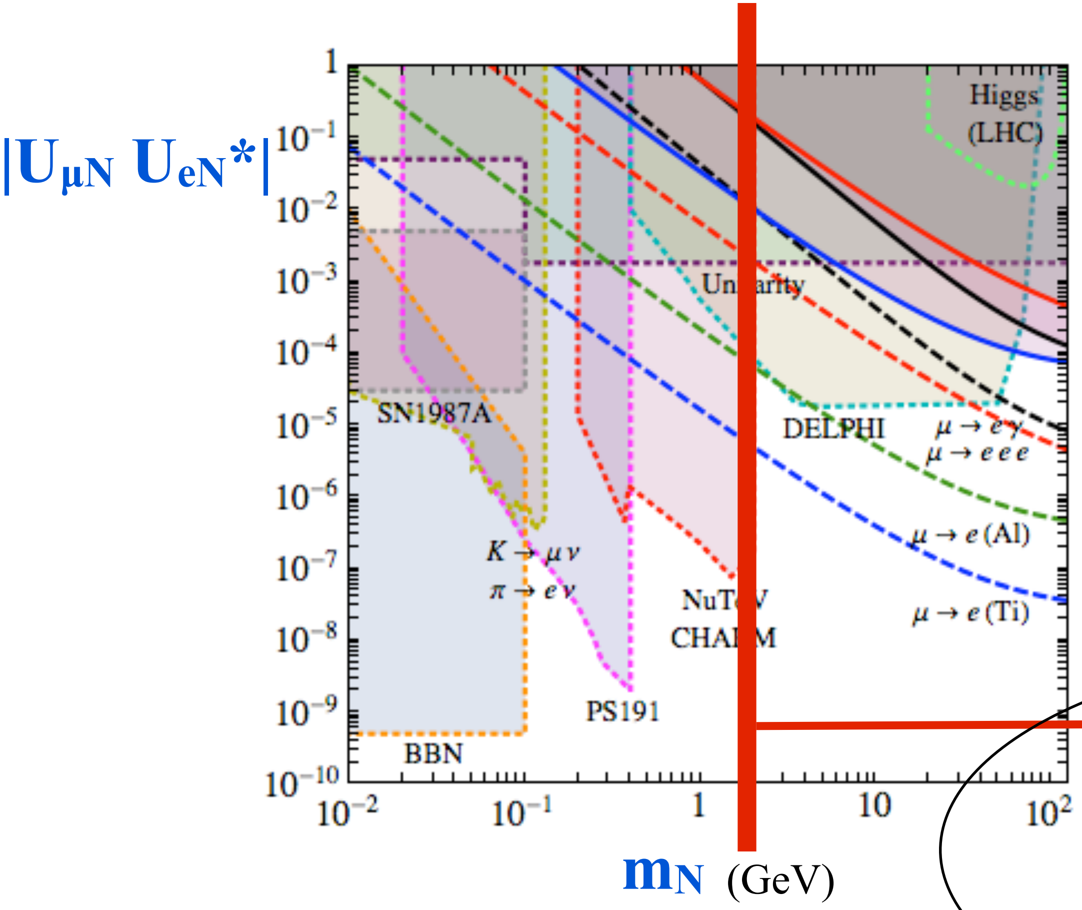
* **μ--> e conversion vanishes for masses in the 2-7 TeV mass regime**



* **we also considered the low mass region,**
  sweeping over eV< m_N < thousands GeV

(Alonso, Dhen, Hambye, B.G.)

In summary

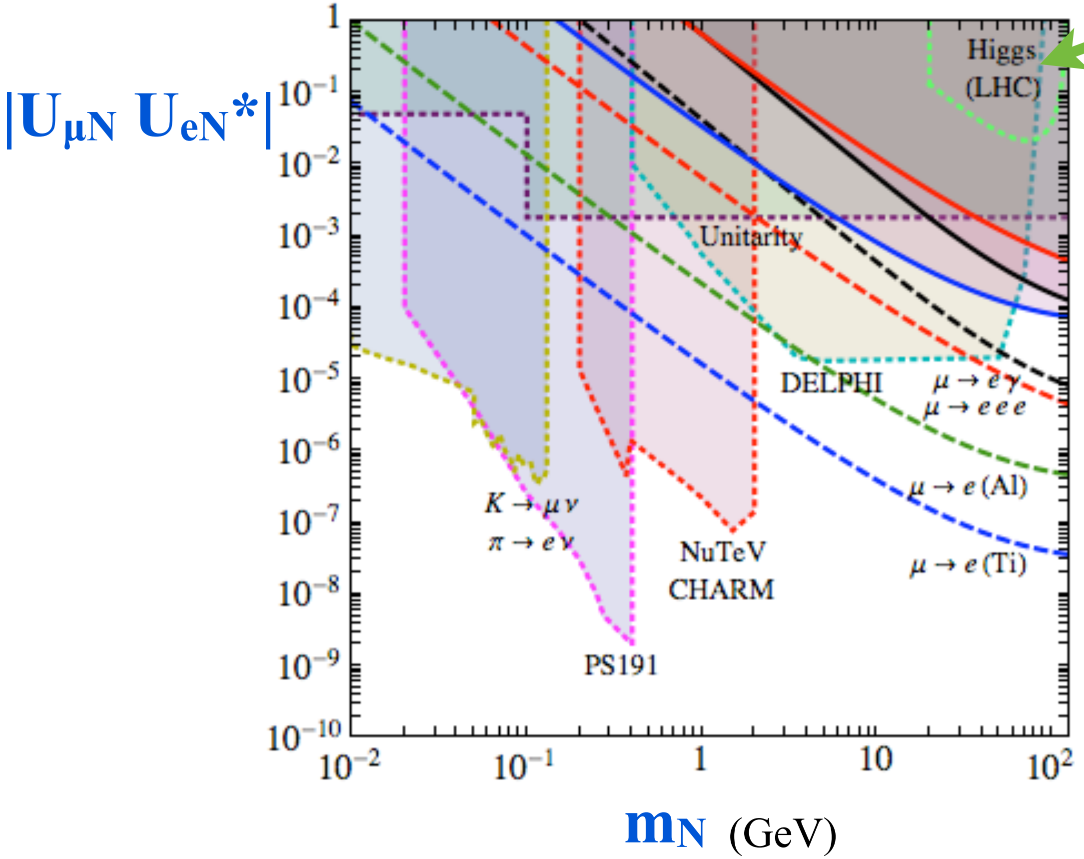


future sensitivity reach of
μ --> e conversion:

2 Gev --- 6000 TeV !

model-independent

(Alonso, Dhen, Hambye, B.G.)



Absolute bound from the decay of the SM scalar doublet, from absence of

$$H \dashrightarrow \nu\, N$$

$$\searrow$$

$$\nu\, l^+ l^-$$

at LHC:

Br (H -> ν N) < 0.4

(Espinosa, Grojean, Muhlleitner, Trott, 12 Dev+Franceschini+Mohapatra12, Cely+ Ibarra+Molinaro+Petcov 12)

LHC is more competitive for concrete seesaw models:

Low M, large Y is typical of seesaws with approximate Lepton Number conservation

$U(1)_{LN}$

(-> ~ degenerate heavy neutrinos)

These models separate the flavor and the lepton number scale

Wyler+Wolfenstein 83, Mohapatra+Valle 86, Branco+Grimus+Lavoura 89, Gonzalez-Garcia+Valle 89, Ilakovac+Pilaftsis 95, Barbieri+Hambye+Romanino 03, Raidal+Strumia+Turzynski 05, Kersten+Smirnov 07, Abada+Biggio+Bonnet+Gavela+Hambye 07, Shaposhnikov 07, Asaka+Blanchet 08, Gavela+Hambye+D. Hernandez+ P. Hernandez 09

seesaw I with **Just TWO heavy neutrinos**

$$\mathcal{L}_{\mathcal{M}_\nu} = (\bar{\ell}_L\,,\,\bar{N}^c\,,\,\bar{N}'^c)\begin{pmatrix} 0 & vY & vY' \\ vY^T & 0 & \mathbf{M} \\ vY'^T & \mathbf{M} & 0 \end{pmatrix}\begin{pmatrix} \ell_L^c \\ N \\ N' \end{pmatrix}$$

Lepton number scale and flavour scale distinct

Raidal, Strumia, Turszynski
Gavela, Hambye, Hernandez^2

Just TWO heavy neutrinos

$$\mathcal{L}_{\mathcal{M}_\nu} = (\bar{\ell}_L\,,\, \bar{N}^c\,,\, \bar{N}'^c) \begin{pmatrix} 0 & vY & vY' \\ vY^T & 0 & \mathbf{M} \\ vY'^T & \mathbf{M} & 0 \end{pmatrix} \begin{pmatrix} \ell_L^c \\ N \\ N' \end{pmatrix}$$

$$m_\nu = \mathbf{Y} \frac{v^2}{M} \mathbf{Y}'^{\,T} \qquad\qquad \mathbf{U}_{\mathrm{lN}} \sim \frac{\mathbf{Y}}{M}$$

--> Lepton number conserved conserved if either Y or Y’ vanish:

Raidal, Strumia, Turszynski
Gavela, Hambye, Hernandez[2]

Just TWO heavy neutrinos

$$\mathcal{L}_{\mathcal{M}_\nu} = (\bar{\ell}_L\,,\,\bar{N}^c\,,\,\bar{N}'^c)\begin{pmatrix} 0 & vY & vY' \\ vY^T & 0 & \mathbf{M} \\ vY'^T & \mathbf{M} & 0 \end{pmatrix}\begin{pmatrix} \ell_L^c \\ N \\ N' \end{pmatrix}$$

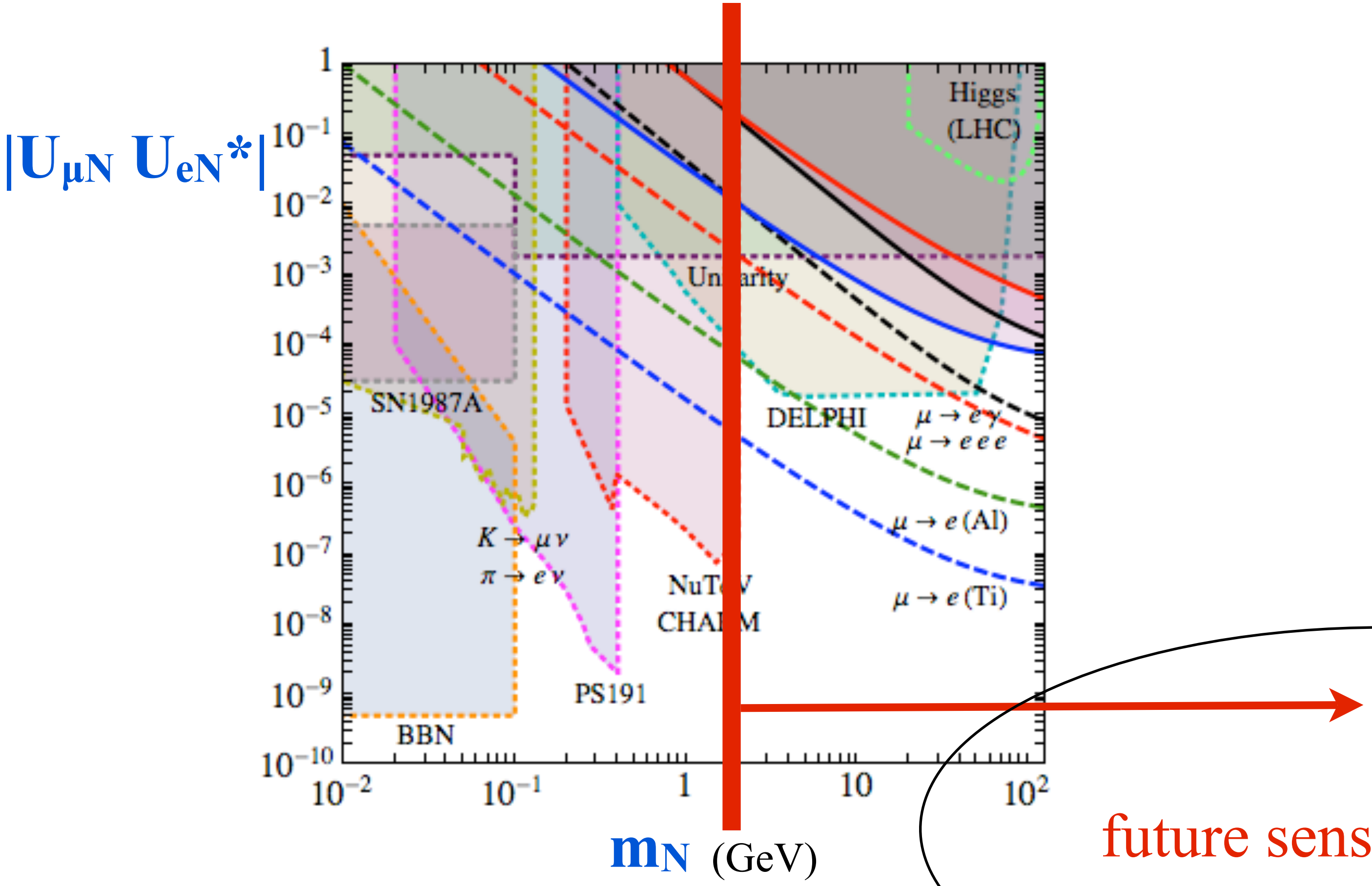
--> One massless neutrino and only one Majorana phase α

the Yukawas are determined up to their overal magnitude

$$\text{N.H.} \qquad Y = \frac{y}{\sqrt{m_{\nu_2} + m_{\nu_3}}}\, U_{PMNS}\begin{pmatrix} 0 \\ -i\sqrt{m_{\nu_2}}e^{-i\alpha} \\ \sqrt{m_{\nu_3}}e^{i\alpha} \end{pmatrix}$$

Gavela, Hambye, Hernandez^2
Raidal, Strumia, Turszynski

In summary



future sensitivity reach of
μ --> e conversion:

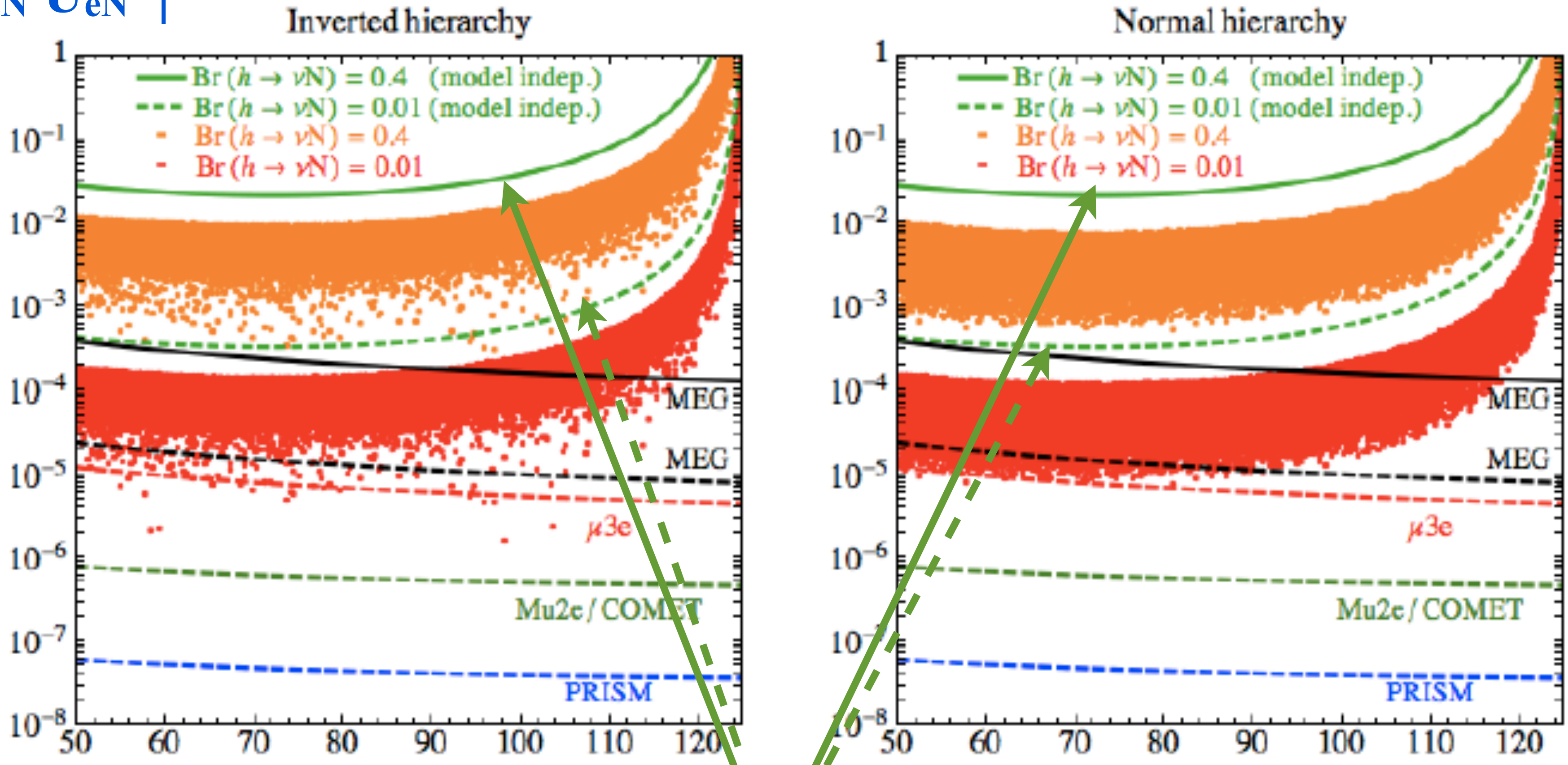
model-independent

2 Gev --- #1000 TeV !

(Alonso, Dhen, Hambye, B.G.)

Varying the CP phases α and δ, we get:

$|U_{\mu N}\, U_{eN}^*|$



m_N (GeV) m_N (GeV)

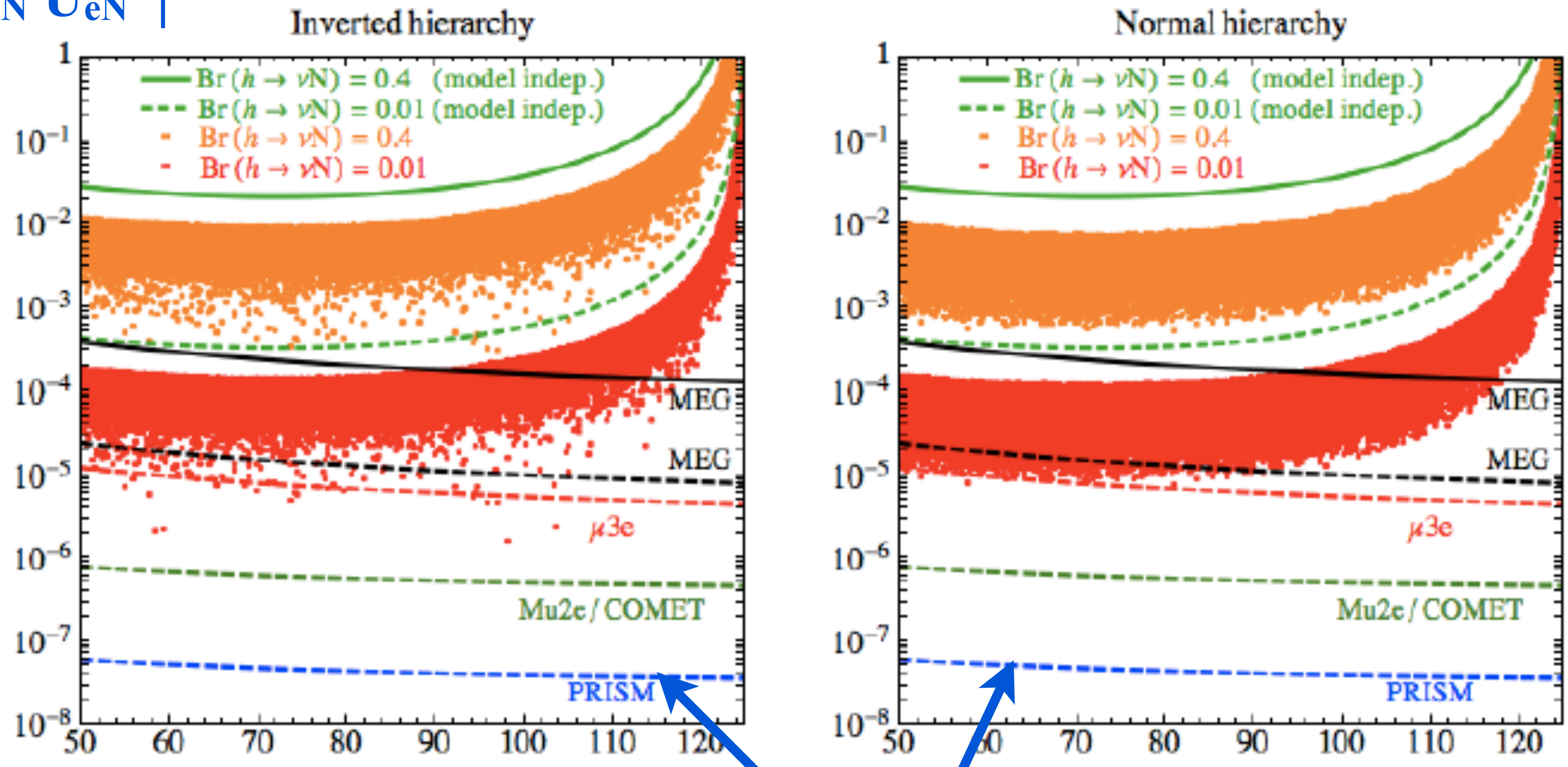
Absolute bound in previous plot

$|U_{\mu N}\, U_{eN}^*|$ versus m_N

~ consistent with Cely et al. 2012, for α~0, δ~ 0

Varying the CP phases α and δ, we get:

$|U_{\mu N}\, U_{eN}^*|$

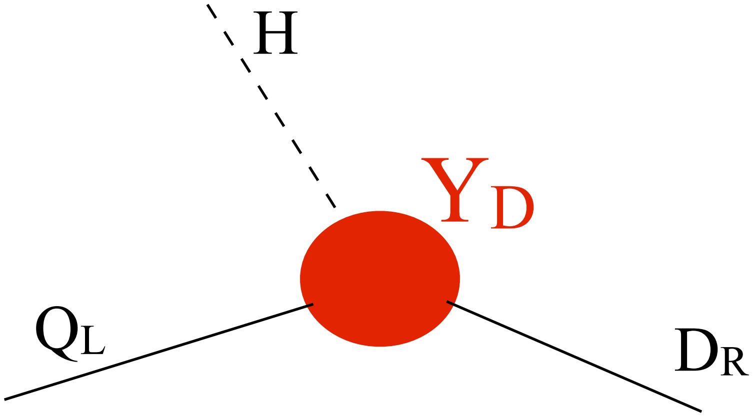


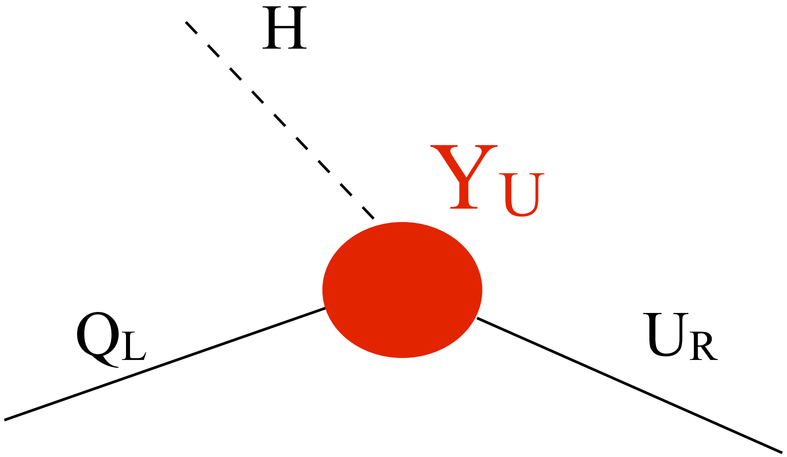
m_N (GeV) m_N (GeV)

In any case, LHC expected sensitivity negligible compared with that of future μ--> e conversion expts.

- Dynamical Yukawas

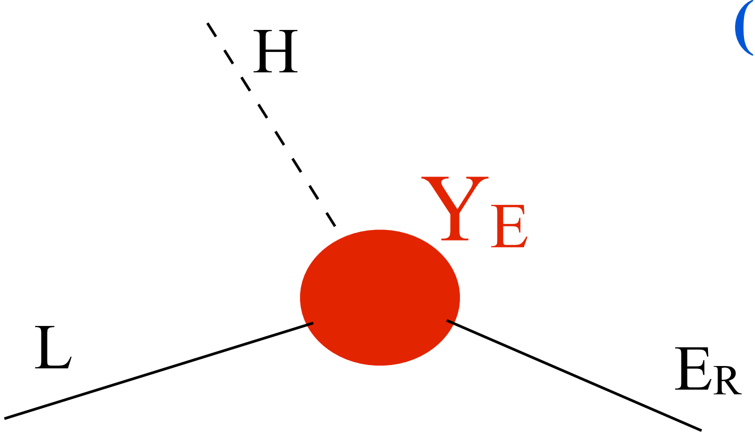
Yukawa couplings are the source of flavour in the SM

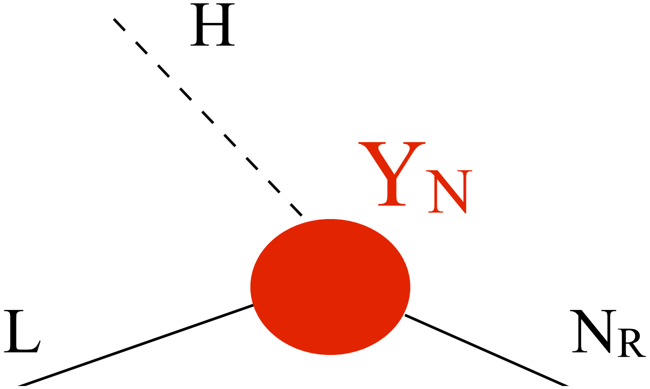




Yukawa couplings are a source of flavour in the ν-SM

(i.e. Seesaw type-I)





$$\mathcal{L} = \mathcal{L}_{SM} + i\overline{N_R}\partial\!\!\!/ N_R - \left[\overline{N_R} Y_N \tilde{\phi}^\dagger \ell_L + \frac{1}{2}\overline{N_R} M N_R{}^c + h.c.\right]$$

May they correspond to
dynamical fields
(e.g. vev of fields that carry flavor) ?

In many BSM the Yukawas do not come from dynamical fields:

Some good ideas:

D.B. Kaplan-Georgi in the 80's proposed a light SM scalar because being a (quasi) goldstone boson: ***composite Higgs***

(D.B. Kaplan, Georgi, Dimopoulos, Banks, Dugan, Galison.......Contino, Nomura, Pomarol; Agashe, Contino, Pomarol; Giudice, Pomarol, Ratazzi, Grojean; Contino, Grojean, Moretti; Azatov, Galloway, Contino... Frigerio, Pomarol, Riva, Urbano...)

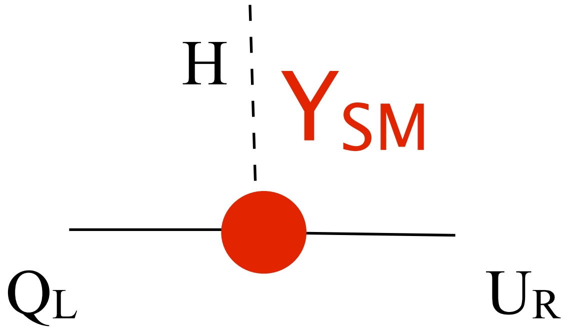
Some good ideas:

D.B. Kaplan-Georgi in the 80's proposed a light SM scalar because being a (quasi) goldstone boson: ***composite Higgs***

Flavour "Partial compositeness" D.B Kaplan 91:

A sort of **"seesaw for quarks"**

(nowadays sometimes justified from extra-dim physics)



$m_q = v\, Y_{SM}$

(D.B Kaplan 91; Redi, Weiler; Contino, Kramer, Son, Sundrum; da Rold, Delauney, Grojean, G. Perez; Contino, Nomura, Pomarol, Agashe, Giudice, Perez, Panico, Redi, Wulzer...)

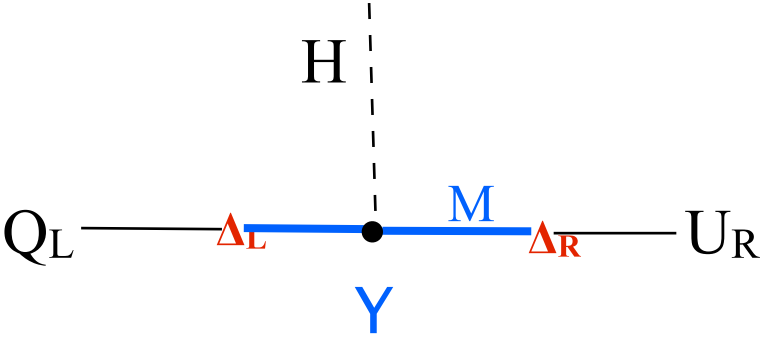
Some good ideas:

D.B. Kaplan-Georgi in the 80's proposed a Higgs light because being a (quasi) goldstone boson: ***composite Higgs***

"Partial compositeness":

A sort of **"seesaw for quarks"**

(nowadays sometimes justified from extra-dim physics)



$$Y_{SM} = Y \Delta_L \Delta_R / M^2$$

$$m_q = v\, Y_{SM}$$

(D.B Kaplan 91; Redi, Weiler; Contino, Kramer, Son, Sundrum; da Rold, Delauney, Grojean, G. Perez; Contino, Nomura, Pomarol, Agashe, Giudice, Perez, Panico, Redi, Wulzer...)

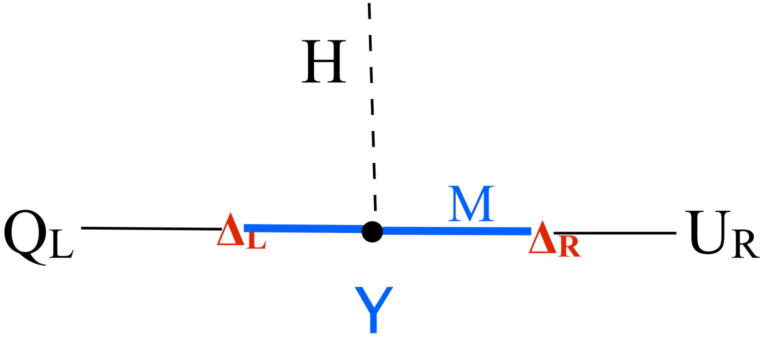
Some good ideas:

D.B. Kaplan-Georgi in the 80’s proposed a Higgs light because being a (quasi) goldstone boson: ***composite Higgs***

“Partial compositeness”:

A sort of **“seesaw for quarks”**

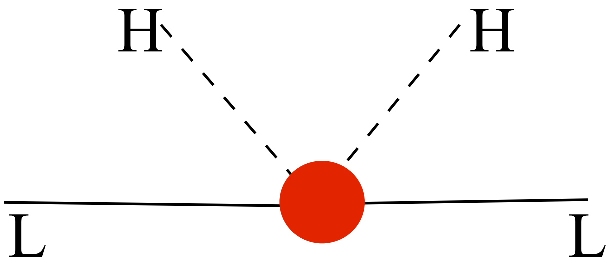
(nowadays sometimes justified from extra-dim physics)



$$Y_{SM} = Y \Delta_L \Delta_R / M^2$$

$$m_q = v\, Y_{SM}$$

Neutrino masses:



d=5 Weinberg operator

(D.B Kaplan 91; Redi, Weiler; Contino, Kramer, Son, Sundrum; da Rold, Delauney, Grojean, G. Perez; Contino, Nomura, Pomarol, Agashe, Giudice, Perez, Panico, Redi, Wulzer...)

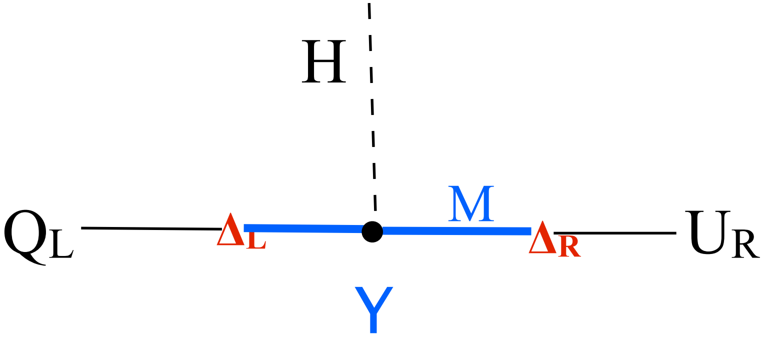
Some good ideas:

D.B. Kaplan-Georgi in the 80’s proposed a Higgs light because being a (quasi) goldstone boson: ***composite Higgs***

“Partial compositeness”:

A sort of **“seesaw for quarks”**

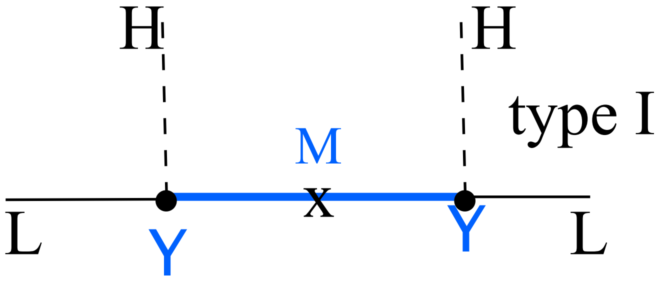
(nowadays sometimes justified from extra-dim physics)



$$Y_{SM} = Y \Delta_L \Delta_R / M^2$$

$$m_q = v\, Y_{SM}$$

Neutrino masses:



$$m_\nu = Y\, v^2/M\, Y^T$$

(D.B Kaplan 91; Redi, Weiler; Contino, Kramer, Son, Sundrum; da Rold, Delauney, Grojean, G. Perez; Contino, Nomura, Pomarol, Agashe, Giudice, Perez, Panico, Redi, Wulzer...)

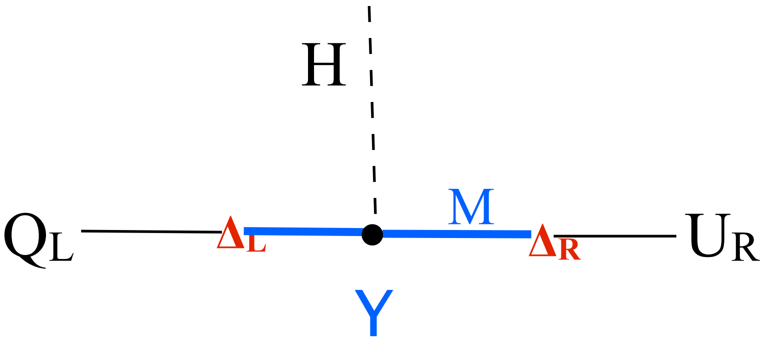
Some good ideas:

D.B. Kaplan-Georgi in the 80's proposed a Higgs light because being a (quasi) goldstone boson: ***composite Higgs***

"Partial compositeness":

A sort of **"seesaw for quarks"**

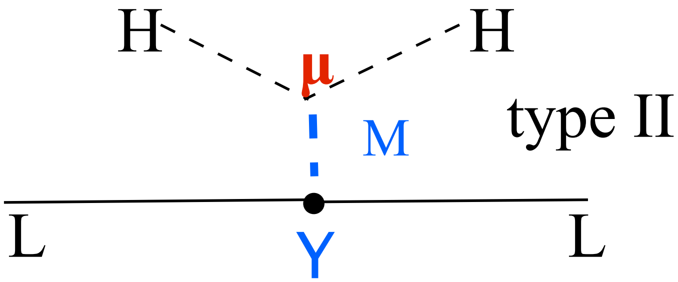
(nowadays sometimes justified from extra-dim physics)



$$Y_{SM} = Y \Delta_L \, \Delta_R / M^2$$

$$m_q = v \, Y_{SM}$$

Neutrino masses:



$$m_\nu = Y \, \mu \, v^2 / M^2$$

(D.B Kaplan 91; Redi, Weiler; Contino, Kramer, Son, Sundrum; da Rold, Delauney, Grojean, G. Perez; Contino, Nomura, Pomarol, Agashe, Giudice, Perez, Panico, Redi, Wulzer...)

In other BSM Yukawas do correspond to dynamical fields:

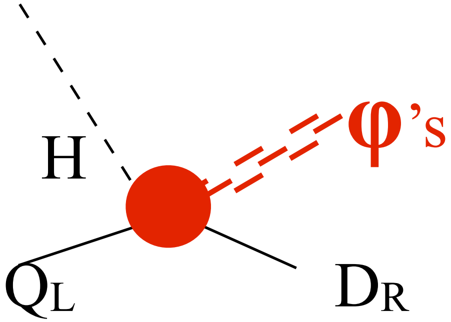
Discrete symmetry ideas:

The Yukawas are indeed explained in terms of dynamical fields.
And they do not need to worry about goldstone bosons.

In spite of θ_{13} not very small, some activity.
For instance, combine generalized CP (Bernabeu, Branco, Gronau 80s) with discrete Z_2 groups in the neutrino sector : maximal θ_{23}, strong constraints on values of CP phases
(Feruglio, Hagedorn and Ziegler 2013; Holthausen, Lindner and Schmidt 2013)

- Discrete approaches do not relate mixing to spectrum
- Difficulties to consider both quarks and leptons

Some good ideas:



Frogatt-Nielsen '79: $\mathbf{U(1)_{flavour}}$ symmetry

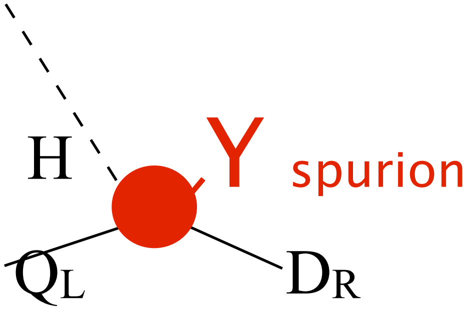
- Yukawa couplings are effective couplings,
- Fermions have $U(1)_{flavour}$ charges

$$\left(\frac{<\varphi>}{\Lambda}\right)^n Q\, H\, q_R \quad , \quad Y \sim \left(\frac{<\varphi>}{\Lambda}\right)^n$$

e.g. n=0 for the top, n large for light quarks, etc.

--> FCNC ?

Some good ideas:



Minimal Flavour Violation:

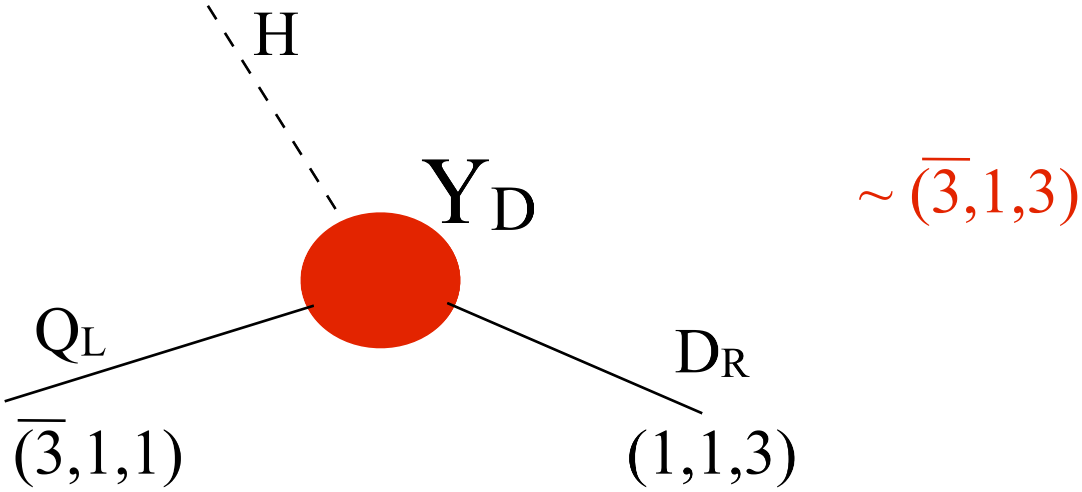
- Use the flavour symmetry of the SM in the limit of massless fermions (Georgi+Chivukula)

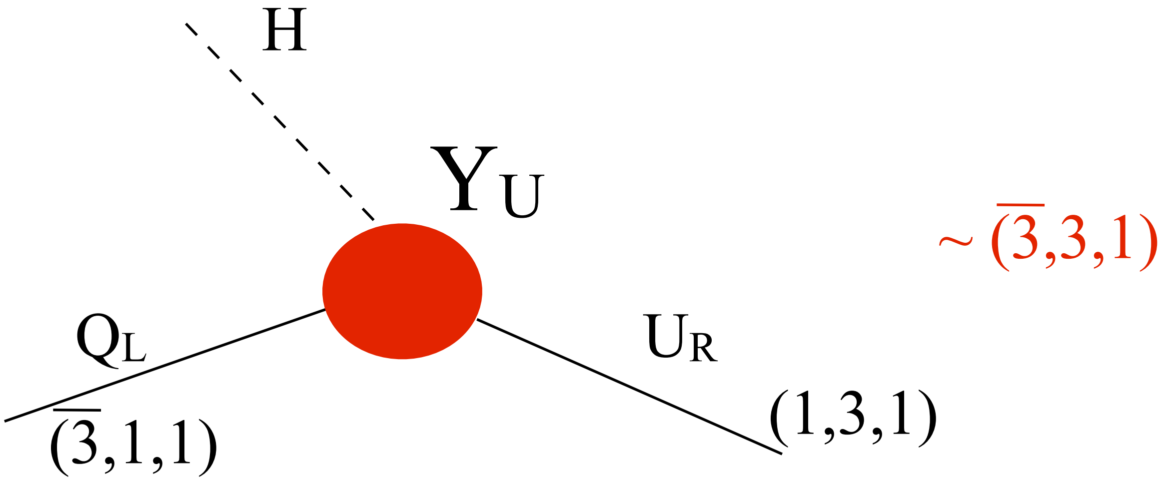
quarks: $G_{flavour} = U(3)_{QL} \times U(3)_{UR} \times U(3)_{DR}$

The non-abelian part of the flavour symmetry of the SM:

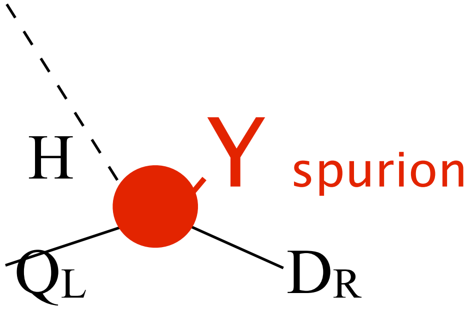
$$G_f = SU(3)_{Q_L} \text{ x } SU(3)_{U_R} \text{ x } SU(3)_{D_R}$$

broken by Yukawas:





Some good ideas:



Minimal Flavour Violation:

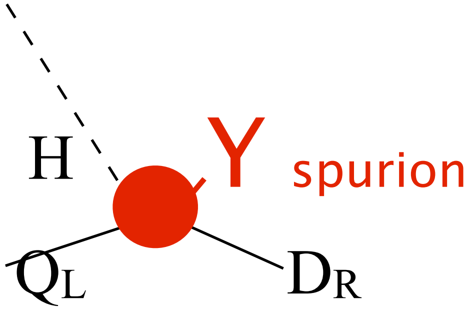
- Use the flavour symmetry of the SM in the limit of massless fermions (**Chivukula+ Georgi**)

quarks: $\mathbf{G_{flavour} = U(3)_{QL} \times U(3)_{UR} \times U(3)_{DR}}$

- Assume that Yukawas are the only source of flavour in the SM and beyond

$$\frac{1}{\Lambda_{flavour}^2} \overline{Q_\alpha} \gamma_\mu Q_\beta \overline{Q_\gamma} \gamma^\mu Q_\delta$$

Some good ideas:



Minimal Flavour Violation:

- Use the flavour symmetry of the SM in the limit of massless fermions (**Chivukula+ Georgi**)

quarks: $G_{flavour} = U(3)_{Q_L} \times U(3)_{U_R} \times U(3)_{D_R}$

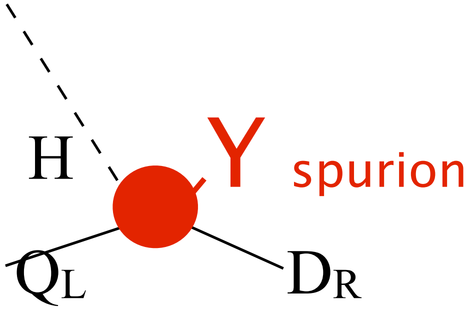
- Assume that Yukawas are the only source of flavour in the SM and beyond

$$\frac{Y_{\alpha\beta}^{+} Y_{\delta\gamma}}{\Lambda_{flavour}^{2}} \overline{Q}_{\alpha} \gamma_{\mu} Q_{\beta} \overline{Q}_{\gamma} \gamma^{\mu} Q_{\delta}$$

... agrees with flavour data being aligned with SM
... allows to bring down $\Lambda_{flavour}$ --> TeV

D'Ambrosio+Giudice+Isidori+Strumia;
Cirigliano+Isidori+Grinstein+Wise

Some good ideas:



Minimal Flavour Violation:

- Use the flavour symmetry of the SM in the limit of massless fermions (Chivukula+ Georgi)

quarks: $G_{flavour} = U(3)_{QL} \text{ x } U(3)_{UR} \text{ x } U(3)_{DR}$

- Assume that Yukawas are the only source of flavour in the SM and beyond

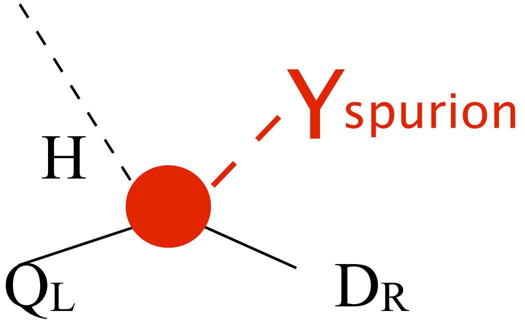
$$\frac{Y_{\alpha\beta}^{+} Y_{\delta\gamma}}{\Lambda_{flavour}^{2}} \overline{Q}_{\alpha} \gamma_{\mu} Q_{\beta} \overline{Q}_{\gamma} \gamma^{\mu} Q_{\delta}$$

... agrees with flavour data being aligned with SM
... allows to bring down $\Lambda_{flavour}$ --> TeV

(Chivukula+Georgi 87; Hall+Randall; D'Ambrosio+Giudice+Isidori+Strumia; Cirigliano+Isidori+Grisntein +Wise; Davidson+Pallorini; Kagan+G. Perez + Volanski+Zupan,...)

Lalak, Pokorski, Ross; Fitzpatrick, Perez, Randall; Grinstein, Redi, Villadoro

Some good ideas:



Related to MFV:

- Use the flavour symmetry of the SM in the limit of massless fermions

quarks: $G_{\text{flavour}} = U(3)_{Q_L} \text{ x } U(3)_{U_R} \text{ x } U(3)_{D_R}$

Hybrid dynamical-non-dynamical Yukawas:

U(2) (Pomarol, Tomasini; Barbieri, Dvali, Hall, Romanino...)....

$U(2)^3$ (Craig, Green, Katz; Barbieri, Isidori, Jones-Peres, Lodone, Straub.. ..Sala)

Sequential ideas (Feldman, Jung, Mannel; Berezhiani+Nesti; Ferretti et al., Calibbi et al. ...)

One step further

For this talk:

each Y_{SM} -- >one single field $\mathcal{Y}$

$$Y_{SM} \sim \frac{<\mathcal{Y}>}{\Lambda_{fl}}$$

transforming under the SM flavour group

Anselm+Berezhiani 96; Berezhiani+Rossi 01; Alonso+Gavela+Merlo+Rigolin 11...

For this talk:

each Y_{SM} -- >one single field $\mathcal{Y}$

$$Y_{SM} \sim \frac{< \mathcal{Y} >}{\Lambda_{fl}}$$

Can it shed light on why quark and neutrino mixings are so different?

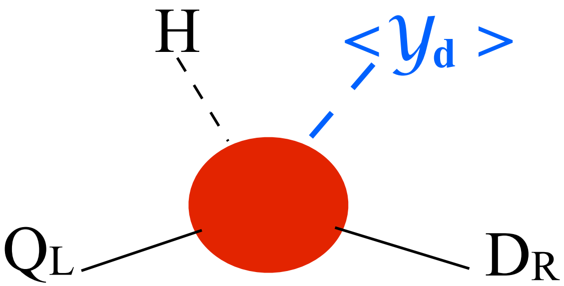
Alonso, B.G., D. Hernandez, L. Merlo, Rigolin

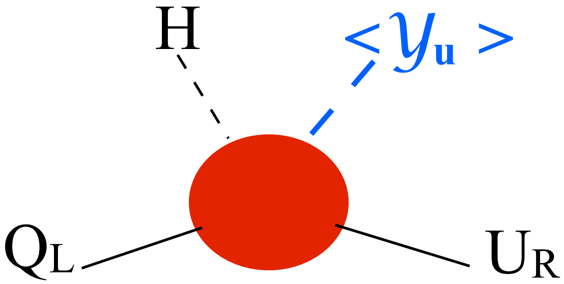
For this talk:

each Y_{SM} -- >one single field $\mathcal{Y}$

$$Y_{SM} \sim \frac{< \mathcal{Y} >}{\Lambda_{fl}}$$

quarks:

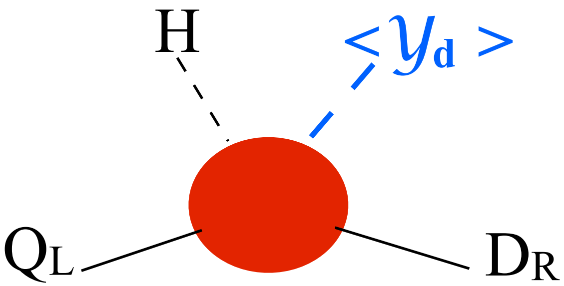


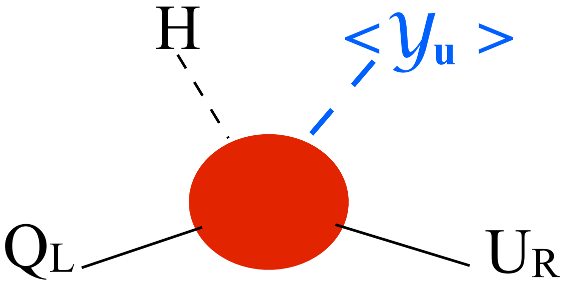


For this talk:

each Y_{SM} -- >one single field $\mathcal{Y}$

$$Y_{SM} \sim \frac{< \mathcal{Y} >}{\Lambda_{fl}}$$



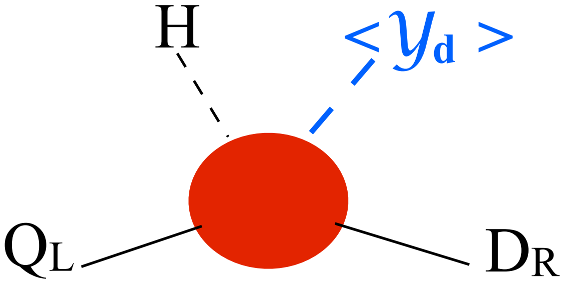


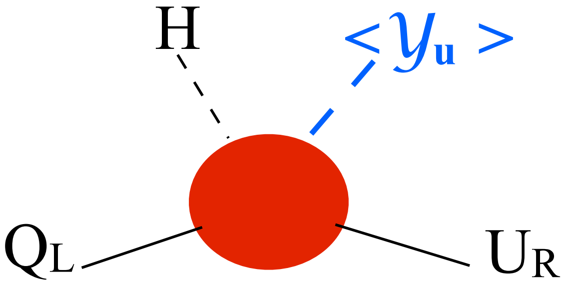
¿$V(\mathcal{Y}_d, \mathcal{Y}_u)$?

For this talk:

each Y_{SM} -- >one single field $\mathcal{Y}$

$$Y_{SM} \sim \frac{< \mathcal{Y} >}{\Lambda_{fl}}$$





*** Does the minimum of the scalar potential justify the observed masses and mixings?**

$$\mathbf{V}(\mathcal{Y}_{\mathbf{d}}, \ \mathcal{Y}_{\mathbf{u}})$$

* Invariant under the SM gauge symmetry

* Invariant under its global flavour symmetry $\mathbf{G}_{\mathbf{flavour}}$

$$\mathbf{G}_{\mathbf{flavour}} = \mathbf{U(3)}_{\mathbf{Q_L}} \ \mathbf{x} \ \mathbf{U(3)}_{\mathbf{U_R}} \ \mathbf{x} \ \mathbf{U(3)}_{\mathbf{D_R}}$$

The basis of the game is to find the minima of the invariants that you can construct out of Yukawa couplings

L. Michel+Radicati 70, Cabibbo+Maiani71 for the spectrum of masses

List of possible invariants: Hanani, Jenkins, Manohar 2010

$V(\mathcal{Y}_d, \mathcal{Y}_u)$

Construction of the Potential

* 5 invariants at d=4 level:

(Feldman, Jung, Mannel)

$$\mathrm{Tr}\,(\,\mathcal{Y}_u\,\mathcal{Y}_u^+\,) \qquad \mathrm{Tr}\,(\,\mathcal{Y}_u\,\mathcal{Y}_u^+\,)^2$$

$$\mathrm{Tr}\,(\,\mathcal{Y}_d\,\mathcal{Y}_d^+\,) \qquad \mathrm{Tr}\,(\,\mathcal{Y}_d\,\mathcal{Y}_d^+\,)^2$$

$$\mathrm{Tr}\,(\,\mathcal{Y}_u\,\mathcal{Y}_u^+\,\mathcal{Y}_d\,\mathcal{Y}_d^+\,)$$

* results following general; for this talk we will illustrate in 2-generation

(Alonso, Gavela, Merlo, Rigolin, arXiv 11; Nardi 11, Espinosa, Fong, Nardi 13)

$V(\mathcal{Y}_d, \mathcal{Y}_u)$

Construction of the Potential

* 5 invariants at d=4 level:

(Feldman, Jung, Mannel)

$$\mathrm{Tr}\,(\mathcal{Y}_u\,\mathcal{Y}_u^+) \qquad \mathrm{Tr}\,(\mathcal{Y}_u\,\mathcal{Y}_u^+)^2$$

$$\mathrm{Tr}\,(\mathcal{Y}_d\,\mathcal{Y}_d^+) \qquad \mathrm{Tr}\,(\mathcal{Y}_d\,\mathcal{Y}_d^+)^2$$

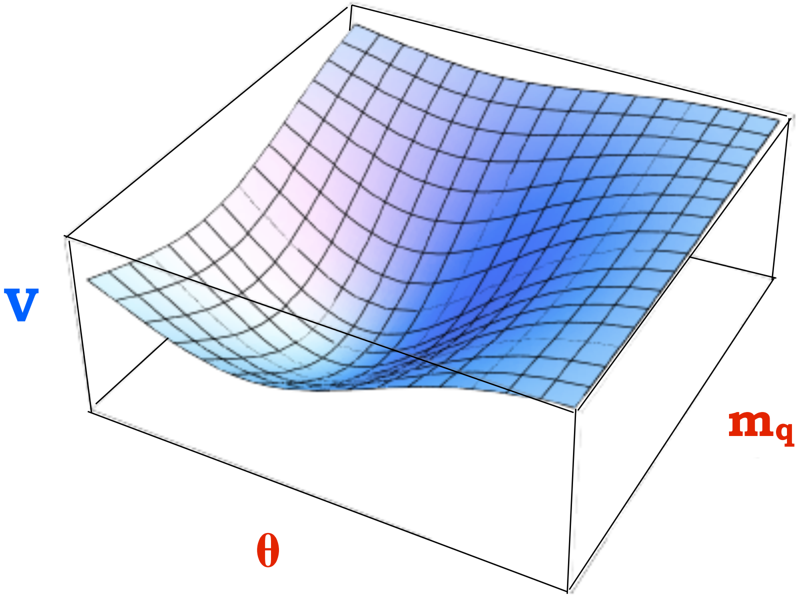
$$\mathrm{Tr}\,(\mathcal{Y}_u\,\mathcal{Y}_u^+\,\mathcal{Y}_d\,\mathcal{Y}_d^+)$$ <-- **mixing**

(Alonso, Gavela, Merlo, Rigolin, arXiv 11; Nardi 11, Espinosa, Fong, Nardi 13)

e.g. for the case of two families:

$$\mathrm{Tr}(\mathcal{Y}_u \mathcal{Y}_u^{+} \mathcal{Y}_d \mathcal{Y}_d^{+}) \propto (m_c^2 - m_u^2)(m_s^2 - m_d^2)\cos 2\theta$$

at the minimum: $\boxed{(m_c^2 - m_u^2)(m_s^2 - m_d^2)\sin 2\theta = 0}$



-> NO MIXING

same conclusion for 3 families

Berezhiani-Rossi; Anselm, Berezhiani; Alonso, Gavela, Merlo, Rigolin

And what happens for leptons ?

Any difference with Majorana neutrinos?

Alonso, B.G., D. Hernandez, Merlo, Rigolin.

Leptons

Just TWO heavy neutrinos

$$\mathcal{L}_{\mathcal{M}_\nu} = (\bar{\ell}_L\,,\,\bar{N}^c\,,\,\bar{N}'^c)\begin{pmatrix} 0 & vY & vY' \\ vY^T & 0 & \mathbf{M} \\ vY'^T & \mathbf{M} & 0 \end{pmatrix}\begin{pmatrix} \ell_L^c \\ N \\ N' \end{pmatrix}$$

the Yukawas are determined up to their overal magnitude

$$\text{N.H.} \qquad Y = \frac{y}{\sqrt{m_{\nu_2} + m_{\nu_3}}}\, U_{PMNS} \begin{pmatrix} 0 \\ -i\sqrt{m_{\nu_2}}e^{-i\alpha} \\ \sqrt{m_{\nu_3}}e^{i\alpha} \end{pmatrix}$$

(Alonso, Gavela, D. Hernandez, Merlo, Rigolin)

Leptons

Just TWO heavy neutrinos

$$\mathcal{L}_{\mathcal{M}_\nu} = (\bar{\ell}_L\,,\,\bar{N}^c\,,\,\bar{N}'^c)\begin{pmatrix} 0 & vY & vY' \\ vY^T & 0 & \mathbf{M} \\ vY'^T & \mathbf{M} & 0 \end{pmatrix}\begin{pmatrix} \ell_L^c \\ N \\ N' \end{pmatrix}$$

the Yukawas are determined up to their overal magnitude

$$\text{N.H.} \qquad Y = \frac{y}{\sqrt{m_{\nu_2}+m_{\nu_3}}}\,U_{PMNS}\begin{pmatrix} 0 \\ -i\sqrt{m_{\nu_2}}e^{-i\alpha} \\ \sqrt{m_{\nu_3}}e^{i\alpha} \end{pmatrix}$$

The flavour symmetry is $G_f =$ $U(3)_{\ell_L} \times U(3)_{E_R} \times O(2)_N$

(Alonso, Gavela, D. Hernandez, Merlo, Rigolin)

Just TWO heavy neutrinos

$$\mathcal{L}_{\mathcal{M}_\nu} = (\bar{\ell}_L\,,\,\bar{N}^c\,,\,\bar{N}'^c)\begin{pmatrix} 0 & vY & vY' \\ vY^T & 0 & \mathbf{M} \\ vY'^T & \mathbf{M} & 0 \end{pmatrix}\begin{pmatrix} \ell_L^c \\ N \\ N' \end{pmatrix}$$

the Yukawas are determined up to their overal magnitude

$$\text{N.H.} \qquad Y = \frac{y}{\sqrt{m_{\nu_2}+m_{\nu_3}}}\, U_{PMNS}\begin{pmatrix} 0 \\ -i\sqrt{m_{\nu_2}}e^{-i\alpha} \\ \sqrt{m_{\nu_3}}e^{i\alpha} \end{pmatrix}$$

The flavour symmetry is $G_f = \quad U(3)_{\ell_L} \times U(3)_{E_R} \times O(2)_N$

(Alonso, Gavela, D. Hernandez, Merlo, Rigolin)

The flavour symmetry is $G_f = U(3)_{\ell_L} \times U(3)_{E_R} \times O(2)_N$

adds a new invariant for the lepton sector, in total:

$$\mathrm{Tr}\,(\,\mathcal{Y}_E\,\mathcal{Y}_E^+\,) \qquad \mathrm{Tr}\,(\,\mathcal{Y}_E\,\mathcal{Y}_E^+\,)^2$$

$$\mathrm{Tr}\,(\,\mathcal{Y}_\nu\,\mathcal{Y}_\nu^+\,) \qquad \mathrm{Tr}\,(\,\mathcal{Y}_\nu\,\mathcal{Y}_\nu^+\,)^2$$

$$\mathrm{Tr}\,(\,\mathcal{Y}_E\,\mathcal{Y}_E^+\,\mathcal{Y}_\nu\,\mathcal{Y}_\nu^+\,) \longleftarrow \textbf{mixing}$$

$$\mathrm{Tr}\,(\,\mathcal{Y}_\nu\,\sigma_2\,\mathcal{Y}_\nu^+\,)^2 \;\text{<--}\; \mathbf{O(2)}_N$$

$O(2)_N$ is simply associated to Lepton Number

e.g., **for 2 generations**, the mixing terms in $\mathbf{V}(\mathcal{Y}_{\mathbf{E}}, \mathcal{Y}_{\boldsymbol{\nu}})$ is :

Leptons

$$\mathrm{Tr}(\mathcal{Y}_E\, \mathcal{Y}_E^+\, \mathcal{Y}_\nu\, \mathcal{Y}_\nu^+) \propto$$

$$(m_\mu^2 - m_e^2)\left[(y^2 + y'^2)(m_{\nu_2} - m_{\nu_1})\cos 2\theta + (y^2 - y'^2)2\sqrt{m_{\nu_2} m_{\nu_1}}\sin 2\alpha \sin 2\theta\right]$$

where $U_{PMNS} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} e^{-i\alpha} & 0 \\ 0 & e^{i\alpha} \end{pmatrix}$

Quarks

$$\mathrm{Tr}(\mathcal{Y}_u\, \mathcal{Y}_u^+\, \mathcal{Y}_d\, \mathcal{Y}_d^+) \propto (m_c^2 - m_u^2)(m_s^2 - m_d^2)\cos 2\theta$$

e.g., **for 2 generations**, the mixing terms in $\mathbf{V}(\mathcal{Y}_{\mathbf{E}}, \mathcal{Y}_{\mathbf{\nu}})$ is :

Leptons

$$\mathrm{Tr}(\mathcal{Y}_E \mathcal{Y}_E^+ \mathcal{Y}_\nu \mathcal{Y}_\nu^+) \propto$$

$$(m_\mu^2 - m_e^2)\left[(y^2 + y'^2)(m_{\nu_2} - m_{\nu_1})\cos 2\theta + (y^2 - y'^2)2\sqrt{m_{\nu_2} m_{\nu_1}}\sin 2\alpha \sin 2\theta\right]$$

Mixing term unphysical if either "up" or "down" fermions degenerate

Mixing physical even with degenerate neutrino masses, if Majorana phase non-trivial

Quarks

$$\mathrm{Tr}(\mathcal{Y}_u \mathcal{Y}_u^+ \mathcal{Y}_d \mathcal{Y}_d^+) \propto (m_c^2 - m_u^2)(m_s^2 - m_d^2)\cos 2\theta$$

e.g., for 2 generations, the mixing terms in **V**($\mathcal{Y}_E$, $\mathcal{Y}_\nu$) is :

Minimisation

Tr($\mathcal{Y}_E \mathcal{Y}_E^+ \mathcal{Y}_\nu \mathcal{Y}_\nu^+$)

* $(y^2 - y'^2)\sqrt{m_{\nu_2} m_{\nu_1}} \sin 2\theta \cos 2\alpha = 0$ $\longrightarrow$ $\boxed{\alpha = \pi/4 \text{ or } 3\pi/4}$

* $$\boxed{\mathrm{tg}2\theta = 2\frac{y^2 - y'^2}{y^2 + y'^2} \sin 2\alpha \frac{\sqrt{m_{\nu_2} m_{\nu_1}}}{m_{\nu_2} - m_{\nu_1}}}$$

Large angles correlated with degenerate masses

Maximal Majorana phase

What makes the difference?

- The Majorana character?
- The flavour group?
- The particular model?

Let us try to generalize to any model

-- for 2 families
-- for 3 families

* Generalize to arbitrary seesaw model

in progress: Alonso, D. Hernandez, Merlo, Rigolin, B.G.

diagonal eigenvalues

Use Casas-Ibarra parametrization $\mathbf{Y}_{\nu}= \mathbf{U}_{\mathrm{PMNS}}\, \mathbf{m}_{\nu}^{1/2}\, \mathbf{R}\, \mathbf{M}_{\mathbf{N}}^{1/2}$

complex orthogonal matrix

* Generalize to arbitrary seesaw model

in progress: Alonso, D. Hernandez, Merlo, Rigolin, B.G.

diagonal eigenvalues

Use Casas-Ibarra parametrization $Y_\nu = U_{PMNS}\, m_\nu^{1/2}\, R\, M_N^{1/2}$

complex orthogonal matrix

The mixing invariant shown before:

$$\mathrm{Tr}(\mathcal{Y}_E\, \mathcal{Y}_E^{+}\, \mathcal{Y}_\nu\, \mathcal{Y}_\nu^{+}) = \mathrm{Tr}(m_i^{1/2}\, U^{+}\, m_l^{2}\, U\, m_i^{1/2}\, R^{+}\, M_N\, R)$$

define $P = (R^{+} M_N R) \xrightarrow[\text{2 fam.}]{}$

$$* \quad \sqrt{m_1 m_2}\,|P_{12}|\,\sin\left[2\alpha - arg(P_{12})\right] = 0$$

$$* \quad \mathrm{tg}2\theta = 2|P_{12}|\sin 2\alpha \frac{\sqrt{m_{\nu_2} m_{\nu_1}}}{m_{\nu_1} P_{11} - m_{\nu_2} P_{22}}$$

* Generalize to arbitrary seesaw model

in progress: Alonso, D. Hernandez, Merlo, Rigolin, B.G.

diagonal
eigenvalues

Use Casas-Ibarra parametrization $\mathbf{Y_\nu = U_{PMNS}\, m_\nu^{1/2}\, R\, M_N^{1/2}}$

complex
orthogonal
matrix

The mixing invariant shown before:

$$\mathrm{Tr}(\mathcal{Y}_E\, \mathcal{Y}_E^+\, \mathcal{Y}_\nu\, \mathcal{Y}_\nu^+) = \mathrm{Tr}(m_i^{1/2}\, U^+ m_l^2\, U\, m_i^{1/2}\, R^+ \mathrm{M_N}\, R)$$

define P=($R^+ \mathrm{M_N} R$) $\xrightarrow[\text{2 fam.}]{}$

$$* \quad \sqrt{m_1 m_2}\,|P_{12}|\,\sin\left[2\alpha - arg(P_{12})\right] = 0$$

$$* \quad \mathrm{tg}2\theta = 2|P_{12}|\sin 2\alpha \frac{\sqrt{m_{\nu_2} m_{\nu_1}}}{m_{\nu_1} P_{11} - m_{\nu_2} P_{22}}$$

*** In degenerate limit of heavy neutrinos $M_{N_1}=M_{N_2}=M$**

$$\mathbf{R} = \begin{pmatrix} \mathrm{ch}\,\boldsymbol{\omega} & -i\,\mathrm{sh}\,\boldsymbol{\omega} \\ i\,\mathrm{sh}\,\boldsymbol{\omega} & \mathrm{ch}\,\boldsymbol{\omega} \end{pmatrix}$$ with ω real,

$$\mathrm{tg}2\theta = \sin 2\alpha \frac{2\sqrt{m_{\nu_2} m_{\nu_1}}}{m_{\nu_2} - m_{\nu_1}}\ \mathbf{tgh}\ \mathbf{2\omega}$$

$$\boxed{\alpha = \pi/4 \text{ or } 3\pi/4}$$

* Generalize to arbitrary seesaw model

in progress: Alonso, D. Hernandez, Merlo, Rigolin, B.G.

diagonal
eigenvalues

Use Casas-Ibarra parametrization $\mathbf{Y_\nu = U_{PMNS}\, m_\nu^{1/2}\, R\, M_N^{1/2}}$

complex
orthogonal
matrix

The mixing invariant shown before:

$$\mathrm{Tr}(\mathcal{Y}_E\, \mathcal{Y}_E^+\, \mathcal{Y}_\nu\, \mathcal{Y}_\nu^+) = \mathrm{Tr}(m_l^{1/2}\, U^+\, m_l^2\, U\, m_l^{1/2}\, R^+\, M_N\, R)$$

define P=($R^+\, M_N\, R$) $\xrightarrow[\text{2 fam.}]{}$

* $\sqrt{m_1 m_2}\,|P_{12}|\,\sin\left[2\alpha - arg(P_{12})\right] = 0$
* $tg2\theta = 2|P_{12}|\sin 2\alpha \dfrac{\sqrt{m_{\nu_2} m_{\nu_1}}}{m_{\nu_1}P_{11} - m_{\nu_2}P_{22}}$

* In degenerate limit of heavy neutrinos $M_{N_1}=M_{N_2}=M$

$$\mathbf{R} = \begin{pmatrix} \mathrm{ch}\,\omega & -i\,\mathrm{sh}\,\omega \\ i\,\mathrm{sh}\,\omega & \mathrm{ch}\,\omega \end{pmatrix}$$ with ω real,

e.g. in Previous model

$$tg2\theta = \sin 2\alpha \frac{2\sqrt{m_{\nu_2} m_{\nu_1}}}{m_{\nu_2} - m_{\nu_1}}\ \frac{y^2-y'^2}{y^2-y'^2}$$

$\boxed{\alpha = \pi/4 \text{ or } 3\pi/4}$

* What is the role of the neutrino flavour group?

Leptons: $G_{\text{flavour}} = U(2)_L \times U(2)_{E_R} \times ?$

O(2), SU(n), O(n) .... ?

Inmediate results using for both quark and leptons

$$Y = U_L\, y^{\text{diag}}\, U_R$$

* What is the role of the neutrino flavour group?

$$U(n)$$

* What is the role of the neutrino flavour group?

$$U(n)$$

i.e.: $U(3)_L \times U(3)_{E_R} \times U(2)_{N_R}$

or: $U(3)_L \times U(3)_{E_R} \times U(3)_{N_R}$

* What is the role of the neutrino flavour group?

e.g. $U(n)_{N_R}$ ... leptons

e.g. generic seesaw

$$\mathcal{L} = \mathcal{L}_{SM} + i\overline{N_R}\not\partial N_R - \left[\overline{N_R} Y_N \tilde{\phi}^\dagger \ell_L + \frac{1}{2}\overline{N_R}\mathbf{M}{N_R}^c + h.c.\right]$$

with **M** carrying flavour → **M** spurion

More invariants in this case:

$\mathrm{Tr}\,(\mathcal{Y}_E\,\mathcal{Y}_E^+)$ $\quad$ $\mathrm{Tr}\,(\mathcal{Y}_E\,\mathcal{Y}_E^+)^2$ $\quad$ $\mathrm{Tr}\,(\mathcal{Y}_E\,\mathcal{Y}_E^+\,\mathcal{Y}_\nu\,\mathcal{Y}_\nu^+)$

$\mathrm{Tr}\,(\mathcal{Y}_\nu\,\mathcal{Y}_\nu^+)$ $\quad$ $\mathrm{Tr}\,(\mathcal{Y}_\nu\,\mathcal{Y}_\nu^+)^2$

$\mathrm{Tr}\,(\mathcal{M}_N\,\mathcal{M}_N^+)$ $\quad$ $\mathrm{Tr}\,(\mathcal{M}_N\,\mathcal{M}_N^+)^2$ $\quad$ $\mathrm{Tr}\,(\mathcal{M}_N\,\mathcal{M}_N^+\,\mathcal{Y}_\nu^+\mathcal{Y}_\nu)$

Result: no mixing for flavour groups U(n)

$\mathbf{O(n)}$

* What is the role of the neutrino flavour group?

$O(2)_{NR}$

e.g. two families

$$m_\nu \sim Y_\nu \frac{v^2}{M} Y_\nu^T = y_1\, y_2 \frac{v^2}{M} \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

$$\longrightarrow \left\{ \begin{array}{l} U_{PMNS} = \boxed{\frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ -1 & 1 \end{pmatrix} \begin{pmatrix} e^{i\pi/4} & 0 \\ 0 & e^{-i\pi/4} \end{pmatrix}} \\ \text{Degenerate neutrino masses} \end{array} \right.$$

Genericallly, O(2) allows :

- **one mixing angle maximal**
- **one relative Majorana phase of $\pi/2$**
- **two degenerate light neutrinos**

*3 families with $O(2)_{N_R}$:

- 3 light + 2 heavy N degenerate: bad θ_{12} quadrant. It cannot accomodate data!

- 3 light + 3 heavy N : **OK for θ_{23} maximal and spectrum**

experimentally $\sin^2\theta_{23}=$ 0.41 +-0.03 or 0.59+-0.02

Gonzalez-Garcia, Maltoni, Salvado, Schwetz Sept. 2012

T2K -> 45º in 2-fam.

*What about the other angles?

3 families: Leading order

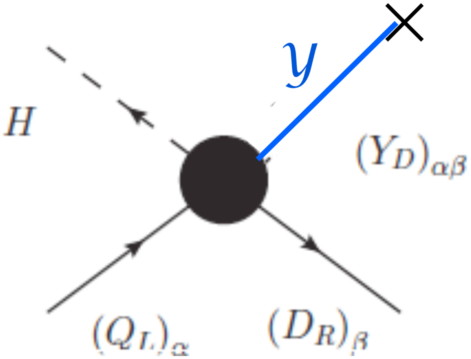
* **Quarks : no mixing**

* **Leptons** (with 3 heavy N),**$O(2)_{N_R}$:**
one ~maximal angle +
one sizeable one +
large Majorana phase

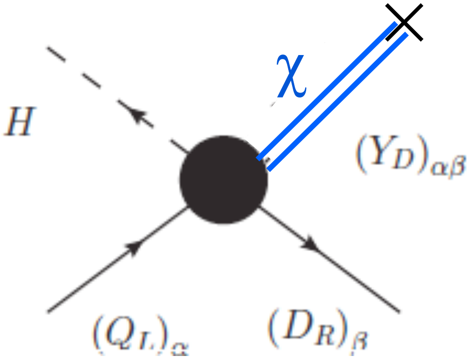
*a good possibility for the other angles :

Yukawas --> add fields in the fundamental of the flavour group

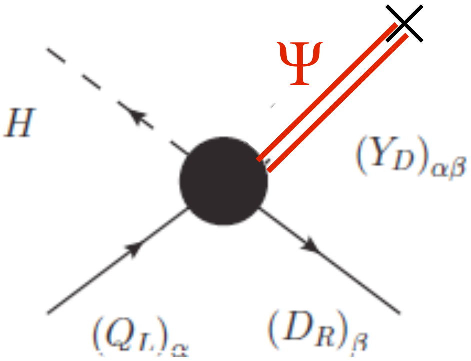
1) Y -- > one single scalar $\mathcal{Y} \sim (3, 1, \bar{3})$



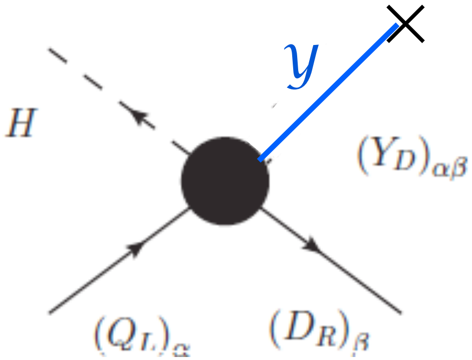
2) Y -- > two scalars $\chi\,\chi^+ \sim (3, 1, \bar{3})$



3) Y -- > two fermions $\bar{\Psi}\Psi \sim (3, 1, 3)$

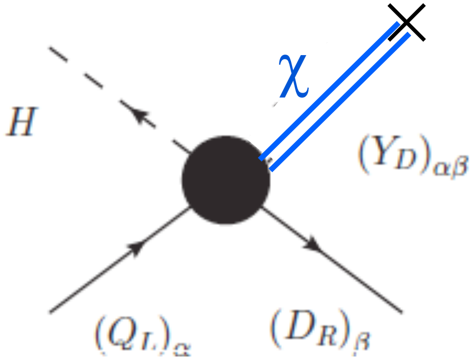


1) Y -- > one single scalar $y \sim (3, 1, \bar{3})$

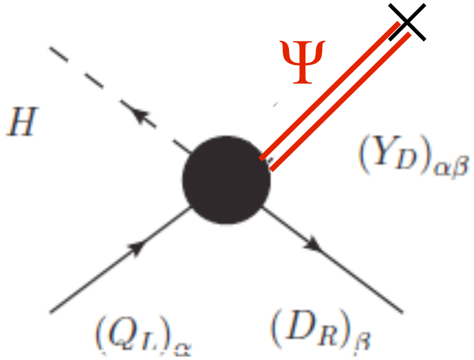


2) Y -- > two scalars $\chi\,\chi^+ \sim (3, 1, \bar{3})$

$\chi \sim (3, 1, 1)$

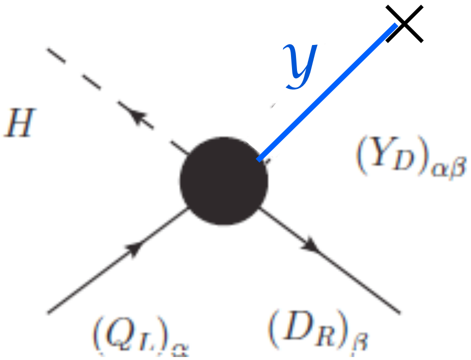


3) Y -- > two fermions $\bar{\Psi}\Psi \sim (3, 1, 3)$



1) Y -- > one single scalar $\mathcal{Y} \sim (3, 1, \bar{3})$

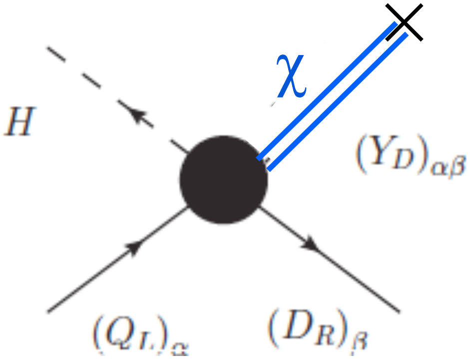
d=5 operator



2) Y -- > two scalars $\chi\,\chi^{+} \sim (3, 1, \bar{3})$

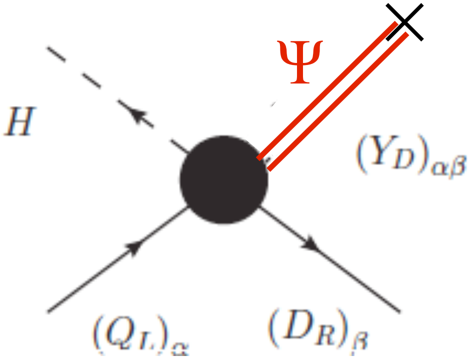
d=6 operator

$\chi \sim (3, 1, 1)$



3) Y -- > two fermions $\bar{\Psi}\Psi \sim (3, 1, 3)$

d=7 operator



i.e. for quarks, a possible path:

*** At leading (renormalizable) order:**

$$Y_u \equiv \frac{<\mathcal{Y}_u>}{\Lambda_f} + \frac{\langle\chi_u^L\rangle\,\langle\chi_u^{R\dagger}\rangle}{\Lambda_f^2} = \begin{pmatrix} 0 & \sin\theta_c\, y_c & 0 \\ 0 & \cos\theta_c\, y_c & 0 \\ 0 & 0 & y_t \end{pmatrix},$$

$$Y_d \equiv \frac{<\mathcal{Y}_d>}{\Lambda_f} + \frac{\langle\chi_d^L\rangle\,\langle\chi_d^{R\dagger}\rangle}{\Lambda_f^2} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & y_s & 0 \\ 0 & 0 & y_b \end{pmatrix}.$$

without unnatural fine-tunings

*** The masses of the first family and the other angles from non-renormalizable terms or other corrections or replicas ?**

....and analogously for leptonic mixing ?

Conclusions

* Exciting experimental windows ahead into neutrino(and/or DM) physics:

μ-e conversion will test SM-singlet fermions in the **2 GeV- 6000 TeV** mass range !

* A **dynamical origin for the Yukawa** couplings, based on the continuous flavour symmetry of the SM allows to tackle flavour for both quarks and leptons, and for both masses and mixings

Simplest case explored here, with approximate $U(1)_{LN}$ seesaws:

-- **Quarks: vanishing mixing at leading order**
-- **Majorana neutrinos with $O(2)_{NR}$:**
  ***one angle maximal (and another sizeable at leading order)**
  ***large/small mixings <--> degenerate/hierarchical masses**

$O(2)_{NR}$ singled out

Conclusions

* Exciting experimental windows ahead into neutrino(and/or DM) physics:

μ-e conversion will test SM-singlet fermions in the **2 GeV- 6000 TeV** mass range !

* A **dynamical origin for the Y** couplings, based on the continuou f avour for both

The extrema of the Yukawa invariants, at leading order gives:

- no mixing for quarks,
- for leptons: allows maximal Majorana phase and maximal/large mixing correlated with the mass spectra....

Food for thought

der)

al masses

$O(2)_{N_R}$ singled out

Back-up slides

Higgs decay (LHC)

e.g. H --> ν N

Pilaftsis92....Chen et al.10, Dev+Franceschini+Mohapatra 12, Cely+Ibarra+Molinaro+Petcov

We get for the model-independent rate:

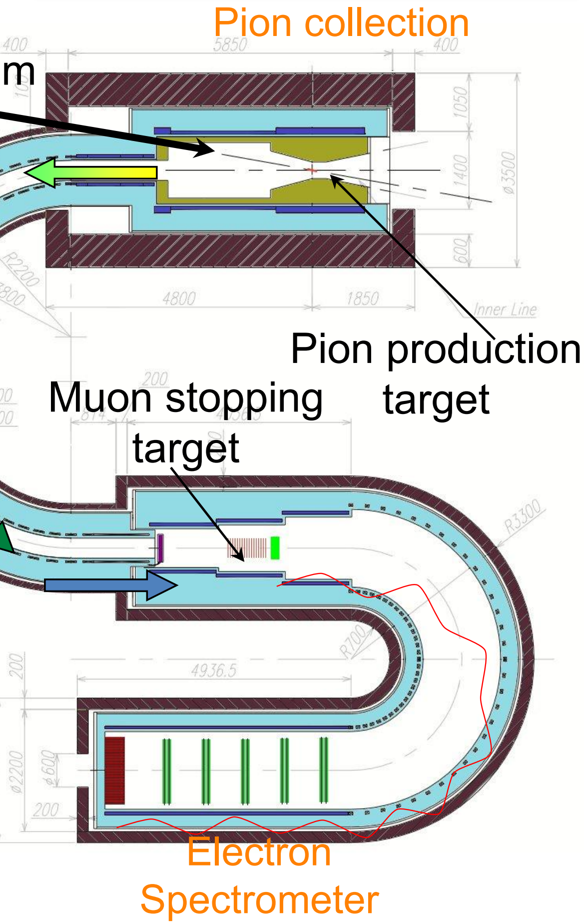
$$Br(h \to \nu N) = \frac{\alpha_W}{8 M_W^2 \Gamma_h^{tot}} \sum_i^k \left(|U_{eN_i}|^2 + |U_{\mu N_i}|^2 + |U_{\tau N_i}|^2\right) m_h m_{N_i}^2 \left(1 - \frac{m_{N_i}^2}{m_h^2}\right)^2$$

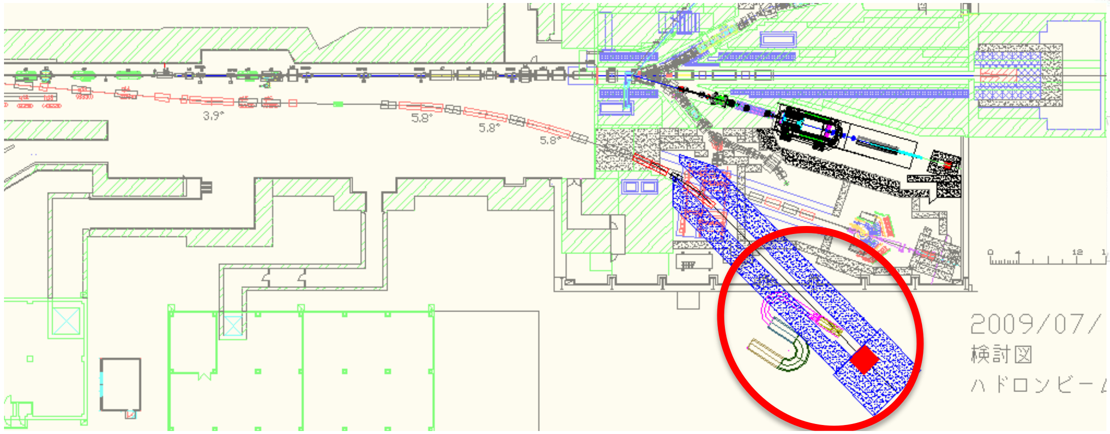
and using $|\Sigma_i U_{eN_i} U^*_{\mu N_i}| < \Sigma_{i,\alpha} |U_{\alpha N_i}|^2$

COMET μ-e conv. search

Phase-I phys run in 2017
Full COMET run in 2021-2022

- Search for cLFV mu-e conv.
  - **10^{-16}** sensitivity (Target S.E.S. 2.6 × 10^{-17})
  - Improve **$O(10^4)$** than present upper bound such as SINDRUM-II BR[$\mu^- + Au \rightarrow e^- + Au$] < 7 × 10^{-13}
- Signature: 105MeV monochromatic electron
- Beam requirement
  - 8GeV bunched slow extraction
  - $1.6\text{x}10^{21}$ pot needed to reach goal
  - 7 uA (56kW) x 4 SN year ($4\text{x}10^7$sec)
  - Extinction <10^{-9}





courtesy of Yoshi Kuno

μ-->e conversion

We performed an exact one-loop calculation, but for obvious approximations:

-- m_e=0 compared to m_μ

-- $m_{\nu 1} = m_{\nu 2} = m_{\nu 3} = 0$ compared to heavy neutrino masses (that is, assume $m_N > \text{eV}$)

-- higher orders in the external momentum neglected versus M_W, as usual

We did many checks to our results, e.g.:

* For “dipole” for factors .... check with b --> s l^+ l^-

* For the other form factors .... agreement with μ --> eee form factors

....

example of check: Decoupling limits

* Large mass $m_N >> m_W$

In the seesaw, for m_N-> ∞ the remaining theory is renormalizable (SM) --> rate must vanish then.
Our results do decouple for $x_N = m_N^2/M_W^2 \gg 1$

$$\Gamma \sim (\log x_N)^2/x_N^2 \,, \qquad \text{for } \mu \to \text{eee} \qquad \text{and} \qquad \mu \to \text{e conversion} \,,$$

$$\Gamma \sim 1/x_N^2 \,, \qquad \text{for } \mu \to e\gamma \,.$$

* Low mass $m_N << m_W$

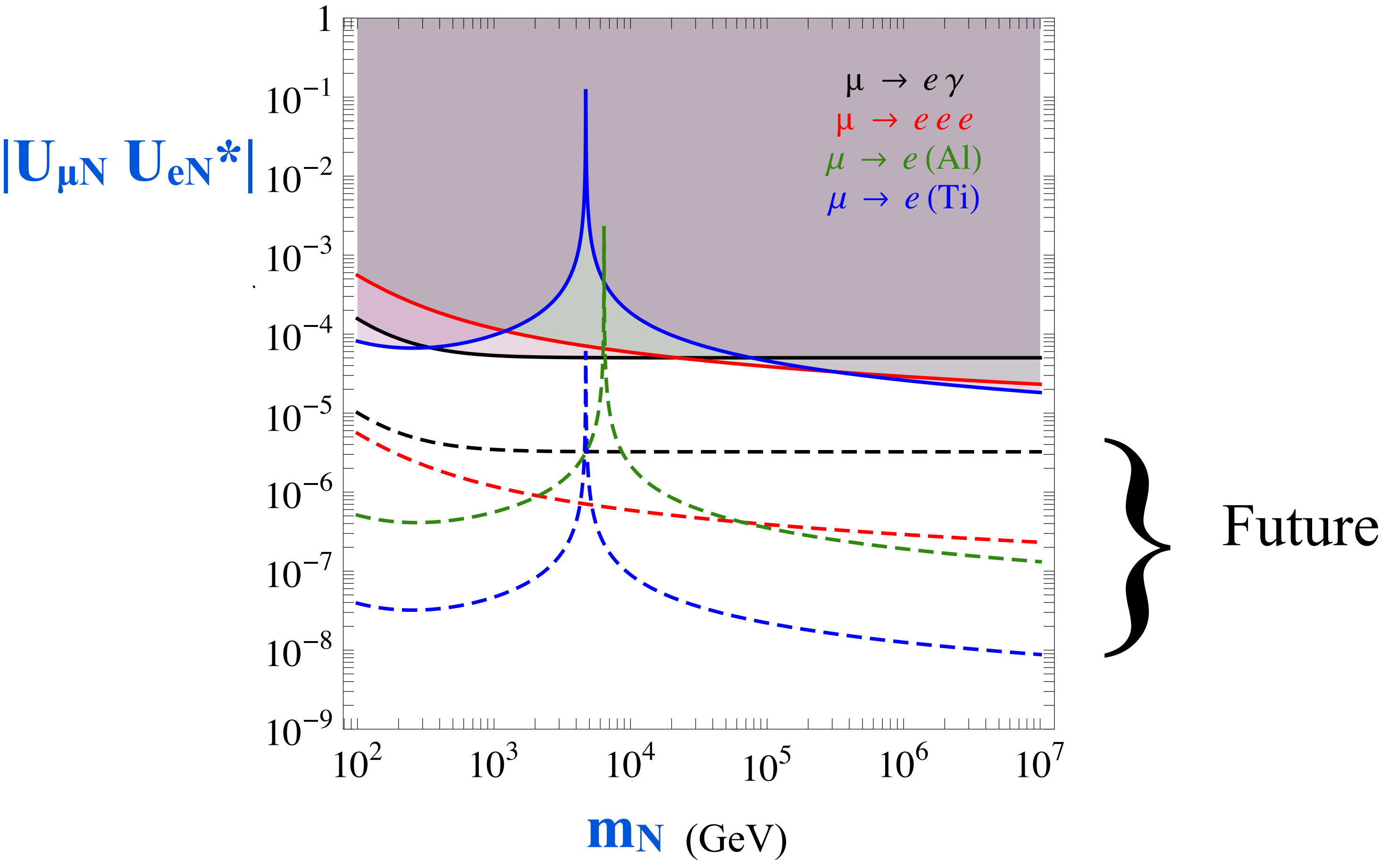
they also vanish for m_N --> 0

$$x_N = m_N^2/M_W^2 \ll 1$$

$$\Gamma \sim x_N^2 (\log x_N)^2 \,, \qquad \text{for } \mu \to \text{eee} \qquad \text{and} \qquad \mu \to \text{e conversion} \,;$$

$$\Gamma \sim x_N^2 \,, \qquad \text{for } \mu \to e\gamma \,.$$

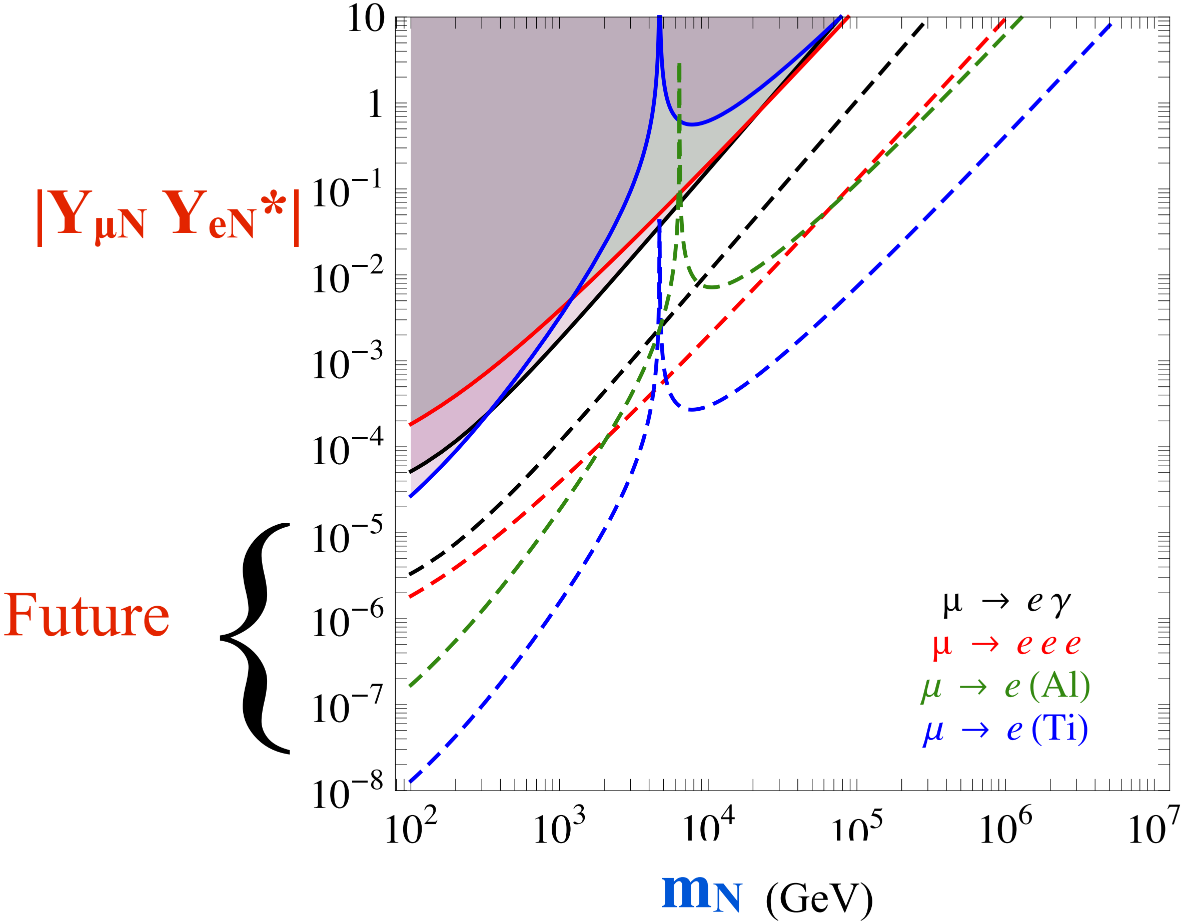
$|U_{\mu N}\, U_{eN}^*|$ versus m_N



Sensitivity up to $m_N \sim 6000$ TeV for Ti

For the particular case of seesaw I : $U_{lN} \sim Y\, v/ M$

$|Y_{\mu N}\, Y_{eN}{}^*|$ versus m_N



Sensitivity up to $m_N \sim 6000$ TeV for Ti

* Large mass $m_N >> m_W$

When one m_N scale dominates (e.g. degenerate heavy neutrinos or hierarchical) the ratio of any two μ-e transitions only depends on m_N (Chu, Dhen, Hambye 11)

Besides, μ-e conversion vanishes at some large m_N

(Dinh, Ibarra, Molinaro, Petcov 12)

For instance, we find that for light nuclei (αZ<<1), it vanishes as

$$m_N^2\Big|_0 = M_W^2 \exp\left(\frac{\frac{9}{8}(A-Z)+\left(\frac{9}{8}+\frac{31 s_W^2}{12}\right)Z}{\frac{3}{8}(A-Z)+\left(\frac{4 s_W^2}{3}-\frac{3}{8}\right)Z}\right)$$

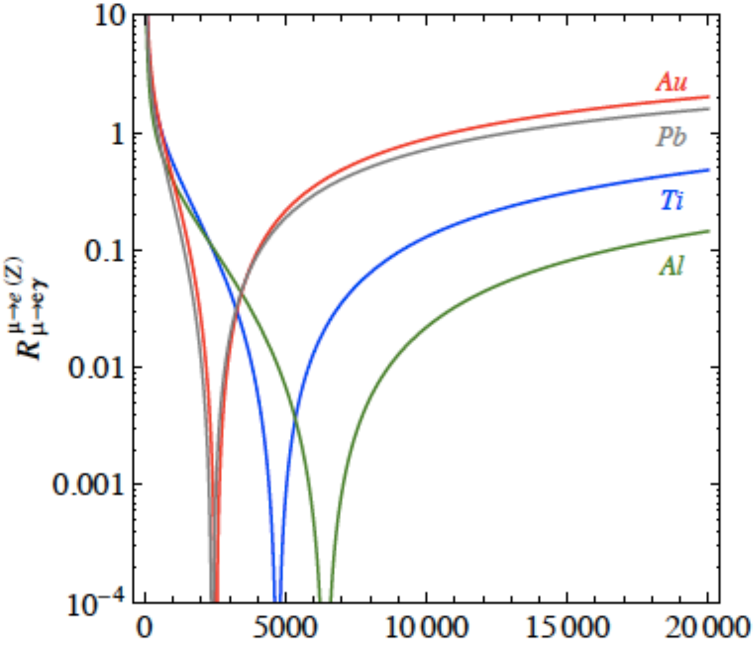
(Alonso, Dhen, Hambye, B.G.)

exponential sensitivity

The ratios of two e-μ transitions....

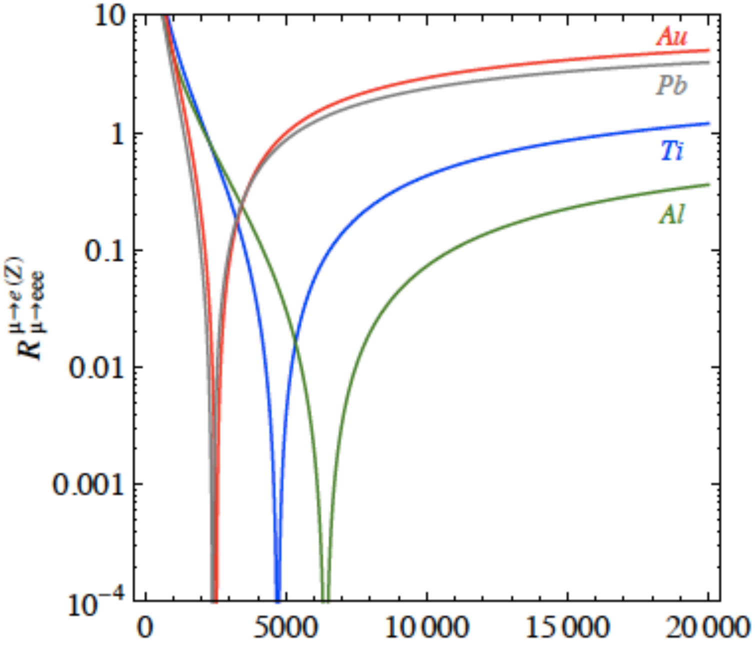
we obtain:

μ-e conversion
μ--> e γ



m_N (GeV)

μ-e conversion
μ--> e e e



m_N (GeV)

...typically vanishes for m_N in 2- 7 TeV range

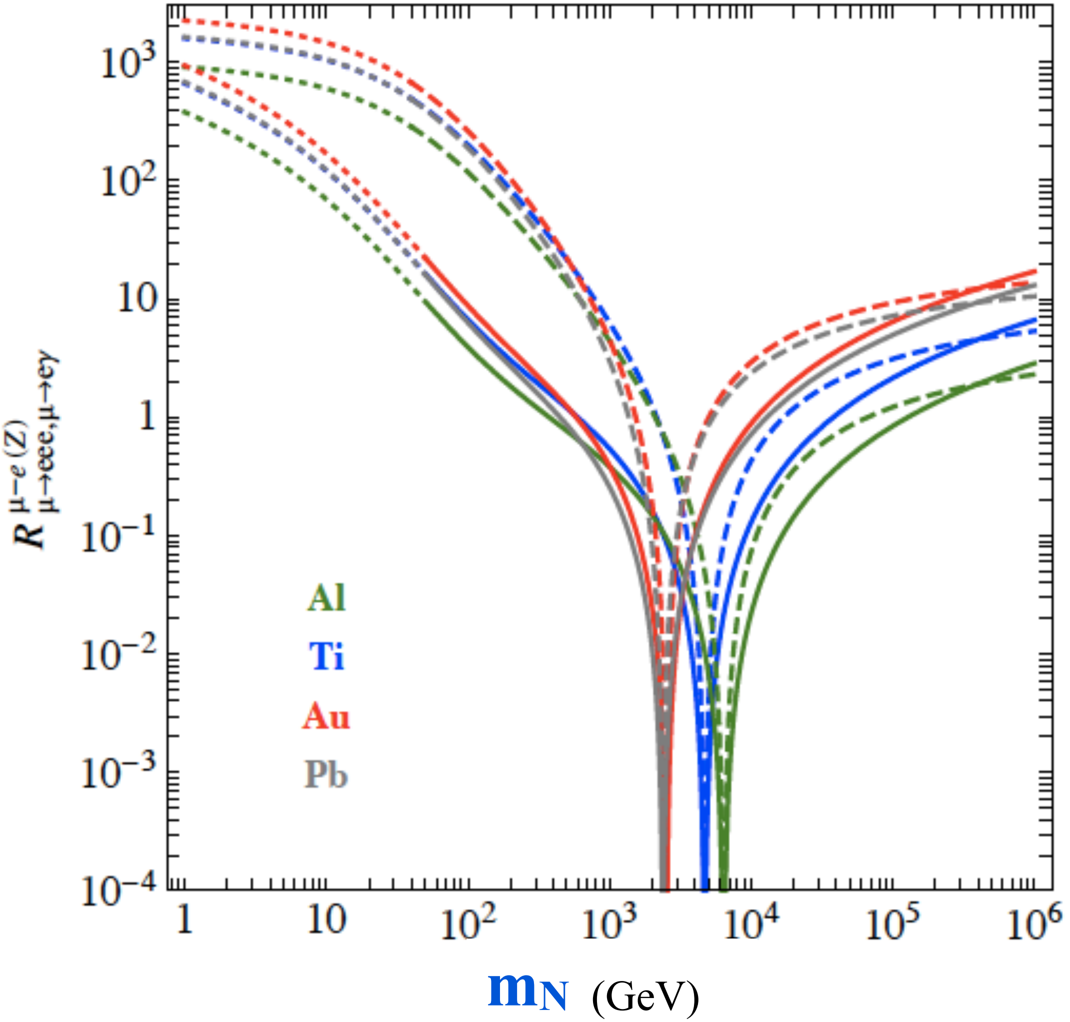
(Alonso, Dhen, Hambye, B.G.)

* Low mass regime eV << m_N << m_W

.... de Gouvea 05...

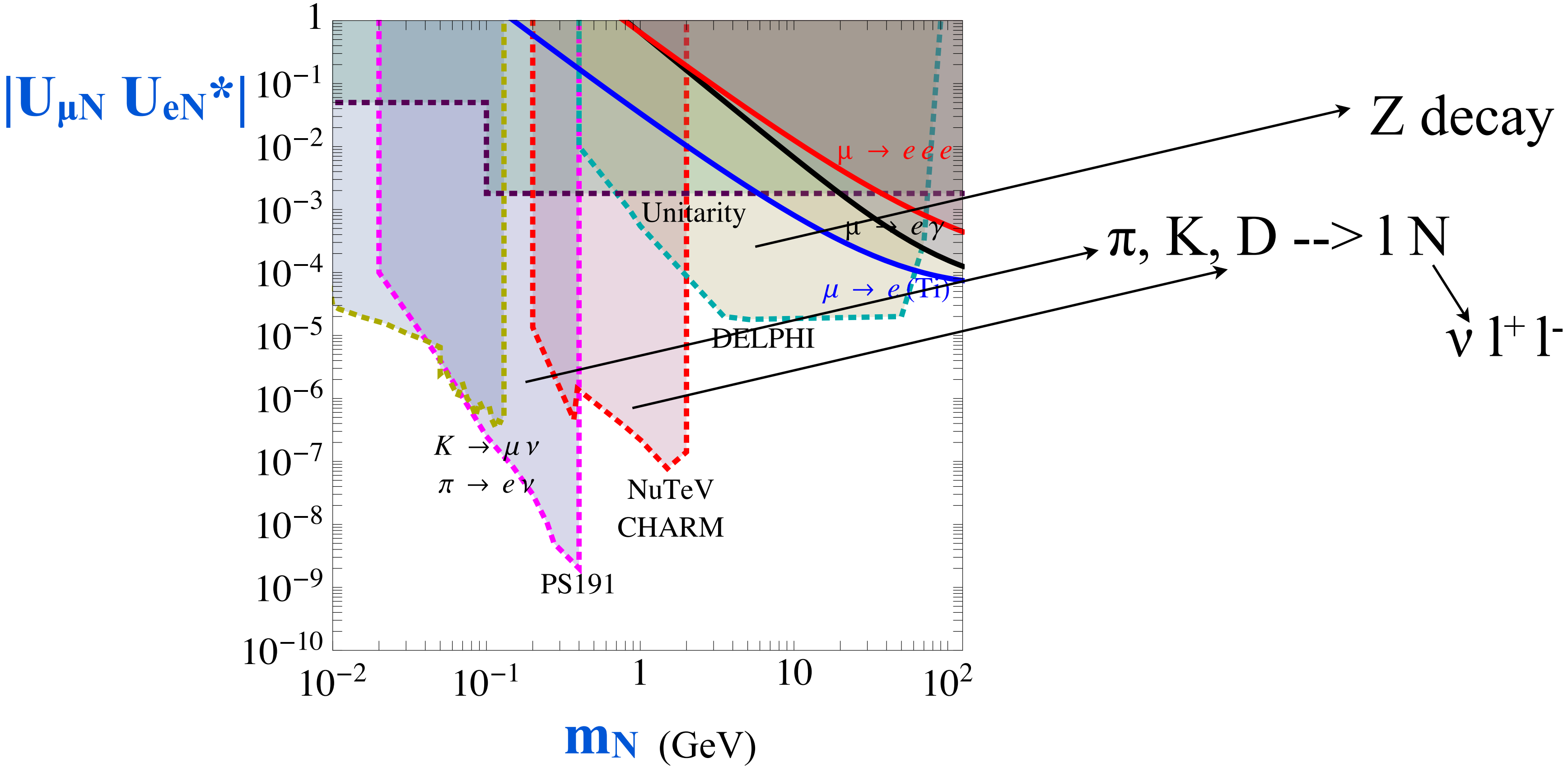
* Low mass regime eV << m_N << m_W

μ--> e conversion does not vanish for low mass



(Alonso, Dhen, Hambye, B.G.)

* Low mass regime eV << m_N << m_W



Z decay

π, K, D --> l N

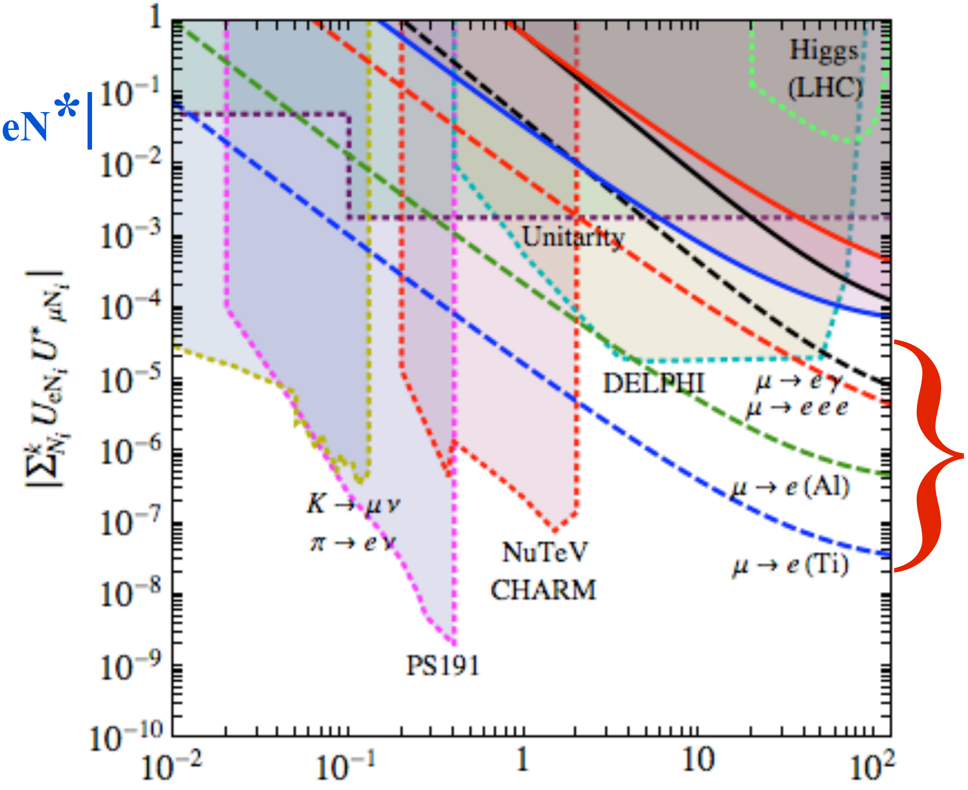
$\nu\ l^+ l^-$

Peak decays+PS191+NuTev/CHARM+Delphi: Atre+Han+Pascoli+Zhang 09...... Richayskiy+Ivashko 12

Unitarity: Antusch+Biggio+Fdez-Martinez+Gavela+Lopez-Pavon 06; Antusch+Bauman+ Fedez-Martinez 09

* Low mass regime eV << m_N << m_W

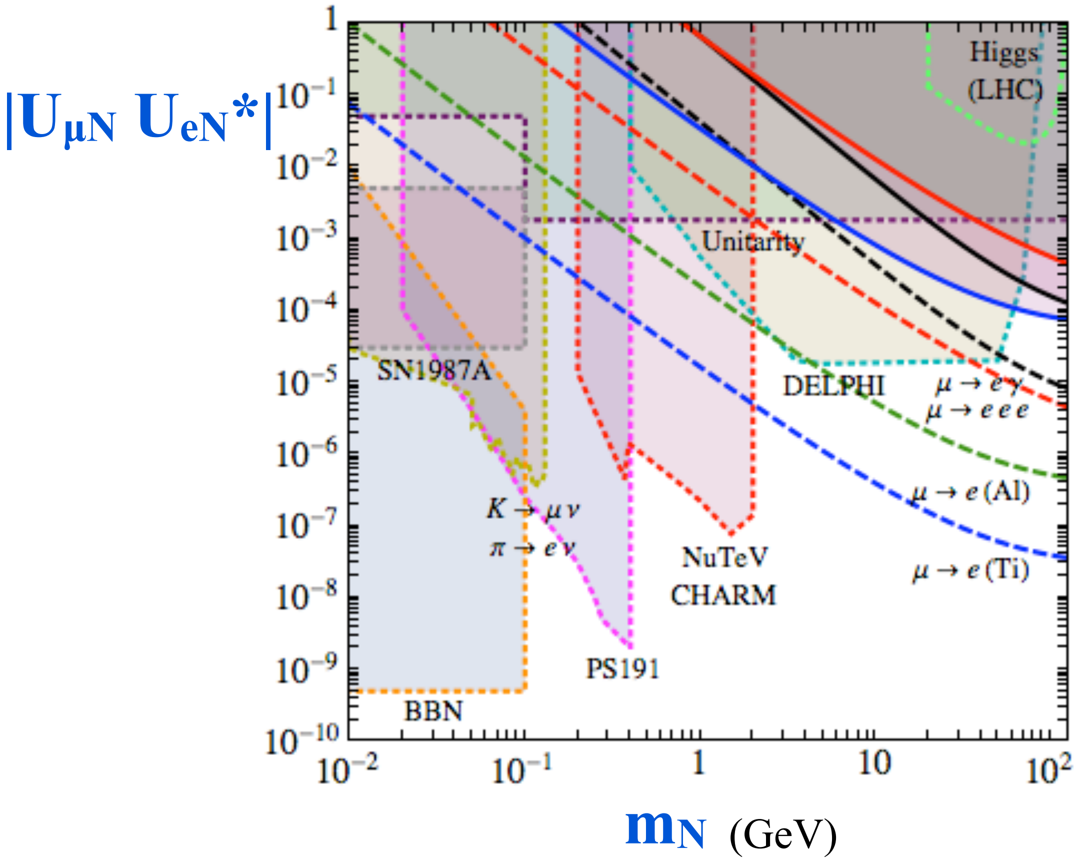
$|U_{\mu N}\, U_{eN}{}^*|$



Future

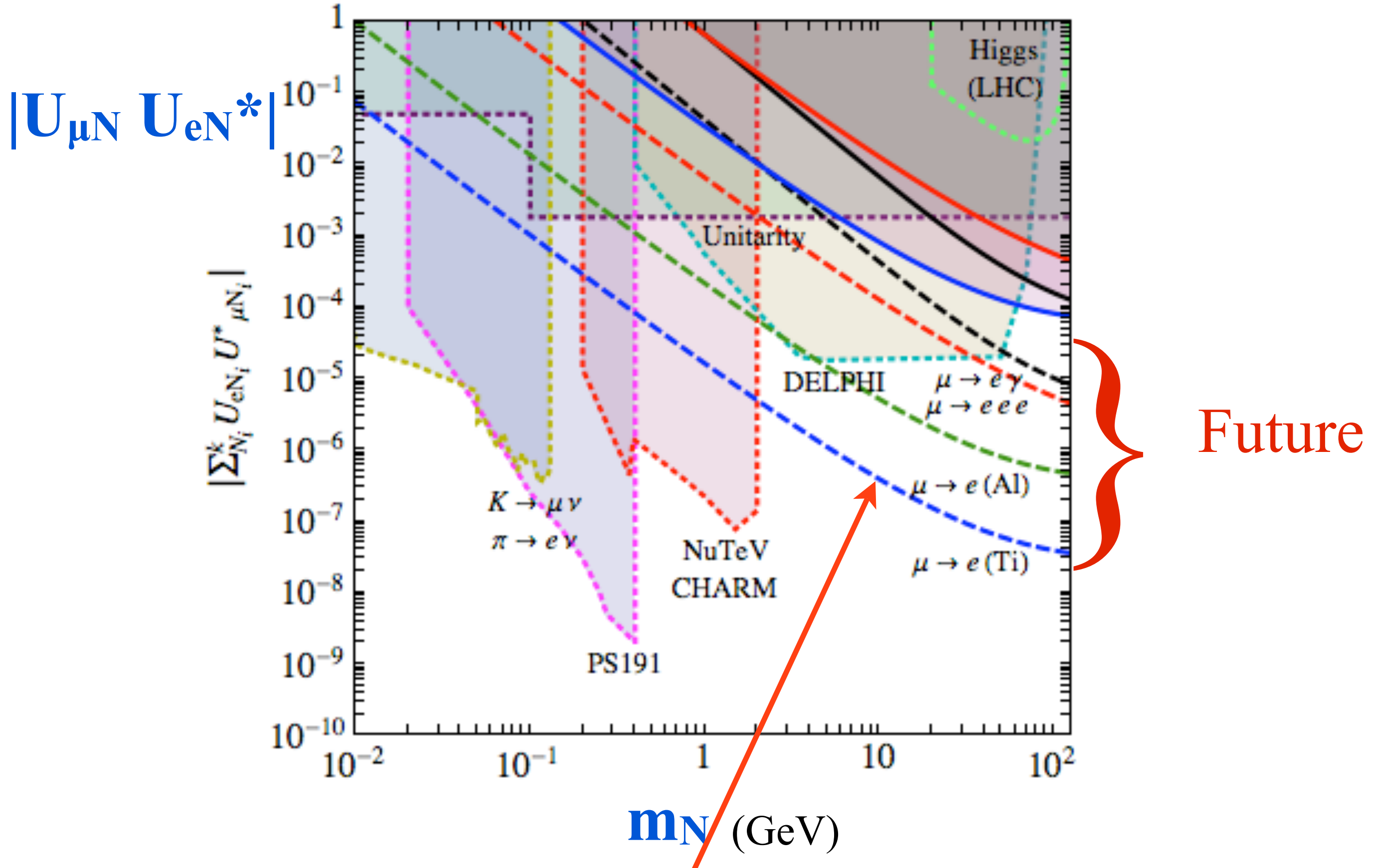
m_N (GeV)

* Low mass regime eV $<<$ m_N $<<$ m_W



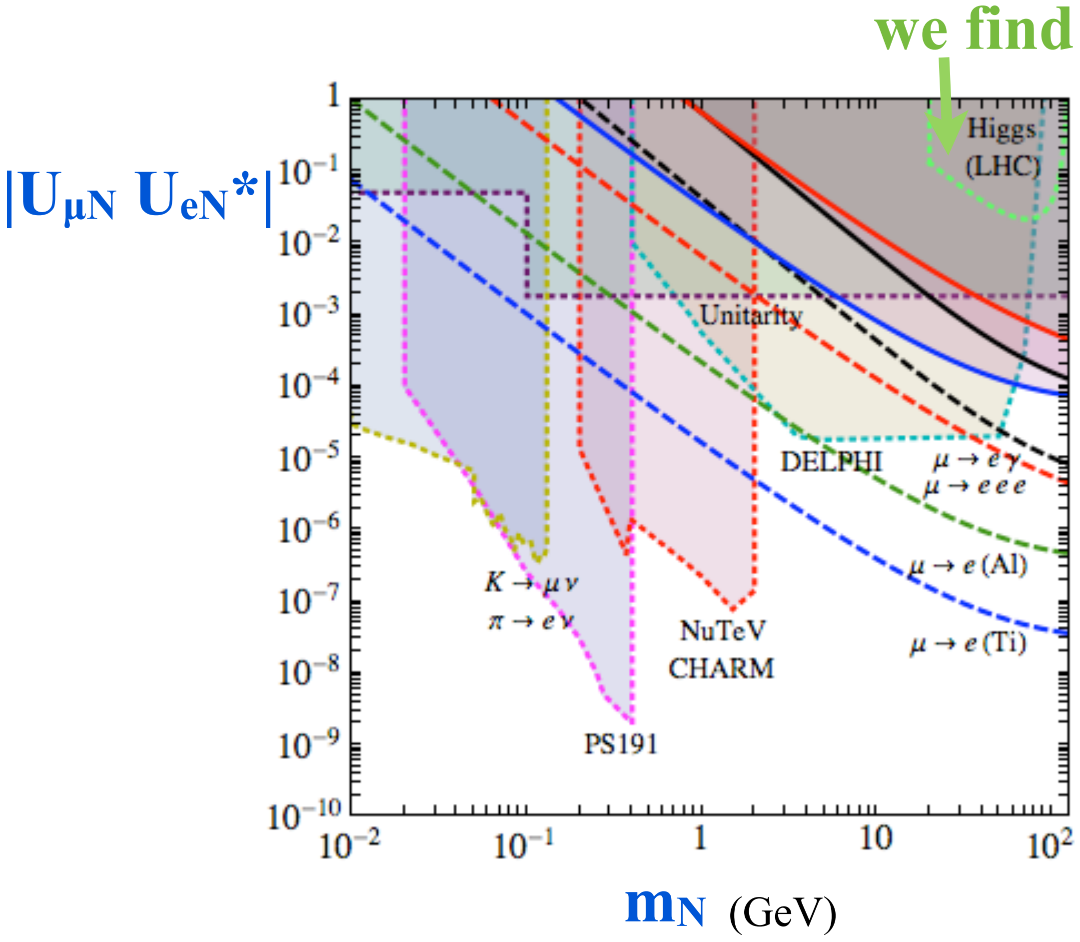
BBN and SN: Kainulainen+Maalampi+Peltoniemi91, Kusenko+Pascoli+Semikoz 05, Mangano+Serpico 11, Ruchaysiliy +Ivashko 12,
Kufflick+McDermott+Zurek 12

* Low mass regime eV << m_N << m_W



This experiment (considered alone) will probe masses down to m_N=2MeV

* Low mass regime eV $<<$ m_N $<<$ m_W



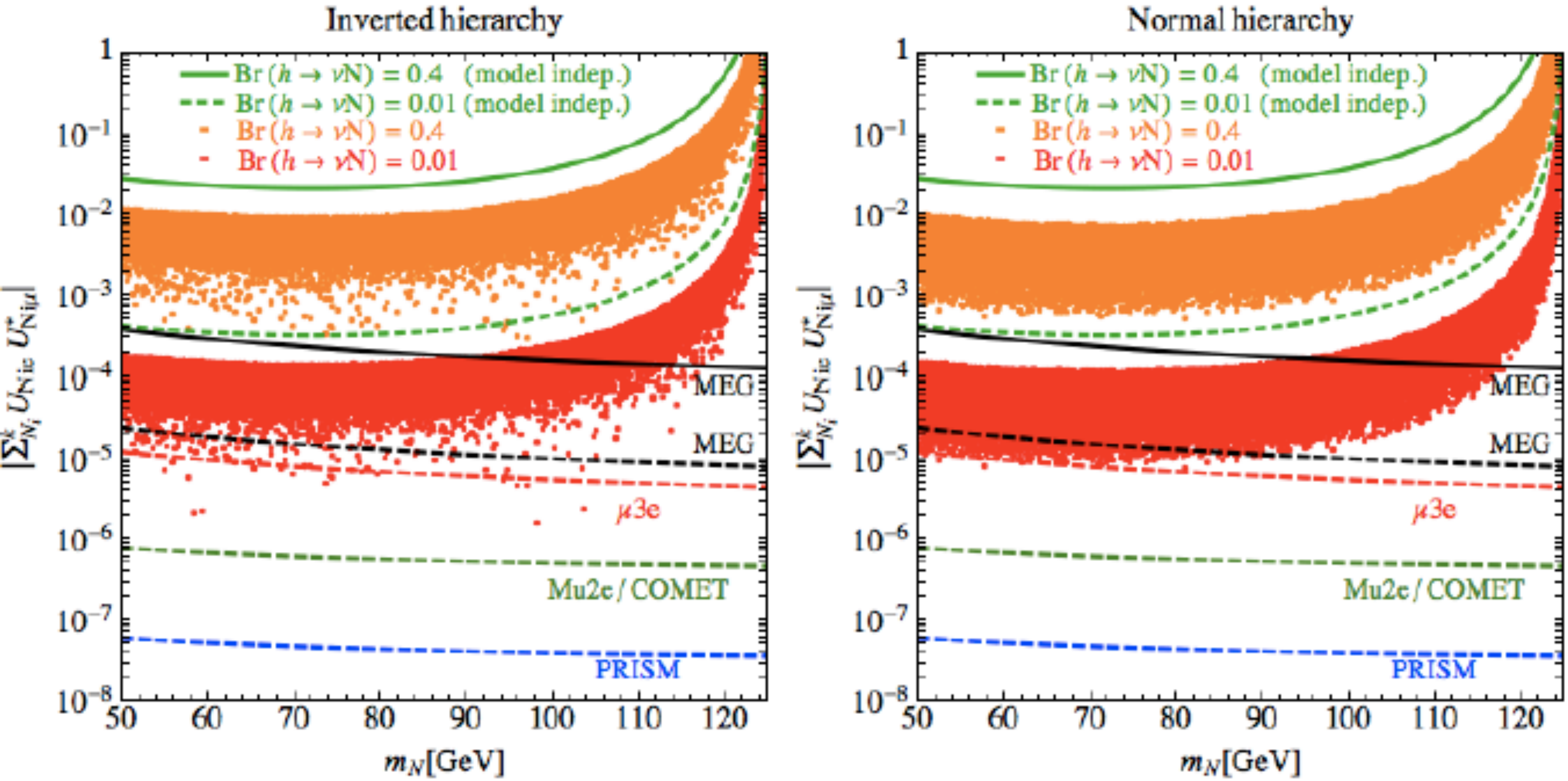
Absolute bound from Higgs, from absence of

$$H \dashrightarrow \nu N, \quad N \to \nu\, l^+ l^-$$

at LHC:
Br (H -> ν N) < 0.4

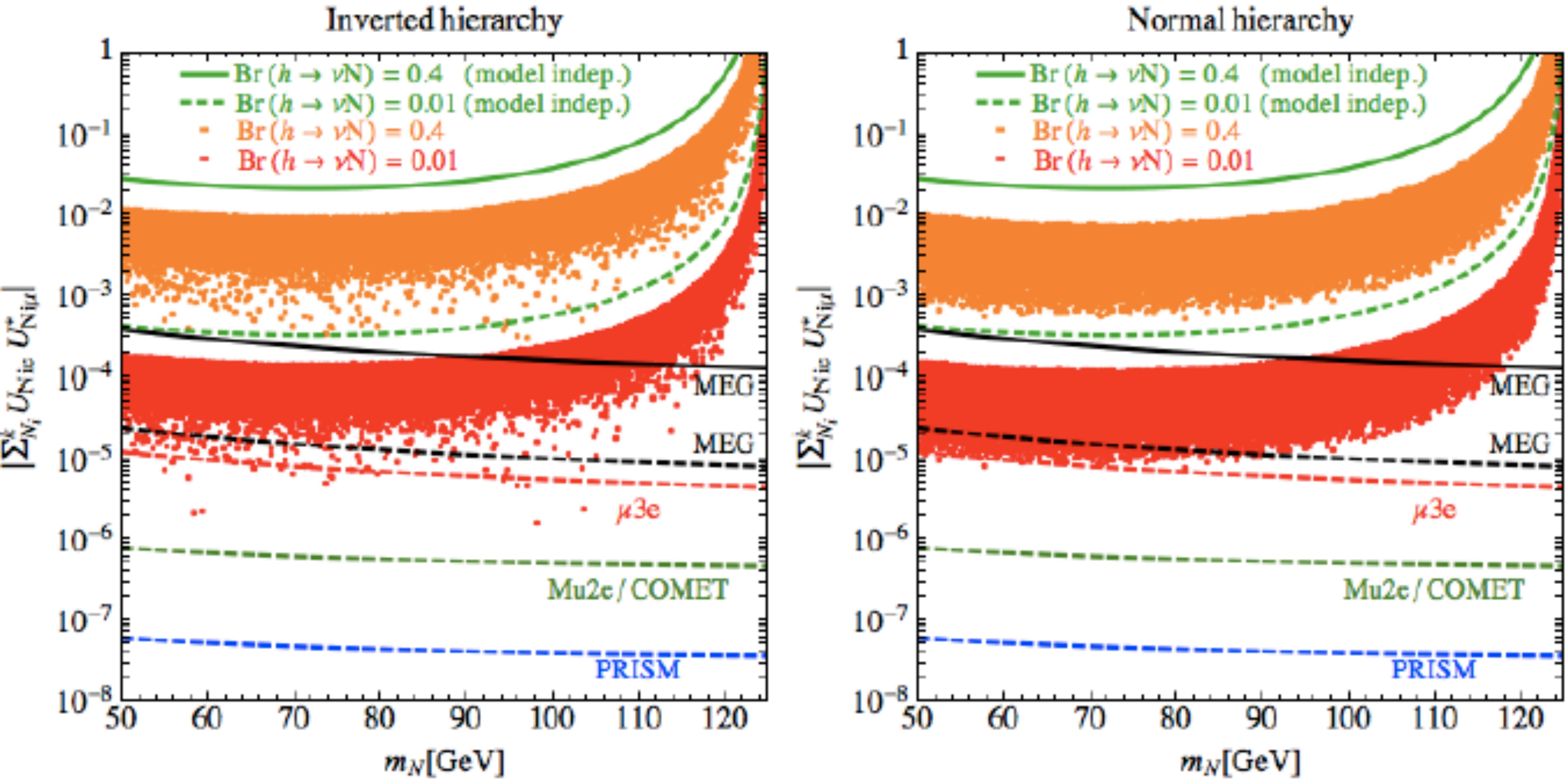
(Espinosa, Grojean, Muhlleitner, Trott, 12
Dev+Franceschini+Mohapatra12, Cely+
Ibarra+Molinaro+Petcov 12)

Varying the CP phases α and δ, we get:



$|U_{\mu N}\, U_{eN}^*|$ versus m_N

Varying the CP phases α and δ, we get:



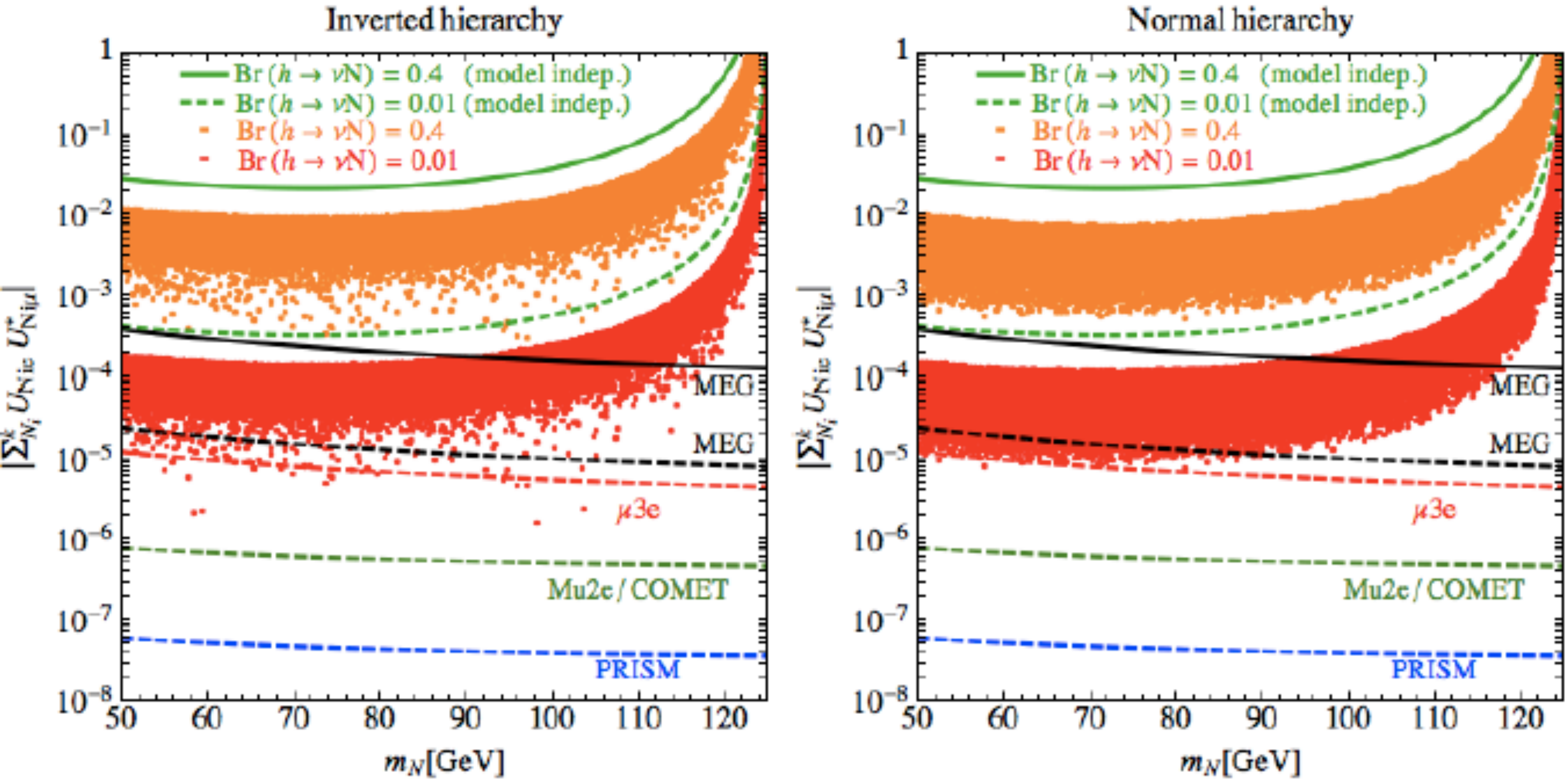
Orange and red model-dependent bounds limited by:

upper boundary: (α= π/2, δ= 0) (α=π, δ=3π/2)

lower boundary: ~(α=-π/2, δ= 0) (α=-π/4, δ=0)

~ it could be consistent with Cely et al. 12, for α~0, δ~ 0

Varying the CP phases α and δ, we get:



For inverted hierarchy: some very low points for which μ-->e very small, because the Yukawas involved ---> 0 for particular values of α and δ (Alonso et al. 09, Alonso 08, Chu+Dhen+Hambye 11....)

For instance, in the minimal seesaw I, Lepton number scale and flavour scale linked:

$$\mathscr{L}_{M_\nu} = \begin{pmatrix} 0 & Y^T v \\ Y v & M_N \end{pmatrix}$$

$$-\mathcal{L}_{\text{seesaw I}} = \overline{L} H Y_E E_R + \overline{L} \tilde{H} Y N + M \overline{N} N^c + h.c.$$

$$m_\nu = Y \frac{v^2}{M} Y^T \qquad U_{lN} \sim \frac{Y}{M}$$

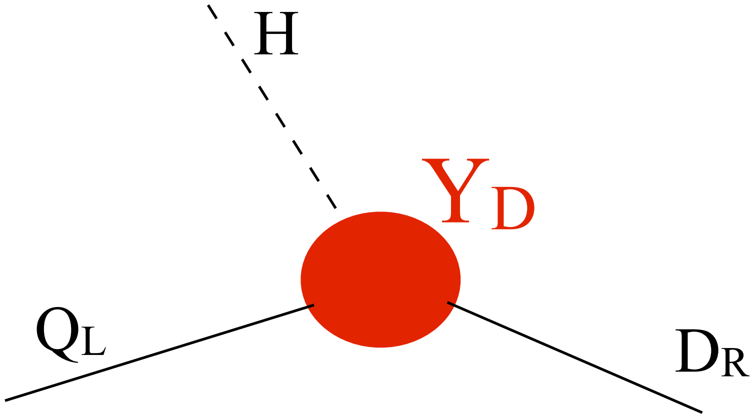
Why quark and neutrino mixings are so different?

May dynamical Yukawas shed light on it?

Assume that the Yukawa couplings have a dynamical origin at high energies …….

(L. Michel+Radicati 70, Cabibbo+Maiani71…Anselm+Berezhiani 96; Berezhiani+Rossi 01)

$Y_{SM} \sim <\varphi>$ or $Y_{SM} \sim 1/<\varphi>$ or …… $<(\varphi\,\chi)^n>$

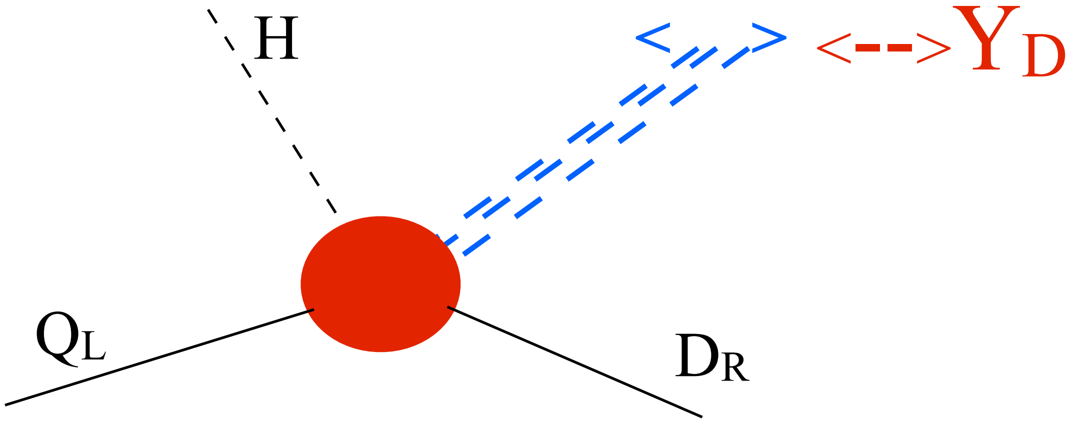


(Alonso+Gavela+Merlo+Rigolin 11)

Assume that the Yukawa couplings have a dynamical origin at high energies ....

(L. Michel+Radicati 70, Cabibbo+Maiani71...Anselm+Berezhiani 96; Berezhiani+Rossi 01)

$Y_{SM} \sim < \varphi >$ or $Y_{SM} \sim 1/< \varphi >$ or ...... $< (\varphi\ \chi)^n>$



(Alonso+Gavela+Merlo+Rigolin 11)

$\mathcal{Y}_d \sim (3, \bar{3}, 1)$ $\mathcal{Y}_u \sim (3, 1, \bar{3})$

$$\boxed{\frac{<\mathcal{Y}_d>}{\Lambda_f} = Y_D = V_{CKM} \begin{pmatrix} y_d & 0 & 0 \\ 0 & y_s & 0 \\ 0 & 0 & y_b \end{pmatrix}}, \quad \boxed{\frac{<\mathcal{Y}_u>}{\Lambda_f} = Y_U = \begin{pmatrix} y_u & 0 & 0 \\ 0 & y_c & 0 \\ 0 & 0 & y_t \end{pmatrix}}.$$

* What is the role of the neutrino flavour group?

To analyze this in general, use common parametrization for quarks and leptons:

$$\mathbf{Y} = U_L \; y^{diag.} \, U_R$$

* **Quarks**, for instance: U_R unphysical, U_L --> U_{CKM}

$$\mathbf{Y_D} = \mathbf{U_{CKM}} \, \mathrm{diag}(y_d, y_s, y_b) \quad ; \quad \mathbf{Y_U} = \mathrm{diag}(y_u, y_c, y_t)$$

* **Leptons**:

$$\mathbf{Y_E} = \mathrm{diag}(y_e, y_\mu, y_\tau) \quad ; \quad \mathbf{Y_\nu} = U_L \; y^{diag.} \, U_R$$

U_{PMNS} diagonalize

$$\boxed{m_\nu \sim \mathbf{Y_\nu} \frac{v^2}{M} \mathbf{Y_\nu}^{\mathbf{T}} = U_L \, y_\nu^{diag.} \, U_R \frac{v^2}{M} U_R^T \, y_\nu^{diag.} \, U_L^T}$$

SU(n)

* What is the role of the neutrino flavour group?

e.g. $SU(n)_{NR}$ ... leptons

e.g. generic seesaw

$$\mathcal{L} = \mathcal{L}_{SM} + i\overline{N_R}\partial\!\!\!/ N_R - \left[\overline{N_R} Y_N \tilde{\phi}^\dagger \ell_L + \frac{1}{2}\overline{N_R}\mathbf{M}N_R{}^c + h.c.\right]$$

with **M** carrying flavour → **M** spurion

More invariants in this case:

$Tr\,(\mathcal{Y}_E\,\mathcal{Y}_E^+)$ $\quad Tr\,(\mathcal{Y}_E\,\mathcal{Y}_E^+)^2$ $\quad Tr\,(\mathcal{Y}_E\,\mathcal{Y}_E^+\,\mathcal{Y}_\nu\,\mathcal{Y}_\nu^+)$

$Tr\,(\mathcal{Y}_\nu\,\mathcal{Y}_\nu^+)$ $\quad Tr\,(\mathcal{Y}_\nu\,\mathcal{Y}_\nu^+)^2$

$Tr\,(\mathcal{M}_N\,\mathcal{M}_N^+)$ $\quad Tr\,(\mathcal{M}_N\,\mathcal{M}_N^+)^2$ $\quad Tr\,(\mathcal{M}_N\,\mathcal{M}_N^+\,\mathcal{Y}_\nu^+\mathcal{Y}_\nu)$

At the minimum:

* $Tr\,(\mathcal{Y}_\nu\,\mathcal{Y}_\nu^+\,\mathcal{Y}_E\,\mathcal{Y}_E^+) = Tr\,(U_L\,y_\nu^{diag.\,2}\,U_L{}^+\,y_l^{diag.\,2}) \longrightarrow U_L=1$

* $Tr\,(\mathcal{M}_N\,\mathcal{M}_N^+\,\mathcal{Y}_\nu\,\mathcal{Y}_\nu^+) = Tr\,(U_R\,y_\nu^{diag.\,2}\,U_R{}^+\,M_i^{diag.\,2}) \longrightarrow U_R=1$

same conclusion for 3 families of quarks:

$$\mathbf{Y} = U_L \; y^{\text{diag.}} \, U_R$$

* **Quarks**, for instance: U_R unphysical, U_L --> U_{CKM}

$$\mathbf{Y_D} = \mathbf{U_{CKM}} \, \text{diag}(y_d, y_s, y_b) \quad ; \quad \mathbf{Y_U} = \text{diag}(y_u, y_c, y_t)$$

$$\text{Tr} \, (\, \mathcal{Y}_u \, \mathcal{Y}_u^+ \, \mathcal{Y}_d \, \mathcal{Y}_d^+ \,) = \text{Tr} \, (\, U_L \, y_u^{\text{diag. } 2} \, U_L^{\,+} \, y_d^{\text{diag. } 2})$$

$$\longrightarrow \quad U_L = U_{\,CKM} \sim 1 \text{ at the minimum}$$

NO MIXING

$$\mathrm{Tr}(\mathcal{Y}_{\mathrm{u}}\,\mathcal{Y}_{\mathrm{u}}^{+}\,\mathcal{Y}_{\mathrm{d}}\,\mathcal{Y}_{\mathrm{d}}^{+}) \propto \sum_{i,j} |\mathbf{V}_{CKM}^{ij}|^2 m_{u_i}^2 m_{d_j}^2$$

e.g. for the case of two families:

$$\mathrm{Tr}(\mathcal{Y}_{\mathrm{u}}\,\mathcal{Y}_{\mathrm{u}}^{+}\,\mathcal{Y}_{\mathrm{d}}\,\mathcal{Y}_{\mathrm{d}}^{+}) \propto (m_c^2 - m_u^2)(m_s^2 - m_d^2)\cos 2\theta$$

The mixing term in **V** is now:

$$\mathrm{Tr}(y_E\, y_E^+\, y_\nu\, y_\nu^+)=$$

$$(y^2+y'^2)\sum_{l,i}|U^{li}_{PMNS}|^2 m_l^2 m_{\nu_i} + (y^2-y'^2)\Big[\;\mathrm{i}\sum_{l,i<j}(U^{li}_{PMNS})^* U^{lj} m_l^2 \sqrt{m_{\nu_i} m_{\nu_j}} + c.c.$$

while for quarks it was:

$$\mathrm{Tr}(y_u\, y_u^+\, y_d\, y_d^+) \propto \sum_{i,j}|U^{ij}_{CKM}|^2 m_{u_i}^2 m_{d_j}^2$$

The mixing term in **V** is now:

$$\mathrm{Tr}(y_E\, y_E^+\, y_\nu\, y_\nu^+) \propto$$

$$(y^2+y'^2)\sum_{l,i}|U^{li}_{PMNS}|^2 m_l^2 m_{\nu_i} + (y^2-y'^2)\left[\; i \sum_{l,i<j}(U^{li}_{PMNS})^* U^{lj} m_l^2 \sqrt{m_{\nu_i} m_{\nu_j}} + c.c. \right]$$

extra term because of Majorana character

while for quarks it was:

$$\mathrm{Tr}(y_u\, y_u^+\, y_d\, y_d^+) \propto \sum_{i,j}|U^{ij}_{CKM}|^2 m_{u_i}^2 m_{d_j}^2$$

* What is the role of the neutrino flavour group?

e.g. $SU(n)_{NR}$ ... leptons

e.g. generic seesaw

$$\mathcal{L} = \mathcal{L}_{SM} + i\overline{N_R}\not{\partial}N_R - \left[\overline{N_R}Y_N\tilde{\phi}^\dagger\ell_L + \frac{1}{2}\overline{N_R}\mathbf{M}N_R{}^c + h.c.\right]$$

with **M** carrying flavour → **M** spurion

More invariants in this case:

$\mathrm{Tr}\,(\mathcal{Y}_E\,\mathcal{Y}_E^+)$ $\quad$ $\mathrm{Tr}\,(\mathcal{Y}_E\,\mathcal{Y}_E^+)^2$ $\quad$ $\mathrm{Tr}\,(\mathcal{Y}_E\,\mathcal{Y}_E^+\,\mathcal{Y}_\nu\,\mathcal{Y}_\nu^+)$

$\mathrm{Tr}\,(\mathcal{Y}_\nu\,\mathcal{Y}_\nu^+)$ $\quad$ $\mathrm{Tr}\,(\mathcal{Y}_\nu\,\mathcal{Y}_\nu^+)^2$

$\mathrm{Tr}\,(\mathcal{M}_N\,\mathcal{M}_N^+)$ $\quad$ $\mathrm{Tr}\,(\mathcal{M}_N\,\mathcal{M}_N^+)^2$ $\quad$ $\mathrm{Tr}\,(\mathcal{M}_N\,\mathcal{M}_N^+\,\mathcal{Y}_\nu^+\mathcal{Y}_\nu)$

At the minimum:

* $\mathrm{Tr}\,(\mathcal{Y}_\nu\,\mathcal{Y}_\nu^+\,\mathcal{Y}_E\,\mathcal{Y}_E^+) = \mathrm{Tr}\,(U_L\,y_\nu^{diag.\,2}\,U_L{}^+\,y_l^{diag.\,2}) \longrightarrow U_L=1$

* $\mathrm{Tr}\,(\mathcal{M}_N\,\mathcal{M}_N^+\,\mathcal{Y}_\nu\,\mathcal{Y}_\nu^+) = \mathrm{Tr}\,(U_R\,y_\nu^{diag.\,2}\,U_R{}^+\,M_i^{diag.\,2}) \longrightarrow U_R=1$

$$G_f = U(3)_Q \times U(3)_U \times U(3)_D$$

$$V(\mathcal{Y}_u, \mathcal{Y}_u) = \Sigma_i [- \mu_i^2 \,\mathrm{Tr}(\mathcal{Y}_i \mathcal{Y}_i^+) - \lambda_i \,\mathrm{Tr}(\mathcal{Y}_i \mathcal{Y}_i^+)^2]$$

$$+ \Sigma_{i \neq j} [\lambda_{ij} \,\mathrm{Tr}(\mathcal{Y}_i \mathcal{Y}_i^+ \mathcal{Y}_j \mathcal{Y}_j^+)] + \ldots$$

it only relies on G_f symmetry and SM gauge symmetry

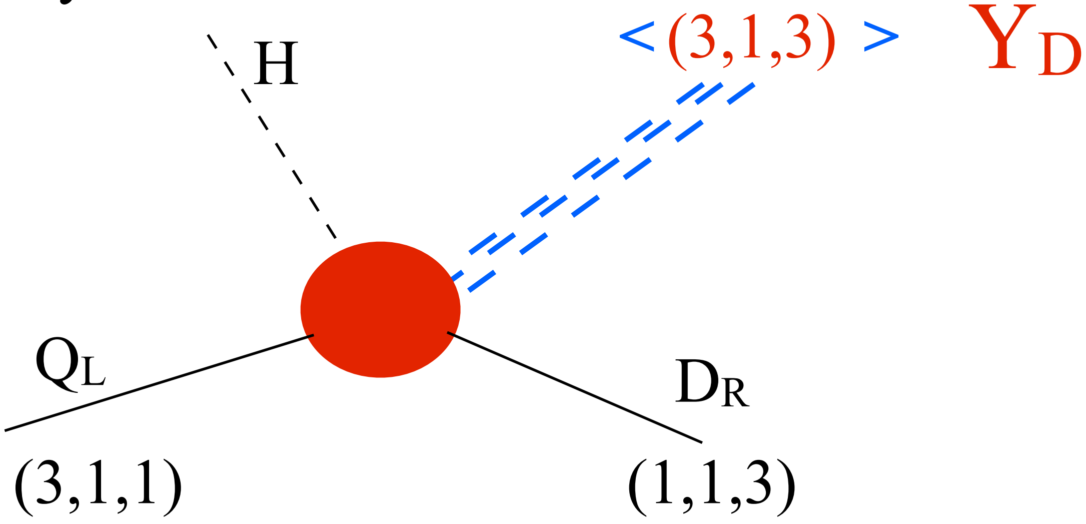
It allows for either (too) hierarchical or degenerate spectrum

(Alonso, Gavela, Merlo, Rigolin 11; Nardi 11; Espinosa, Fong, Nardi 13)

Use the flavour symmetry of the SM with masless fermions:

$$G_f = U(3)_{Q_L} \times U(3)_{U_R} \times U(3)_{D_R}$$

replace Yukawas by fields:



Spontaneous breaking of flavour symmetry dangerous

--> i.e. gauge it (Grinstein, Redi, Villadoro, 2010)
(Feldman, 2010)
(Guadagnoli, Mohapatra, Sung, 2010)

$$G_f = U(3)_Q \;\; \mathrm{x} \;\; U(3)_U \;\; \mathrm{x} \;\; U(3)_D$$

$$V\,(\,\mathcal{Y}_{\mathbf{u}},\; \mathcal{Y}_{\mathbf{u}}\,) = \Sigma_i\, [\, -\, \mu_i^2\, \mathrm{Tr}\,(\, \mathcal{Y}_{\mathbf{i}}\, \mathcal{Y}_{\mathbf{i}}^{+}\,) - \lambda_i\, \mathrm{Tr}\,(\, \mathcal{Y}_{\mathbf{i}}\, \mathcal{Y}_{\mathbf{i}}^{+}\,)^2\,]$$

$$+\, \Sigma_{i \neq j}\, [\, \lambda_{ij}\, \mathrm{Tr}\,(\, \mathcal{Y}_{\mathbf{i}}\, \mathcal{Y}_{\mathbf{i}}^{+}\, \mathcal{Y}_{\mathbf{j}}\, \mathcal{Y}_{\mathbf{j}}^{+}\,)\,] + \;\ldots$$

it only relies on G_f symmetry and SM gauge symmetry

and analyzed its minima

(Alonso, Gavela, Merlo, Rigolin, arXiv 11; Nardi 11, Espinosa, Fong, Nardi 13)

Can its minimum correspond **naturally** to the observed masses and mixings?

i.e. with all dimensionless λ’s ~ 1

and dimensionful μ´s $\leqq \Lambda_f$

Y --> one single field Σ

Spectrum for flavons Σ in the bifundamental:

* 3 generations: for the largest fraction of the parameter space, the stable solution is a degenerate spectrum

$$\begin{pmatrix} y_u & & \\ & y_c & \\ & & y_t \end{pmatrix} \sim \begin{pmatrix} y & & \\ & y & \\ & & y \end{pmatrix}$$

instead of the observed hierarchical spectrum, i.e.

$$\begin{pmatrix} y_u & & \\ & y_c & \\ & & y_t \end{pmatrix} \sim \begin{pmatrix} 0 & & \\ & 0 & \\ & & y \end{pmatrix}$$

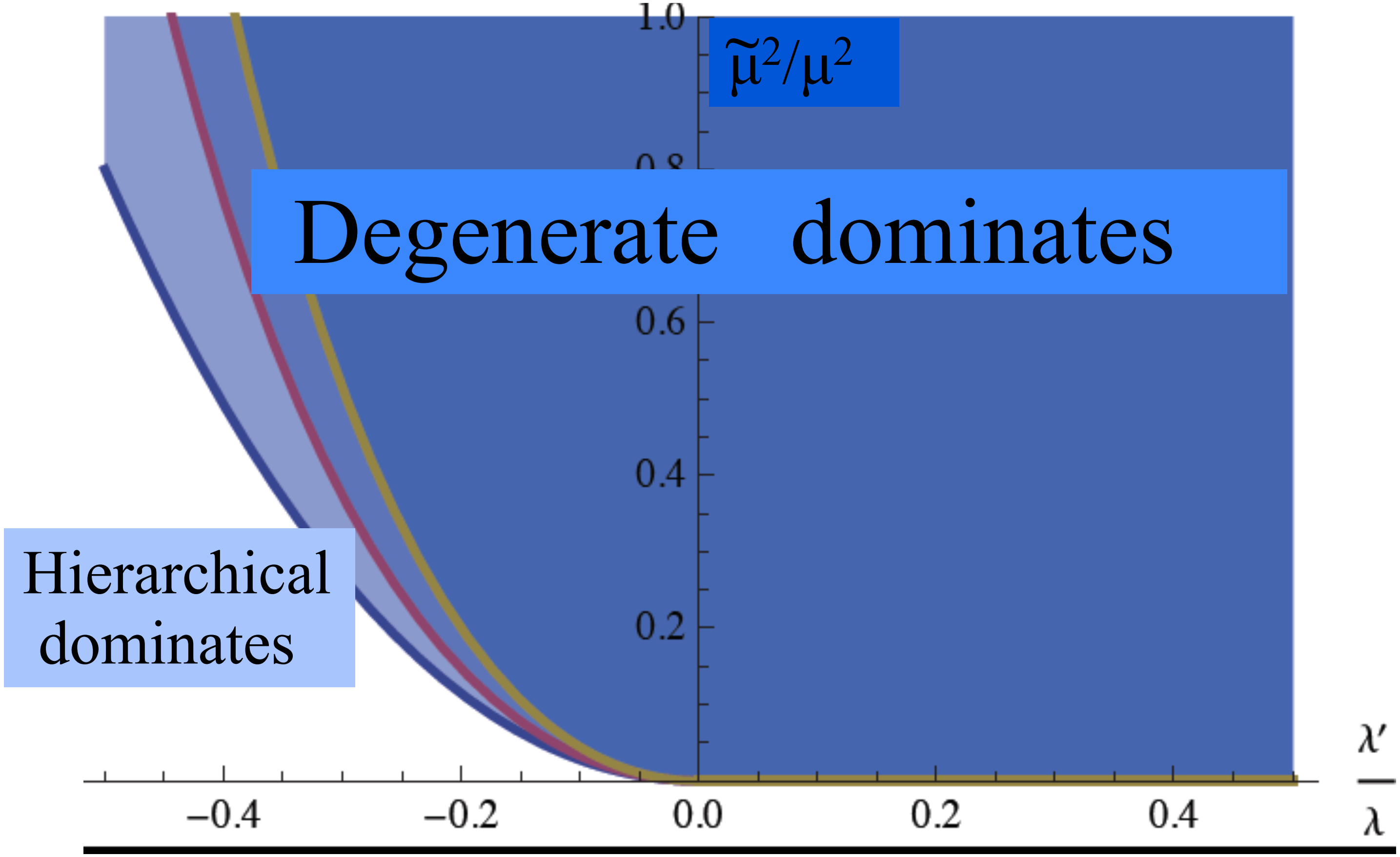
(at leading order)

Spectrum: the hierarchical solution is unstable in most of the parameter space.

Stability: $\boxed{\frac{\tilde{\mu}^2}{\mu^2} < \frac{2\lambda'^2}{\lambda}}$

$$V^{(4)} = \sum_{i=u,d} \left(-\mu_i^2 A_i + \tilde{\mu}_i B_i + \lambda_i A_i^2 + \lambda_i' A_{ii}\right) + g_{ud} A_u A_d + \lambda_{ud} A_{ud} \,.$$

ie, the u-part: $V^{(4)} = -\mu_u^2 A_u + \tilde{\mu}_u B_u + \lambda_u A_u^2 + \lambda_u' A_{uu}$

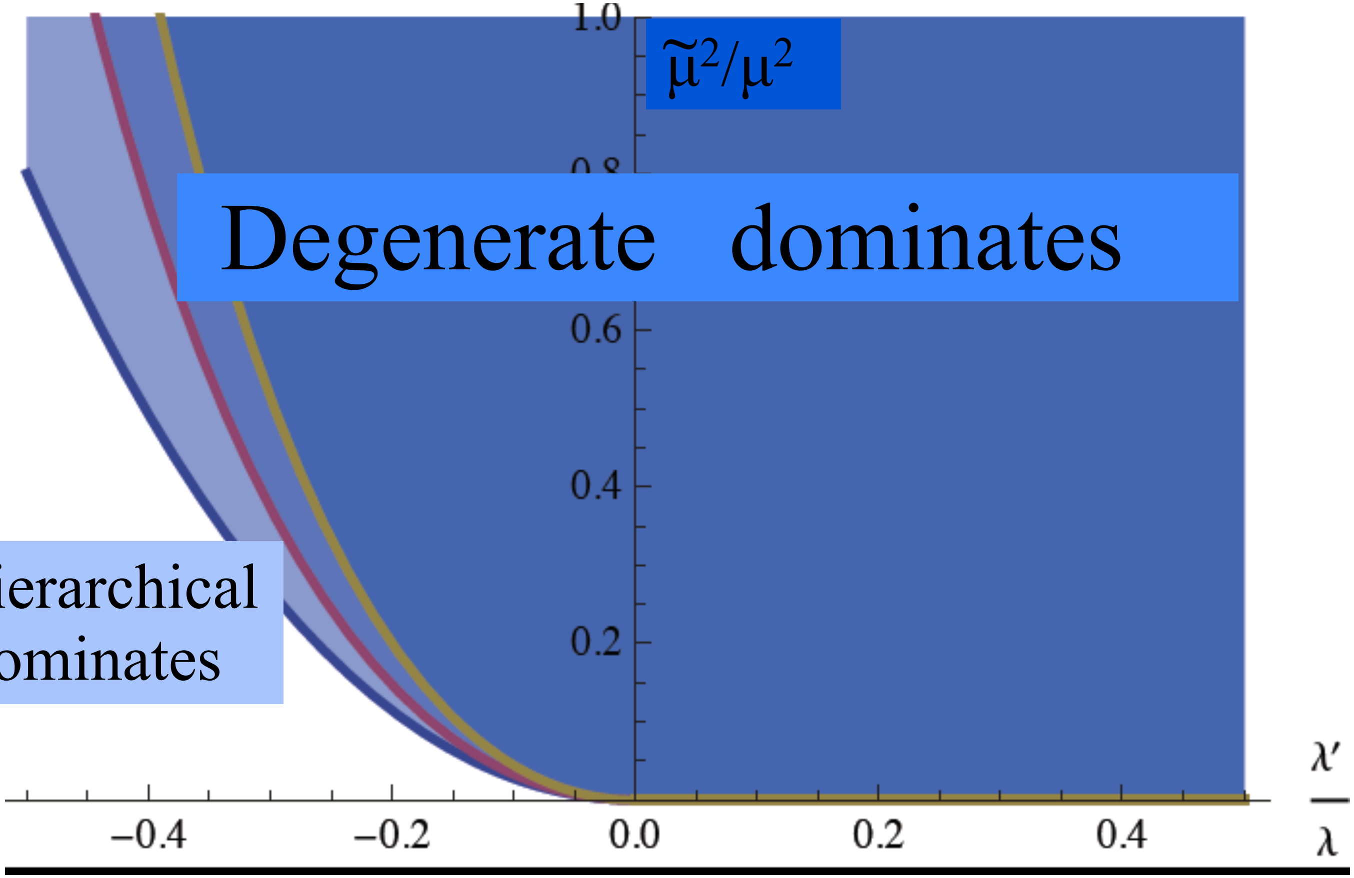


Spectrum: the hierarchical solution is unstable in most of the parameter space.

Stability: $\boxed{\frac{\tilde{\mu}^2}{\mu^2} < \frac{2\lambda'^2}{\lambda}}$

$$V^{(4)} = \sum_{i=u,d} \left(-\mu_i^2 A_i + \tilde{\mu}_i B_i + \lambda_i A_i^2 + \lambda_i' A_{ii}\right) + g_{ud} A_u A_d + \lambda_{ud} A_{ud} \,.$$

ie, the u-part: $V^{(4)} = -\mu_u^2 A_u + \tilde{\mu}_u B_u + \lambda_u A_u^2 + \lambda_u' A_{uu}$



Degenerate dominates

Hierarchical dominates

Nardi emphasized this solution (and extended the analysis to include also U(1) factors)

Normal hierarchy:

Up to terms of $\mathcal{O}(\sqrt{r}, s_{13})$, we find

$$r = \frac{|\Delta m^2_{12}|}{|\Delta m^2_{13}|}$$

$$Y_N^T \simeq y \begin{pmatrix} e^{i\delta} s_{13} + e^{-i\alpha} s_{12} r^{1/4} \\ s_{23}\left(1 - \frac{\sqrt{r}}{2}\right) + e^{-i\alpha} r^{1/4} c_{12} c_{23} \\ c_{23}\left(1 - \frac{\sqrt{r}}{2}\right) - e^{-i\alpha} r^{1/4} c_{12} s_{23} \end{pmatrix} .$$

Inverted hierarchy:

$$Y_N^T \simeq \frac{y}{\sqrt{2}} \begin{pmatrix} c_{12} e^{i\alpha} + s_{12} e^{-i\alpha} \\ c_{12}\left(c_{23} e^{-i\alpha} - s_{23} s_{13} e^{i(\alpha-\delta)}\right) - s_{12}\left(c_{23} e^{i\alpha} + s_{23} s_{13} e^{-i(\alpha+\delta)}\right) \\ -c_{12}\left(s_{23} e^{-i\alpha} + c_{23} s_{13} e^{i(\alpha-\delta)}\right) + s_{12}\left(s_{23} e^{i\alpha} - c_{23} s_{13} e^{-i(\alpha+\delta)}\right) \end{pmatrix}$$

Y --> one single field Σ

The invariants can be written in terms of masses and mixing

* two families:

$< \Sigma_d > = \Lambda_f \,.\, \mathrm{diag}\,(y_d)\,;\qquad < \Sigma_u > = \Lambda_f \,.V_{\mathrm{Cabibbo}}\,\mathrm{diag}(y_u)$

$$Y_D = \begin{pmatrix} y_d & 0 \\ 0 & y_s \end{pmatrix}\,, \qquad Y_U = \mathcal{V}_C^\dagger \begin{pmatrix} y_u & 0 \\ 0 & y_c \end{pmatrix} \qquad V_{\mathrm{Cabibbo}} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}$$

$<\mathrm{Tr}\,(\,\Sigma_u\,\Sigma_u^+\,)> = \Lambda_f^{\,2}\;(y_u^2 + y_c^2)\,;\quad <\det\,(\,\Sigma_u\,)> = \Lambda_f^{\,2}\,y_u y_c$

$<\mathrm{Tr}\,(\,\Sigma_u\,\Sigma_u^+\,\Sigma_d\,\Sigma_d^+\,)>=\Lambda_f^4\,[(\,y_c^2 - y_u^2)\,(\,y_s^2 - y_d^2)\,\cos 2\theta + \ldots]/2$

(Alonso, Gavela, Merlo, Rigolin, arXiv 1103.2915)

Y --> one single field Σ

Minimum of the Potential

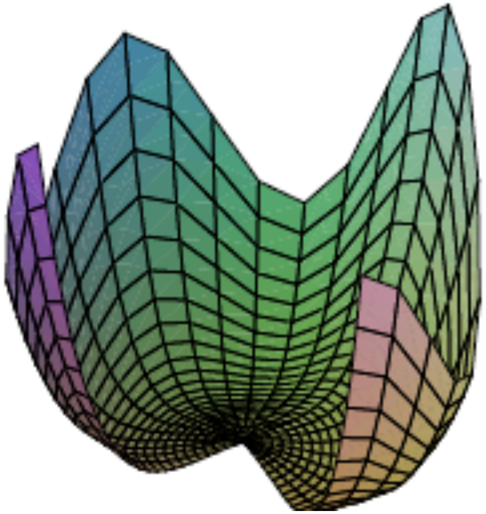
Dimension 5 Yukawa Operator

The minimum of the Potential is given by:

$$\frac{\partial V}{\partial y_i} = 0 \qquad \frac{\partial V}{\partial \theta_i} = 0$$

Take the angle for example:

$$\frac{\partial V}{\partial \theta_c} \propto (y_c^2 - y_u^2)\,(y_s^2 - y_d^2)\,\sin 2\theta_c = 0$$



Non-degenerate masses $\longrightarrow$ $\sin 2\theta_c = 0$ No mixing !

Notice also that $\dfrac{\partial V^{(4)}}{\partial \theta} \sim \sqrt{J}$ (Jarlskog determinant)

(Alonso, Gavela, Merlo, Rigolin, arXiv 1103.2915)

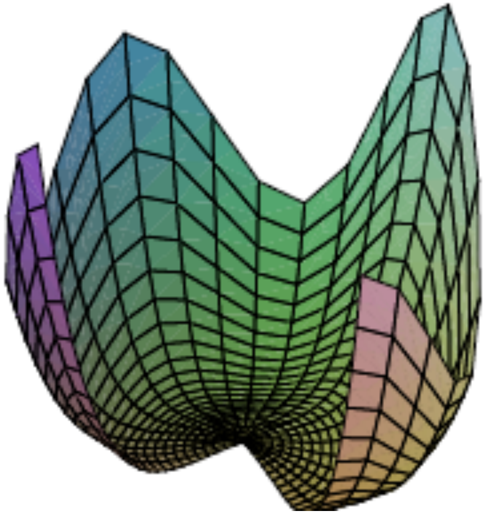
Y --> one single field Σ

Minimum of the Potential

Dimension 5 Yukawa Operator

The minimum of the Potential is given by:

$$\frac{\partial V}{\partial y_i} = 0 \qquad \frac{\partial V}{\partial \theta_i} = 0$$



Take the angle for example:

$$\frac{\partial V}{\partial \theta_c} \propto (y_c^2 - y_u^2)\,(y_s^2 - y_d^2)\,\sin 2\theta_c = 0$$

Non-degenerate masses $\longrightarrow$ $\sin 2\theta_c = 0$ No mixing !

Can the actual masses and mixings fit naturally in the minimum of the Potential? e.g. adding non-renormalizable terms...

NO

(Alonso, Gavela, Merlo, Rigolin, arXiv 1103.2915)

Y --> one single field Σ

Minimum of the Potential

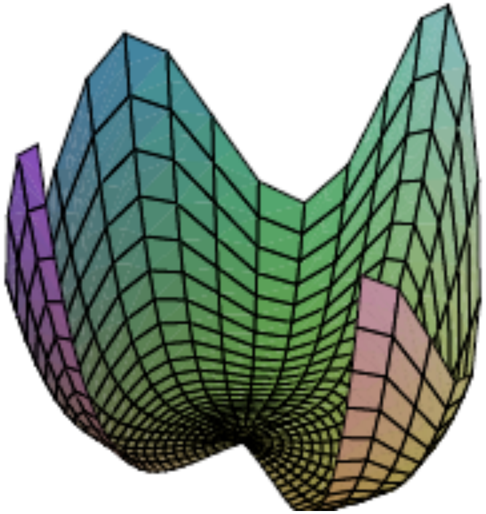
Dimension 5 Yukawa Operator

The minimum of the Potential is given by:

$$\frac{\partial V}{\partial y_i} = 0 \qquad \frac{\partial V}{\partial \theta_i} = 0$$

Take the angle for example:

$$\frac{\partial V}{\partial \theta_c} \propto \left(y_c^2 - y_u^2\right)\left(y_s^2 - y_d^2\right) \sin 2\theta_c = 0$$



Non-degenerate masses $\sin 2\theta_c = 0$ No mixing !

Can the actual masses and mixings fit naturally in the minimum of the Potential? e.g. adding non-renormalizable terms... **NO**

* Without fine-tuning, for two families the spectrum is degenerate

* To accomodate realistic mixing one must introduce wild fine tunnings of O(10^{-10}) and nonrenormalizable terms of dimension 8

(Alonso, Gavela, Merlo, Rigolin)

Y --> one single field Σ

three families

* at renormalizable level: 7 invariants instead of the 5 for two families

$$\mathrm{Tr}\left(\Sigma_u\Sigma_u^\dagger\right) \overset{vev}{=} \Lambda_f^2\left(y_t^2+y_c^2+y_u^2\right)\,, \qquad Det\left(\Sigma_u\right) \overset{vev}{=} \Lambda_f^3 y_u y_c y_t\,,$$

$$\mathrm{Tr}\left(\Sigma_d\Sigma_d^\dagger\right) \overset{vev}{=} \Lambda_f^2\left(y_b^2+y_s^2+y_d^2\right)\,, \qquad Det\left(\Sigma_d\right)\Big| \overset{vev}{=} \Lambda_f^3 y_d y_s y_b\,,$$

$$\mathrm{Tr}\left(\Sigma_u\Sigma_u^\dagger\Sigma_u\Sigma_u^\dagger\right) \overset{vev}{=} \Lambda_f^4\left(y_t^4+y_c^4+y_u^4\right)\,,$$

$$\mathrm{Tr}\left(\Sigma_d\Sigma_d^\dagger\Sigma_d\Sigma_d^\dagger\right) \overset{vev}{=} \Lambda_f^4\left(y_b^4+y_s^4+y_d^4\right)\,,$$

$$\mathrm{Tr}\left(\Sigma_u\Sigma_u^\dagger\Sigma_d\Sigma_d^\dagger\right) \overset{vev}{=} \Lambda_f^4\left(P_0+P_{int}\right)\,,$$

Interesting angular dependence:

$$P_0 \equiv -\sum_{i<j}\left(y_{u_i}^2-y_{u_j}^2\right)\left(y_{d_i}^2-y_{d_j}^2\right)\sin^2\theta_{ij}\,,$$

$$\begin{aligned}
P_{int} \equiv \sum_{i<j,k}&\left(y_{d_i}^2-y_{d_k}^2\right)\left(y_{u_j}^2-y_{u_k}^2\right)\sin^2\theta_{ik}\sin^2\theta_{jk}+\\
&-\left(y_d^2-y_s^2\right)\left(y_c^2-y_t^2\right)\sin^2\theta_{12}\sin^2\theta_{13}\sin^2\theta_{23}+\\
&+\frac{1}{2}\left(y_d^2-y_s^2\right)\left(y_c^2-y_t^2\right)\cos\delta\,\sin 2\theta_{12}\sin 2\theta_{23}\sin\theta_{13}\,,
\end{aligned}$$

The real, unavoidable, problem is again mixing:

* Just one source:

$$\mathrm{Tr}\,(\Sigma_u\,\Sigma_u^+\,\Sigma_d\,\Sigma_d^+) = \Lambda_f^4\,(P_0 + P_{int})$$

P_0 and P_{int} encode the angular dependence,

$$P_0 \equiv -\sum_{i<j}\left(y^2_{u_i} - y^2_{u_j}\right)\left(y^2_{d_i} - y^2_{d_j}\right)\sin^2\theta_{ij}\,,$$

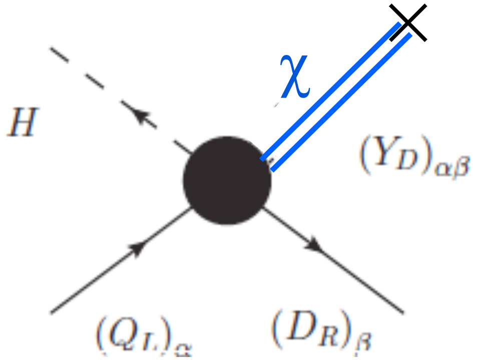
$$\begin{aligned} P_{int} \equiv \sum_{i<j,k} & (y^2_{d_i} - y^2_{d_k})\left(y^2_{u_j} - y^2_{u_k}\right)\sin^2\theta_{ik}\sin^2\theta_{jk} + \\ & - (y_d^2 - y_s^2)\,(y_c^2 - y_t^2)\sin^2\theta_{12}\sin^2\theta_{13}\sin^2\theta_{23} + \\ & + \frac{1}{2}\,(y_d^2 - y_s^2)\,(y_c^2 - y_t^2)\cos\delta\,\sin 2\theta_{12}\sin 2\theta_{23}\sin\theta_{13} \end{aligned}$$

Sad conclusions as for 2 families:

needs non-renormalizable + super fine-tuning

Y --> quadratic in fields χ

$$Y \sim \frac{< \chi \chi^{\dagger} >}{\Lambda_f^2}$$

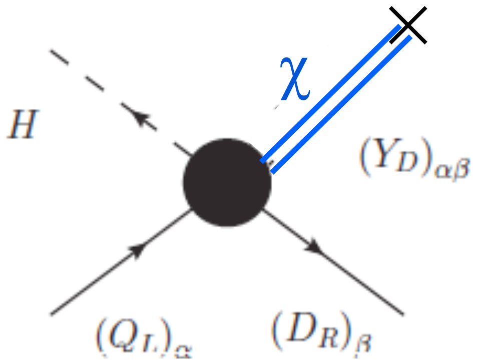


→ **Automatic strong mass hierarchy and one mixing angle already at the renormalizable level**

Holds for 2 and 3 families !

2) Y --> quadratic in fields χ

$$Y \sim \frac{<\chi\chi^{\dagger}>}{\Lambda_f^2}$$



→ i.e. $Y_D \sim \dfrac{\chi^L{}_d\ (\chi^R{}_d)^+}{\Lambda_f^2} \sim (3, 1, 1)\ (1, 1, \overline{3}) \sim (3, 1, \overline{3})$

Y --> quadratic in fields χ

It is very simple:

- a square matrix built out of 2 vectors

$$\begin{pmatrix} d \\ e \\ f \\ \vdots \end{pmatrix} (a , b , c \ldots\ldots)$$

has only one non-vanishing eigenvalue

→ strong mass hierarchy at leading order:

-- only 1 heavy “up” quark

-- only 1 heavy “down” quark

only $|\chi|$´s relevant for scale

Y --> quadratic in fields χ

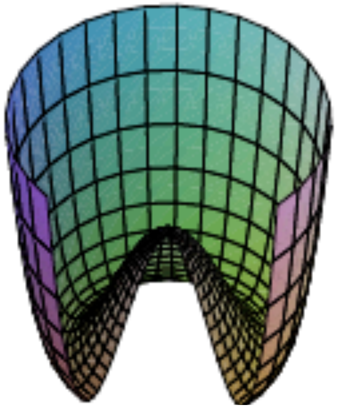
Minimum of the Potential

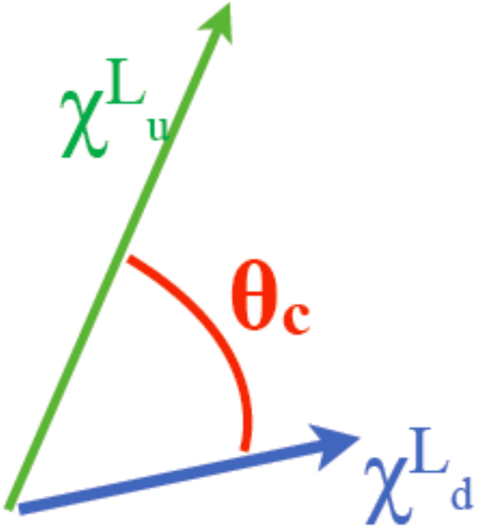
Dimension 6 Yukawa Operator

The invariants are:

$$\chi_u^{L\dagger}\chi_u^L\,, \qquad \chi_u^{R\dagger}\chi_u^R\,, \qquad \chi_d^{L\dagger}\chi_d^L\,,$$

$$\chi_d^{R\dagger}\chi_d^R\,, \qquad \chi_u^{L\dagger}\chi_d^L = \left|\chi_u^L\right|\left|\chi_d^L\right|\cos\theta_c\,.$$





θ_c is the angle between up and down L vectors

Y --> quadratic in fields χ

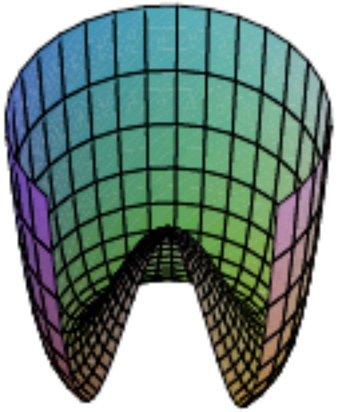
Minimum of the Potential

Dimension 6 Yukawa Operator

The invariants are:

$$\chi_u^{L\dagger}\chi_u^L, \qquad \chi_u^{R\dagger}\chi_u^R, \qquad \chi_d^{L\dagger}\chi_d^L,$$

$$\chi_d^{R\dagger}\chi_d^R, \qquad \chi_u^{L\dagger}\chi_d^L = \left|\chi_u^L\right| \left|\chi_d^L\right| \cos\theta_c .$$



We can fit the angle and the masses in the Potential; as an example:

$$V' = \lambda_u \left(\chi_u^{L\dagger}\chi_u^L - \frac{\mu_u^2}{2\lambda_u}\right)^2 + \lambda_d \left(\chi_d^{L\dagger}\chi_d^L - \frac{\mu_d^2}{2\lambda_d}\right)^2$$

$$+\lambda_{ud} \left(\chi_u^{L\dagger}\chi_d^L - \frac{\mu_{ud}^2}{2\lambda_{ud}}\right)^2 + \cdots$$

Whose minimum sets (2 generations):

$$y_c^2 = \frac{\mu_u^2}{2\lambda_u\Lambda_f^2} \qquad y_s^2 = \frac{\mu_d^2}{2\lambda_d\Lambda_f^2} \qquad \cos\theta = \frac{\mu_{ud}^2\sqrt{\lambda_u\lambda_d}}{\mu_u\mu_d\lambda_{ud}}$$

Y --> quadratic in fields χ

Towards a realistic 3 family spectrum

e.g. replicas of χ^L , χ^R_u , χ^R_d

???

Suggests sequential breaking:

$$\mathbf{SU(3)^3} \xrightarrow[\mathbf{mt,\ mb}]{} \mathbf{SU(2)^3} \xrightarrow[\mathbf{mc,\ ms,\ \theta_C}]{} \ldots\ldots\ldots\ldots$$

$$Y_u \equiv \frac{\langle \chi^L \rangle \langle \chi_u^{R\dagger} \rangle}{\Lambda_f^2} + \frac{\langle \chi_u'^L \rangle \langle \chi_u'^{R\dagger} \rangle}{\Lambda_f^2} = \begin{pmatrix} 0 & \sin\theta\, y_c & 0 \\ 0 & \cos\theta\, y_c & 0 \\ 0 & 0 & y_t \end{pmatrix}$$

$$Y_d \equiv \frac{\langle \chi^L \rangle \langle \chi_d^{R\dagger} \rangle}{\Lambda_f^2} + \frac{\langle \chi_d'^L \rangle \langle \chi_d'^{R\dagger} \rangle}{\Lambda_f^2} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & y_s & 0 \\ 0 & 0 & y_b \end{pmatrix} .$$

Y --> quadratic in fields χ

Towards a realistic 3 family spectrum

e.g. replicas of χ^L , χ^R_u , χ^R_d

???

Suggests sequential breaking:

$$\mathbf{SU(3)^3} \xrightarrow[\text{mt, mb}]{} \mathbf{SU(2)^3} \xrightarrow[\text{mc, ms, }\theta_C]{} \cdots\cdots\cdots$$

$$\frac{\langle\chi^L\rangle\,\langle\chi^{R\dagger}\rangle}{\Lambda_f^2} \quad \frac{\langle\chi'^L\rangle\,\langle\chi'^{R\dagger}\rangle}{\Lambda_f^2} \quad \begin{pmatrix} 0 & \sin\theta\, y_c & 0 \\ & & \\ 0 & 0 & y_t \end{pmatrix}$$

Maybe some connection to: Berezhiani+Nesti; Ferretti et al., Calibbi et al. ??

$$Y_d \equiv \frac{\langle\chi^L\rangle\,\langle\chi_d^{R\dagger}\rangle}{\Lambda_f^2} + \frac{\langle\chi_d'^L\rangle\,\langle\chi_d'^{R\dagger}\rangle}{\Lambda_f^2} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & y_s & 0 \\ 0 & 0 & y_b \end{pmatrix}.$$

* From bifundamentals: $<y_u> = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & y_t \end{pmatrix}$

$$<y_d> = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & y_b \end{pmatrix}$$

* From fundamentals χ : y_c, y_s and θ_C

Y --> linear + quadratic in fields

Towards a realistic 3 family spectrum

Combining fundamentals and bi-fundamentals

i.e. combining d=5 and d =6 Yukawa operators

$$\Sigma_u \sim (3,\overline{3},1)\ , \qquad \Sigma_d \sim (3,1,\overline{3})\ , \qquad \Sigma_R \sim (1,3,\overline{3})\ ,$$

$$\chi_u^L \in (3,1,1)\ , \qquad \chi_u^R \in (1,3,1)\ , \qquad \chi_d^L \in (3,1,1)\ , \qquad \chi_d^R \in (1,1,3)\ .$$

The Yukawa Lagrangian up to the second order in $1/\Lambda_f$ is given by:

$$\mathscr{L}_Y = \overline{Q}_L \left[\frac{\Sigma_d}{\Lambda_f} + \frac{\chi_d^L \chi_d^{R\dagger}}{\Lambda_f^2} \right] D_R H + \overline{Q}_L \left[\frac{\Sigma_u}{\Lambda_f} + \frac{\chi_u^L \chi_u^{R\dagger}}{\Lambda_f^2} \right] U_R \tilde{H} + \text{h.c.}\ ,$$

* What is the role of the neutrino flavour group?

e.g. $O(2)_{NR}$ ... leptons

e.g. seesaw with approximately conserved lepton number

$$\mathcal{L}_{\mathcal{M}_\nu} = (\bar{\ell}_L\,,\, \bar{N}^c\,,\, \bar{N}'^c) \begin{pmatrix} 0 & vY & vY' \\ vY^T & 0 & \mathbf{M}^\mathrm{T} \\ vY'^T & \mathbf{M} & 0 \end{pmatrix} \begin{pmatrix} \ell_L^c \\ N \\ N' \end{pmatrix}$$

* What is the role of the neutrino flavour group?

e.g. O(2)NR ... leptons

e.g. seesaw with approximately conserved lepton number

$$\mathcal{L}_{mass} = \bar{\ell}_L \phi Y_E E_R + \bar{\ell}_L \tilde{\phi} \tilde{Y}_\nu (N_1, N_2)^T + M(\overline{N}_1 N_1^c + \overline{N}_2 N_2^c) + h.c.$$

$$\tilde{Y}_\nu = \frac{1}{\sqrt{2}} U_{PMNS}\, f_{m_\nu} \begin{pmatrix} y + y' & -i(y - y') \\ i(y - y') & y + y' \end{pmatrix}$$

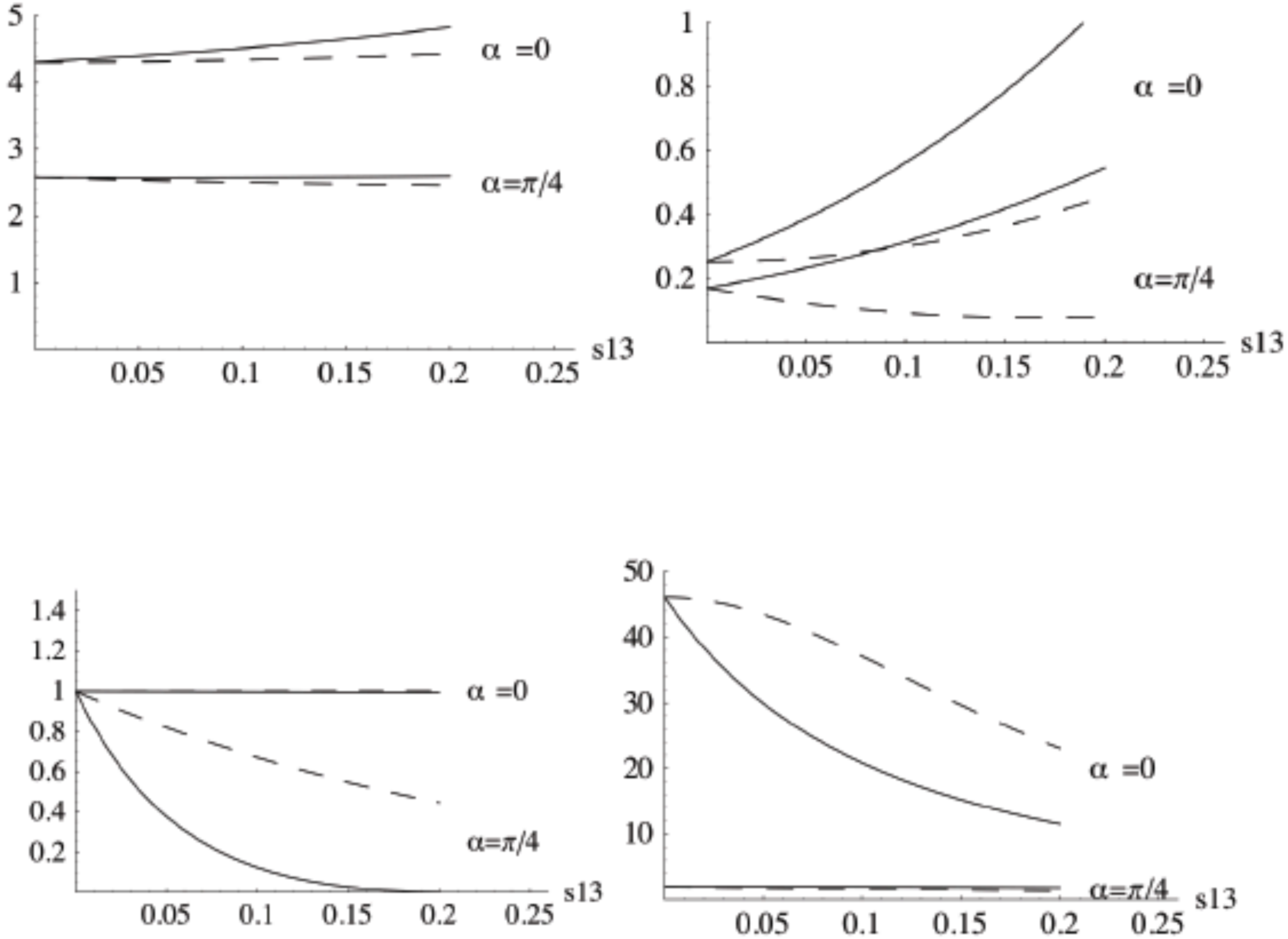
$$U(3)_{\ell_L} \times U(3)_{E_R} \times O(2)_N$$

$$Y_E = \frac{<\mathcal{Y}_E>}{\Lambda_f} \sim (3, \bar{3}, 1); \quad (Y, Y') = \frac{<\mathcal{Y}_\nu>}{\Lambda} \sim (3, 1, 2)$$

$$<\mathcal{Y}_E> \propto \begin{pmatrix} m_e & 0 & 0 \\ 0 & m_\mu & 0 \\ 0 & 0 & m_\tau \end{pmatrix} \quad <\mathcal{Y}_\nu> \propto U_{PMNS} \begin{pmatrix} 0 & 0 \\ \sqrt{m_{\nu_2}} & 0 \\ 0 & \sqrt{m_{\nu_3}} \end{pmatrix} \begin{pmatrix} -iy & iy' \\ y & y' \end{pmatrix}$$

* e-μ, μ-τ etc. oscillations and rare decays studied:

Gavela, Hambye, Hernandez^2 ; .....



$Br(\mu \to e\gamma)/Br(\tau \to e\gamma)$

$Br(\mu \to e\gamma)/Br(\tau \to \mu\gamma)$

NH

IH

Gavela, Hambye, Hernandez^2 ;

Degeneracy in the Majorana phase α

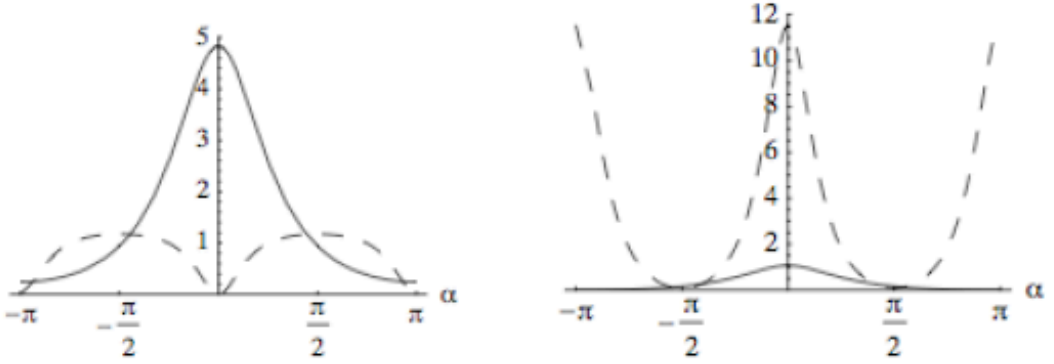
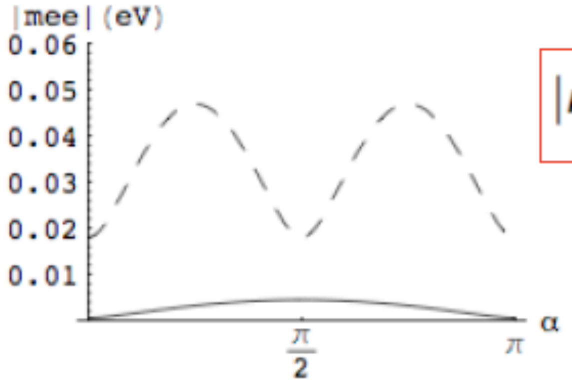


Figure 3: Left: Ratio $B_{e\mu}/B_{e\tau}$ for the normal hierarchy (solid) and the inverse hierarchy (dashed) as a function of α for $(\delta, s_{13}) = (0, 0.2)$. Right: the same for the ratio $B_{e\mu}/B_{\mu\tau}$.



$$\boxed{|m_{ee}|_{IH} \simeq |s_{12}^2 e^{-2i\alpha} - c_{12}^2 e^{2i\alpha}|}$$

Figure 5: m_{ee} as a function of α for the normal (solid) and inverted (dashed) hierarchies, for $(\delta, s_{13}) = (0, 0.2)$.

Gavela, Hambye, Hernandez^2 ;

$$B_{\mu\to e\gamma} \propto |Y_{N_e} Y_{N_\mu}|^2$$

cancellations
for large θ$_{13}$

i.e.

$$Y_N^T \simeq y \begin{pmatrix} e^{i\delta}s_{13} + e^{-i\alpha}s_{12}r^{1/4} \\ s_{23}\left(1-\frac{\sqrt{r}}{2}\right) + e^{-i\alpha}r^{1/4}c_{12}c_{23} \\ c_{23}\left(1-\frac{\sqrt{r}}{2}\right) - e^{-i\alpha}r^{1/4}c_{12}s_{23} \end{pmatrix}$$

$$r = \frac{|\Delta m^2_{12}|}{|\Delta m^2_{13}|}$$

Normal hierarchy

* Alonso + Li, 2010, MINSIS report:
possible suppresion of μ-e transitions for large θ$_{13}$

* e-μ, μ-τ etc. oscillations and rare decays studied:

Gavela, Hambye, Hernandez^2 09 ; .....

* Alonso + Li, 2010: possible suppresion of μ-e transitions

->important impact of ν_μ - ν_τ at a near detectors

$$\boxed{B_{\mu\to e\gamma} \propto |Y_{N_e} Y_{N_\mu}|^2}$$

i.e.

$$Y_N^T \simeq y \begin{pmatrix} e^{i\delta} s_{13} + e^{-i\alpha} s_{12} r^{1/4} \\ s_{23}\left(1 - \frac{\sqrt{r}}{2}\right) + e^{-i\alpha} r^{1/4} c_{12} c_{23} \\ c_{23}\left(1 - \frac{\sqrt{r}}{2}\right) - e^{-i\alpha} r^{1/4} c_{12} s_{23} \end{pmatrix} \qquad r = \frac{|\Delta m_{12}^2|}{|\Delta m_{13}^2|}$$

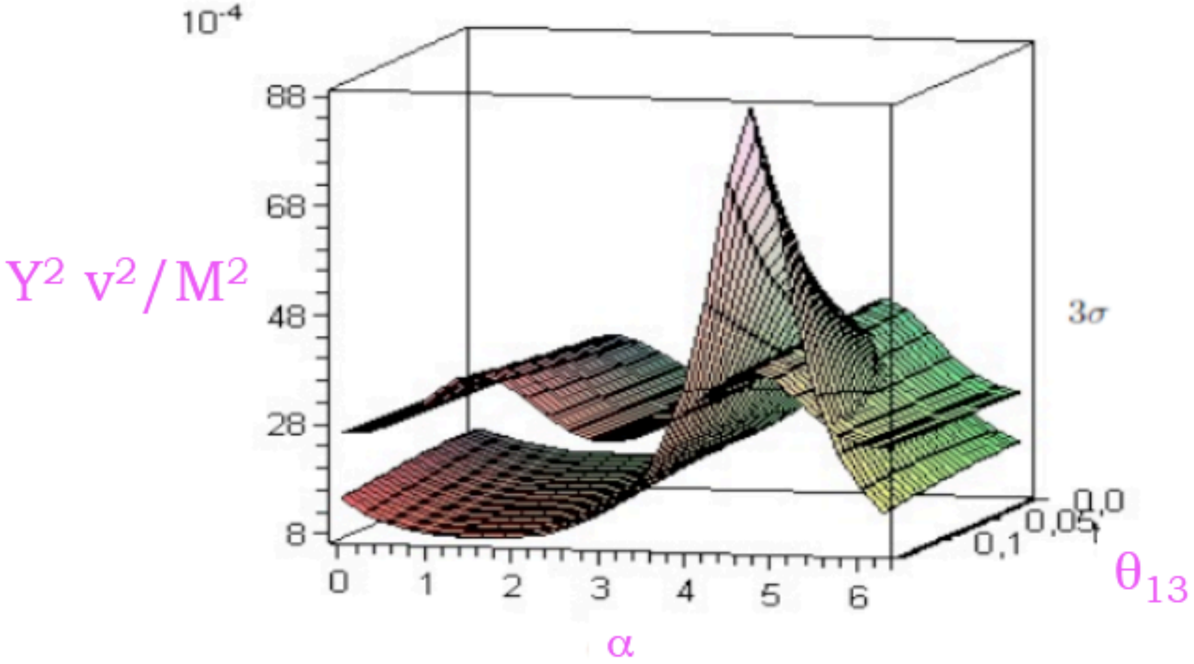
Normal hierarchy

* e-μ, μ-τ etc. oscillations and rare decays studied:

Gavela, Hambye, Hernandez^2 09; .....

* Alonso + Li, 2010: possible suppresion of μ-e transitions

->important impact of ν_μ - ν_τ at a near detectors

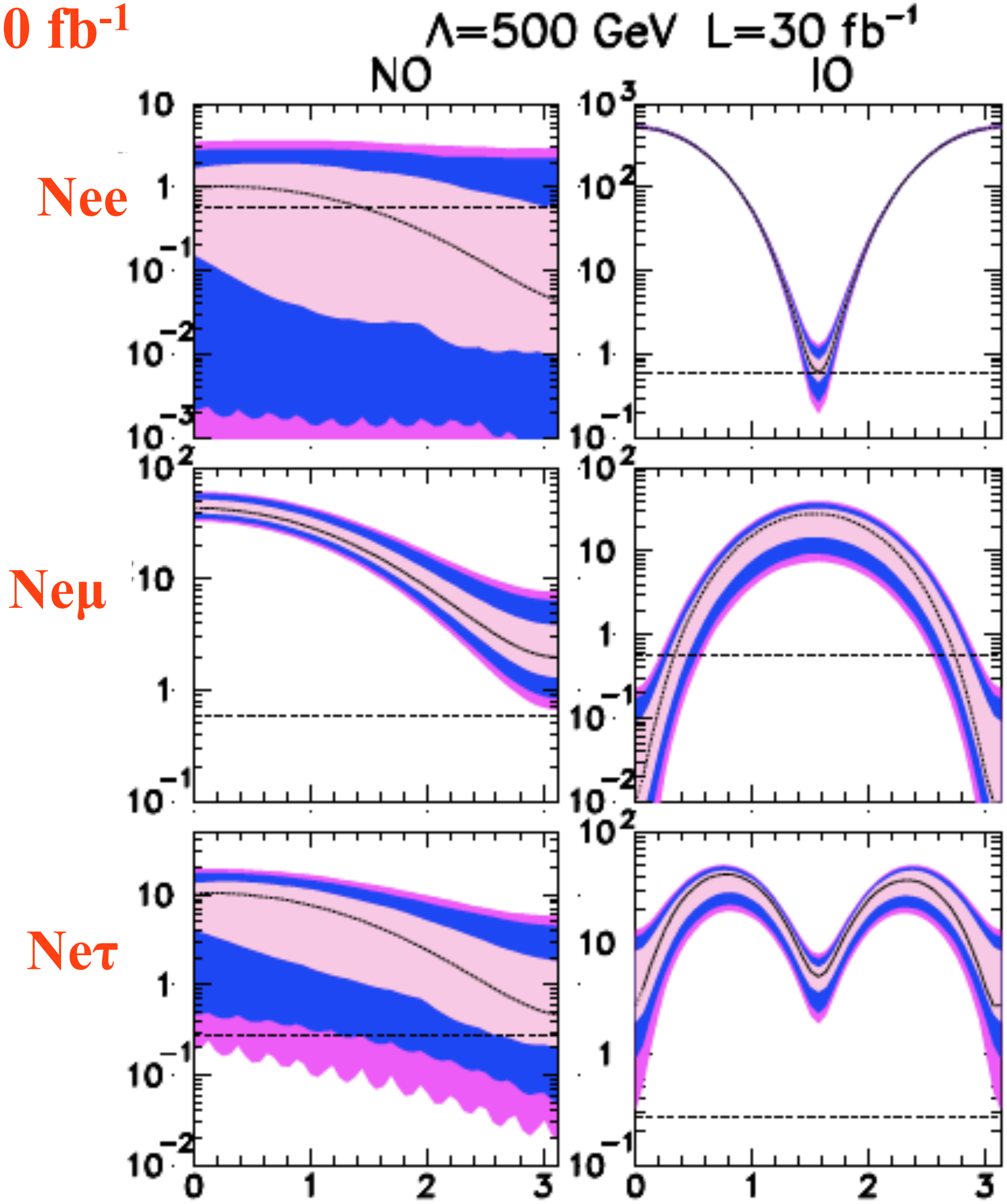


We find that there are regions where an experiment as MINSIS would improve the present bounds on our Model

For type III version of our 2 N model, signals observable at LHC up to Λ~ 500 GeV for 30 fb^{-1}

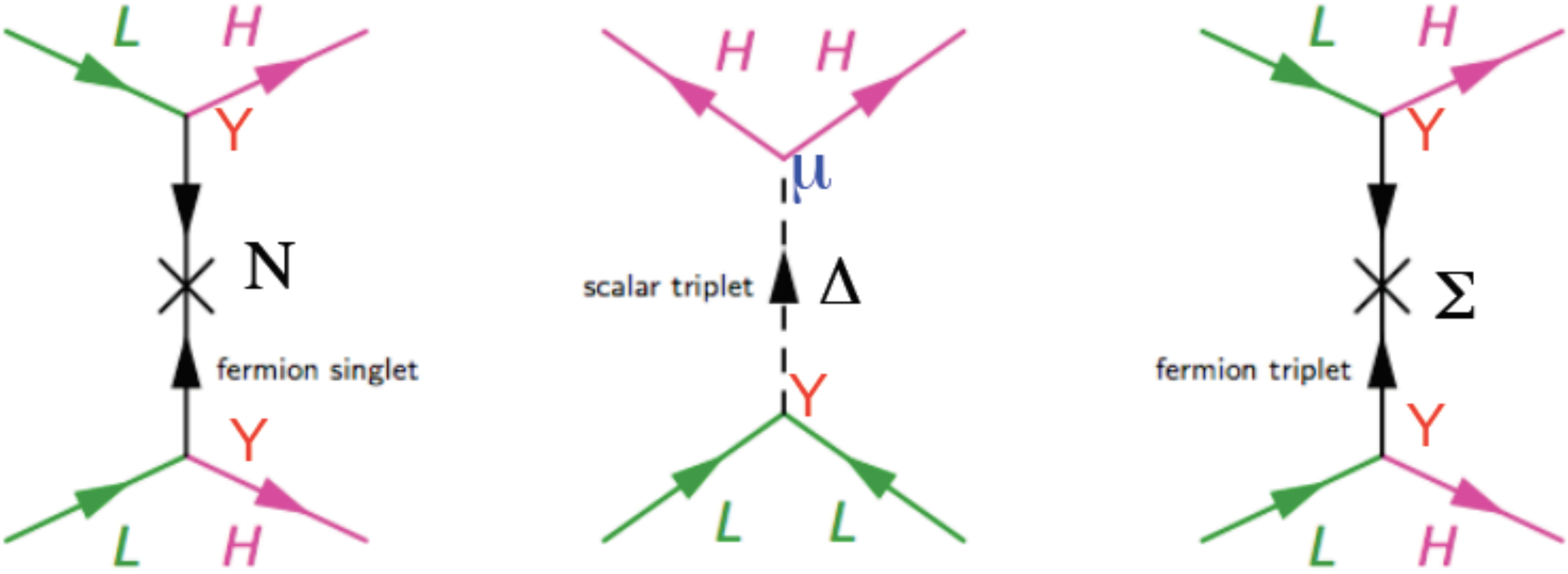
Eboli,
Gonzalez-Fraile,
Gonzalez-Garcia

$$pp \to \ell_a^{\pm}\ell_b^{\mp}jjjj$$



α α <-- Majorana phase

- Three types of models yield the Weinberg operator at tree level



Type I

$$m_\nu \sim v^2 \; Y_N^T \frac{1}{M_N} Y_N$$

Type II

$$m_\nu \sim v^2 \; Y_\Delta \frac{\mu}{M_\Delta{}^2}$$

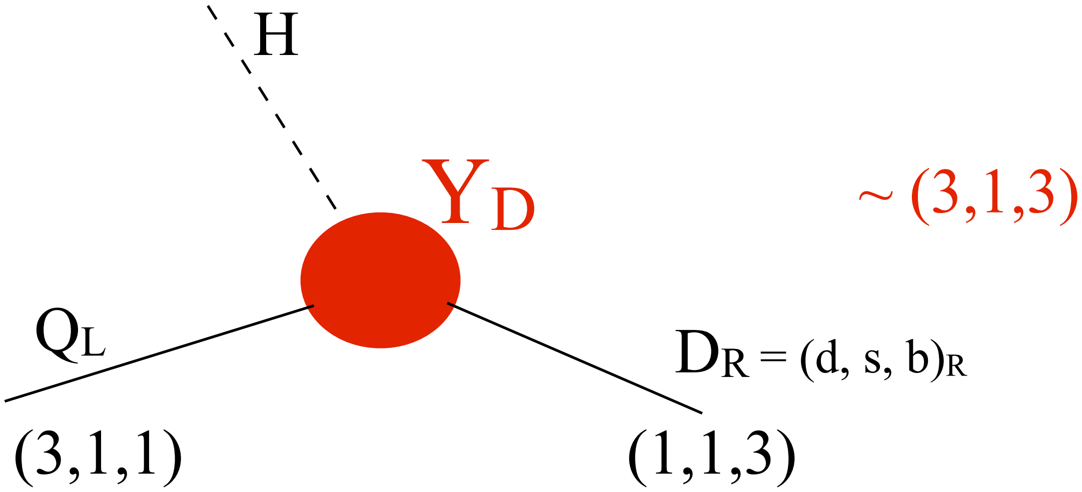
Type III

$$m_\nu \sim v^2 \; Y_\Sigma^T \frac{1}{M_\Sigma} Y_\Sigma$$

Use the flavour symmetry of the SM with masless fermions:

$$G_f = SU(3)_{Q_L} \ \mathrm{x} \ SU(3)_{U_R} \ \mathrm{x} \ SU(3)_{D_R} \ \mathrm{x} \ U(1)s$$

which is broken by Yukawas:



H

Y_D

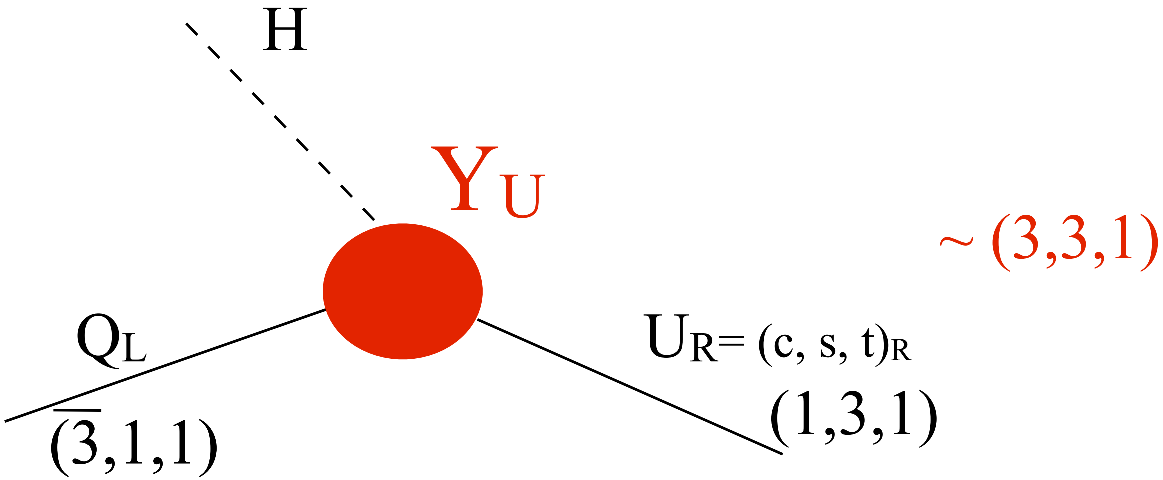
$\sim (3,1,3)$

Q_L

$D_R = (d, s, b)_R$

(3,1,1)

(1,1,3)



H

Y_U

$\sim (3,3,1)$

Q_L

$U_R = (c, s, t)_R$

$(\overline{3},1,1)$

(1,3,1)

*In the O(2)model used before: $\mathrm{tgh}\, 2\omega = \frac{y^2 - y'^2}{y^2 - y'^2}$ and

$$\mathrm{tg}2\theta = \sin 2\alpha \frac{2\sqrt{m_{\nu_2} m_{\nu_1}}}{m_{\nu_2} - m_{\nu_1}} \frac{y^2 - y'^2}{y^2 - y'^2}$$

$$\boxed{\alpha = \pi/4 \text{ or } 3\pi/4}$$

*If we had used instead a flavor SU(2)model $\sinh 2\omega = 0$ -->NO MIXING

Some good ideas:

“Partial compositeness”:

D.B. Kaplan-Georgi in the 80s proposed a composite Higgs:

* **Higgs light because the whole Higgs doublet is multiplet of goldstone bosons**

They explored **SU(5)--> SO(5)**.

Explicit breaking of SU(2)xU(1) symmetry via external gauged U(1)
(Kaplan, Georgi, Dimopoulos, Banks, Dugan, Galison)

Nowadays **SO(5)--> SO(4)** and explicit breaking via SM weak interaction
(Contino, Nomura, Pomarol; Agashe, Contino, Pomarol; Giudice, Pomarol, Ratazzi, Grojean; Contino, Grojean, Moretti; Azatov, Galloway, Contino...)

SO(6) --> SO(5) to get also DM (Frigerio, Pomarol, Riva, Urbano)

Anarchy: alive with not so small θ_{13} and not θ_{23} not maximal

no symmetry in the lepton sector, just random numbers

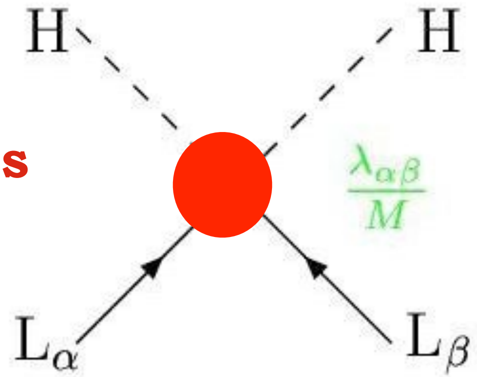
$$m_\nu \sim \begin{pmatrix} \sim 1 & \sim 1 & \sim 1 \\ \sim 1 & \sim 1 & \sim 1 \\ \sim 1 & \sim 1 & \sim 1 \end{pmatrix}$$

- Does not relate mixing to spectrum
- Does not address both quarks and leptons

(Hall, Murayama, Weiner; Haba, Murayama; De Gouvea, Murayama...
Going towards hierarchy: Altarelli, Feruglio, Masina, Merlo)

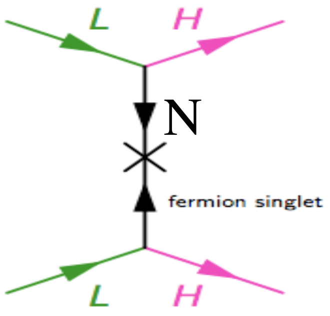
Seesaw models

ν-mass

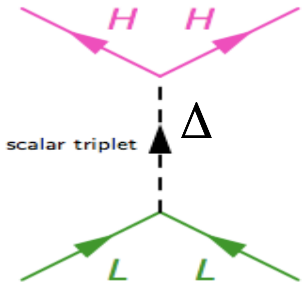


λ/M (L L H H)

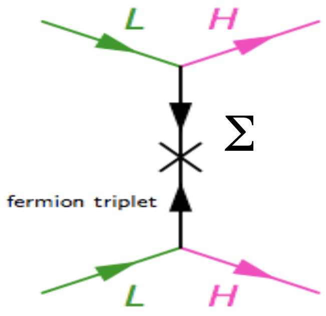
SU(2) xU(1)$_{em}$ inv.



Type I

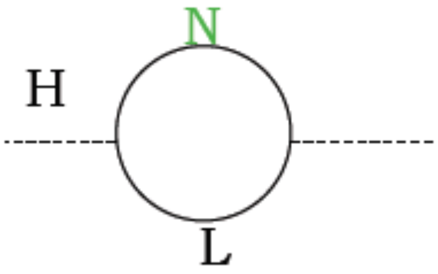


Type II



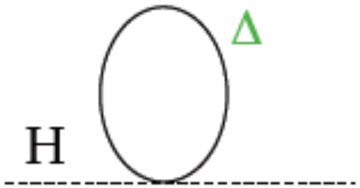
Type III

M~1 TeV is suggested by electroweak hierarchy problem



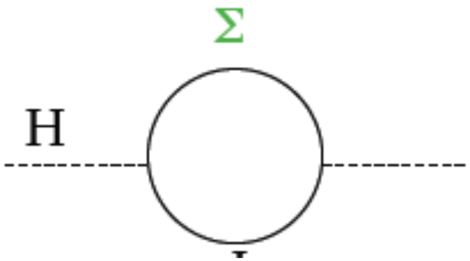
$$\delta m_H^2 = -\frac{Y_N^\dagger Y_N}{16\pi^2}\left[2\Lambda^2 + 2M_N^2 \log\frac{M_N^2}{\Lambda^2}\right]$$

(Vissani, Casas et al., Schmaltz)



$$\delta m_H^2 = -3\frac{\lambda_3}{16\pi^2}\left[\Lambda^2 + M_\Delta^2\left(\log\frac{M_\Delta^2}{\Lambda^2} - 1\right)\right] - \frac{\mu_\Delta^2}{2\pi^2}\log\left(\left|\frac{M_\Delta^2 - \Lambda^2}{M_\Delta^2}\right|\right)$$

(Abada, Biggio, Bonnet, Hambye, M.B.G.)

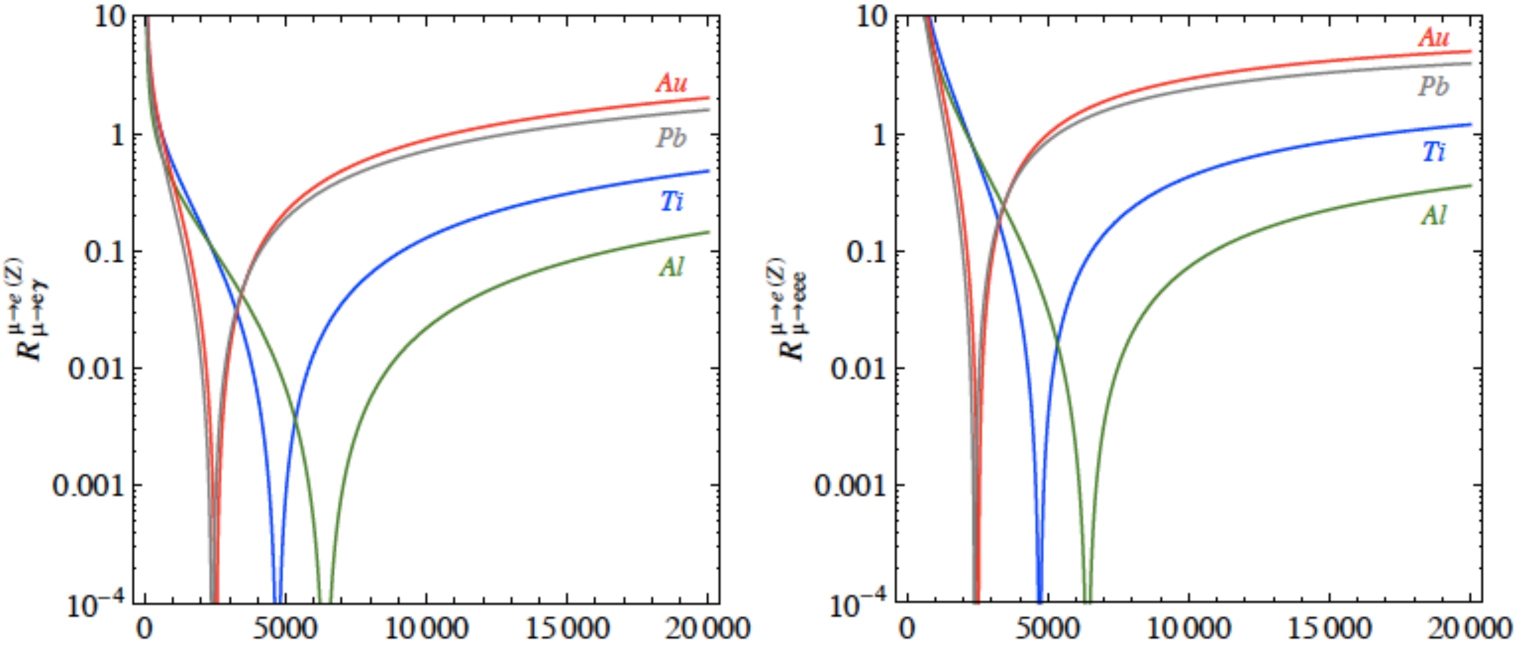


$$\delta m_H{}^2 = -3\frac{Y_\Sigma^\dagger Y_\Sigma}{16\pi^2}\left[2\Lambda^2 + 2M_\Sigma^2 \log\frac{M_\Sigma^2}{\Lambda^2}\right]$$

Ratios of two e-μ transitions may depend only on m_N (Chu, Dhen, Hambye 11) and e-μ conversion may vanish(Dinh et al. 12)

we obtain:

$$m_N^2\Big|_0 = M_W^2 \exp\left(\frac{\frac{9}{8}(A-Z)+\left(\frac{9}{8}+\frac{31 s_W^2}{12}\right)Z}{\frac{3}{8}(A-Z)+\left(\frac{4 s_W^2}{3}-\frac{3}{8}\right)Z}\right) \qquad (\alpha Z << 1)$$



μ-e conversion / μ--> e γ

μ-e conversion / μ--> e e e

m_N (GeV)

m_N (GeV)

...typically vanishes for m_N in 2- 7 TeV range

(Alonso, Dhen, Hambye, B.G.)

*3 families with $O(2)_{N_R}$:

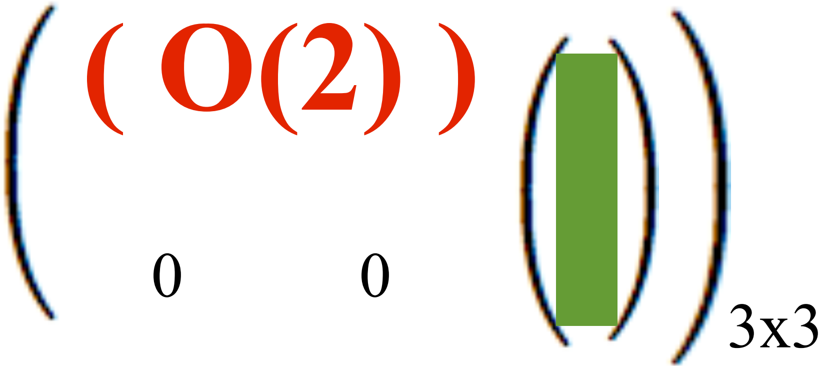
- 3 light + 2 heavy N degenerate: bad θ_{12} quadrant. It cannot accomodate data!

- 3 light + 3 heavy N : **OK for θ_{23} maximal and spectrum**

experimentally $\sin 2\theta_{23}$= 0.41 +-0.03 or 0.59+-0.02

Gonzalez-Garcia, Maltoni, Salvado, Schwetz Sept. 2012

*What about the other angles?



*3 families with $O(2)_{NR}$:

- 3 light + 2 heavy N degenerate: bad θ_{12} quadrant. It cannot accomodate data!

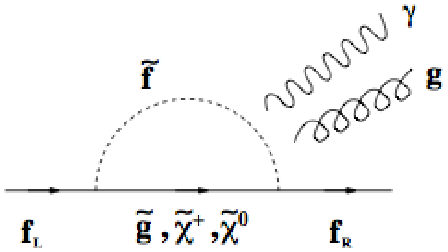
- 3 light + 3 heavy N : **OK for θ_{23} maximal and spectrum**

Moriond this morning, T2K best fit point $\sin^2 2\theta_{23}=1.00_{-0.068}$ 90%CL
-> 45º !

*What about the other angles?

The FLAVOUR WALL for BSM

i) Typically, BSMs have **electric dipole moments** at one loop
i.e susy MSSM:

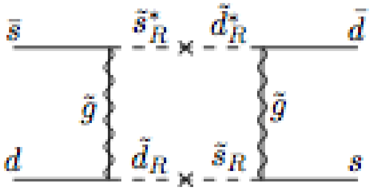


< 1 loop in SM ---> **Best (precision) window of new physics**

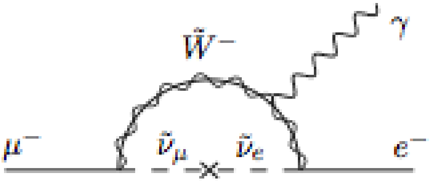
ii) **FCNC**

i.e susy MSSM:

$K^0 - \overline{K}^0$ mixing



$\mu \rightarrow e\gamma$

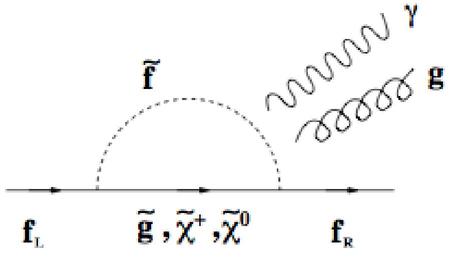


competing with SM at one-loop

The FLAVOUR WALL for BSM

i) Typically, BSMs have **electric dipole moments** at one loop

i.e susy MSSM:

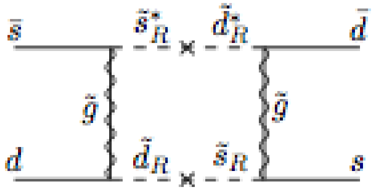


< 1 loop in SM ---> **Best (precision) window of new physics**

ii) **FCNC**

i.e susy MSSM:

$K^0-\overline{K}^0$ mixing



$\mu \to e$ conversion (MEG, μ2e...)

competing with SM at one-loop