Observables in Neutrino Mass Spectroscopy Using Atoms

S. T. Petcov

SISSA/INFN, Trieste, Italy,
IPMU, University of Tokyo, Tokyo, Japan

Neutrino Telescopes
Venice, March 13, 2013

Future Progress

- Determination of the nature - Dirac or Majorana, of ν_j .

- Determination of sgn($\Delta m^2_{\rm atm}$), type of $\nu-$ mass spectrum

$$m_1 < (\ll) m_2 < m_3, \qquad \text{NO (NH)},$$

$$m_3 < (\ll) m_1 < m_2, \qquad \text{IO (IH)},$$

$$m_1 \cong m_2 \cong m_3,\ m^2_{1,2,3} >> \Delta m^2_{\rm atm},\ \text{QD};\ m_j \gtrsim 0.10 \text{ eV}.$$

- Determining, or obtaining significant constraints on, the absolute scale of ν_j-masses, or min(m_j).

- Status of the CP-symmetry in the lepton sector: violated due to δ (Dirac), and/or due to α_{21}, α_{31} (Majorana)?

- High precision determination of $\Delta m^2_\odot$, $\theta_\odot$, $\Delta m^2_{\rm atm}$, θ_{atm}.
- Searching for possible manifestations, other than ν_l−oscillations, of the non-conservation of L_l, $l = e, \mu, \tau$, such as $\mu \rightarrow e + \gamma$, $\tau \rightarrow \mu + \gamma$, etc. decays.
- Understanding at fundamental level the mechanism giving rise to the ν− masses and mixing and to the L_l−non-conservation. Includes understanding

  – the origin of the observed patterns of ν-mixing and ν-masses ;

  – the physical origin of CPV phases in $U_{\rm PMNS}$;

  – Are the observed patterns of ν-mixing and of $\Delta m^2_{21,31}$ related to the existence of a new symmetry?

  – Is there any relations between q−mixing and ν− mixing? Is $\theta_{12} + \theta_c = \pi/4$?

  – Is $\theta_{23} = \pi/4$, or $\theta_{23} > \pi/4$ or else $\theta_{23} < \pi/4$?

  – Is there any correlation between the values of CPV phases and of mixing angles in $U_{\rm PMNS}$?
- Progress in the theory of ν-mixing might lead to a better understanding of the origin of the BAU.

  – Can the Majorana and/or Dirac CPVP in $U_{\rm PMNS}$ be the leptogenesis CPV parameters at the origin of BAU?

All compelling ν–Oscillation Data: 3-ν mixing (a reference scheme)

$$\nu_{l\mathrm{L}} = \sum_{j=1}^{3} U_{lj}\,\nu_{j\mathrm{L}}\,, \quad l = e, \mu, \tau\,; \qquad \nu_j:\ m_j > 0\,.$$

PMNS Matrix: Standard Parametrization

$$U = V\,P\,, \qquad P = \begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{i\frac{\alpha_{21}}{2}} & 0 \\ 0 & 0 & e^{i\frac{\alpha_{31}}{2}} \end{pmatrix},$$

$$V = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix}$$

- $s_{ij} \equiv \sin\theta_{ij}$, $c_{ij} \equiv \cos\theta_{ij}$, $\theta_{ij} = [0, \frac{\pi}{2}]$,
- δ - Dirac CP-violation phase, $\delta = [0, 2\pi]$,
- α_{21}, α_{31} - the two Majorana CP-violation phases.

S.M. Bilenky, J. Hosek, S.T.P., 1980

- $\Delta m^2_{\odot} \equiv \Delta m^2_{21} \cong 7.54 \times 10^{-5}$ eV$^2 > 0$, $\sin^2\theta_{12} \cong 0.307$, $\cos 2\theta_{12} \gtrsim 0.28$ (3σ),
- $|\Delta m^2_{\rm atm}| \equiv |\Delta m^2_{31}| \cong 2.47(2.46) \times 10^{-3}$ eV^2, $\sin^2 2\theta_{23} \cong 0.39$,
- θ_{13} - the CHOOZ angle: $\sin^2 2\theta_{13} = 0.089 \pm 0.010 \pm 0.005$, Daya Bay; $\sin^2\theta_{13} = 0.0241$ (0.0244), Fogli et al.

Fogli, Lisi, Marrone, Montanino, Palazzo, Rotunno, arXiv:1205.5254

- Dirac phase δ: $\nu_l \leftrightarrow \nu_{l'}$, $\bar{\nu}_l \leftrightarrow \bar{\nu}_{l'}$, $l \neq l'$; $A_{CP}^{(l,l')} \propto J_{CP} \propto \sin\theta_{13} \sin\delta$
- Majorana phases α_{21}, α_{31}:
  - $\nu_l \leftrightarrow \nu_{l'}$, $\bar{\nu}_l \leftrightarrow \bar{\nu}_{l'}$ not sensitive;

    S.M. Bilenky, J. Hosek, S.T.P.,1980;
    P. Langacker, S.T.P., G. Steigman, S. Toshev, 1987

  - $|<m>|$ in $(\beta\beta)_{0\nu}$–decay depends on α_{21}, α_{31};
  - $\Gamma(\mu \to e + \gamma)$ etc. in SUSY theories depend on $\alpha_{21,31}$;
  - BAU, leptogenesis scenario: $\alpha_{21,31}$!

Radiative Emission of Neutrino Pair (RENP)

RENP: collective de-excitation of atoms in a metastable level into emission mode of a single photon plus a neutrino pair.

For a single atom the process is:

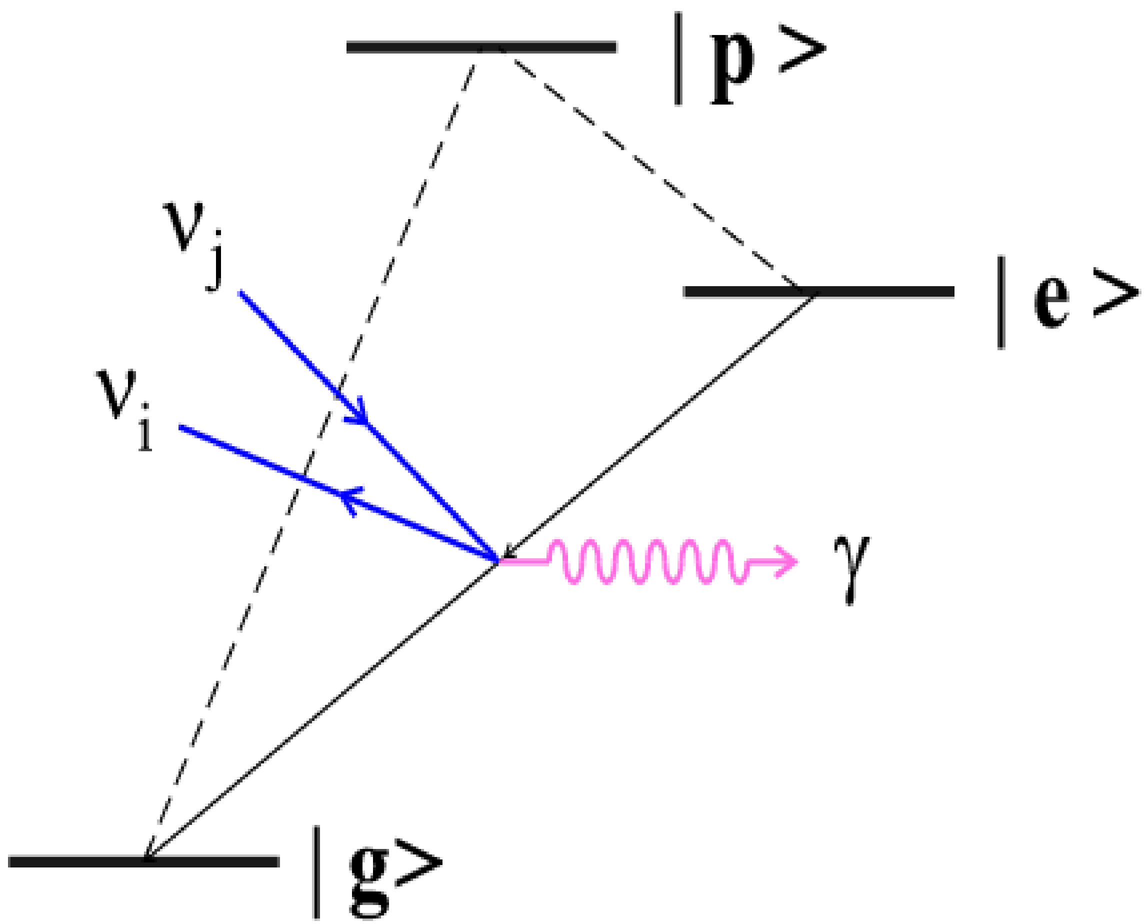
$|e\rangle \to |g\rangle + \gamma + (\nu_i + \nu_j)$,

Dirac ν_i:

$(\nu_i + \nu_j)$: either $(\nu_i + \bar{\nu}_j)$ or $(\bar{\nu}_i + \nu_j)$, if $i \neq j$; $(\nu_i + \bar{\nu}_i)$, for $i = j$;

Majorana ν_i:

$(\nu_i + \nu_j)$ are neutrinos with masses m_i and m_j.



Combined weak and QED process; $|e\rangle - |g\rangle$ transition - of E1×M1 type, with $\Delta J = 2$; $|e\rangle, |g\rangle$ - opposite parities. $|e\rangle$ - metastable with roughly $\tau(|e\rangle) \gtrsim 10^{-3}$ sec.

The problem was investigated in:

M. Yoshimura, Phys. Rev. D75 (2007) 113007.
M. Yoshimura et al., arXiv:0805.1970.
M. Yoshimura et al., Progr. Theor. Phys. 123 (2010) 523.
M. Yoshimura, Phys. Lett. B699 (2011) 123.
M. Yoshimura, N. Sasao, and M. Tanaka, Phys. Rev. A86 (2012) 013812 and arXiv:1203.5394.
D.N. Dinh, STP, N. Sasao, M. Tanaka and M. Yoshimura, Phys. Lett. B (2013) (arXiv:1209.4808).

A Group at Okayama University was created in 2009 in order to study the feasibility of the experiment. They have published recently a detailed report on the status of the research program:

A. Fukumi et al., arXiv:1211.4904.

My talk is based on

D.N. Dinh, STP, N. Sasao, M. Tanaka and M. Yoshimura, Phys. Lett. B (2013) (arXiv:1209.4808), in which the phenomenology of the case of massive Majorana ν_j is treated correctly for the first time (with consequences for the Dirac-Majorana neutrino distinction, for the possible determination of the Majorana phases, etc.).

The RENP rate is extremely small:

$\Gamma \propto G_F^2 \alpha \epsilon_{eg}^5$, $G_F = 10^{-23}$ eV^{-2}; typically $\epsilon_{eg} \sim 1$ eV.

M. Yoshimura et al.: this can be overcomed by "macro-coherence amplification" of the rate, the amplification factor being

$\propto n^2 V$, n (V) - the target number density (volume). If $n \sim N_A/cm^3$, $V \sim 1$ cm^3, the RENP rate becomes observable.

Similar amplification observed experimentally in the case of a single photon emission: "super-radiance" (predicted by Dicke in 1954).

The macro-coherence of interest is developed by irradiation of two trigger lasers of frequencies ω_1, ω_2, satisfying $\omega_1 + \omega_2 = \epsilon_{eg}$. It is a complicated dynamical process. The asymptotic state of fields and target atoms in the latest stage of trigger irradiation is described by a static solution of the master evolution equation. In many cases there is a remanant state consisting of field condensates (of the soliton type) accompanied with a large coherent medium polarisation. This asymptotic target state is stable against two-photon emission, while RENP occurs from any point in the target. The Group at Okayama University is working on the experimental realisation of the macro-coherent RENP.

Proposed Experimental Method:

measure, under irradiation of two counter-propagating trigger lasers, the continuous γ energy spectrum below each of the 6 thresholds energies ω_{ij} corresponding to the production of the six different pairs of neutrinos, $\nu_1\nu_1$, $\nu_1\nu_2, \ldots$, $\nu_3\nu_3$: $\omega < \omega_{ij}$, ω being the photon energy,

$$\omega_{ij} = \omega_{ji} = \frac{\epsilon_{eg}}{2} - \frac{(m_i + m_j)^2}{2\epsilon_{eg}}\,, \quad i,j = 1,2,3, \quad m_{1,2,3} \geq 0\,,$$

ϵ_{eg} being the energy difference between the two relevant atomic levels.

The two laser energies $\omega_{1,2}$: $\omega_1 + \omega_2 = \epsilon_{eg}$, $\omega_1 < \epsilon_{eg}/2$. For a given target atom, the experiment is repeated at different ω_1.

The Spectral Rate (M. Yoshimura et al.):

$$\Gamma_{\gamma 2\nu}(\omega) = \Gamma_0 I(\omega)\eta_\omega(t)$$

$\Gamma_{\gamma 2\nu}(\omega)$: the spectral rate of number of events per unit time at each photon energy;

Γ_0 - determines the scale of the rate;
$I(\omega)$ - dimensionless spectral function containing all the information about the neutrino properties;
$\eta_\omega(t)$ - dimensionless dynamical factor (of order unity).

Example and numerical results for Yb target atom:

$$\text{Yb;}\ |e\rangle = (6s6p)\,{}^3P_0\,,\ |g\rangle = (6s^2)\,{}^1S_0\,,\ |p\rangle = (6s6p)\,{}^3P_1\,.$$

$$\epsilon_{eg} = 2.14349\ \text{eV}\,,\ \epsilon_{pg} = 2.23072\ \text{eV}\,;$$

$$\Gamma_0 \cong 0.37\ \text{mHz}\left(\frac{n}{10^{21}\ \text{cm}^{-3}}\right)^3 \frac{V}{10^2\ \text{cm}^3}\,,$$

The Spectral Function $I(\omega)$:

$$I(\omega) = \frac{1}{(\epsilon_{pg}-\omega)^2}\sum_{ij}|a_{ij}|^2\,\Delta_{ij}(\omega)\left(I_{ij}(\omega) - \delta_M m_i\, m_j\, B^M_{ij}\right),$$

$$B^M_{ij} = \frac{\Re(a^2_{ij})}{|a_{ij}|^2} = \left(1 - 2\,\frac{(\mathrm{Im}(a_{ij}))^2}{|a_{ij}|^2}\right),\;\; a_{ij} = U^*_{ei}U_{ej} - \frac{1}{2}\delta_{ij}\,,$$

$$\Delta_{ij}(\omega) = \frac{1}{\epsilon_{eg}(\epsilon_{eg}-2\omega)}\left\{\left(\epsilon_{eg}(\epsilon_{eg}-2\omega)-(m_i+m_j)^2\right)\left(\epsilon_{eg}(\epsilon_{eg}-2\omega)-(m_i-m_j)^2\right)\right\}^{1/2},$$

$$I_{ij}(\omega) = \left(\frac{1}{3}\epsilon_{eg}(\epsilon_{eg}-2\omega)+\frac{1}{6}\omega^2-\frac{1}{18}\omega^2\Delta^2_{ij}(\omega)-\frac{1}{6}(m_i^2+m_j^2)-\frac{1}{6}\frac{(\epsilon_{eg}-\omega)^2}{\epsilon_{eg}^2(\epsilon_{eg}-2\omega)^2}(m_i^2-m_j^2)^2\right).$$

Dirac ν_j: $\delta_M = 0$; Majorana ν_j: $\delta_M = 1$.

The term $\propto m_i m_j B^M_{ij} = m_i m_j(1-2(\mathrm{Im}(a_{ij}))^2/|a_{ij}|^2)$ is similar to, and has the same physical origin as, the term $\propto M_i M_j$ in the production cross section of two different Majorana neutralinos χ_i and χ_j with masses M_i and M_j in the process of $e^- + e^+ \to \chi_i + \chi_j$. For $i \neq j$, B^M_{ij} depends on the Majorana phases in U.

STP, 1986

$I(\omega) \propto |a_{ij}|^2 = |U^*_{ei} U_{ej} - \frac{1}{2}\delta_{ij}|^2$:

$\lvert a_{11}\rvert^2 = \lvert c^2_{12} c^2_{13} - \frac{1}{2}\rvert^2$	$\lvert a_{12}\rvert^2 = c^2_{12} s^2_{12} c^4_{13}$	$\lvert a_{13}\rvert^2 = c^2_{12} s^2_{13} c^2_{13}$
0.0311	0.2027	0.0162
$\lvert a_{22}\rvert^2 = \lvert s^2_{12} c^2_{13} - \frac{1}{2}\rvert^2$	$\lvert a_{23}\rvert^2 = s^2_{12} s^2_{13} c^2_{13}$	$\lvert a_{33}\rvert^2 = \lvert s^2_{13} - \frac{1}{2}\rvert^2$
0.0405	0.0072	0.2266

To identify the emission of each of the 6 pairs of ν's, the RENP spectral rate, should be measured with a relative precision not worse than $\sim 5 \times 10^{-3}$.

The ordering of $\omega_{ij} = \omega_{ji}$:

$m_1 < m_2 < m_3$ (NO spectrum):

$$\omega_{11} > \omega_{12} > \omega_{22} > \omega_{13} > \omega_{23} > \omega_{33}.$$

$m_3 < m_1 < m_2$ (IO spectrum):

$$\omega_{33} > \omega_{13} > \omega_{23} > \omega_{11} > \omega_{12} > \omega_{22}.$$

The separation of the thresholds:

$$\text{NH}: \quad \omega_{11} - \omega_{12} = \frac{1}{3}(\omega_{12} - \omega_{22}) = \frac{1}{2\epsilon_{eg}}\Delta m^2_{21} \cong 1.759 \times 10^{-5}\ \text{eV},$$

$$\text{NH}: \quad \omega_{13} - \omega_{23} = \frac{1}{2\epsilon_{eg}}(2\sqrt{\Delta m^2_{21}}\sqrt{\Delta m^2_{31}} + \Delta m^2_{21}) \cong 0.219 \times 10^{-3}\ \text{eV},$$

$$\text{NH}: \quad \omega_{22} - \omega_{13} = \frac{1}{2\epsilon_{eg}}(\Delta m^2_{31} - 4\Delta m^2_{21}) \cong 0.506 \times 10^{-3}\ \text{eV},$$

$$\text{NH}: \quad \omega_{23} - \omega_{33} = \frac{1}{2\epsilon_{eg}}(3\Delta m^2_{31} - 2\sqrt{\Delta m^2_{21}}\sqrt{\Delta m^2_{31}} - \Delta m^2_{21}) \cong 1.510 \times 10^{-3}\ \text{eV},$$

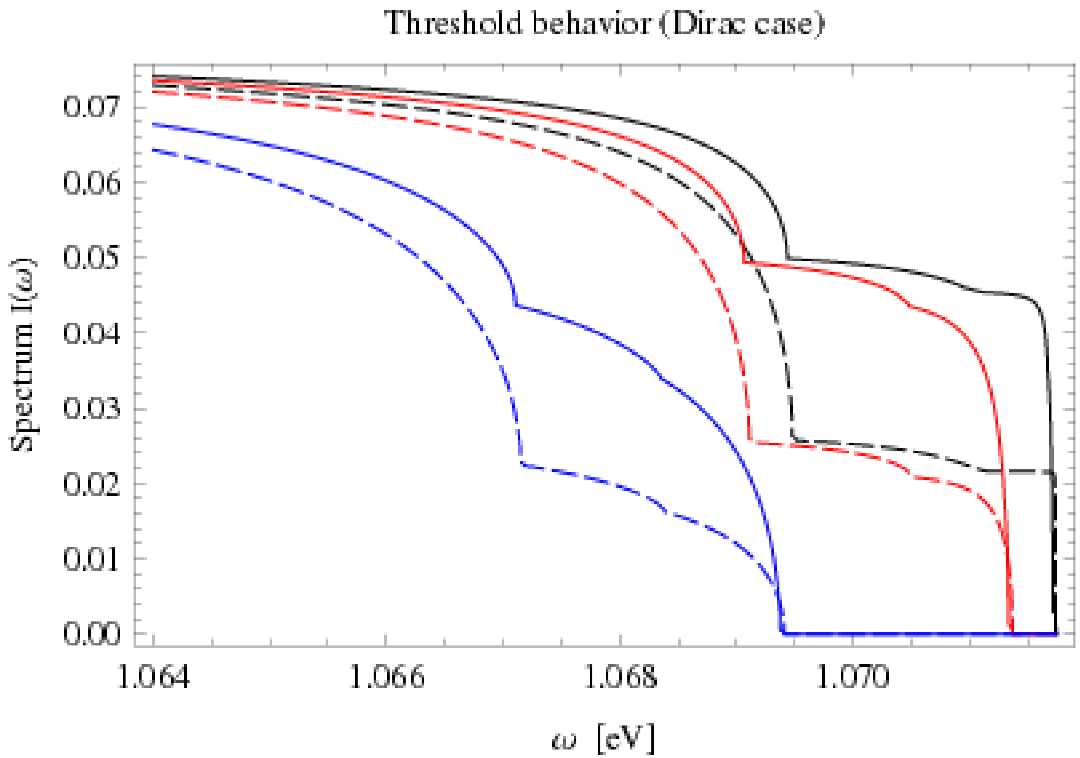
In order to determine all 6 threshold energies ω_{ij}, the photon energy ω should be measured with a precision not worse than $\sim 10^{-5}$ eV (valid also for IO and QD spectra); possible in the RENP experiments since the energy resolution in the spectrum is determined by accuracy of the trigger laser frequency, which is much better than 10^{-5} eV.

Absolute ν Mass Scale

Can be determined from the location of the thresholds ω_{ij}, which depends on min(m_j):
i) the location of the highest threshold (ω_{11} for NO and ω_{33} for IO);
ii) the location of the most prominent kink, which comes from the heavier neutrino pair emission thresholds (ω_{33} in the NO case and ω_{12} in the IO case).

The Neutrino Mass Spectrum (or Hierarchy)

The NH (NO) and IH (IO) spectra distinction: by measuring the ratio of rates below and above the lowest thresholds ω_{33} and ω_{12} (or ω_{11}), respectively.
For, e.g., $m_0 \lesssim 20$ meV,
$\tilde{R}(\omega_{33}(\omega_{12}); NH(IO)) \cong 0.70\ (0.36)$



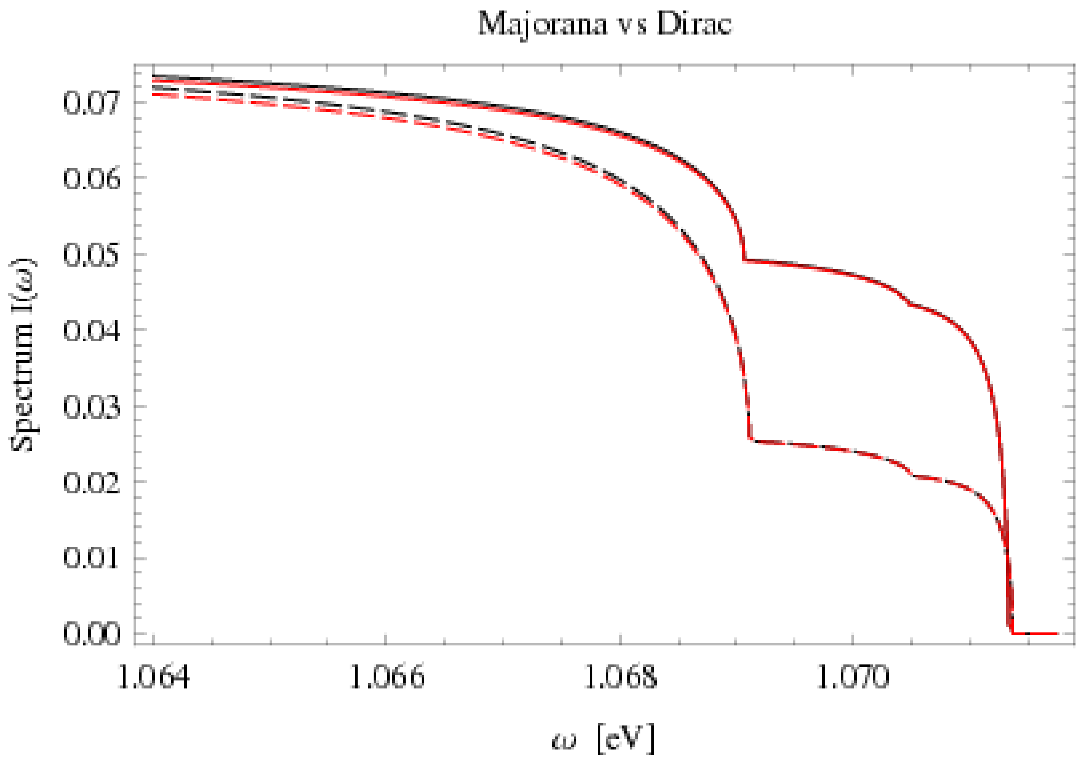
Photon energy spectrum from Yb $^3P_0 \to {}^1S_0$ transitions in the threshold region; NH spectrum (solid lines) and IH spectrum (dashed lines); Dirac ν_j; $\min(m_j) = 2$ meV (black lines), 20 meV (red lines) and 50 meV (blue lines).

The Nature of Massive Neutrinos

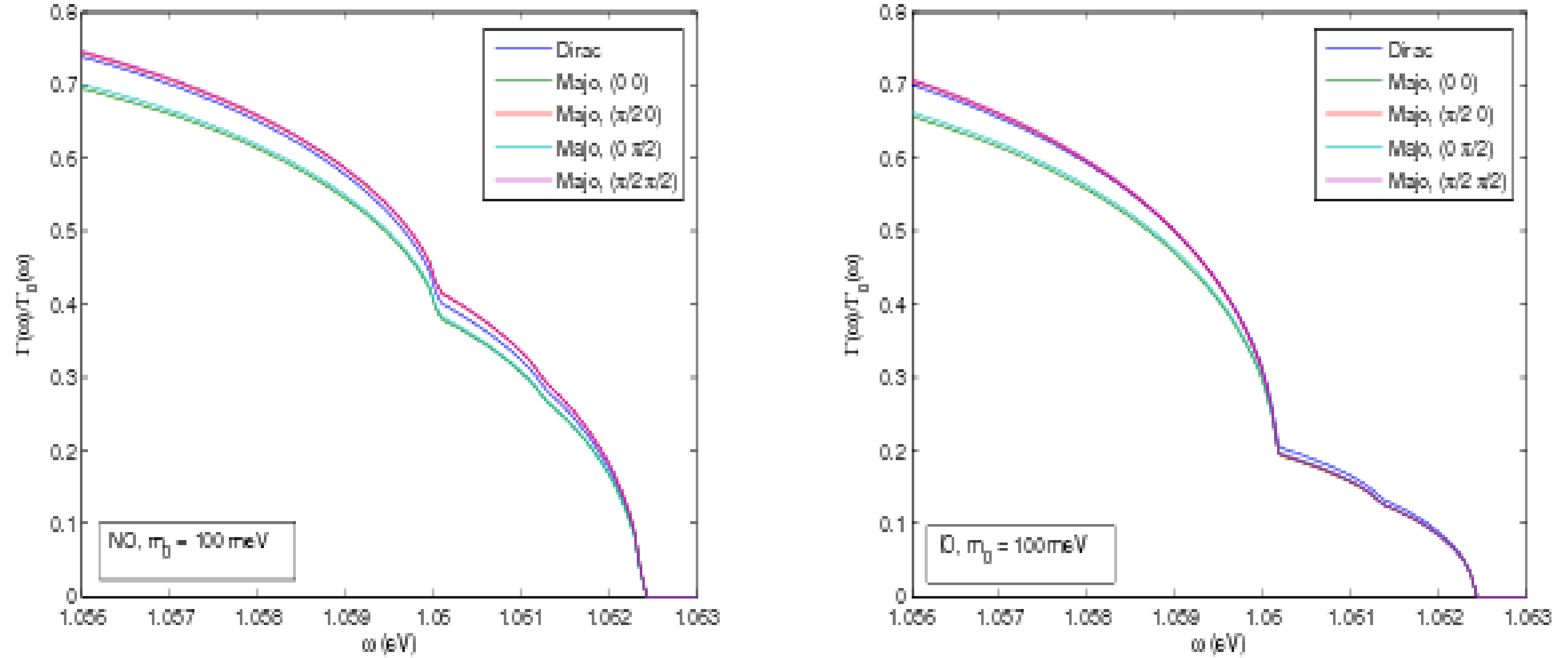
The Majorana vs Dirac neutrino distinction - much more challenging experimentally, if not impossible, with the Yb atom. The difference between the emission of pairs of Dirac and Majorana neutrinos can be noticeable in the case of QD spectrum with $m_0 \sim 100$ meV and for values of the phases $\alpha_{21} \cong 0$.

A lower atomic energy scale $\epsilon_{eg} > 100$ meV, which is closer in value to the largest neutrino mass, would provide more favorable conditions for determination of the nature of massive neutrinos and possibly for getting information about at least some (if not all) of the CPV phases.

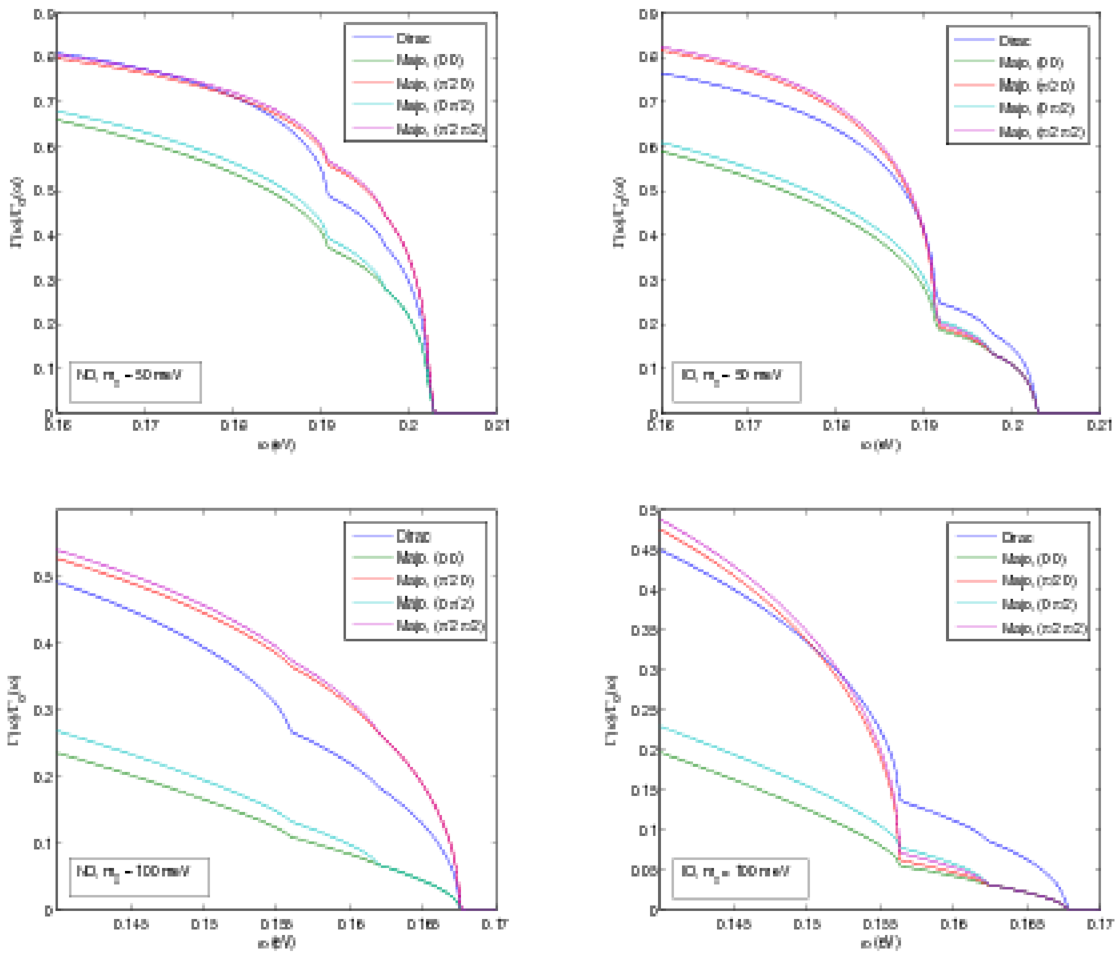
We have considered a hypothetical atom X scaled down in energy by 1/5 from the real Yb, thus $\epsilon_{eg} \sim 0.4$ eV. There may or may not be good candidate atoms/molecules experimentally accessible, having level energy difference of order of the indicated value.



Spectra from Yb $^3P_0 \to {}^1S_0$ transitions in the cases of Dirac neutrinos (black lines) and Majorana neutrinos (red lines) with masses corresponding to $\min(m_j) = 20$ meV, for NH spectrum (solid lines) and IH spectrum (dashed lines), and $(\alpha_{21}/2, \alpha_{31}/2 - \delta) = (0, 0)$.



The ratio $R(\Gamma) \equiv \Gamma_{\gamma 2\nu}(\omega)/\Gamma_{\gamma 2\nu}(\omega; m_i = 0) = I(\omega)/I(\omega; m_i = 0)$ as a function of ω in the case of emission of Dirac and Majorana massive neutrinos having NO (left panels) or IO (right panels) mass spectrum corresponding to $m_0 = 100$ meV, for $\epsilon_{eg} = 2.14$ eV and four values of the CPV phases $(\alpha_{21}/2, \alpha_{31}/2 - \delta)$ in the Majorana case. At $\omega < \omega_{ij}$, for $\alpha_{21} = 0$, the difference can reach 6%.



The same as in the previous figure, but for $\epsilon_{eg} = 0.43$ **eV and** $\min(m_j) \equiv m_0 = 50; 100$ **meV.**

The Majorana Phases

Enter into $I(\omega)$ via $B^M_{ij} = (1 - 2(\mathrm{Im}(a_{ij}))^2/|a_{ij}|^2)$:

$B^M_{ii} = 1$,

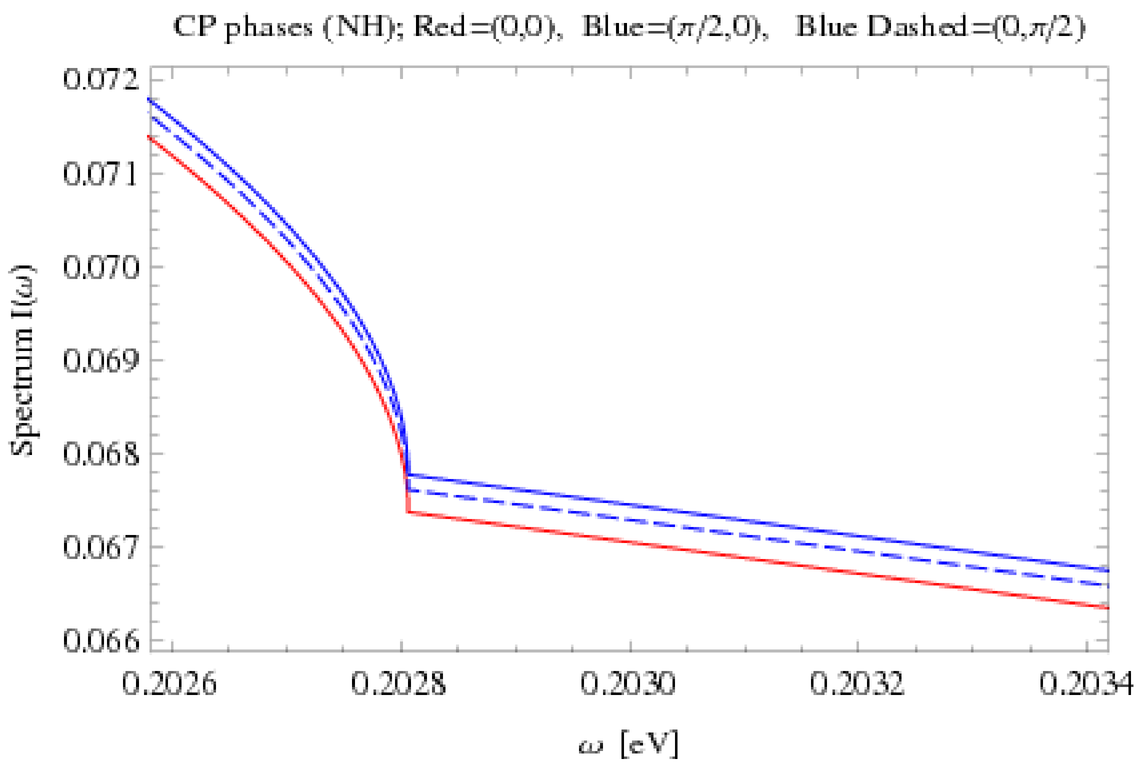
$B^M_{12} = \cos 2\alpha\,,\ B^M_{13} = \cos 2(\beta - \delta)\,,\ B^M_{23} = \cos 2(\alpha - \beta + \delta)\,,$

$\alpha = \alpha_{21}/2,\ \beta = \alpha_{31}/2.$

In the case of CP invariance: $\alpha, \beta = 0, \pi/2, \pi$, $\delta = 0, \pi$, and, correspondingly, $B^M_{ij} = -1$ or $+1$, $i \neq j$.
If CP invariance does not hold, $-1 < B^M_{ij} < 1$.

Moreover, $|a_{13}|^2 = 0.016$, $|a_{23}|^2 = 0.007$, while $|a_{12}|^2 = 0.203$, but for NH spectrum the $m_1 m_2$ factor can be very small.

The CPV phase measurement is challenging, requiring a high statistics. Possible exception: the case of α and IH spectrum where the difference between the spectral rates for $\alpha = 0$ and $\alpha = \pi/2$ can reach 10%. For the NH spectrum, the analogous difference is at most a few percent; observing this case requires large statistics in actual measurements.



The dependence of $I(\omega)$ on the CPV phases α and $(\beta-\delta)$ in the case of NH spectrum with $m_0 = 2$ meV and for $\epsilon_{eg} = 0.43$ eV. The red, solid blue and dashed blue lines are obtained for $(\alpha, \beta-\delta) = (0,0), (\pi/2, 0)$ and $(0, \pi/2)$, respectively.

Conclusions

The RENP experiment(s) have remarkable physics potential:

– Can determine the absolute scale of neutrino masses.

– Can determine the neutrino mass hierarchy.

– Can determine the nature - Dirac or Majorana - of massive neutrinos.

– Can provide information on the Majorana CPV phases.

It is not clear at present whether they are feasible.

However, given the potential the RENP experiments have, their feasibility studies should be further rigorously pursued and supported by our Community.

Supporting Slides

Absolute Neutrino Mass Measurements

The Troitzk and Mainz ^3H β-decay experiments

$$m_{\nu_e} < 2.3 \text{ eV} \qquad (95\% \text{ C.L.})$$

It is expected that the following sensitivity will be reached:

$$\text{KATRIN}: \qquad m_{\nu_e} \sim 0.2 \text{ eV}$$

Cosmological and astrophysical data, the CMB data of the WMAP experiment, combined with supernovae data and data on galaxy clustering imply (depending on the model complexity and the input data used):

$$\sum_j m_j \leq (0.3 - 1.3) \text{ eV} \quad (95\% \text{ C.L.})$$

K.N. Abazajian *et al.*, arXiv:1103.5083

The WMAP and future PLANCK experiments can be sensitive to

$$\sum_j m_j \cong 0.4 \text{ eV}$$

Data on weak lensing of galaxies by large scale structure, combined with data from the WMAP and PLANCK experiments may allow to determine

$$\sum_j m_j : \qquad \delta \cong 0.04 \text{ eV}.$$

Improved β energy resolution requires a ***BIG*** β spectrometer.


KATRIN'S JOURNEY
UNITED KINGDOM
IRELAND
Dublin
London
Birmingham
Cardiff
Manchester
Amsterdam
Rotterdam
Brussels
Celtic Sea
DENMARK
Copenhagen
GERMANY
Hamburg
Berlin
Bremen
Essen
Cologne
Leipzig
Bonn
Frankfurt am Main
Stuttgart
Munich
Luxembourg
POLAND
Warsaw
Poznań
Wrocław
Gdańsk
Kraków
BELARUS
Minsk
Vilnius
UKRAINE
Kiev
Chișinău
Odesa
FRANCE
Paris
Nantes
Bordeaux
Lyon
Toulouse
Strasbourg
Bay of Biscay
CZECH REPUBLIC
Prague
Brno
SLOVAKIA
Bratislava
AUSTRIA
Vienna
HUNGARY
Budapest
ROMANIA
Bucharest
Constanța
Black Sea
Zagreb
Ljubljana
Sarajevo
Belgrade
BULGARIA
Sofia
Varna
Istanbul
Skopje
Tirana
GREECE
Athens
TURKEY
Zurich
Bern
Geneva
Milan
Turin
Genoa
Venice
Florence
Naples
Palermo
Tyrrhenian Sea
Ionian Sea
Sardinia
Sicily
Cagliari
SPAIN
Madrid
Barcelona
Valencia
Bilbao
Sevilla
Malaga
PORTUGAL
Lisbon
Porto
Mediterranean Sea
BALEARIC ISLANDS
Algiers
ALGERIA
Tunis
TUNISIA
MOROCCO
Rabat
Casablanca
Oran
Valletta
MALTA
Scale 1: 19,500,000
Lambert Conformal Conic Projection,
standard parallels 40°N and 56°N