The Dark Matter density

Fabrizio Nesti

Università dell'Aquila, Italy

LNGS, July 5th 2012

w/ C.F. Martins, G. Gentile, P. Salucci

Dark Matter?

- A number of indirect supporting evidences

  (galaxy rotation, cluster velocity dispersion, CMB, LSS)

- Modify Gravity or Matter (or both)

  Modify Gravity: we look still for a healthy theory
  (I'd say still mainly a theoretical activity)

  Dark Matter: still elusive (well, more than Higgs)
  (good to have many search channels)

- Hints (puzzles) from Direct and Indirect searches?

  (DAMA, Cogent, CDMS, CRESST, Fermi line(?))

- Collisionless? (Bullet cluster) or Collisional? (A520 cluster)
  (mistery)

The DM densities

All searches depend on the expected DM density:

- In the Solar System

  Direct laboratory searches at Earth:
  ...depend on the local density at earth ρ_0

  Indirect searches (mainly neutrino annihilation in Sun, Earth)
  ...depend on accumulated DM which in turn is driven by ρ_0

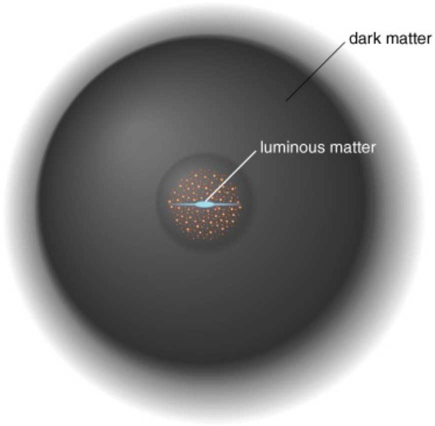
- In the Galaxy

  Looking for decay or annihilation
  ...depend on ρ or ρ^2 along the l.o.s.

Both the Local and Galactic DM density are interesting...

Our galaxy

- Bulge/bar ($10^{10}\ M_\odot$)
- Stellar disk (5–$7 \times 10^{10}\ M_\odot$)
- Dark Matter halo ($10^{11–12}\ M_\odot$)



and subleading

- Thick disk (older stars up to $z \sim$ kpc)
- Stellar halo (globular clusters, old BHB, red, brown dwarfs, etc) (at least up to 80 kpc)

The DM Density profile

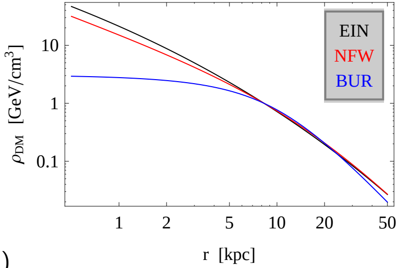
Component profiles

- DM profiles, Einasto, NFW, Burkert, cusped or cored

$$\rho_{EIN} = \rho_H \, e^{-(2/\alpha)(x^\alpha - 1)} \quad (\alpha = 0.17)$$

$$\rho_{NFW} = \frac{\rho_H}{x(1+x)^2}$$

$$\rho_{BUR} = \frac{\rho_H}{(1+x)(1+x^2)}$$



(with $x = r/R_H$, scale radius R_H)
Triaxiality? small [OBrien+ '10]. Smooth?

- Bulge: pointlike (as seen from $r > 2\,\text{kpc}$!)

$$M_B = 1.2\text{–}1.7 \times 10^{10}\, M_\odot$$

- Disk: biexponential, $\Sigma_D = (M_D/2\pi R_D^2)e^{-r/R_D}$ [PR04,juric08,robin08,reyle09]

$$z_0 = 240\text{pc}$$
$$M_D = 5\text{–}7 \times 10^{10}\, M_\odot$$
$$R_D = 2.5 \pm 0.2\,\text{kpc}$$

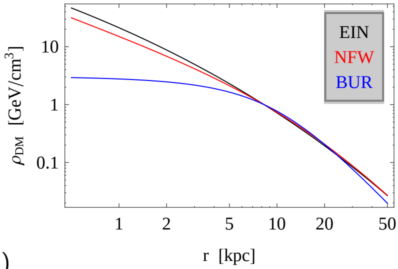
Component profiles

- DM profiles, Einasto, NFW, Burkert, cusped or cored

$$\rho_{EIN} = \rho_H \, \mathrm{e}^{-(2/\alpha)(x^\alpha - 1)} \quad (\alpha = 0.17)$$

$$\rho_{NFW} = \frac{\rho_H}{x(1+x)^2}$$

$$\rho_{BUR} = \frac{\rho_H}{(1+x)(1+x^2)}$$



(with $x = r/R_H$, scale radius R_H)
Triaxiality? small [OBrien+ '10]. Smooth?

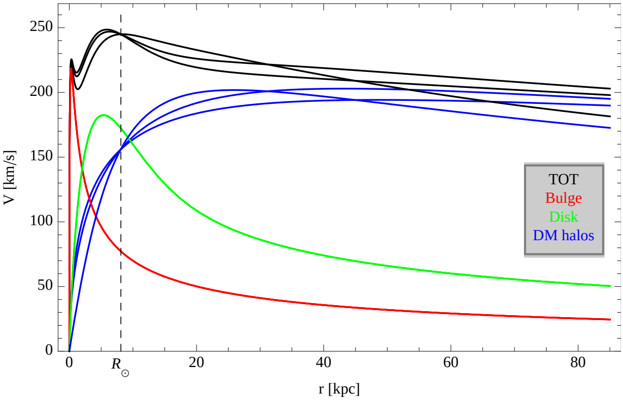
- Bulge: pointlike (as seen from $r > 2\,$kpc!)

$$M_B = 1.2\text{–}1.7 \times 10^{10}\, M_\odot$$

- Disk: biexponential, $\Sigma_D = (M_D/2\pi R_D^2)\mathrm{e}^{-r/R_D}$
[PR04,juric08,robin08,reyle09]

$$z_0 = 240\text{pc}$$
$$M_D = 5\text{–}7 \times 10^{10}\, M_\odot$$
$$R_D = 2.5 \pm 0.2\,\text{kpc}$$

All together



Would like to constrain $V(r)$ to constrain ρ_{DM}.

Unlike other galaxies, where we can measure V(r) quite well...
...here situation is much harder.

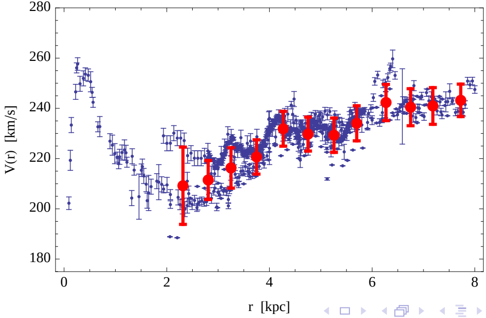
The Inner rotational velocities

Rotating HI gas in the inner region

- Doppler gives relative speed along the l.o.s.
- Maximum at the tangential point, terminal velocities V_T:

$$V(r) = V_T(r/R_\odot) + V_\odot \; r/R_\odot$$

- Inside $\sim$ 1–2 kpc the bulge/bar structure prevents analysis.
- between 2 and 8 kpc a lot of measures along the arms, with systematic variations

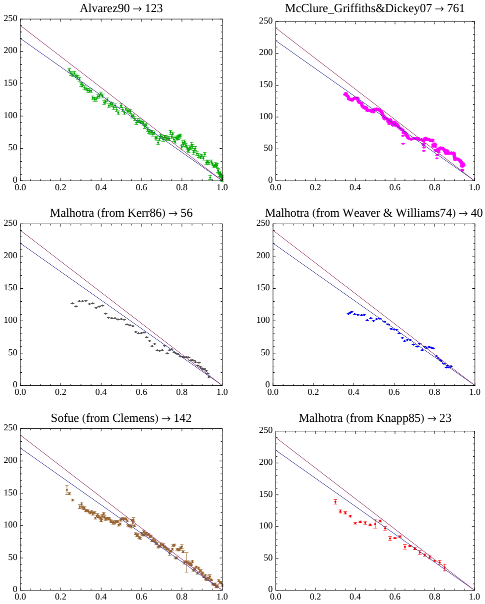


The Inner rotational velocities

Real data,

relative speed

$V_T(r/R_\odot)$



The Outer rotational velocities

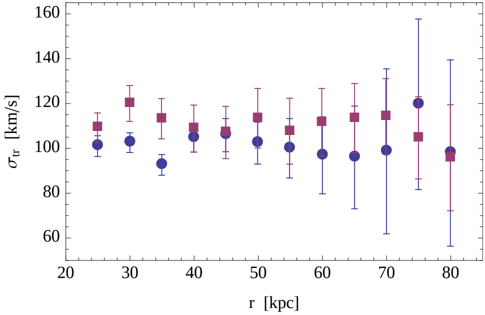
Out to $\sim 80\,\mathrm{kpc}$, survey of 'old' halo stars, moving randomly...

Only l.o.s. speed... need to rely on virial equilibrium

- $\sim$3000 Tracers
- Eliminate the ouliers ($|v| > 500\,\mathrm{km/s}$)
- Velocity dispersion $\sim 110\,\mathrm{km/s}$

Binned:

[Brown '10, Xue '08]



The Outer rotational velocities

Out to $\sim 80\,\text{kpc}$, survey of ‘old’ halo stars, moving randomly...

Only l.o.s. speed... need to rely on virial equilibrium

- $\sim$3000 Tracers
- Eliminate the ouliers ($|v| > 500\,\text{km/s}$)
- Velocity dispersion $\sim 110\,\text{km/s}$

Binned:

[Brown ’10, Xue ’08]

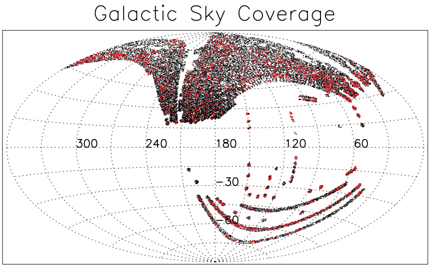


Fig. 11.— The Galactic sky coverage of the observed BHB stars (red dots) and selected simulated stars (black dots), drawn from Simulation I.

The Outer rotational velocities cont'd

Each population of tracers, has a measured density $\rho_i \propto r^{-\gamma_i}$,

- Consider (?) virial equilibrium and use Jeans' Equation:

$$V^2 = \sigma_i^2 \left[\gamma_i - 2\beta_i - \frac{\partial \ln \sigma_i^2}{\partial \ln r} \right]$$

- Unknown velocity anisotropy β_i (maybe r dependent)
- $\gamma_i \simeq 3.5$–4, for observed populations.

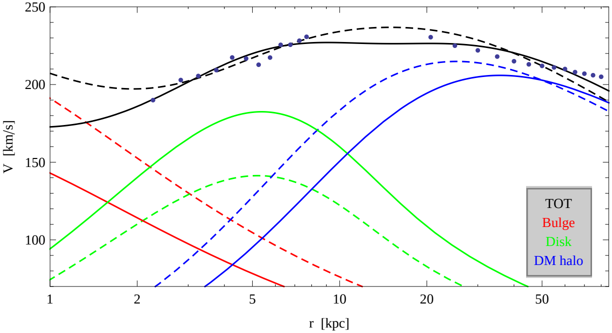
We can integrate Jeans' equation, for each model:

$$\{V^{model}(r), \beta_i\} \to \sigma_i^{model}(r)\,,$$

and compare σ_i^{model} with data for that population.

(Traditionally: derive pseudo-measures of V , w/ great uncertainties.)

Until 2011: the degeneration



- Inner: Bulge-Disk compensation
- Middle: Disk-DM Halo compensation
- Outer: DM Halo ρ_H-R_H flat direction
- and, $V_\odot$ not fixed $\rightarrow$ shift up/down.

Masers in Star forming regions

Parallax from ground based arrays: (angular precision 0.01 mas!)

Able to constrain:

$V_\odot/R_\odot \simeq 30.2 \pm 0.3\,\mathrm{km/s\,kpc}$

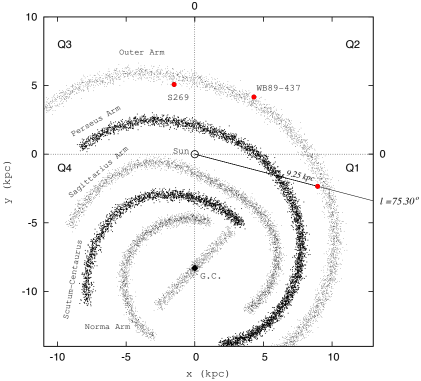
$V_\odot \simeq 239 \pm 7\,\mathrm{km/s}$

[Brunthaler+ '11]

$V(r \simeq 10\mathrm{kpc}) \simeq 240 \pm 5\,\mathrm{km/s}$

$V(r \simeq 13\mathrm{kpc}) \simeq 244 \pm 4\,\mathrm{km/s}$

[Sanna+ '11]



First results only.
In the near future more extensive surveys from BeSSeL and VERA.

Masers in Star forming regions

Parallax from ground based arrays: (angular precision 0.01 mas!)

Able to constrain:

$V_\odot / R_\odot \simeq 30.2 \pm 0.3\,\mathrm{km/s\,kpc}$

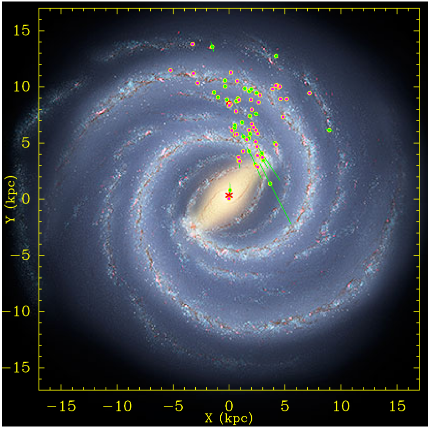
$V_\odot \simeq 239 \pm 7\,\mathrm{km/s}$

[Brunthaler+ '11]

$V(r \simeq 10\mathrm{kpc}) \simeq 240 \pm 5\,\mathrm{km/s}$

$V(r \simeq 13\mathrm{kpc}) \simeq 244 \pm 4\,\mathrm{km/s}$

[Sanna+ '11]



First results only.
In the near future more extensive surveys from BeSSeL and VERA.

Fitting (work in progress)

Model parameters, giving $V_{circ}(r)$ and integrated dispersion $\sigma(r)$:

- Sun: $R_\odot$, $V_\odot$ (related)
- Bulge: M_B
- Disk: M_D, R_D
- DM Halo: ρ_H, R_H
- Anisotropy for each population of tracers $\beta_i(r)$

Fitted against data: $V_T(x_i)$, $\sigma(r_i)$ and $V_{maser}(r_i)$.

Not all parameters relevant. (i.e. R_D and β r-dependence)

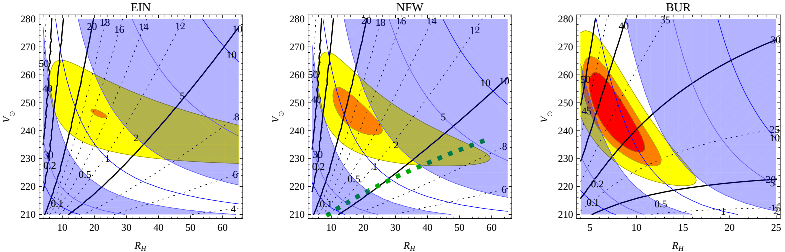
Also, bulge and disk are preferred as light as possible, extremal:
$M_B \simeq 1 \times 10^{10}\, M_\odot$, $M_D \simeq 5 \times 10^{10}\, M_\odot$.

Also, anisotropy of tracers are in tension among two populations:
even β_i required to be somehow extremal.

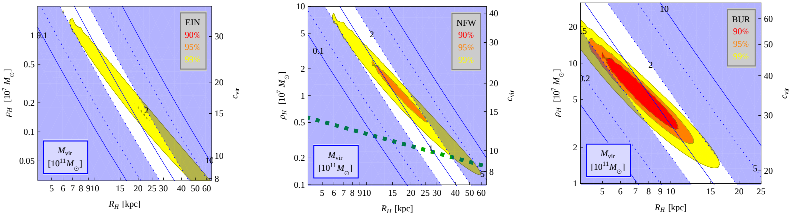
Most important are thus ρ_H, R_H...

...which can be traded for $V_\odot$, R_H.

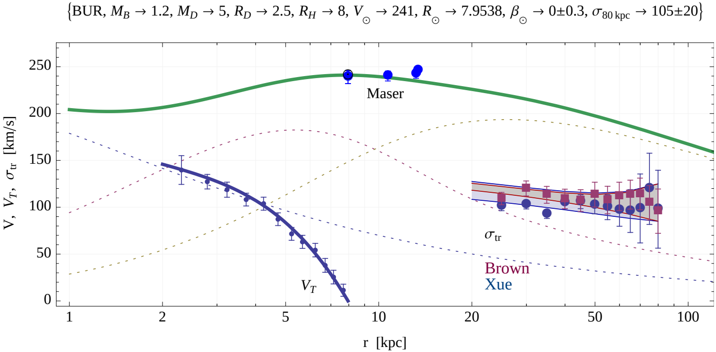
Consistency with data at 90%, 95%, 99%



Black lines mark $C_{vir} = 4$–50, green dots are cosmological simulations.
Blue lines mark $M_{vir}[10^{12}\ M_\odot]$ and region disfavored by MW total mass.
Same, ρ_H–R_H: [from Dehnen+'96, to Deason+'12]



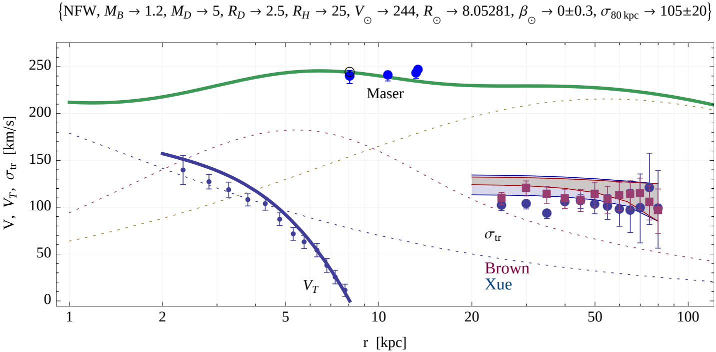
One fit (Burkert)



All fits require minimal Disk and minimal Bulge.

	ρ_H	r_H	$V_\odot$	$R_\odot$	$\rho_\odot$	M_{50}	M_{100}	M_{vir}	c_{vir}
	$[10^7 M_\odot / \text{kpc}^3]$	[kpc]	[km/s]	[kpc]	$[\text{GeV/cm}^3]$	$[10^{12}\text{Ms}]$	$[10^{12}\text{Ms}]$	$[10^{12}\text{Ms}]$	$[\Delta=100]$
EIN	0.165	22.0	246.	8.12	0.391	0.448	0.831	1.75	15.4
NFW	0.881	20.0	245.	8.09	0.419	0.477	0.849	1.71	16.8
BUR	5.48	8.00	245.	8.09	0.511	0.425	0.641	0.985	34.9

One fit (NFW)

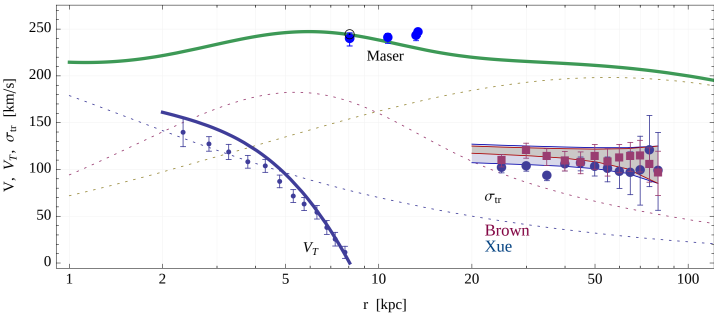


All fits require minimal Disk and minimal Bulge.

	ρ_H	r_H	$V_\odot$	$R_\odot$	$\rho_\odot$	M_{50}	M_{100}	M_{vir}	c_{vir}
	$[10^7 M_\odot/\text{kpc}^3]$	[kpc]	[km/s]	[kpc]	$[\text{GeV}/\text{cm}^3]$	$[10^{12}\text{Ms}]$	$[10^{12}\text{Ms}]$	$[10^{12}\text{Ms}]$	$[\Delta=100]$
EIN	0.165	22.0	246.	8.12	0.391	0.448	0.831	1.75	15.4
NFW	0.881	20.0	245.	8.09	0.419	0.477	0.849	1.71	16.8
BUR	5.48	8.00	245.	8.09	0.511	0.425	0.641	0.985	34.9

One fit (Einasto)

$\{\text{EIN}, M_B \to 1.2, M_D \to 5, R_D \to 2.5, R_H \to 25, V_\odot \to 244, R_\odot \to 8.05281, \beta_\odot \to 0\pm0.3, \sigma_{80\,\text{kpc}} \to 105\pm20\}$

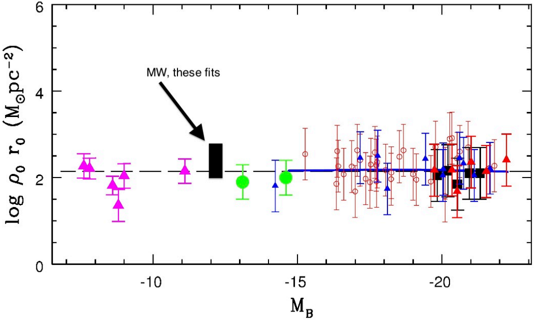


All fits require minimal Disk and minimal Bulge.

	ρ_H	r_H	$V_\odot$	$R_\odot$	$\rho_\odot$	M_{50}	M_{100}	M_{vir}	c_{vir}
	$[10^7 M_\odot / \text{kpc}^3]$	[kpc]	[km/s]	[kpc]	$[\text{GeV/cm}^3]$	$[10^{12}\text{Ms}]$	$[10^{12}\text{Ms}]$	$[10^{12}\text{Ms}]$	$[\Delta{=}100]$
EIN	0.165	22.0	246.	8.12	0.391	0.448	0.831	1.75	15.4
NFW	0.881	20.0	245.	8.09	0.419	0.477	0.849	1.71	16.8
BUR	5.48	8.00	245.	8.09	0.511	0.425	0.641	0.985	34.9

Comparing

Comparing the best (Burkert) fits with other galaxies



MW fits well, despite the large uncertainties.

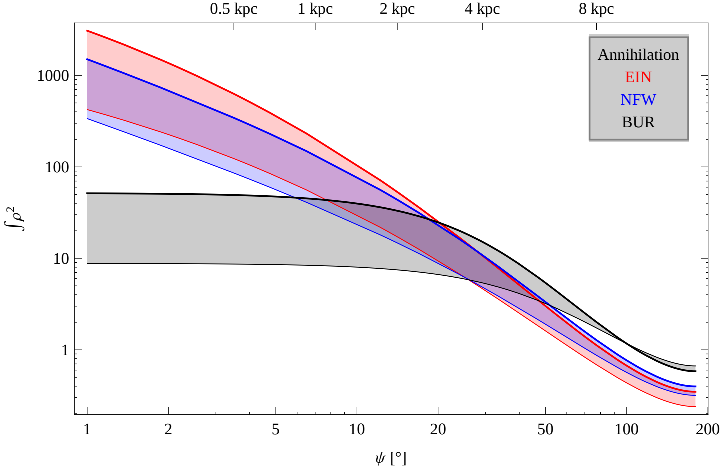
Conclusions for global fit of galaxy DM profile

- Some degeneracy removed thanks to masers.
- Mild core-cusp discrimination, with preference for cored.
  The terminal velocities are responsible for the core preference.
- Unlike in external galaxies, MW uncertainties are still large: Can not rule out 'cuspy' profile, but
- For NFW the $c_{vir} \sim 20$ is at odds with ΛCDM simulations.
  (Adiabatic contraction could raise c_{vir} in simulations but would make them even more cusped)
  The high $V_\odot \sim 250$km/s is responsible for the large c_{vir}.
- A preference for more radial velocity dispersion in BHB halo tracers, with respect to DR6 ones.
- Total mass of the galaxy is large for EIN and NFW, ok for BUR.

What about DM annihilation?

The DM annihilation angular profile

At 90% CL:



...hard to discriminate, need to mess with the Center.

The DM Density at the sun's location

The Local DM density

Curiously no dedicated estimate before 2009.
(only very old guesses using outdated data, DM profile)
Main estimates using a global profile modeling, which is very uncertain, or cosmological simulations (even more uncertain)

- In 2009 Catena Ullio, by global modeling, claim

  $$\rho_\odot = 0.389 \pm 0.02\,\text{GeV}/\text{cm}^3$$ [Catena+ '10]

  Criticised by [Weber+ '10] and others, still global modeling.

- More recently ESO survey of z-motions claim no DM!?

  $$\rho_\odot = 0 \pm 0.05\,,\text{GeV}/\text{cm}^3$$ [Moni-Bidin+ '12]

  Criticized first by [Tremaine+ '12], on the velocity assumptions. Other criticisms may be advanced.

- Our work to assess the uncertainties finds

  $$\rho_\odot = 0.43 \pm 0.1 \pm 0.1\,,\text{GeV}/\text{cm}^3$$ [Salucci, FN+ '10]

  still the most accurate, and halo model independent.

A new method for the Local DM density

Decompose radial acceleration as due to Bulge + Disk + DM Halo

$$V^2/r = a_B + a_D + a_H \,,$$

Use Gauss law for the DM Halo: $\rho_H r^2 \propto \partial_r (r^2 a_H)$

$$\begin{aligned}\rho_H(r) &= \frac{1}{4\pi G}\frac{1}{r^2}\frac{d}{dr}\left[r^2\left(\frac{V^2(r)}{r} - a_D(r) - a_B(r)\right)\right] X_q \,,\\ &= \frac{1}{4\pi G}\frac{V^2}{r^2}\left[\left(1+2\frac{d\ln V}{d\ln r}\right) - \frac{V_D^2}{V^2} f\left(\frac{r}{R_D}\right) X_{z_0}\right] X_q \,.\end{aligned}$$

with f a known analytic function, for thin disk.

Notes:

- At $R_\odot$ the contribution of Bulge is negligible
- $X_q \simeq 1.0$–1.05 corrects spherical Gauss law, for oblateness.
- $X_{z_0} \simeq 0.95 \pm 0.01$ corrects for nonzero disk thickness.

The Local DM density, cont'd

$$\rho_\odot = 1.2 \times 10^{-27} \frac{\mathrm{g}}{\mathrm{cm}^3} \left(\frac{\omega_\odot}{\mathrm{km/s\,kpc}} \right)^2 X_q \left[(1 + 2\alpha_\odot) - \beta\, f(r_{\odot D})\, X_{z_0} \right],$$

Result depends on

- $\omega_\odot \equiv (V_\odot / R_\odot)$, angular speed, (very well known)
- $\alpha_\odot \equiv d \ln V / d \ln r|_\odot$, RC slope (uncertain)
- $\beta \equiv (V_D / V_\odot)^2$ (constrained)
- $\rho_{\odot D} \equiv R_\odot / R_D$. (constrained)

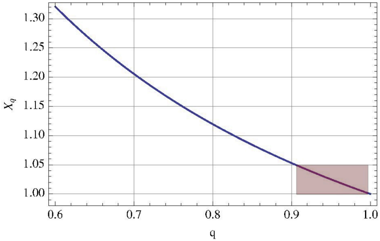


Fig. A.1. Effect of the DM halo oblateness q.

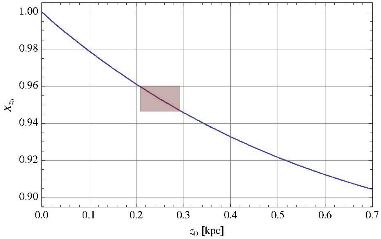


Fig. B.1. Effect of the disk thickness z_0.

The Sun Galactic Radius and Angular Velocity

- $R_\odot$
  Gillessen 2009:
  $8.33 \pm 0.3\,\text{kpc}$

  Ghez et al 2009 (using orbits):
  $8.0 \pm 0.6\,\text{kpc}$
  $8.4 \pm 0.4\,\text{kpc}$(assuming stationary BH)

  Bovy et al 2009 (a global average) [0907.5423v2]

$$R_\odot = (8.2 \pm 0.5)\,\text{kpc}$$

- $V_\odot/R_\odot$ is measured with a very high accuracy and much better than $V_\odot$ and $R_\odot$ separately:

$$V_\odot/R_\odot = (30.3 \pm 0.3)\,\text{km/s/kpc}$$

[MB+09,reid+09,Brunthaler+11]

The Sun Galactic Radius and Angular Velocity

- $R_\odot$
  Gillessen 2009:
  $8.33 \pm 0.3\,\text{kpc}$

  Ghez et al 2009 (using orbits):
  $8.0 \pm 0.6\,\text{kpc}$
  $8.4 \pm 0.4\,\text{kpc}$(assuming stationary BH)

  Bovy et al 2009 (a global average) [0907.5423v2]

$$R_\odot = (8.2 \pm 0.5)\,\text{kpc}$$

- $V_\odot/R_\odot$ is measured with a very high accuracy and much better than $V_\odot$ and $R_\odot$ separately:

$$V_\odot/R_\odot = (30.3 \pm 0.3)\,\text{km/s/kpc}$$

[MB+09,reid+09,Brunthaler+11]

The Slope and Disk contribution $R_\odot$

- Circular velocity slope $\alpha(r) = \frac{d \ln V(r)}{d \ln r}$
  It is limited but uncertain from 2 to 8 kpc:

$$\alpha(2\,\text{kpc} < r < 8\,\text{kpc}) \simeq 0.1\text{–}0$$

(also slightly correlated with $R_\odot$ through the terminal velocities)
At $R_\odot$ we can take the broad range

$$\alpha_\odot = 0. \pm 0.1$$

(confirmed by the global profile fits, above)

- Contribution of disk to sun's rotation, $\beta = V_D/V_\odot$
  The disk can neither contribute totally, nor negligibly.
  A broad conservative range is

$$0.65 < \beta < 0.77$$

The Slope and Disk contribution $R_\odot$

- Circular velocity slope $\alpha(r) = \frac{d\ln V(r)}{d\ln r}$
  It is limited but uncertain from 2 to 8 kpc:

  $$\alpha(2\,\mathrm{kpc} < r < 8\,\mathrm{kpc}) \simeq 0.1\text{–}0$$

  (also slightly correlated with $R_\odot$ through the terminal velocities)
  At $R_\odot$ we can take the broad range

  $$\alpha_\odot = 0. \pm 0.1$$

  (confirmed by the global profile fits, above)

- Contribution of disk to sun's rotation, $\beta = V_D/V_\odot$
  The disk can neither contribute totally, nor negligibly.
  A broad conservative range is

  $$0.65 < \beta < 0.77$$

Result

An analytical formula:

$$\rho_\odot = 0.43\frac{\text{GeV}}{\text{cm}^3}\Big[1 + 2.9\,\alpha_\odot - 0.64\left(\beta - 0.72\right) + 0.45\left(r_{\odot D} - 3.4\right) - 0.1\left(\frac{z_0}{\text{kpc}} - 0.25\right) + 0.10\left(q - 0.95\right) + 0.07\left(\frac{\omega}{\text{km/s kpc}} - 30.3\right)\Big].$$

Good also for the future.

Today, using central values and present uncertainties:

$$\rho_\odot = \left(0.43 \pm 0.094_{(\alpha_\odot)} \mp 0.016_{(\beta)} \pm 0.096_{(r_{\odot D})}\right)\frac{\text{GeV}}{\text{cm}^3},$$

Result

An analytical formula:

$$\rho_\odot = 0.43\,\frac{\text{GeV}}{\text{cm}^3}\Big[1 + 2.9\,\alpha_\odot - 0.64\left(\beta - 0.72\right) + 0.45\left(r_{\odot D} - 3.4\right) - 0.1\left(\frac{z_0}{\text{kpc}} - 0.25\right) + 0.10\left(q - 0.95\right) + 0.07\left(\frac{\omega}{\text{km/s kpc}} - 30.3\right)\Big].$$

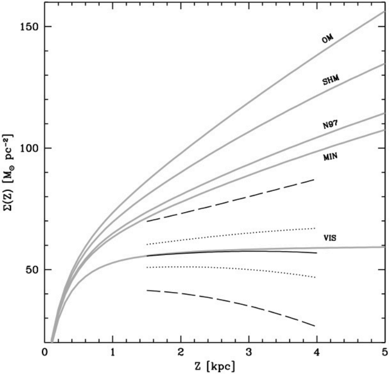
Good also for the future.

Today, using central values and present uncertainties:

$$\rho_\odot = \left(0.43 \pm 0.094_{(\alpha_\odot)} \mp 0.016_{(\beta)} \pm 0.096_{(r_{\odot D})}\right)\frac{\text{GeV}}{\text{cm}^3}\,,$$

Claim of no local DM!?

- ESO claim [Mona-Bidin+'12]
  using thick disk stars, with $|z| < 4\,\mathrm{kpc}$
  (This is a lot above or below the disk.)
  Measures l.o.s. velocity dispersion
  Assume 'circular' velocity is z and R independent
  Use vertical Jeans equation to find the gravitational potential
  $\rightarrow$ local DM surface density =0



Claim of no local DM!?

- ESO claim [Mona-Bidin+'12]
  using thick disk stars, with $|z| < 4\,\text{kpc}$
  (This is a lot above or below the disk.)
  Measures l.o.s. velocity dispersion
  Assume 'circular' velocity is z and R independent
  Use vertical Jeans equation to find the gravitational potential
  $\rightarrow$ local DM surface density =0
- Tremaine refutes (nonconstant velocity at higher z)
  Finds $\rho_0 \simeq 0.3 \pm 0.1$.
- Garbari et al refine the analysis and finds $0.9\,\text{GeV}/\text{cm}^3$
  But using simulation of the z dynamics and MCMC.
- Also consistency of the sample can be questioned.
- More generally,
  it is hard to estimate the vertical dynamics.
  Maybe with GAIA - increasing statistics and precision.

Claim of no local DM!?

- ESO claim [Mona-Bidin+'12]
  using thick disk stars, with $|z| < 4$ kpc
  (This is a lot above or below the disk.)
  Measures l.o.s. velocity dispersion
  Assume 'circular' velocity is z and R independent
  Use vertical Jeans equation to find the gravitational potential
  $\rightarrow$ local DM surface density =0

- Tremaine refutes (nonconstant velocity at higher z)
  Finds $\rho_0 \simeq 0.3 \pm 0.1$.
- Garbari et al refine the analysis and finds 0.9 GeV/cm^3
  But using simulation of the z dynamics and MCMC.
- Also consistency of the sample can be questioned.

- More generally,
  it is hard to estimate the vertical dynamics.
  Maybe with GAIA - increasing statistics and precision.

Conclusions

Dark Matter in our Galaxy:

- Galaxy profile intrinsecally uncertain, observations hard.
- Still, it appears consistent with similar galaxies.
- Preference for cored profile, down to 2 kpc.
- At odds with ΛCDM simulations.
- Hard to discriminate profiles, need to look inside 1 kpc.

Dark matter near the sun:

- $\rho_\odot = 0.4 \pm 0.2$ is still the proper estimate.
- Uncertainties can not be reduced, at present.
- $r_D/R_\odot$, and the RC slope $\alpha_\odot$ are driving the uncertainty,

Conclusions

Dark Matter in our Galaxy:

- Galaxy profile intrinsecally uncertain, observations hard.
- Still, it appears consistent with similar galaxies.
- Preference for cored profile, down to 2 kpc.
- At odds with ΛCDM simulations.
- Hard to discriminate profiles, need to look inside 1 kpc.

Dark matter near the sun:

- $\rho_\odot = 0.4 \pm 0.2$ is still the proper estimate.
- Uncertainties can not be reduced, at present.
- $r_D/R_\odot$, and the RC slope $\alpha_\odot$ are driving the uncertainty,

Thanks.

Conclusions

Dark Matter in our Galaxy:

- Galaxy profile intrinsecally uncertain, observations hard.
- Still, it appears consistent with similar galaxies.
- Preference for cored profile, down to 2 kpc.
- At odds with ΛCDM simulations.
- Hard to discriminate profiles, need to look inside 1 kpc.

Dark matter near the sun:

- $\rho_\odot = 0.4 \pm 0.2$ is still the proper estimate.
- Uncertainties can not be reduced, at present.
- $r_D/R_\odot$, and the RC slope $\alpha_\odot$ are driving the uncertainty,

Thanks.

Conclusions

Dark Matter in our Galaxy:

- Galaxy profile intrinsecally uncertain, observations hard.
- Still, it appears consistent with similar galaxies.
- Preference for cored profile, down to 2 kpc.
- At odds with ΛCDM simulations.
- Hard to discriminate profiles, need to look inside 1 kpc.

Dark matter near the sun:

- $\rho_\odot = 0.4 \pm 0.2$ is still the proper estimate.
- Uncertainties can not be reduced, at present.
- $r_D/R_\odot$, and the RC slope $\alpha_\odot$ are driving the uncertainty,

Thanks.