Elastic nucleon-deuteron scattering and breakup with chiral forces

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Plan:

1. Formalism and calculational scheme
2. Results for nd reactions – low energies and standard NN potentials
3. Results for nd reactions – higher energies and standard NN potentials
4. Relativistic effects
5. Chiral effective field theory potentials
6. Summary
• Present day models of nucleon-nucleon potentials (semi-phenomenological: AV18, CD-Bonn, NijmI and II) provide very good description of 2-nucleon bound state (deuteron) and very good description of NN scattering data.

• Bound state of three nucleons ($^3$H, $^3$He) calculated with NN interactions only is underbound → **IMPORTANT INFORMATION**: three-nucleon Hamiltonian $H$ must contain also a three-nucleon potential – a term which cannot be reduced to interactions of pairs of nucleons:

\[
H = H_0 + V_{12} + V_{23} + V_{31} + V_4
\]

\[
V_4 = V_4^{(1)} + V_4^{(2)} + V_4^{(3)}
\]

splitting of 3N potential to three parts with proper symmetry

• **Can we describe the data in 3N continuum with such three-nucleon Hamiltonian?**
First model of three-nucleon force (3NF) came from Japan:
J.Fujita and H.Miyazawa, Prog. Theor. Phys. 17, 360 (1957)).

Other important 3N potential models:

- Model Tucson-Melbourne based on pion-nucleon scattering amplitude
(S.A.Coon, W.Glöckle, PRC23, 1790 (1981))

- Urbana IX 3NF model is based on $2\pi$-exchange with $\Delta$-resonance excitation in intermediate state; there is also spin- and isospin-independent short-range term (B.S.Pudliner et al., PRC56, 1720 (1997))
What equations must be solved?

Theoretical description of 3N continuum (elastic nucleon-deuteron scattering and the deuteron breakup reaction) requires solution of the following Faddeev-type equation for $T|\phi_d>$ state:

$$T|\phi_d\rangle = tP|\phi_d\rangle + \left(1 + tG_0\right)V_4^{(1)}(1 + P)|\phi_d\rangle$$

$$+ tPG_0T|\phi_d\rangle + \left(1 + tG_0\right)V_4^{(1)}(1 + P)G_0T|\phi_d\rangle,$$

Free 3N propagator 2N transition operator generated from L-S equation

$$t = V_{23} + V_{23}G_0^{2N}t$$

Part of $V_4$ symmetrical with respect to exchange of nucleons 2 and 3

$|\phi_d\rangle \equiv |\phi_d\rangle|\tilde{q}_0\rangle$ is composed of the deuteron internal wave function and the state of the relative nucleon-deuteron motion.

These equations can be solved numerically precisely for any 2N potential and 3N-force.
The transition amplitude for elastic scattering is given by:

\[
\langle \phi_d' | U | \phi_d \rangle = \langle \phi_d' | P G_0^{-1} | \phi_d \rangle + \langle \phi_d' | V_4^{(1)} (1 + P) | \phi_d \rangle \\
+ \langle \phi_d' | P T | \phi_d \rangle + \langle \phi_d' | V_4^{(1)} (1 + P) G_0 T | \phi_d \rangle
\]

Transition amplitude for the deuteron breakup is given by:

\[
\langle \phi_0 | U_0 | \phi_d \rangle = \langle \phi_0 | (1 + P) T | \phi_d \rangle
\]

The singularities of operator $G_0$ and pole of operator $t$ in calculations for positive energies, makes treatment of the 3N continuum much more difficult than of the 3N bound state!
Quite good description of low energy data (below about 30 MeV): it seems that at these energies one does not need a 3NF!
Exception: $A_y$ puzzle

REM bok mark: quite different effects of TM99 and UIX 3NF’s (in spite of the fact that both are of $2\pi$-exchange origin. That can imply that consistency in derivation of NN- and 3N-forces is important.
Higher energies: large discrepancies between data and theory based only on NN forces starts to appear

Elastic scattering $d(p,p)d$

- NN only (AV18, CD Bonn, Nijm1, Nijm2)
- NN+3NF TM99

Total nd cross section: (W.P. Abfalterer et al. PRL 81(1998)57)
- up to ~ 50 MeV good agreement with predictions based on 2N forces
- adding 3NF provides explanation of the disagreement up to ~ 150 MeV
- at even larger energies a clear disagreement which increases with energy

Data 70: K. Sekiguchi et al., PR C65, 034003 (2002)

Data 250:
- $x\text{ nd}$ – Y. Maeda et al., PR C76, 014004 (2007)
- $o\text{ pd}$ – K. Hatanaka et al., PR C 66, 044002 (2002)
what is responsible for large differences between theory and data in 250 MeV cross section and in the total nd cross section even after inclusion of 2π-exchange 3NF ?

it is evident, that in the applied dynamics something is wrong or missing

one possibility is that omitted short-range components of 3NF become more and more important with increasing energy

however, increasing energy means also a transition to a region, where relativity could be important → relativistic Faddeev calculations
Relativistic Faddeev calculations

The formal structure of the equations remains the same but the ingredients change

- Form of the free Hamiltonian $H_0$ (and $G_0$) changes:

$$H_0 = \sqrt{2\sqrt{m^2 + k^2}^2 + q^2 + \sqrt{m^2 + \bar{q}^2}}$$

- Interacting $2N$ subsystem (2-3) has nonzero total momentum $-q$ in the $3N$ c.m. system, what leads to the boosted potential $V$:

$$V(\bar{q}) \equiv \sqrt{\left[2\sqrt{m^2 + \bar{k}^2 + v}\right]^2 + \bar{q}^2} - \sqrt{\left[2\sqrt{m^2 + k^2}\right]^2 + \bar{q}^2}$$

- $V(q=0)$ reduces to the relativistic potential $v$ defined in the $2N$ c.m. system. From $V(q)$ the boosted t-matrix is obtained

- The Lorentz transformation from $2N$ to $3N$ c.m. is performed along the total momentum of the $2N$ subsystem, which in general is not parallel to momenta of these nucleons. This leads to Wigner rotation of spin states. When defining $3N$ partial wave states care must be taken about spin states
Faddeev approach with both 3NF and relativity included
elastic scattering: \( d(p,p)d \)

- only NN forces (CDBonn):
  - nrel solid red
  - rel dashed blue

- NN + 3NF (TM99):
  - nrel dotted blue

- effects of relativity seen only in Nd elastic scattering backward cross section!
- relativistic effects are not responsible for large discrepancies in elastic Nd scattering cross section
- interplay of relativity and 3NF’s leads to a slight increase of the cross section at angles larger than \( \theta_{\text{cm}} \sim 100^\circ \)

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data 250: 
- x nd – Y.Maeda et al., PR C76, 014004 (2007)
It follows that:

- relativistic effects are **not responsible** for large discrepancies in elastic Nd scattering

- those discrepancies must come from **neglection of short-range 3NF components** which become active at higher energies

**Challenge:** apply NN and 3NF’s derived consistently in the framework of chiral perturbation theory
LENPIC (Low Energy Nuclear Physics International Collaboration): to understand nuclear structure and reactions with chiral forces.
Few remarks on chiral forces:

- In order to reproduce properly 2N data up to about 250 MeV N3LO order of chiral expansion is required
  - About 2 years ago: NN interaction up to N3LO, 3NF interaction up to N4LO (N3LO used in preliminary calculations at low energies)
  - nonlocal momentum space regularization has been applied:
    \[ V \rightarrow f(p',\Lambda) V(p',p) f(p,\Lambda) \text{ with } f(p,\Lambda) \equiv e^{-p^6/\Lambda^6} \]
    \[ V^4 \rightarrow f(p',q',\Lambda) V^4(p',q',p',q) f(p,q,\Lambda) \text{ with } f(p,q,\Lambda) \equiv e^{-(p^2+0.75q^2)^3/\Lambda^6} \]
    what leads to finite cut-off artefacts (problems when applied to higher energy Nd scattering)
- New, improved chiral force, presented by Bochum-Bonn group in 2014:
  - Local regularization in the coordinate space \( V_{lr}(r) \rightarrow V_{lr}(r)f(r) \) with \( f(r) \equiv \left(1 - e^{-r^2/R^2}\right)^n \)
  - R=0.8–1.2 fm what corresponds to \( \Lambda=330-500 \) MeV
  - Such regularization preserves more long-range OPE and TPE physics
  - All LECs in the long-range part are taken from pion-nucleon scattering without fine tuning
  - Very good description of the deuteron properties, phase shifts etc.
Various topologies contributing to the 3NF up to and including N⁴LO

- N²LO: (a) + (d) + (f) (E.Epelbaum et al., PR C66, 064001 (2002))

- N³LO: (a) + (b) + (c) + (d) + (e) + (f) + rel
  V.Bernard et al., PR C77, 064004 (2008) - long range contributions (a), (b), (c)
  V.Bernard et al., PR C84, 054001 (2011) - short range terms (e)
  and leading relativistic corrections

**N³LO contributions do not involve any unknown low energy constants!**

The full N³LO 3NF depends on two parameters c_D and c_E coming with (d) and (f) terms, respectively. They are adjusted to two chosen 3N observables.

- N⁴LO (longest range contributions): (a) + (b) + (c) + (d) + (e) + (f) (H.Krebs et al., arXiv:1203.0067)

\[ X(p) \text{- observable and } X^{(i)}, i = 0, 2, 3, \ldots \text{prediction at order } Q^{(i)} \text{ in the chiral expansion} \]

Order-\( Q^{(i)} \) correction:

\[ \Delta X^{(2)} = X^{(2)} - X^{(0)} \]
\[ \Delta X^{(i)} = X^{(i)} - X^{(i-1)} \quad i \geq 3 \]

The chiral expansion for \( X \) takes the form:

\[ X^{(i)} = X^{(0)} + \Delta X^{(2)} + \ldots + \Delta X^{(i)} \quad \text{Size of corrections is expected to be } \Delta X^{(i)} = O(Q^{(i)} X^{(0)}) \]

Quantitative estimates of the theoretical uncertainty \( \delta X^{(i)} \) of the prediction \( X^{(i)} \) is made using the expected and actual sizes of higher-order contributions:

\[ \delta X^{(0)} = Q^2 |X^0| \]
\[ \delta X^{(2)} = \max \left( Q^3 |X^0|, Q^1 |\Delta X^{(2)}| \right) \]
\[ i \geq 3: \quad \delta X^{(i)} = \max \left( Q^{i+1} |X^{(0)}|, Q^{i-1} |\Delta X^{(2)}|, Q^{i-2} |X^{(3)}| \right) \]
\[ \delta X^{(2)} \geq Q \delta X^{(0)}, \quad \delta X^{(i \geq 3)} \geq Q \delta X^{(i-1)} \]

\( Q = \max(p/\Lambda_b, m_\pi/\Lambda_b) \) with \( \Lambda_b = 600, 500 \) and \( 400 \) MeV for \( R = 0.8-1.0 \) fm, \( R = 1.1 \) fm and \( R = 1.2 \) fm, respectively.
- NN developed up to N4LO: E.Epelbaum et al. arXiv:1412.4623 [nucl-th]
- Novel way of quantifying the theoretical uncertainty due to the truncation of the chiral expansion: E.Epelbaum et al. arXiv:1412.0142 [nucl-th]

Theoretical uncertainty grows with energy and decreases with increasing order
Exclusive breakup reaction: quasi free pp scattering

![Graphs showing exclusive breakup reaction results for different energies and angles.](image-url)
Exclusive breakup reaction: symmetric space-star configuration

Big challenge: application of full N3LO chiral force: $\text{NN} + 3\text{NF}$
Summary:

- Nd elastic scattering and deuteron breakup reaction reveal large sensitivity to underlying nuclear forces --> good tools to test nuclear Hamiltonian

- It is clear, that based on the present day NN forces additional term in nuclear Hamiltonian is needed --> 3NF

- 3NF models, derived independently from NN potentials, can account in some cases for discrepancies between theory and data

- Call for consistency between 2N and 3N forces: support and guidance --> chiral perturbation theory

- Big challenge: application of chiral N³LO forces (2- and 3-body) to 3N continuum at higher energies