Impact of pairing on thermodynamical properties of stellar matter

Nucleus Nucleus 2015

Dipartimento di Fisica e Astronomia and INFN - LNS

Catania, 21 - 26 June 2015

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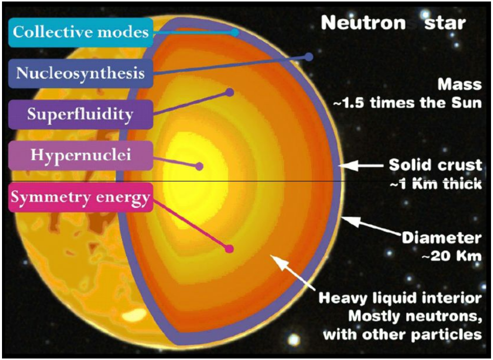
Introduction: main Neutron Stars (NS) properties



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  wide range of ρ, y_p, T
- **Inner crust** structure:
  - lattice of nuclear **clusters**
  - ultrarelativistic electron gas
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- **Superfluidity** effects:
  - giant **glitches**
  - **cooling** process

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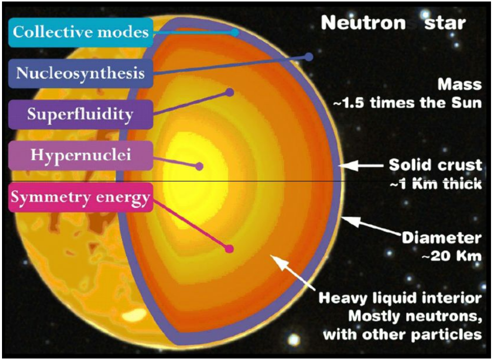

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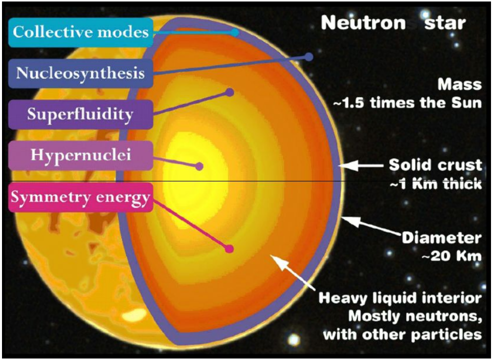

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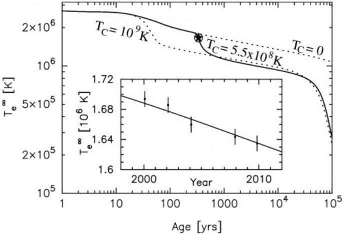
Recent observations

Cassiopeia: strong evidence for superfluidity

[Page, D. et al. Physical Review Letters 106.8 (2011)]



Clusters in the inner crust

- NS **cooling** simulations ⇒ description of **inhomogeneous** crust
- Self-consistent **mean-field** approaches:
  - **Microscopic** calculations (sometimes too computationally expensive!)
  - **Phenomenological** model (ex. **N**uclear **S**tatistical **E**quilibrium)
- **Wigner-Seitz (WS)** approximation:

  non-interacting and electrically **neutral** spherical cells
- **T = 0**: min[E_{WS}/V_{WS}] → one single nucleus (**SNA**)
- **Finite T**: beyond SNA → **statistical distribution** of nuclei
- **NSE** model: nucleons and nuclei in thermal and chemical equilibrium

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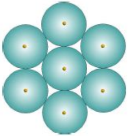

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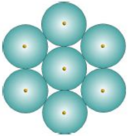

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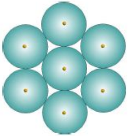

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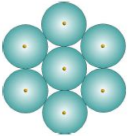

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The NSE model

- **Unbound nucleons** (gas): non-relativistic density functional

$$\mathcal{E}_{gas} = 2 \sum_{q=n,p} \int_0^\infty \frac{d\mathbf{p}}{h^3} f_q \frac{p^2}{2m_q^*} + \mathscr{E}_{Sky}^{pot} \qquad \text{(SLy4 Skyrme effective interaction)}$$

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  [P. Danielewicz and J. Lee, Nucl. Phys. A 818 (2009)]

- **Light clusters underbound** by formula ⇒ two parameterizations adopted:
  - LDM, using functional for each cluster
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$$V_\pi(\mathbf{r}_i, \mathbf{r}_j) = \frac{1}{2}(1 - P_\sigma) v_\pi(\rho_n) \delta(\mathbf{r}_{ij})$$

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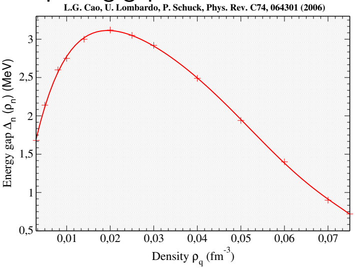

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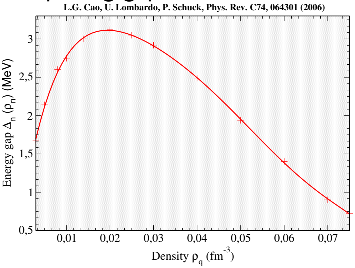

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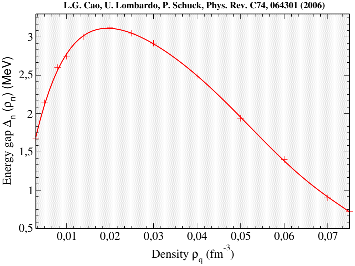

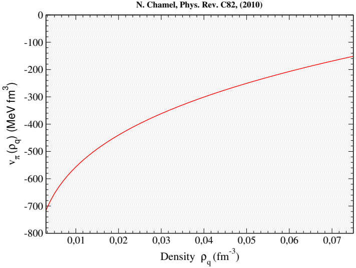

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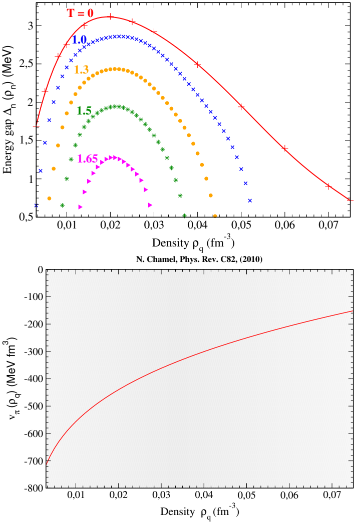

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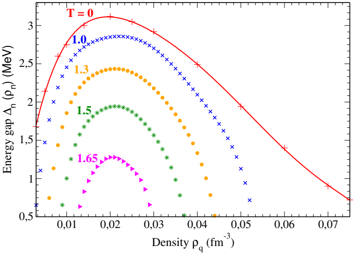

- **Superfluid** $\xleftrightarrow{T=T_c}$ **normal** (2^{nd} order)
- Variation on **energy density**

[Burrello S., Colonna M., Matera F., 2014, PRC 89]

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$$\rho_n = \frac{(2m_n^*)^{3/2}}{4\pi^2\hbar^3} \int_0^{\mu_n^*+\epsilon_\Lambda} d\epsilon \sqrt{\epsilon} \left[1 - \frac{\xi}{E_\Delta} \tanh\left(\frac{E_\Delta}{2T}\right)\right]$$

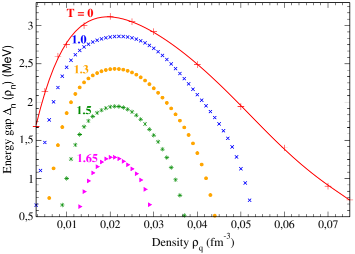
$$1 = -v_\pi(\rho_n) \frac{(2m_n^*)^{3/2}}{8\pi^2\hbar^3} \int_0^{\mu_n^*+\epsilon_\Lambda} d\epsilon \frac{\sqrt{\epsilon}}{E_\Delta} \tanh\left(\frac{E_\Delta}{2T}\right)$$

$$E_\Delta = \sqrt{\xi^2 + \Delta^2}, \qquad \xi = \epsilon - \mu_n^*, \qquad \mu_n^* = \mu_n - U_n$$

- T $= 0 \Rightarrow$ **strength** $v_\pi(\rho_n)$

$$v_\pi(\rho_n) = V_\pi^\Lambda \left[1 - \eta \left(\frac{\rho_n}{\rho_0}\right)^\alpha\right]$$

- $T \neq 0 \Rightarrow \Delta(T)$



- **Superfluid** $\xleftrightarrow{T=T_c}$ **normal** (2^{nd} order)
- Variation on **energy density**

[Burrello S., Colonna M., Matera F., 2014, PRC 89]

$$\mathcal{E}_{gas}^\pi = 2 \sum_{q=n,p} \int_0^\Lambda \frac{d\mathbf{p}}{h^3} f_q^\pi \frac{p^2}{2m_q^*} + \mathcal{E}_\pi + \mathcal{E}_{Sky}^{pot}$$

Self-consistent NSE model with pairing

- **Starting point**: given thermodynamic condition (ρ_B, y_p, T)
  - 10 representative values for baryonic density: $10^{-5} \leq \rho_B \leq 10^{-1}$ fm^{-3} [Negele, J. W., Vautherin D. Nuclear Physics A (1973).]
  - Neutrinoless β-**equilibrium**: $\mu_n = \mu_p + \mu_e \Rightarrow$ fixed y_p
  - Low temperature: $0.1 \leq T \leq 2$ MeV
- **Aim of the work:** analyze how wide distribution of nuclear species and β-equilibrium affect superfluid properties of crust
- Comparison with microscopic **FT-HFB calculations**

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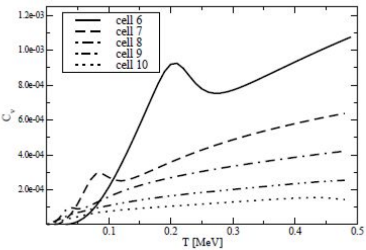
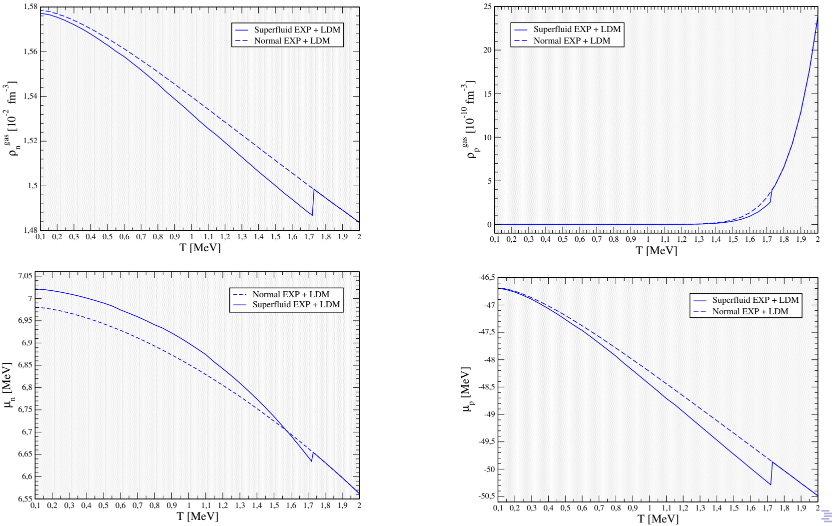


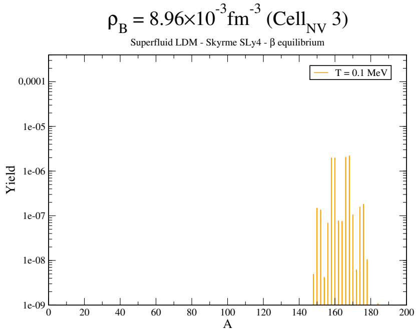
Main results: inner crust composition

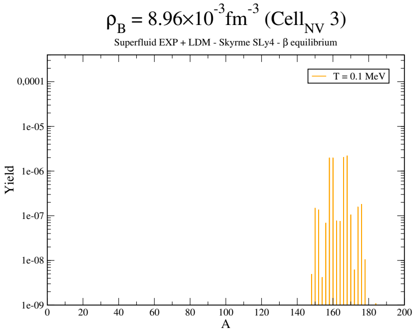
$\rho_B = 2.03 \times 10^{-2}$ fm^{-3} (Cell$_{\mathrm{NV}}$ 2): **Gas densities and chemical potentials**



Main results: clusters distribution (LDM vs. EXP)

- $T \approx 0 \rightarrow$ Single Nucleus Approximation
- Higher $T \rightarrow$ wide clusters distribution
- EXP + LDM: **sharp** transition heavy/light clusters dominance
- LDM: smooth transition heavy/light clusters dominance





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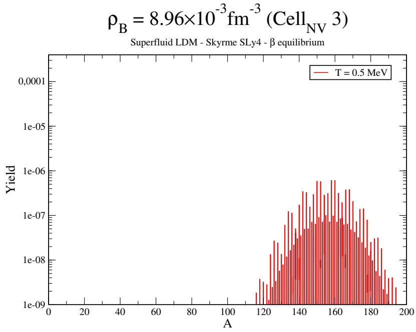

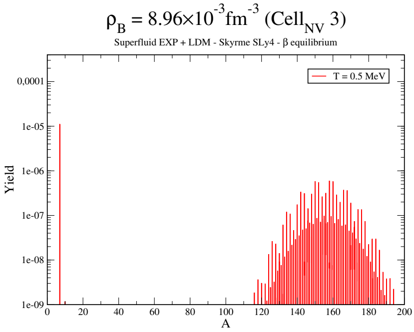

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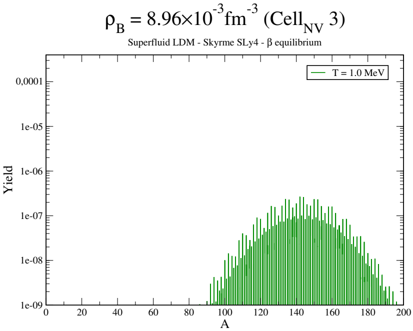

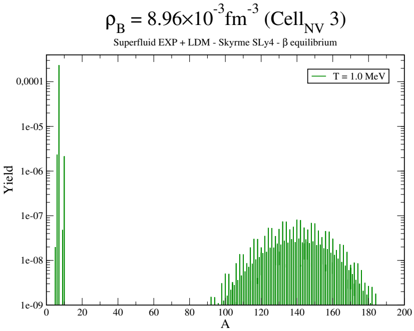

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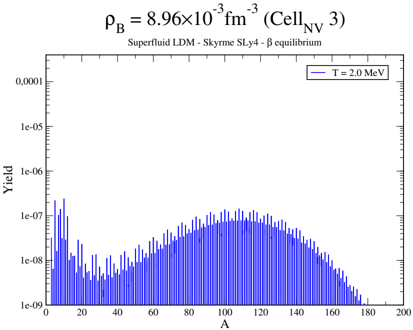

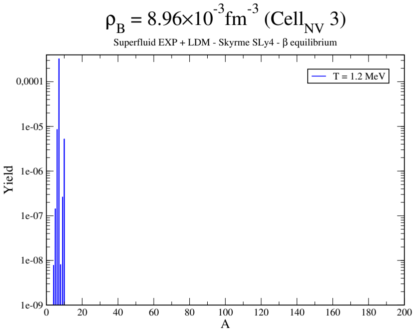

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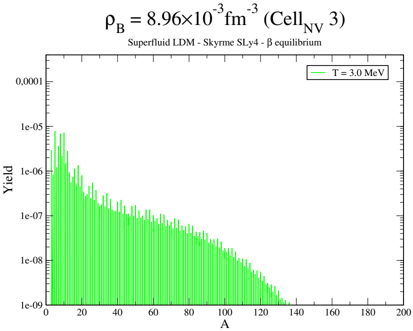

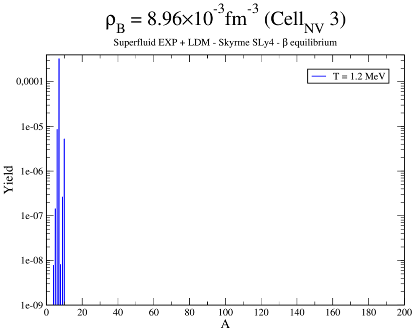

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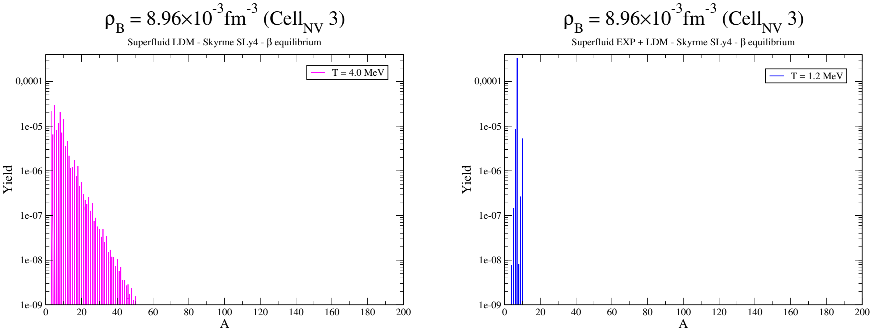

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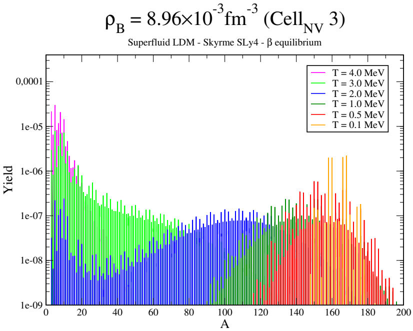
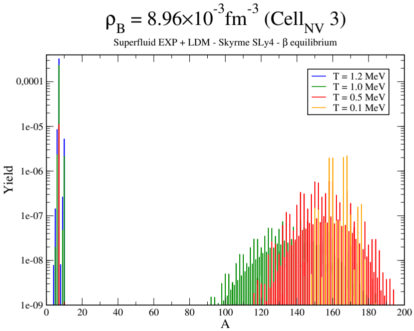
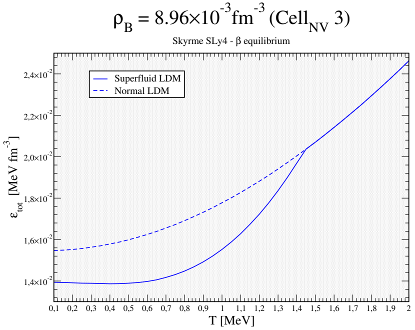




Main results: energy and specific heat

Total energy density

LDM only



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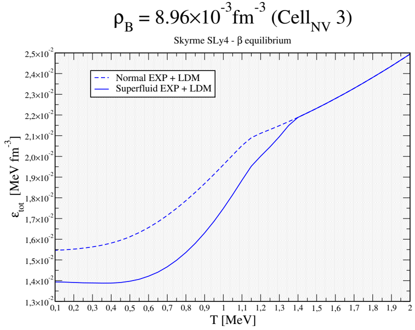

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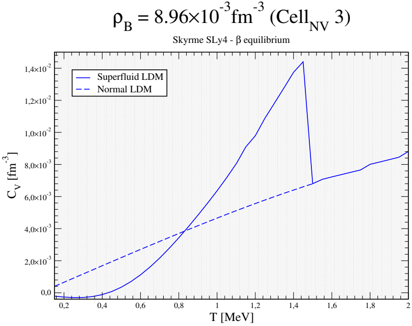
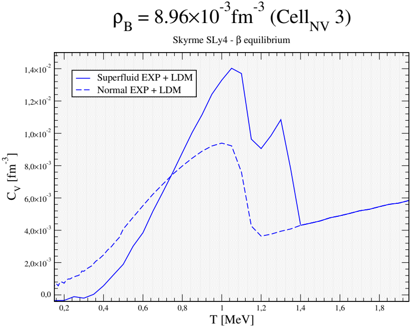


One jump

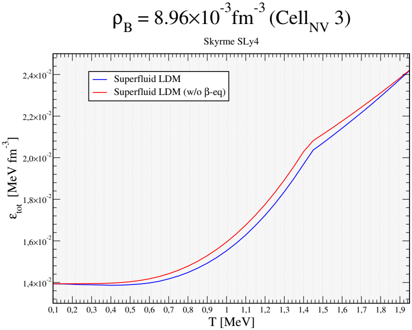
EXP + LDM

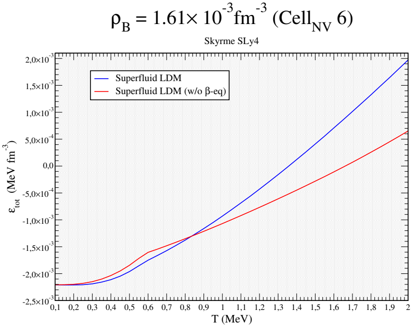


Two jumps

Main results: importance of β-equilibrium

Total energy density



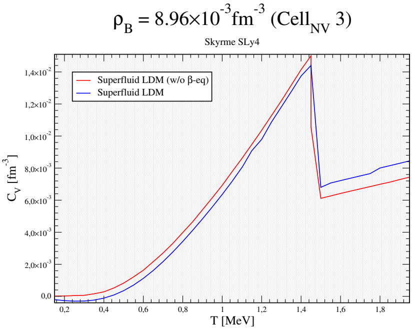


Small effect at higher ρ_B

Large effect at lower ρ_B (higher T)

Main results: importance of β-equilibrium

Specific heat



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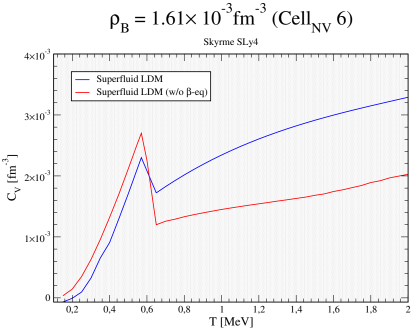
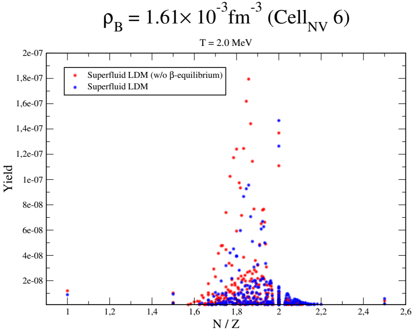
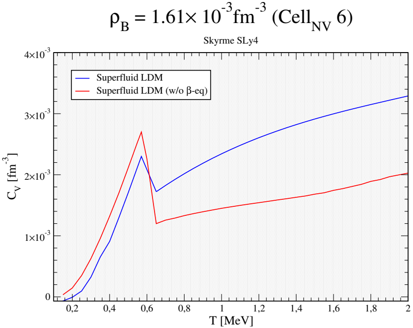


Large effect at lower ρ_B (higher T)

Main results: importance of β-equilibrium

Specific heat and isotopic distribution



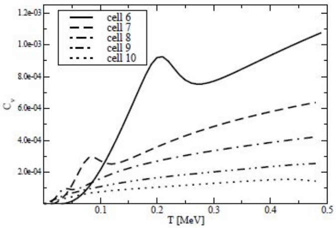


More asymmetric clusters with β eq.

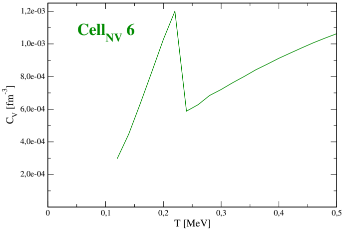
Large effect at lower ρ_B (higher T)

Comparison with HFB calculations

HFB calculations



Extended NSE model



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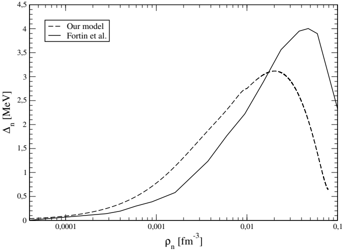

- Different gap but **good agreement**
- Smooth transition ⇒**in-medium effects**
- Importance of **gap** and **beta-equilibrium**

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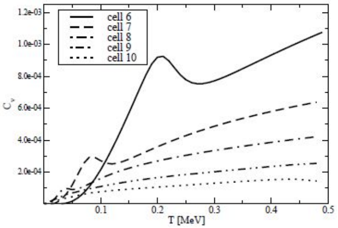

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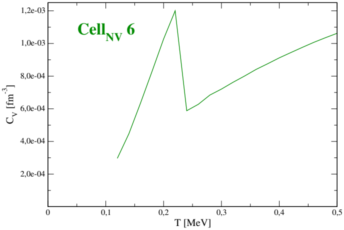
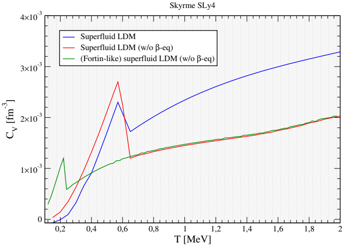


$\rho_B = 1.61\times 10^{-3}\text{fm}^{-3}$ ($Cell_{NV}$ 6)



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Final remarks and conclusions

Summary

- Heat capacity in the NS inner crust
- Complete distribution of nuclear species in thermal and β-equilibrium
- Pairing contribution of unbound neutrons in BCS approximation
- Clusters distribution: - heavy/light dominance transition
  - exotic neutron-rich resonant states
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THANK YOU!

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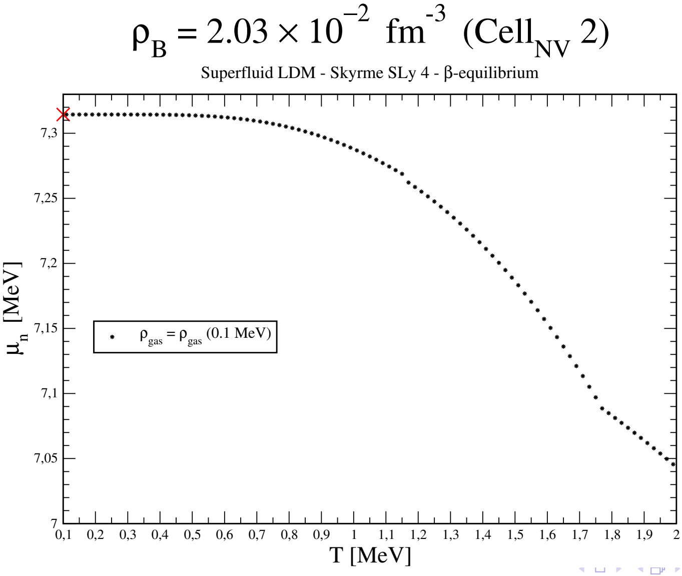

Discontinuity on chemical potential: **(pseudo)1^{st}order** phase transition

$\mu_q\ (T)|_{T=T_c}$ **continuous** at fixed ρ_{gas}!

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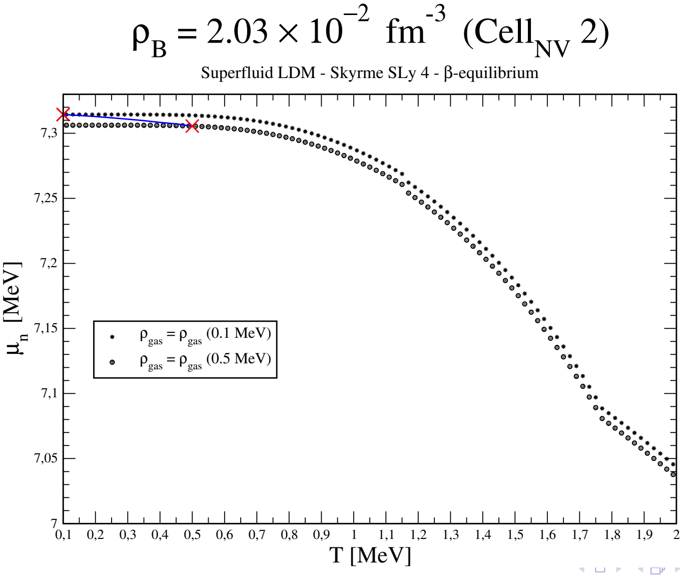

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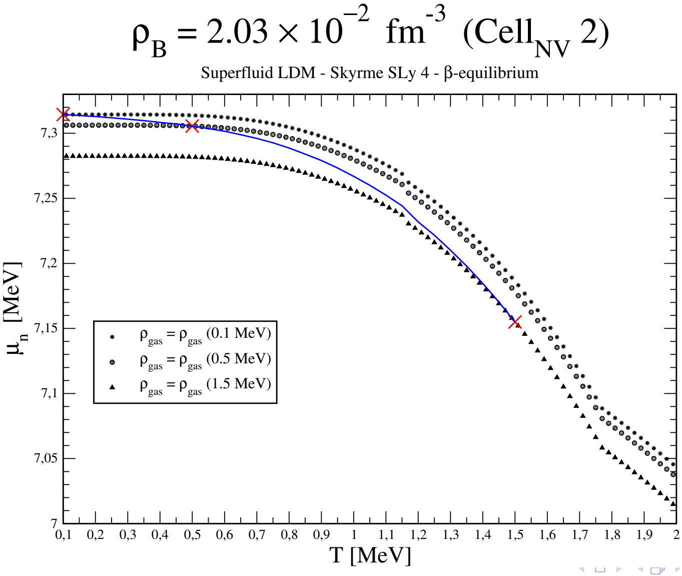

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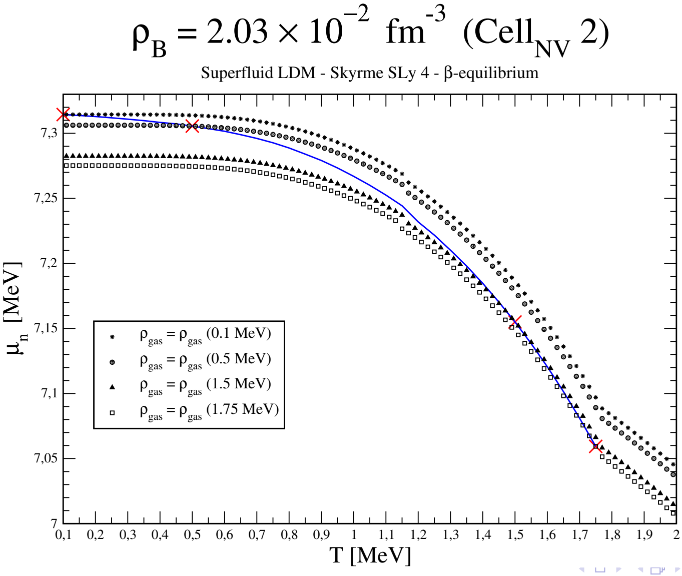

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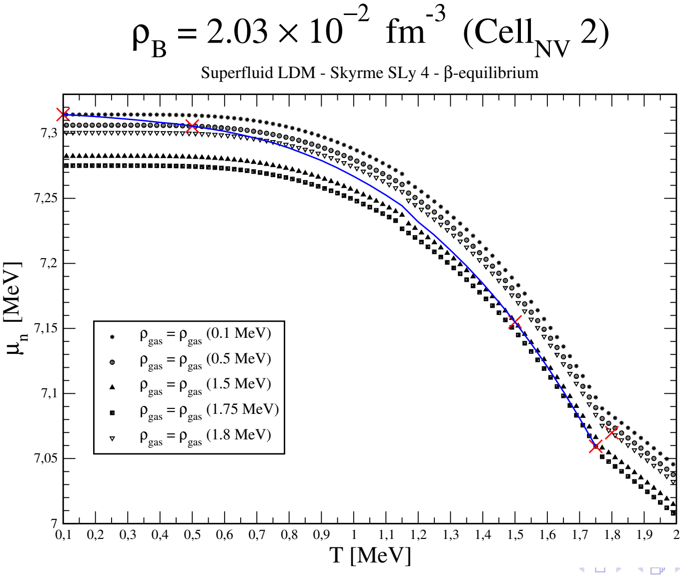

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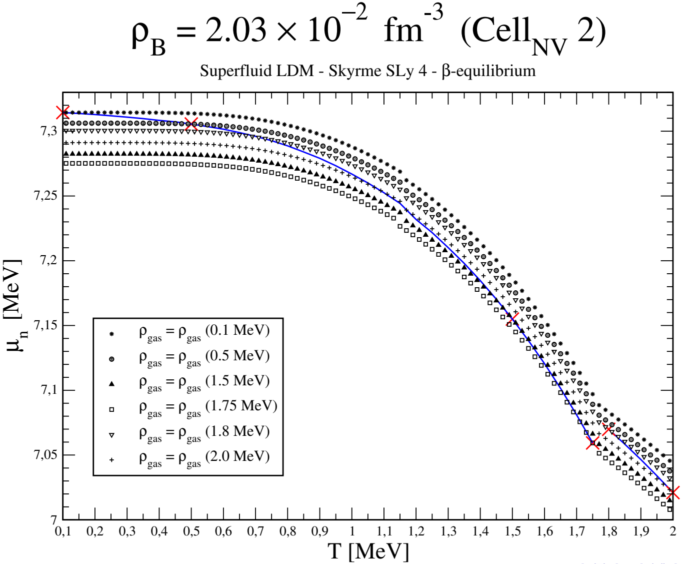

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