Four-body effects on $^{9}Be+^{208}Pb$ scattering around the Coulomb barrier

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In collaboration with

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1. Introduction
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3. ^{9}Be description (3-body $\alpha+\alpha+n$)
4. $^{9}Be+^{208}Pb$ elastic scattering
5. $^{9}Be+^{208}Pb$ breakup and fusion
6. Conclusion

1. Introduction

Many CDCC studies (Continuum Discretzed Coupled Channel)

Essentially

- Three-body problems (two-body projectiles)
- Elastic scattering
- Fusion (few papers)

Problem for weakly bound projectiles:

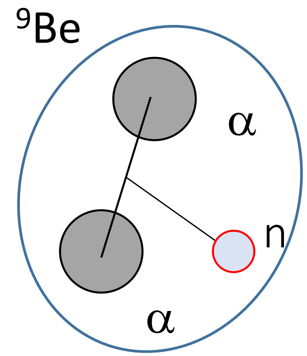
- very slow convergence (partial wave inside the projectile) of the cross sections (essentially around the Coulomb barrier)
- ^{11}Be+^{64}Zn: up to L=5 (in ^{11}Be) is necessary around the Coulomb barrier
  Ref. T. Druet and P.D., EPJA 48 (2012) 147

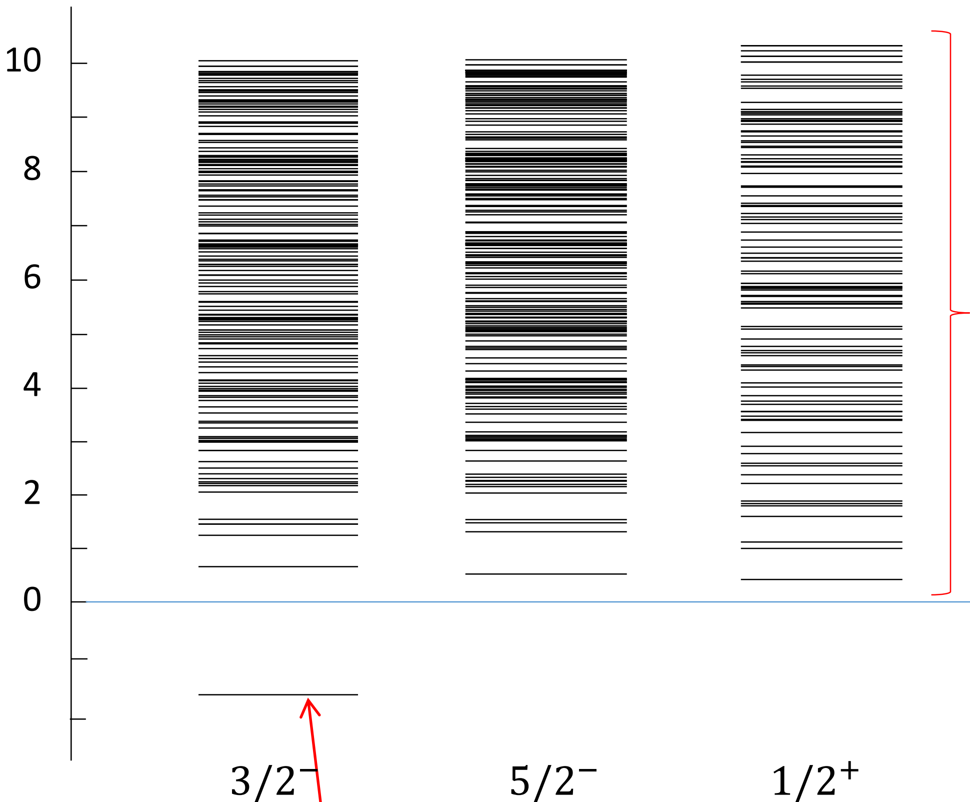
Here: ^{9}Be+^{208}Pb system (many experimental data are available)

- ^{9}Be described by a 3-body structure $\alpha+\alpha$+n
- Simultaneous description of:
  - Elastic scattering
  - Breakup
  - Fusion

2. The CDCC method

Example: ${}^9Be=\alpha+\alpha+n$





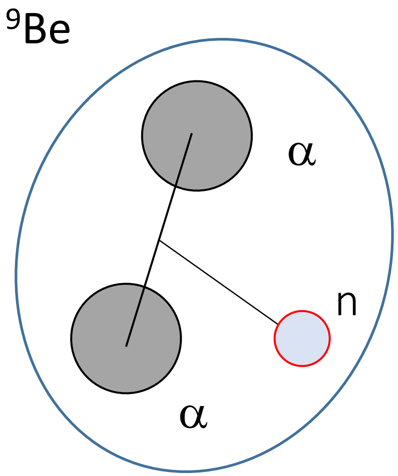
α+α+n

- Pseudostates :E_n>0
- Simulate breakup effects
- Depend on the basis

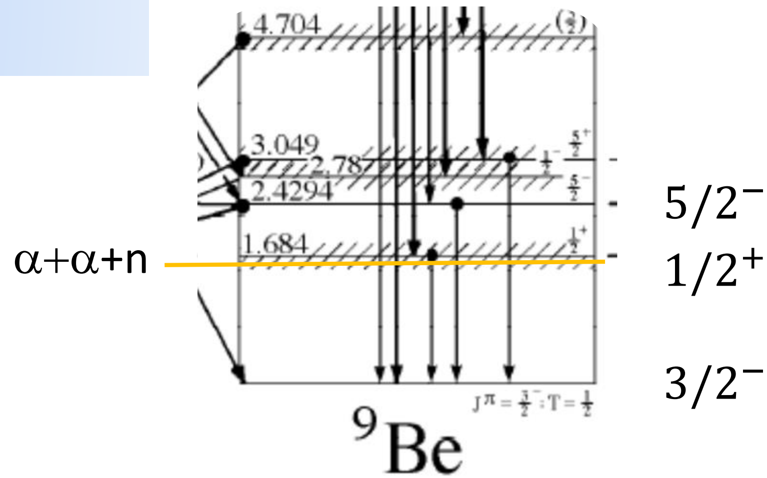
- Ground state: E_n<0
- Independent of the basis

^{9}Be description (3-body α+α+n)

3. ^{9}Be $\alpha+\alpha+n$ description



Three-body $\alpha+\alpha+n$ model



Description of ^{9}Be=$\alpha+\alpha+n$

- **$\alpha+\alpha$ potential**: Buck et al. NPA275 (1977) 246 (2 forbidden states for L=0, 1 fs for L=2)
- **$\alpha+n$ potential**: Kanada et al. Prog. Theor. Phys. 61 (1979) 1327 (1 forbidden state for L=0)

  → supersymmetry transform to remove the forbidden states

Both reproduce the elastic phase shifts up to ~20 MeV (deep potentials)

3-body method: Hyperspherical coordinates

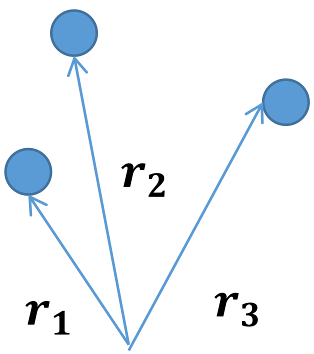
Tests: different Kmax, number of basis functions

3. ^{9}Be α+α+n description

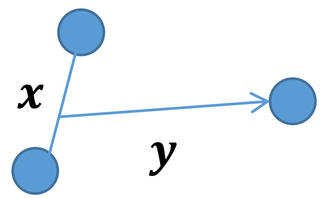
Hyperspherical formalism for 3-body nuclei

- essentially spectroscopy, continuum possible (more difficult!)
- Hamiltonian: $H = T_1 + T_2 + T_3 + V_{12}(|\boldsymbol{r}_1 - \boldsymbol{r}_2|) + V_{13}(|\boldsymbol{r}_1 - \boldsymbol{r}_3|) + V_{23}(|\boldsymbol{r}_2 - \boldsymbol{r}_3|)$

Absolute coordinates



Jacobi coordinates
(c.m. removed)



$$\boldsymbol{x} = \boldsymbol{r}_2 - \boldsymbol{r}_1$$
$$\boldsymbol{y} = \boldsymbol{r}_3 - (A_1\boldsymbol{r}_1 + \boldsymbol{A}_2\boldsymbol{r}_2)/A_{12}$$

Hyperspherical coordinates

- Ω_x, Ω_y
- $\rho = \sqrt{x^2 + y^2}$
- $\tan\alpha = \frac{y}{x}$

ρ =hyperradius
α= hyperangle

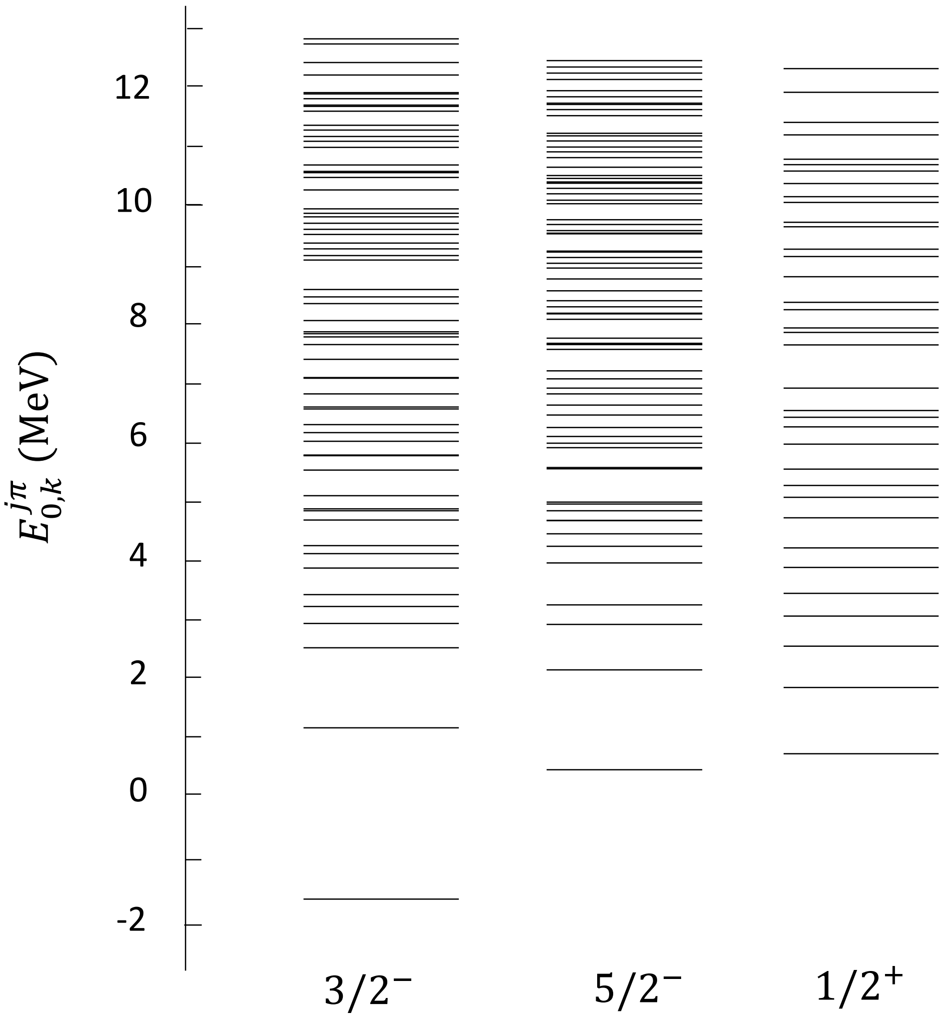
In hyperspherical coordinates: $H = T_\rho + V\left(\rho, \alpha, \Omega_x, \Omega_y\right)$

Eigenstates of T_ρ: **hyperspherical functions** $\mathcal{Y}^J_{KSl_xl_y}\left(\alpha, \Omega_x, \Omega_y\right) = \mathcal{Y}^J_{K\gamma}(\Omega_5)$

known functions (analytical)
extension of spherical harmonics $Y_l^m(\Omega)$ in 2-body problems
K=hypermoment

3. ^{9}Be α+α+n description

Discretization of the three-body α+α+n continuum



- j=1/2, 3/2, 5/2
- Kmax=20
- Lagrange functions: N=20
- Maximum Emax=12.5 MeV

➔ many pseudostates
Many channels in the CDCC calculation

Need to solve the coupled-channel equations

$$\left[-\frac{\hbar^2}{2\mu}\left(\frac{d^2}{dR^2}-\frac{L(L+1)}{R^2}\right)+E_c-E\right]u_c^{J\pi}(R)+\sum_{c'}V_{cc'}^{J\pi}(R)u_{c'}^{J\pi}(R)=0$$

At large distances

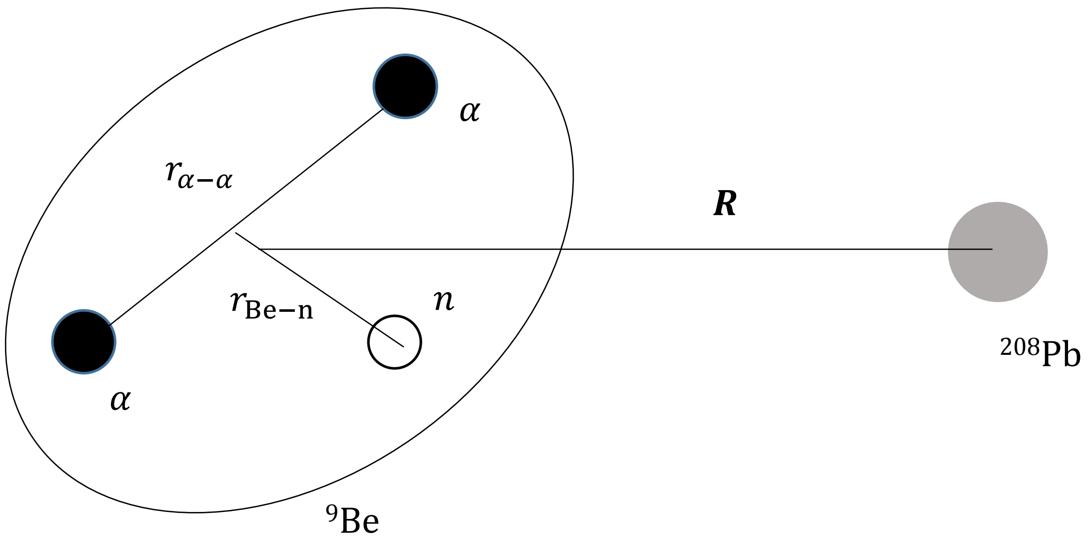
- Nuclear potential negligible, only Coulomb remains
- Wave function $u_c^{J\pi}(R) \rightarrow I_c(k_cR)\delta_{\omega c} - U_{\omega c}^{J\pi}\, O_c(k_cR)$
  with ω=entrance channel
  $I_c(x), O_c(x)$ = incoming and outgoing Coulomb functions
  $U_{\omega c}^{J\pi}$=scattering matrix → various cross sections (elastic, breakup, etc)

Procedure

- Calculation of the 3-body wave functions
- Calculation of the coupling potentials $V_{cc'}^{J\pi}(R)$
- Solving the coupled-channel system (R matrix)
- Determining the scattering matrices $U_{\omega c}^{J\pi}$ and cross sections

$^{9}Be+^{208}Pb$ elastic scattering

4. ^{9}Be+^{208}Pb elastic scattering



α-^{208}Pb potential: Goldring et al. Phys. Lett. B 32, 465 (1970).
n-^{208}Pb potential: Koning Delaroche, Nucl. Phys. A 713, 231 (2003).

4. ^{9}Be+^{208}Pb elastic scattering

- Experimental data (E_{lab}~38-60 MeV, E_{cm} ~36-58 MeV, V_{coul}~38 MeV)
  - R. J. Woolliscroft et al. Phys. Rev. C 69, 044612 (2004).
  - N. Yu et al., J. Phys. G37, 075108 (2010).

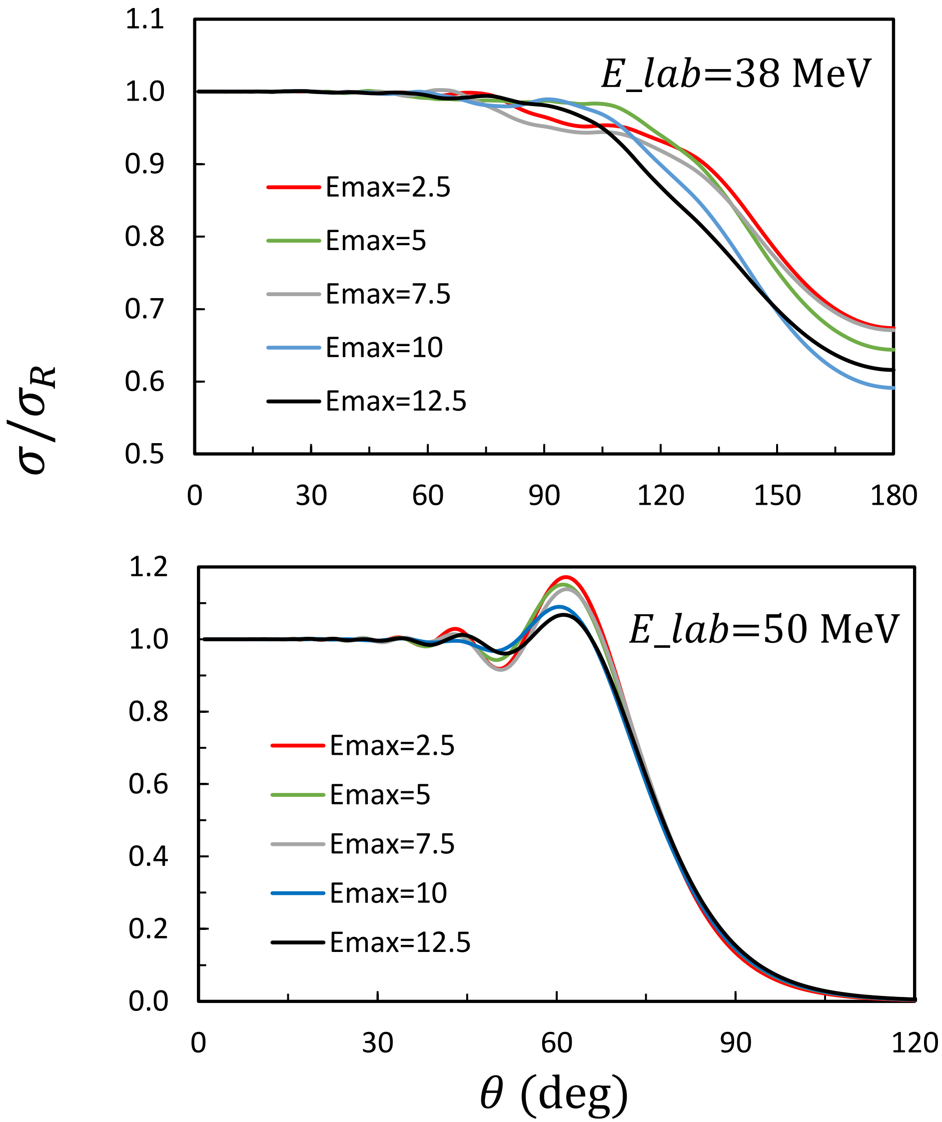
- Various convergence tests
  - Emax, number of basis functions: specific to ^{9}Be
  - Partial waves in ^{9}Be j^π: $3/2^\pm$,$5/2^\pm$,$1/2^\pm$

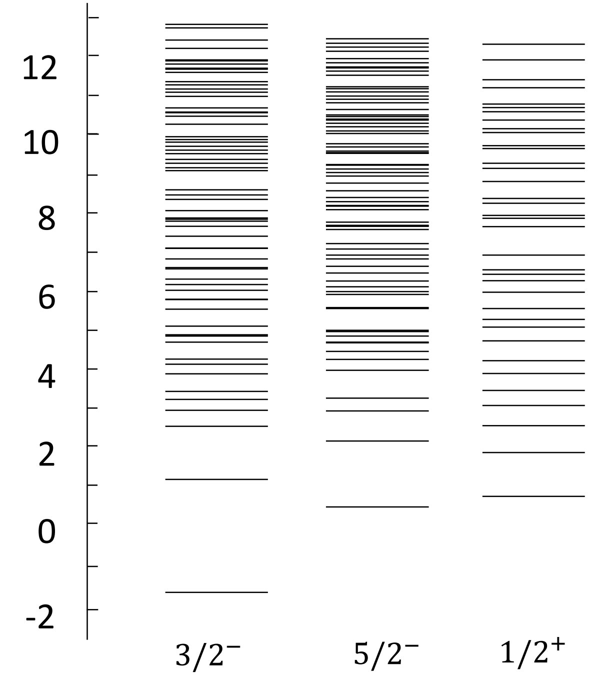
- Calculation (with the same conditions) of
  - Elastic scattering
  - Breakup
  - Fusion

- Known problematic issues
  - from ^{11}Be+^{64}Zn, T. Druet and P.D., EPJA 48 (2012) 147, Di Pietro et al., PRC 85, 054607 (2012)
  - Convergence (^{11}Be partial waves) slower at low energies
  - Improved when the binding energy is larger (^{11}Be: 0.5 MeV, ^{9}Be: 1.57 MeV)

4. $^{9}Be+^{208}Pb$ elastic scattering

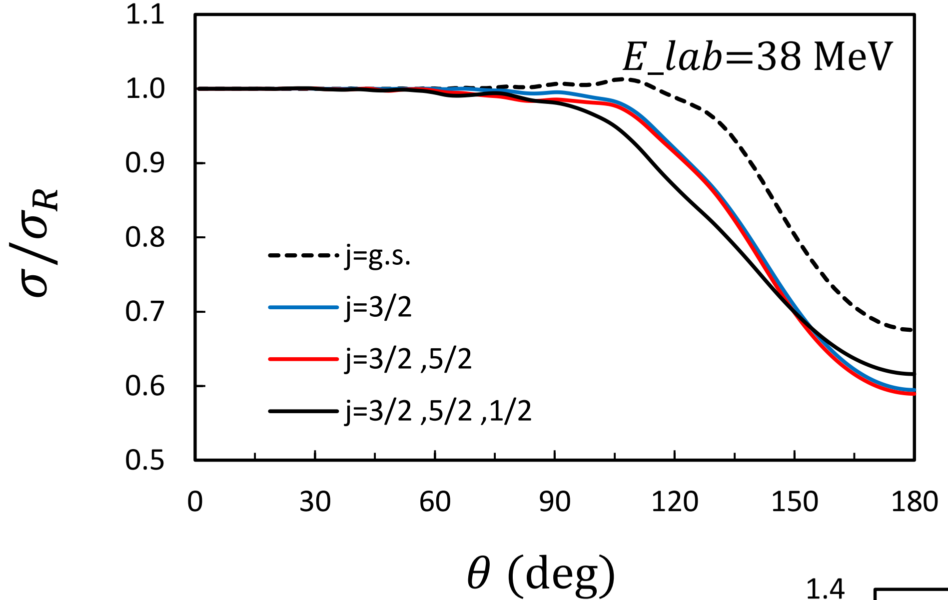
Convergence with E_{max} (cut off energy in ^{9}Be)

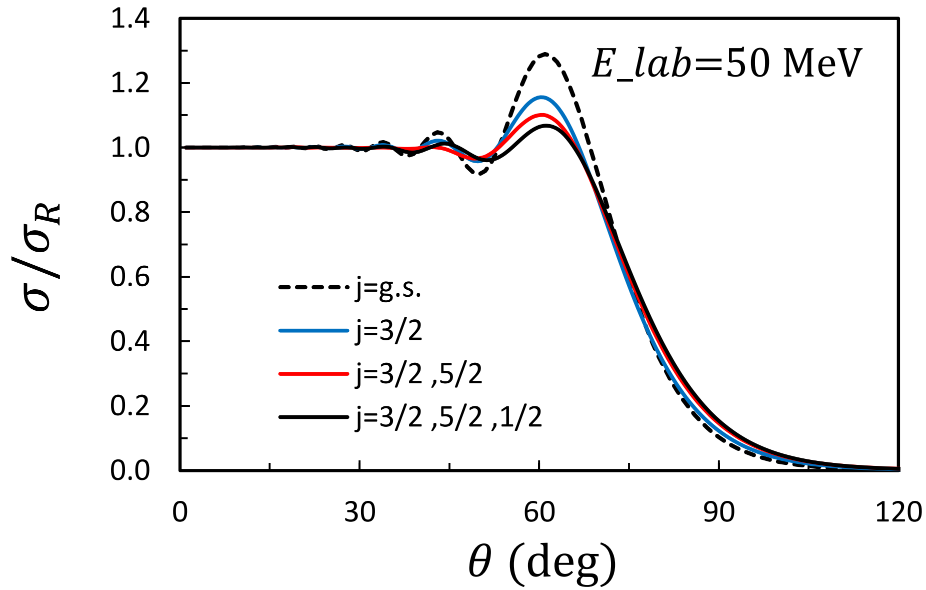




4. ^{9}Be+^{208}Pb elastic scattering

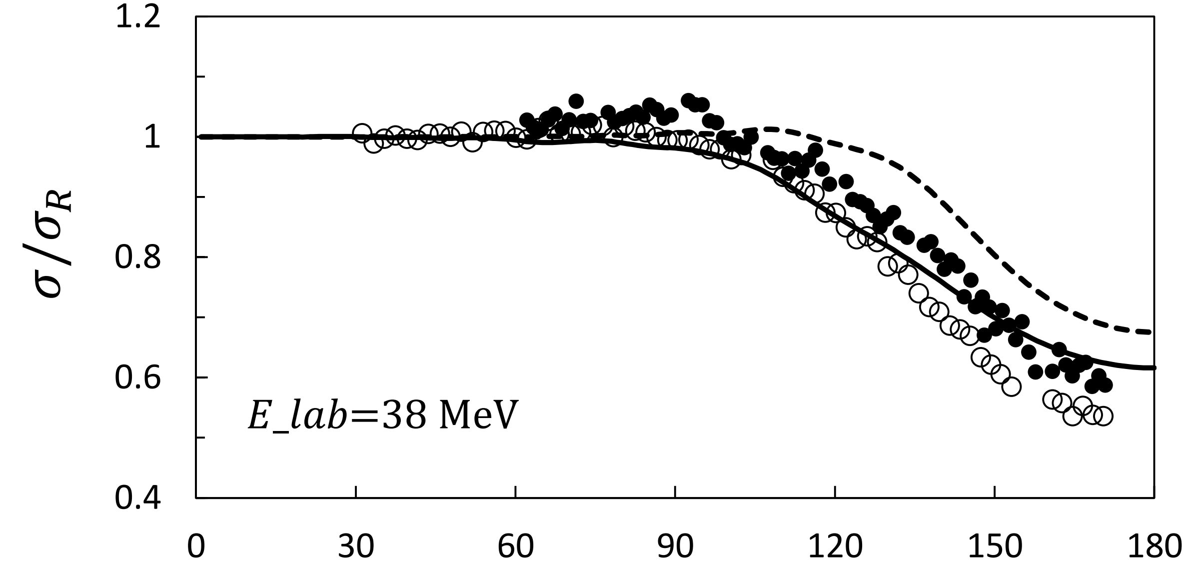
Convergence with *j* values (^{9}Be partial waves)

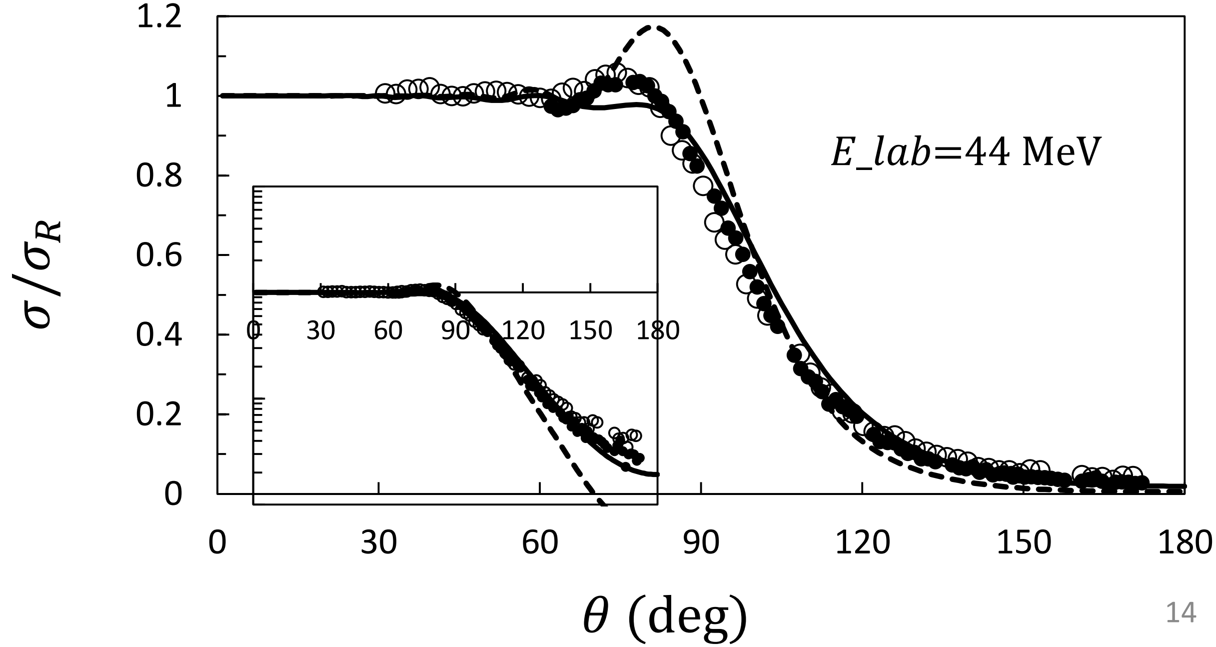




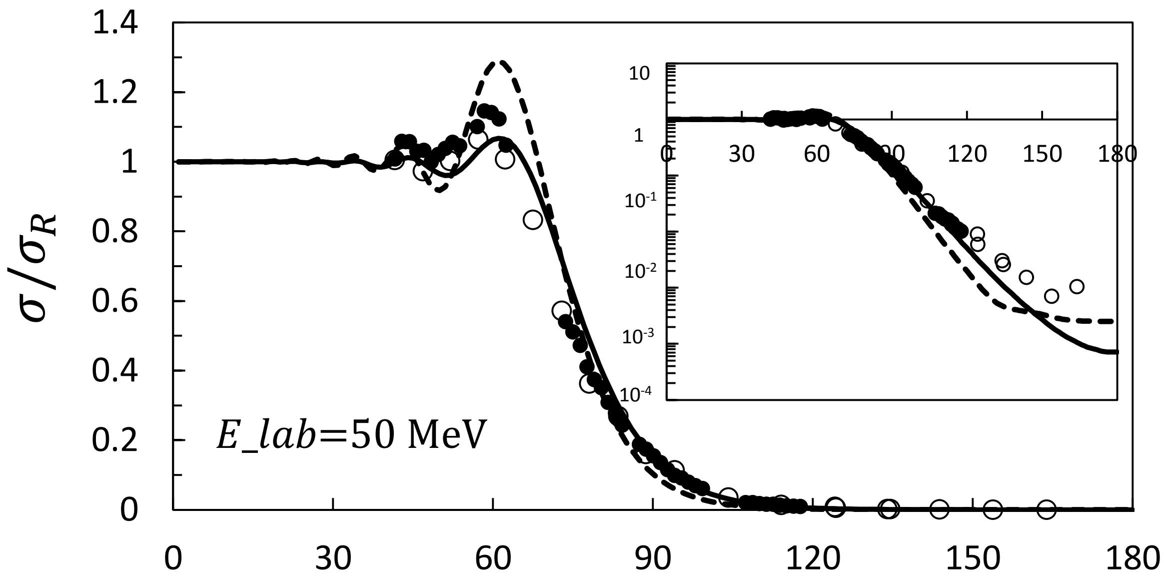
4. $^{9}Be+^{208}Pb$ elastic scattering

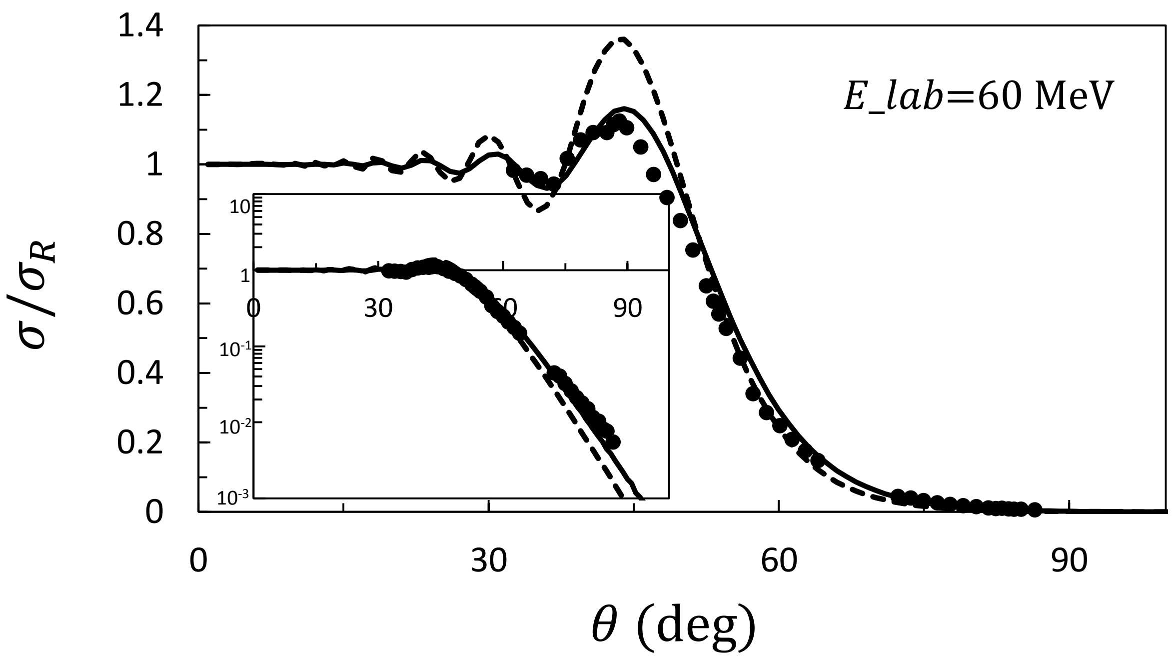
Comparison with experiment





4. $^{9}Be+^{208}Pb$ elastic scattering



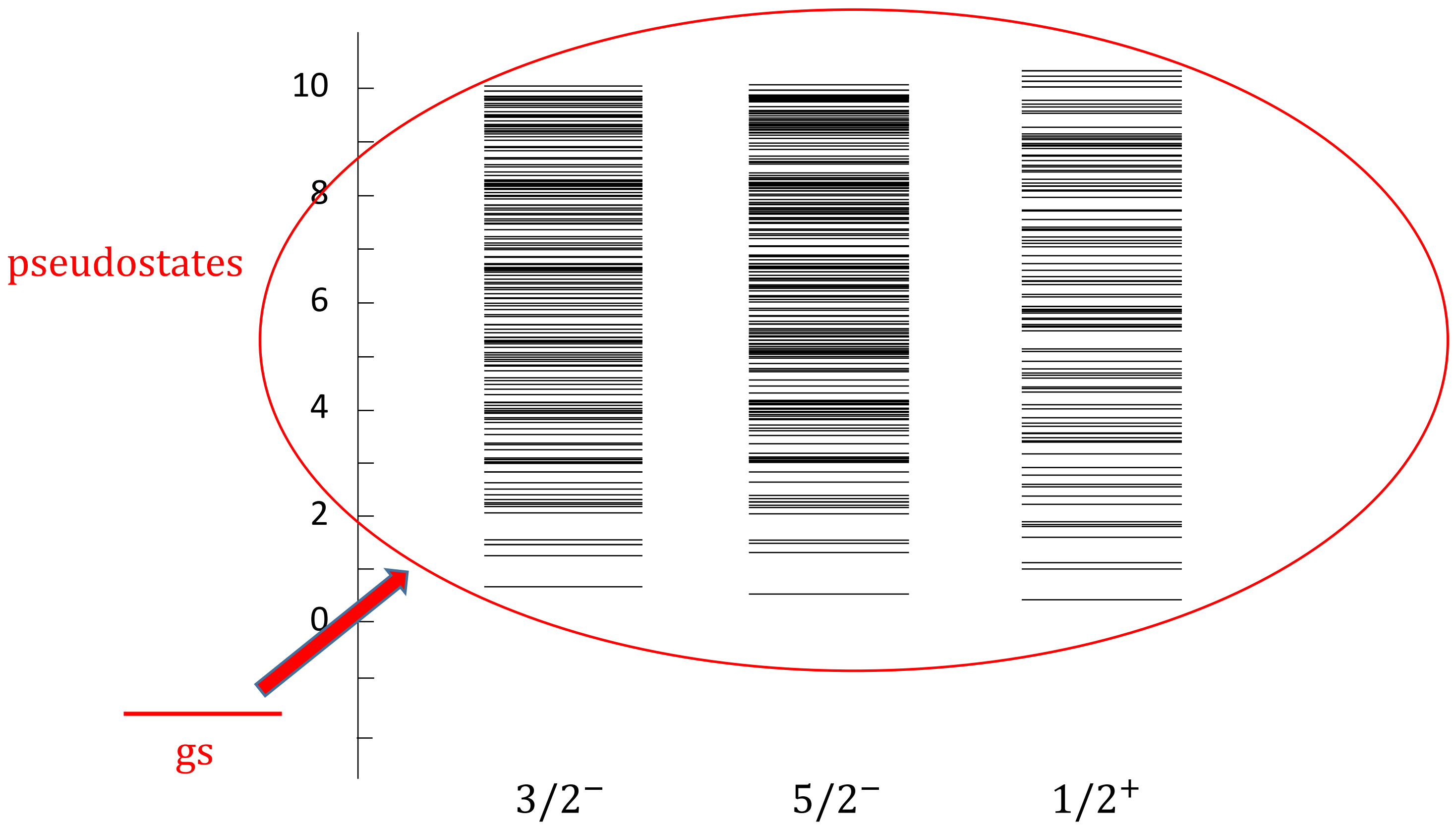


$^{9}Be+^{208}Pb$ breakup and fusion

5. $^{9}Be+^{208}Pb$ breakup and fusion

Breakup cross sections: simulated by transitions to pseudostates

→ Equivalent to « inelastic channels »



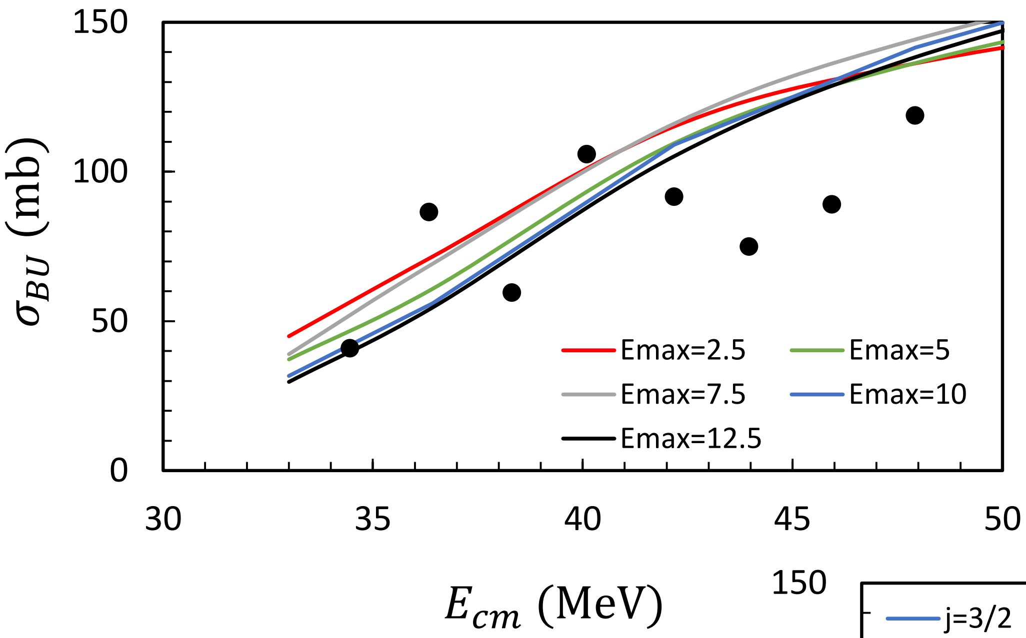
Breakup cross section: $\sigma_{BU}(E) = \frac{\pi}{4k^2}\sum_{J\pi\ell f\ell'}(2J+1)U^{J\pi}_{1\ell,f\ell'}(E)$

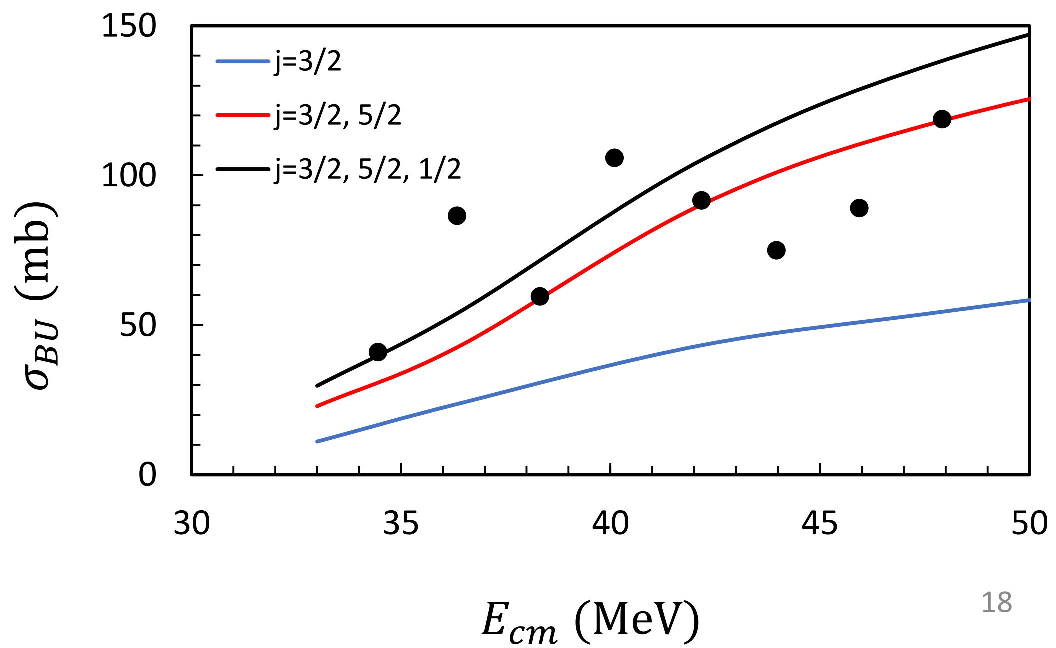
f=final states

$U^{J\pi}_{1\ell,f\ell'}(E)$=scattering matrix (non diagonal terms gs→ pseudo states f)

5. ^{9}Be+^{208}Pb breakup and fusion

Data: R. J. Woolliscroft et al., Phys. Rev. C 68, 014611 (2003).





5. ^{9}Be+^{208}Pb breakup and fusion

Fusion cross section: 2 processes

- **Complete fusion CF**: the full projectile is captured
- **Incomplete fusion IF**: the projectile breaks up, and one constituent is captured

Total fusion: sum of both processes: $\sigma_F = \sigma_{CF} + \sigma_{IF}$

Previous CDCC calculations: Many calculations with two-body projectiles

- A. Diaz-Torres and I. J. Thompson, PRC65 (2002) 024606: ^{11}Be+^{208}Pb
- V. Jha et al., PRC89 (2014) 034605 : ^{9}Be +heavy targets
- Many others

Present work:

- CDCC with three-body projectiles
- Total fusion only

5. $^9Be+^{208}Pb$ breakup and fusion

Calculation from scattering matrices

For spin and parity J^π: matrix $U^{J\pi}(E) = \begin{pmatrix} U_{11} & U_{12} & \dots & U_{1N} \\ U_{21} & U_{22} & \dots & U_{2N} \\ \vdots & \vdots & \vdots & \vdots \\ U_{N1} & U_{N2} & \dots & U_{NN} \end{pmatrix}$

Entrance channel: 1
Peudostates (breakup) channels: 2→N

- Elastic scattering cross section: $\sigma_{el}(E) \leftarrow U_{11}$
- Reaction cross section: $\sigma_R(E) \leftarrow 1 - |U_{11}|^2$
- Breakup cross section: $\sigma_{BU}(E) \leftarrow \sum_{i=2}^{N} |U_{1i}|^2$
- Total fusion cross section: $\sigma_F(E) = \sigma_R(E) - \sigma_{BU}(E)$

  Note: if real potential: $\sigma_F(E) = 0$ (from unitarity of the scattering matrix $\sum_{i=1}^{N} |U_{1i}|^2 = 1$)

5. $^{9}Be+^{208}Pb$ breakup and fusion

From the scattering matrices (sum over J^{π})

$$\sigma_F(E) = \sigma_R(E) - \sigma_{BU}(E)$$

Alternative definition of the fusion cross section

$$\sigma_F(E) \leftarrow \frac{4k}{E} \sum_{c,c\prime} \int u_c(r) W_{c,c'}(r) u_{c'}(r) dr$$

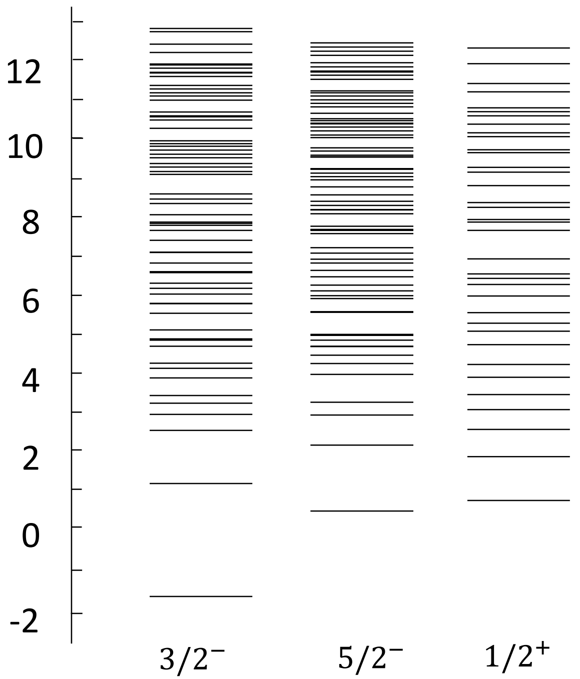
- $W_{c,c'}(r)$=imaginary potential (computed from $\alpha+^{208}Pb$ and $n+^{208}Pb$ potentials)
- $u_c(r)$=wave function in channel c

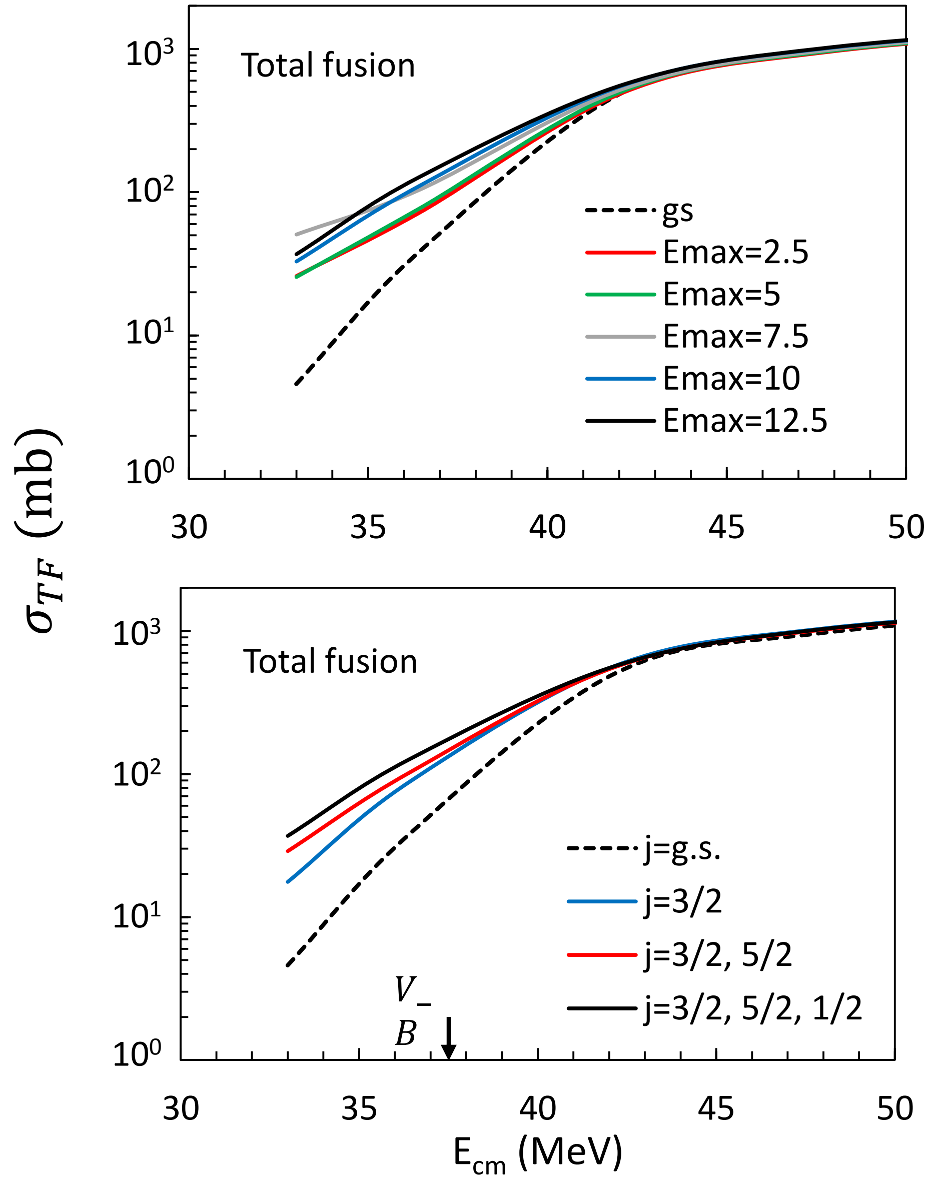
- $\sigma_F(E) = 0$ if $W_{c,c'}(r) = 0$ (as expected)

- Stricly equivalent (→ strong numerical test!)
  ☹ more difficult to use (requires the wave function $u_c(r)$)
  ☺ More accurate at low energies ($\sigma_R(E) \approx \sigma_{BU}(E)$)
  ☺ Allows a decomposition into the different channels

5. $^{9}Be+^{208}Pb$ breakup and fusion

Total fusion: convergence with E_{max} and j

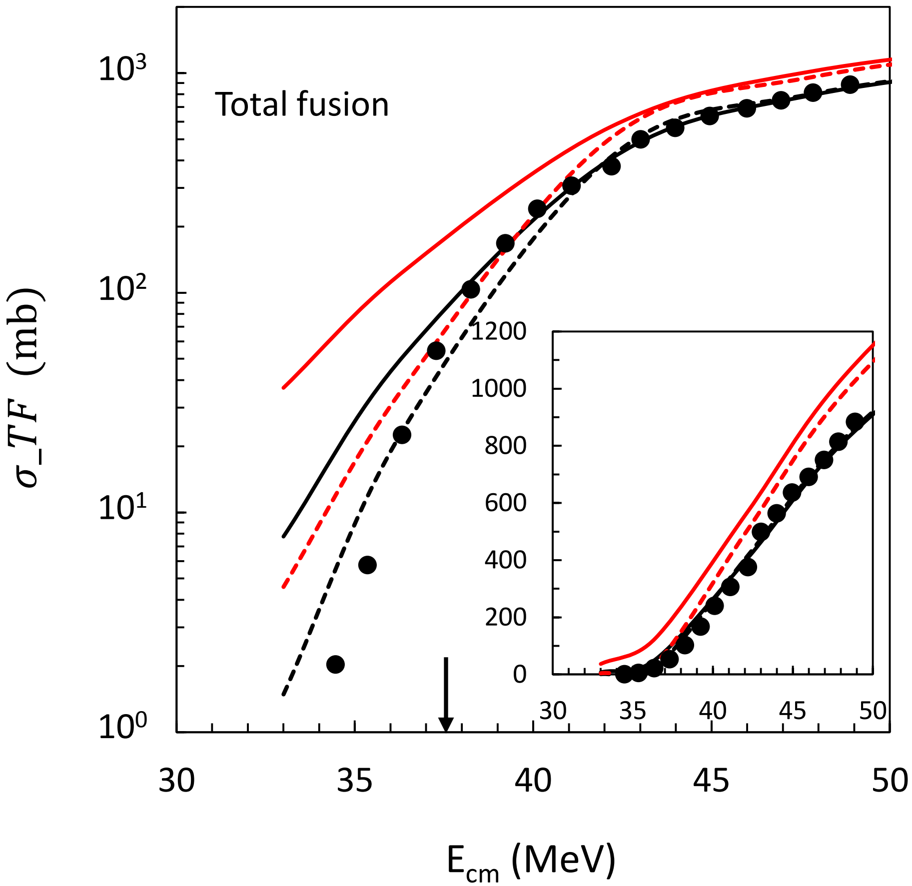




5. ^{9}Be+^{208}Pb breakup and fusion

Total fusion: comparison with experiment

Data from M. Dasgupta et al., PRC 70, 024606 (2004), PRL82 (1999) 1395



- Red lines: capture of α and n
- Black lines: capture of α only

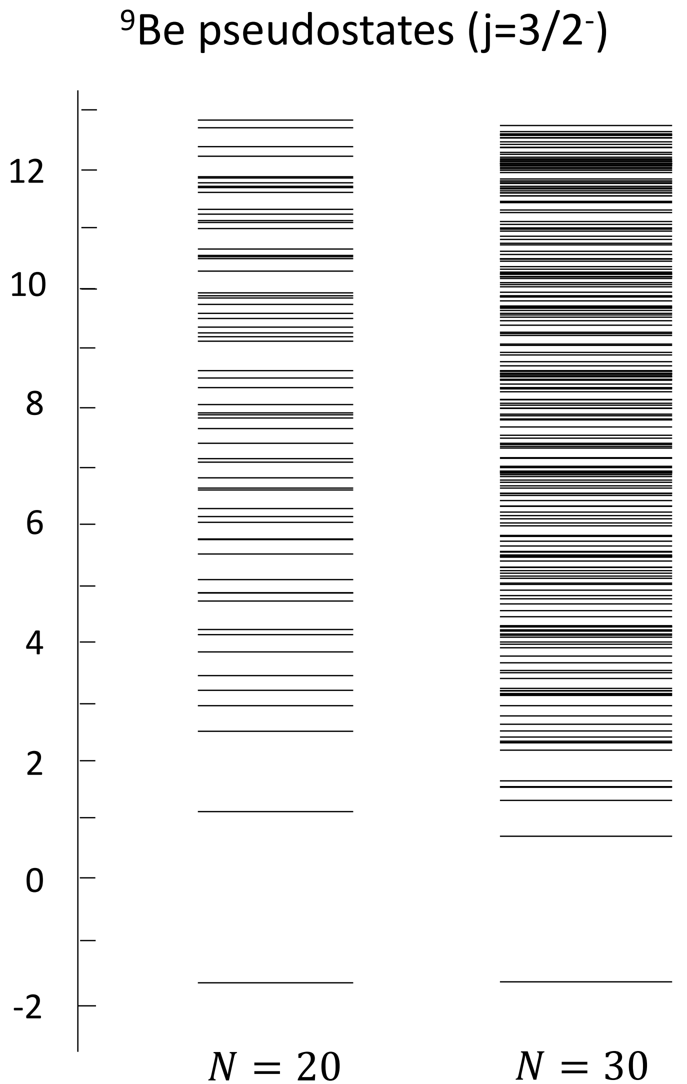
$$\sigma_F(E) \leftarrow \frac{4k}{E}\sum_{c,c\prime}\int u_c(r)W_{c,c\prime}(r)u_{c\prime}(r)dr$$

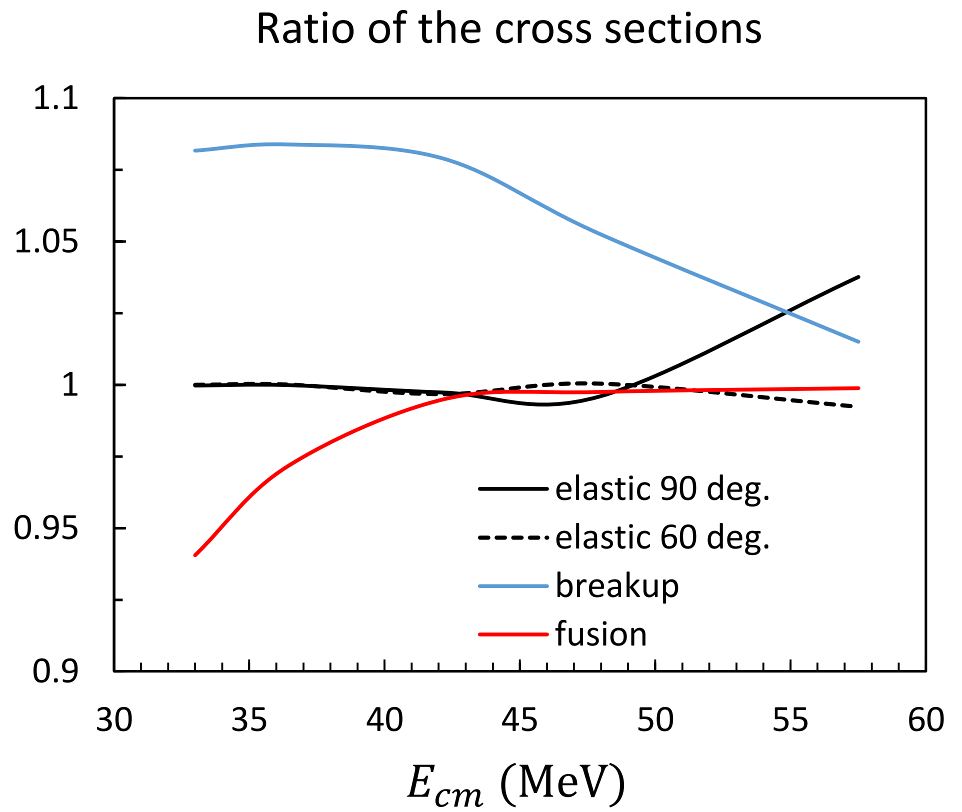
n-^{208}Pb imaginary potential set to 0

- Neglecting neutron capture is necessary
- Good agreement with the data except at the lowest energies

6. Numerical issues

Role of the ^{9}Be basis: calculation limited to j=3/2$^-$ with N=20 and N=30

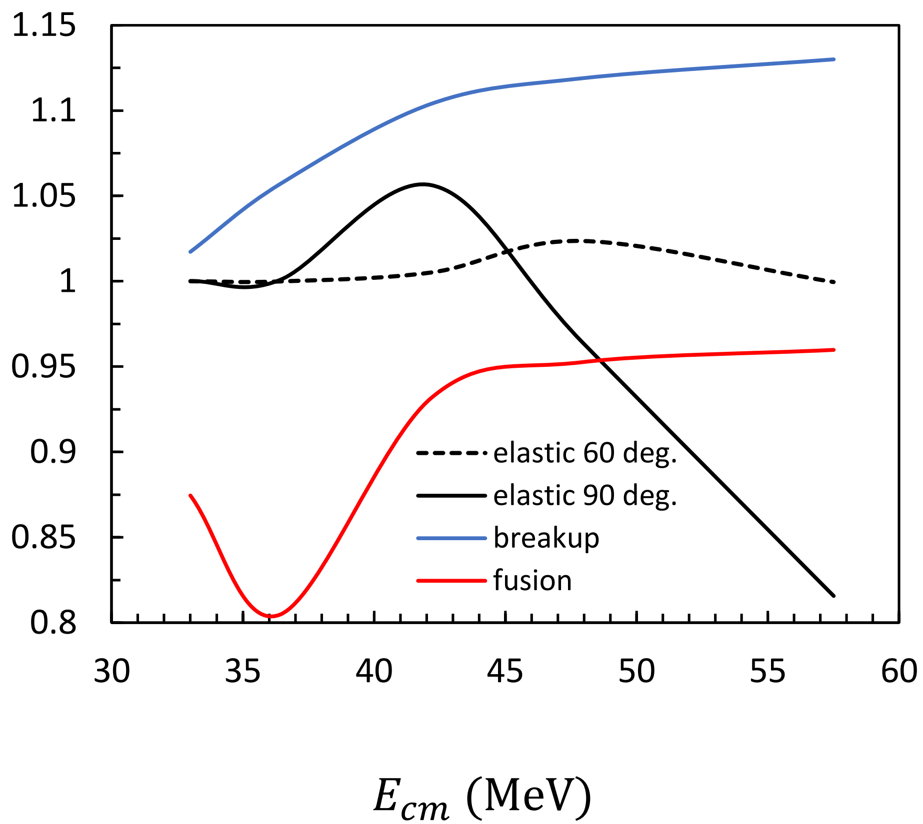




6. Numerical issues

Role of the α-^{208}Pb potential

1. G. Goldring et al., Phys. Lett. B **32**, 465 (1970).
2. A. R. Barnett and J. S. Lilley, Phys. Rev. C **9**, 2010 (1974).



Effect more important in breakup and fusion

6. Numerical issues

Main problem: large number of channels

→ high accuracy is required

→ Very long computer times

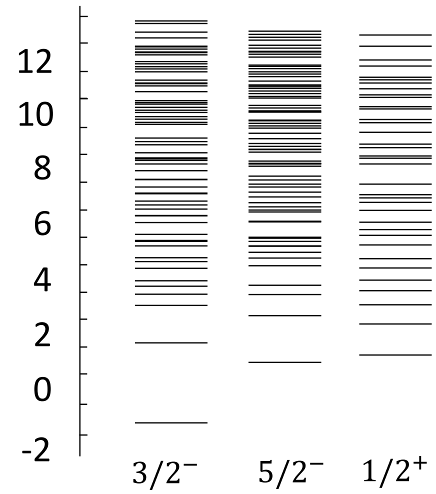
j=1/2+: 36, j=1/2-: 37,

j=3/2+: 59, j=3/2-: 63,

j=5/2+: 71, j=5/2-: 71

+ different L values, $|J-j| \leq L \leq J+j$

+ J_{max} large (~301/2)



→ Statistical CDCC??

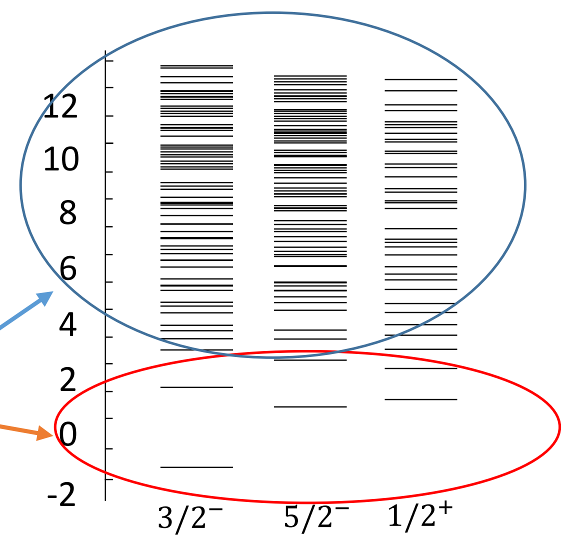
M.S. Hussein

C. Bertulani,

P.D.

B. Carlsson

- Explicit treatment of some « important » channels
- Statistical treatment of the other pseudostates



- C. A. Bertulani, P. D., M. S. Hussein
  EPJ Web of Conferences 69 (2014) 00020

Conclusion

7. Conclusion

- Spectroscopy of ^{9}Be
  - Described by a 3-body $\alpha+\alpha+n$ model
  - Hyperspherical coordinates
  - $\alpha+\alpha$ and $\alpha+n$ potentials reproduced the elastic phase shifts
  - A small 3-body force is necessary to adjust the energy of the ground state
  - Fair energy spectrum, spectroscopic properties

- ^{9}Be+^{208}Pb elastic scattering and breakup
  - $\alpha+^{208}$Pb and n+^{208}Pb taken from literature → no fit
  - Slow convergence with ^{9}Be states (known effect for 2-body nuclei)
  - Good agreement with experiment

- Fusion
  - **Total fusion:** fast convergence and good agreement with the data (except at very low energies: theory is larger than experiment)

  - To be done:
    - Separation between CF and IF?
    - Many pseudostates → Statistical CDCC??