Four-body effects on $^9$Be+$^{208}$Pb scattering around the Coulomb barrier

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1. Introduction

Many CDCC studies (Continuum Discretized Coupled Channel)
Essentially
• Three-body problems (two-body projectiles)
• Elastic scattering
• Fusion (few papers)

Problem for weakly bound projectiles:
• very slow convergence (partial wave inside the projectile) of the cross sections (essentially around the Coulomb barrier)
• $^{11}\text{Be}+^{64}\text{Zn}$: up to $L=5$ (in $^{11}\text{Be}$) is necessary around the Coulomb barrier
  Ref. T. Druet and P.D., EPJA 48 (2012) 147

Here: $^{9}\text{Be}+^{208}\text{Pb}$ system (many experimental data are available)
• $^{9}\text{Be}$ described by a 3-body structure $\alpha+\alpha+n$
• Simultaneous description of:
  • Elastic scattering
  • Breakup
  • Fusion
2. The CDCC method

Example: $^9\text{Be}=\alpha+\alpha+n$

- Pseudostates: $E_n>0$
- Simulate breakup effects
- Depend on the basis

• Ground state: $E_n<0$
• Independent of the basis
\(^{9}\text{Be} \) description (3-body \( \alpha + \alpha + n \))
### 3. $^9$Be $\alpha+\alpha+n$ description

**Description of $^9$Be=$\alpha+\alpha+n$**
- **$\alpha+\alpha$ potential**: Buck et al. NPA275 (1977) 246 (2 forbidden states for $L=0$, 1 fs for $L=2$)
- **$\alpha+n$ potential**: Kanada et al. Prog. Theor. Phys. 61 (1979) 1327 (1 forbidden state for $L=0$) → supersymmetry transform to remove the forbidden states

Both reproduce the elastic phase shifts up to ~20 MeV (deep potentials)

3-body method: Hyperspherical coordinates
Tests: different $K_{\text{max}}$, number of basis functions
3. $^9$Be $\alpha+\alpha+n$ description

**Hyperspherical formalism for 3-body nuclei**
- essentially spectroscopy, continuum possible (more difficult!)
- Hamiltonian: $H = T_1 + T_2 + T_3 + V_{12}(r_1 - r_2) + V_{13}(r_1 - r_3) + V_{23}(r_2 - r_3)$

**Absolute coordinates**
- $r_1, r_2, r_3$

**Jacobi coordinates** (c.m. removed)
- $x = r_2 - r_1$
- $y = r_3 - (A_1 r_1 + A_2 r_2)/A_{12}$

**Hyperspherical coordinates**
- $\Omega_x, \Omega_y$
- $\rho = \sqrt{x^2 + y^2}$
- $\tan \alpha = \frac{y}{x}$

$\rho =$ hyperradius
$\alpha =$ hyperangle

In hyperspherical coordinates: $H = T_\rho + V(\rho, \alpha, \Omega_x, \Omega_y)$

Eigenstates of $T_\rho$: **hyperspherical functions** $Y_{KSLxly}^J(\alpha, \Omega_x, \Omega_y) = Y_{KY}^J(\Omega_5)$

known functions (analytical)
extension of spherical harmonics $Y_l^m(\Omega)$ in 2-body problems
$K =$ hypermoment
3. $^9\text{Be} \alpha+\alpha+n$ description

Discretization of the three-body $\alpha+\alpha+n$ continuum

- $j=1/2, 3/2, 5/2$
- $K_{\text{max}}=20$
- Lagrange functions: $N=20$
- Maximum $E_{\text{max}}=12.5$ MeV

$\Rightarrow$ many pseudostates
Many channels in the CDCC calculation
Need to solve the coupled-channel equations

\[
\left[-\frac{\hbar^2}{2\mu}\left(\frac{d^2}{dR^2} - \frac{L(L + 1)}{R^2}\right) + E_c - E\right]u_c^{J\pi}(R) + \sum_{c'} V_{cc'}^{J\pi}(R)u_{c'}^{J\pi}(R) = 0
\]

At large distances

- Nuclear potential negligible, only Coulomb remains
- Wave function \(u_c^{J\pi}(R) \rightarrow \delta_{\omega c} I_c(k_c R) - U_{\omega c}^{J\pi} O_c(k_c R)\)
  with \(\omega = \text{entrance channel}\)

  \(I_c(x), O_c(x) = \text{incoming and outgoing Coulomb functions}\)

  \(U_{\omega c}^{J\pi} = \text{scattering matrix} \rightarrow \text{various cross sections (elastic, breakup, etc)}\)

Procedure

- Calculation of the 3-body wave functions
- Calculation of the coupling potentials \(V_{cc'}^{J\pi}(R)\)
- Solving the coupled-channel system (R matrix)
- Determining the scattering matrices \(U_{\omega c}^{J\pi}\) and cross sections
$^{9}\text{Be}+^{208}\text{Pb}$ elastic scattering
4. $^9\text{Be} + ^{208}\text{Pb}$ elastic scattering


4. $^9\text{Be}+^{208}\text{Pb}$ elastic scattering

- Experimental data ($E_{\text{lab}} \sim 38-60$ MeV, $E_{\text{cm}} \sim 36-58$ MeV, $V_{\text{coul}} \sim 38$ MeV)

- Various convergence tests
  - $E_{\text{max}}$, number of basis functions: specific to $^9\text{Be}$
  - Partial waves in $^9\text{Be} j^\pi$: $3/2^\pm, 5/2^\pm, 1/2^\pm$

- Calculation (with the same conditions) of
  - Elastic scattering
  - Breakup
  - Fusion

- Known problematic issues
  - from $^{11}\text{Be}+^{64}\text{Zn}$, T. Druet and P.D., EPJA 48 (2012) 147, Di Pietro et al., PRC 85, 054607 (2012)
  - Convergence ($^{11}\text{Be}$ partial waves) slower at low energies
  - Improved when the binding energy is larger ($^{11}\text{Be}$: 0.5 MeV, $^9\text{Be}$: 1.57 MeV)
4. $^9\text{Be} + ^{208}\text{Pb}$ elastic scattering

Convergence with $E_{\text{max}}$ (cut off energy in $^9\text{Be}$)

**$E_{\text{lab}} = 38\text{ MeV}$**

- $E_{\text{max}} = 2.5$
- $E_{\text{max}} = 5$
- $E_{\text{max}} = 7.5$
- $E_{\text{max}} = 10$
- $E_{\text{max}} = 12.5$

**$E_{\text{lab}} = 50\text{ MeV}$**

- $E_{\text{max}} = 2.5$
- $E_{\text{max}} = 5$
- $E_{\text{max}} = 7.5$
- $E_{\text{max}} = 10$
- $E_{\text{max}} = 12.5$
4. $^9$Be+$^{208}$Pb elastic scattering

Convergence with $j$ values ($^9$Be partial waves)

$E_{lab}=38$ MeV

$E_{lab}=50$ MeV
4. $^9$Be+$^{208}$Pb elastic scattering

Comparison with experiment

\[ E_{lab}=38 \text{ MeV} \]

\[ E_{lab}=44 \text{ MeV} \]
4. $^9$Be$^{208}$Pb elastic scattering

\( E_{lab}=50 \text{ MeV} \)

\( E_{lab}=60 \text{ MeV} \)
$^9$Be+$^{208}$Pb breakup and fusion
5. $^{9}\text{Be} + ^{208}\text{Pb}$ breakup and fusion

Breakup cross sections: simulated by transitions to pseudostates

→ Equivalent to « inelastic channels »

Breakup cross section: $\sigma_{BU}(E) = \frac{\pi}{4k^2} \sum_{J,\pi, f, f'} (2J + 1) U_{1, f, f'}^{J\pi}(E)$

$f =$final states

$U_{1, f, f'}^{J\pi}(E) =$scattering matrix (non diagonal terms $gs \rightarrow$ pseudo states $f$)
5. $^9$Be$^{208}$Pb breakup and fusion

5. $^9\text{Be} + ^{208}\text{Pb}$ breakup and fusion

**Fusion cross section**: 2 processes
- **Complete fusion CF**: the full projectile is captured
- **Incomplete fusion IF**: the projectile breaks up, and one constituent is captured

Total fusion: sum of both processes: $\sigma_F = \sigma_{CF} + \sigma_{IF}$

Previous CDCC calculations: Many calculations with two-body projectiles
- A. Diaz-Torres and I. J. Thompson, PRC65 (2002) 024606: $^{11}\text{Be} + ^{208}\text{Pb}$
- V. Jha et al., PRC89 (2014) 034605: $^9\text{Be} +$ heavy targets
- Many others

Present work:
- CDCC with three-body projectiles
- Total fusion only
Calculation from scattering matrices

For spin and parity $J^\pi$: matrix $U^{J^\pi}(E) = \begin{pmatrix} U_{11} & U_{12} & \ldots & U_{1N} \\ U_{21} & U_{22} & \ldots & U_{2N} \\ \vdots & \vdots & \ddots & \vdots \\ U_{N1} & U_{N2} & \ldots & U_{NN} \end{pmatrix}$

Entrance channel: 1
Peudostates (breakup) channels: 2 → N

- Elastic scattering cross section: $\sigma_{el}(E) \leftarrow U_{11}$
- Reaction cross section: $\sigma_R(E) \leftarrow 1 - |U_{11}|^2$
- Breakup cross section: $\sigma_{BU}(E) \leftarrow \sum_{i=2}^{N} |U_{1i}|^2$
- Total fusion cross section: $\sigma_F(E) = \sigma_R(E) - \sigma_{BU}(E)$

Note: if real potential: $\sigma_F(E) = 0$ (from unitarity of the scattering matrix $\sum_{i=1}^{N} |U_{1i}|^2 = 1$)
From the scattering matrices (sum over $J^\pi$)

$$\sigma_F(E) = \sigma_R(E) - \sigma_{BU}(E)$$

Alternative definition of the fusion cross section

$$\sigma_F(E) \leftarrow \frac{4k}{E} \sum_{c,c'} \int u_c(r)W_{c,c'}(r)u_{c'}(r)dr$$

- $W_{c,c'}(r)$=imaginary potential (computed from $\alpha^{208}$Pb and n$^{208}$Pb potentials)
- $u_c(r)$=wave function in channel $c$

- $\sigma_F(E) = 0$ if $W_{c,c'}(r) = 0$ (as expected)

- Strictly equivalent ($\rightarrow$ strong numerical test!)
  - 😞 more difficult to use (requires the wave function $u_c(r)$)
  - 😊 More accurate at low energies ($\sigma_R(E) \approx \sigma_{BU}(E)$)
  - 😊 Allows a decomposition into the different channels
5. $^9\text{Be}+^{208}\text{Pb}$ breakup and fusion

Total fusion: convergence with $E_{\text{max}}$ and $j$
5. \( ^9\text{Be}+^{208}\text{Pb} \) breakup and fusion

**Total fusion: comparison with experiment**

Data from M. Dasgupta et al., PRC 70, 024606 (2004), PRL82 (1999) 1395

- Red lines: capture of \( \alpha \) and n
- Black lines: capture of \( \alpha \) only

\[
\sigma_F(E) \leftarrow \frac{4k}{E} \sum_{c,c'} \int u_c(r) W_{c,c'}(r) u_{c'}(r) dr
\]

- n-\(^{208}\text{Pb}\) imaginary potential set to 0

• Neglecting neutron capture is necessary
• Good agreement with the data except at the lowest energies
6. Numerical issues

Role of the $^9$Be basis: calculation limited to $j=3/2^-$ with $N=20$ and $N=30$

$^9$Be pseudostates ($j=3/2^-$)

<table>
<thead>
<tr>
<th>$^9$Be pseudostates ($j=3/2^-$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$N = 20$</td>
</tr>
<tr>
<td>$N = 30$</td>
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Ratio of the cross sections

- elastic 90 deg.
- elastic 60 deg.
- breakup
- fusion

$E_{cm}$ (MeV)
6. Numerical issues

Role of the $\alpha^{-208}$Pb potential

![Graph showing the effect of $E_{cm}$ on breakup and fusion](image)

Effect more important in breakup and fusion
6. Numerical issues

Main problem: large number of channels
→ high accuracy is required
→ Very long computer times

\[
\begin{align*}
j=1/2+ & : 36, \quad j=1/2- : 37, \\
j=3/2+ & : 59, \quad j=3/2- : 63, \\
j=5/2+ & : 71, \quad j=5/2- : 71
\end{align*}
\]

+ different L values, \( |J - j| \leq L \leq J + j \)
+ \( J_{\text{max}} \) large (~301/2)

→ Statistical CDCC??
M.S. Hussein
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B. Carlsson

• Explicit treatment of some « important » channels
• Statistical treatment of the other pseudostates

• C. A. Bertulani, P. D., M. S. Hussein

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Conclusion
7. Conclusion

- **Spectroscopy of $^9$Be**
  - Described by a 3-body $\alpha+\alpha+n$ model
  - Hyperspherical coordinates
  - $\alpha+\alpha$ and $\alpha+n$ potentials reproduced the elastic phase shifts
  - A small 3-body force is necessary to adjust the energy of the ground state
  - Fair energy spectrum, spectroscopic properties

- **$^9$Be+$^{208}$Pb elastic scattering and breakup**
  - $\alpha+^{208}$Pb and $n+^{208}$Pb taken from literature $\rightarrow$ no fit
  - Slow convergence with $^9$Be states (known effect for 2-body nuclei)
  - Good agreement with experiment

- **Fusion**
  - **Total fusion**: fast convergence and good agreement with the data (except at very low energies: theory is larger than experiment)

- To be done:
  - Separation between CF and IF?
  - Many pseudostates $\rightarrow$ Statistical CDCC??