## Properties of Nuclear Matter & Finite Nuclei with Finite Range Simple Effective Interaction



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#### **Isoscalar part of mean field**

$$\mathbf{u}(k,\rho) = \lim_{Y_p \to 1/2} \frac{\mathbf{u}^n(k,\rho,Y_p) + u^p(k,\rho,Y_p)}{2}$$

#### **Isovector part of mean field**

$$u_{\tau}(k,\rho) = \lim_{\substack{Y_p \to 1/2 \\ k \to \text{momentum,} \quad \rho = \rho_n + \rho_p, \quad Y_p = \frac{\rho_p}{\rho}, \\ \delta = \frac{\rho_n - \rho_p}{1 + \rho_p} = 1 - 2Y_p.$$

$$u^{n(p)}(k,\rho,Y_{p}) = u(k,\rho) \pm (1-2Y_{p})u_{\tau}(k,\rho)^{2}$$

For the purpose,

$$v_d^l(r), v_d^{ul}(r), v_{ex}^l(r) \text{ and } v_{ex}^{ul}(r)$$

 $\rho_n \& \rho_p \text{ densities } \rightarrow \text{ respective Fermi-Dirac}$ distribution functions,  $\rho_{n,p} = \int f_T^{n,p}(\vec{k}) d^3k$ 

The energy density in ANM at temperature T,

$$H_{T}(\rho_{n},\rho_{p}) = \frac{\hbar^{2}}{2m} \int [f_{T}^{n}(\vec{k}) + f_{T}^{p}(\vec{k})] k^{2} d^{3}k + V_{T}(\rho_{n},\rho_{p})$$

$$V_T(\rho_n, \rho_p) = \frac{1}{2} (\rho_n^2 + \rho_p^2) \int v_d^l(r) \, d^3r + \rho_n \rho_p \int v_d^{ul}(r) \, d^3r$$

 $+\frac{1}{2}\iint [f_T^n(\vec{k})f_T^n(\vec{k'}) + f_T^p(\vec{k})f_T^p(\vec{k'})] g_{ex}^l(|\vec{k} - \vec{k'}|) d^3k d^3k'$ 

 $+\frac{1}{2}\iint [f_T^n(\vec{k})f_T^p(\vec{k'})+f_T^p(\vec{k})f_T^n(\vec{k'})] g_{ex}^{ul}(|\vec{k}-\vec{k'}|) d^3k d^3k'.$ 

$$\in_T^{n,p} (k,\rho,Y_p) = \frac{\partial H_T}{\partial [f_T^{n,p}]}$$

$$\begin{aligned} & \in_{T}^{i} (k, \rho, Y_{p}) = \frac{\hbar^{2} k^{2}}{2m} + u_{T}^{i}(k, \rho, Y_{p}), i = n, p \\ & u_{T}^{n}(k, \rho_{n}, \rho_{p}) = \left[ \rho_{n} \int v_{d}^{\ l}(\mathbf{r}) d^{3}\mathbf{r} + \rho_{p} \int v_{d}^{\ ul}(\mathbf{r}) d^{3}\mathbf{r} \right] \\ & + \left[ \int f_{T}^{\ n}(\vec{k}') g_{ex}^{\ l}(|\vec{k} - \vec{k}'|) d^{3}k' + \int f_{T}^{\ p}(\vec{k}') g_{ex}^{\ ul}(|\vec{k} - \vec{k}'|) d^{3}k' \right] \end{aligned}$$

+ rearrangement term

$$u_{T}^{p}(\mathbf{k},\rho_{n},\rho_{p}) = \left[\rho_{p}\int v_{d}^{l}(\mathbf{r})d^{3}\mathbf{r} + \rho_{n}\int v_{d}^{ul}(\mathbf{r})d^{3}\mathbf{r}\right] \\ + \left[\int f_{T}^{p}(\vec{k}')g_{ex}^{l}(|\vec{k}-\vec{k}'|)d^{3}k' + \int f_{T}^{n}(\vec{k}')g_{ex}^{ul}(|\vec{k}-\vec{k}'|)d^{3}k'\right]$$

+ rearrangement term

$$g_{ex}^{l,ul}(|\vec{k} - \vec{k'}|) = \int e^{i(\vec{k} - \vec{k'}) \cdot \vec{r}} v_{ex}^{l,ul}(r) d^3r$$

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In SNM at 
$$T = 0$$
,  $\rho_n = \rho_p = \rho/2$ ,  $k_n = k_p = k_f = \left(\frac{3\pi^2 \rho}{2}\right)^{1/3}$   
 $3\hbar^2 k^2 = \rho^2 \left(y_1^{-l} + y_2^{-ul}\right) = \rho^2 \left(2i(k_r)\right)^2 \left(y_1^{-l} + y_2^{-ul}\right)$ 

$$H(\rho) = \frac{3n \kappa_f}{10m} \rho + \frac{\rho}{2} \int \left(\frac{v_d + v_d}{2}\right) d^3r + \frac{\rho}{2} \int \left(\frac{3J_1(\kappa_n r)}{(\kappa_n r)}\right) \left(\frac{v_{ex} + v_{ex}}{2}\right) d^3r$$

$$\in (k,\rho) = \frac{\hbar^2 k^2}{2m} + u(k,\rho)$$

$$u(k,\rho) = \rho \int \left( \frac{v_d^{\ l} + v_d^{\ ul}}{2} \right) d^3r + \left[ \rho \int j_0(kr) \frac{3j_1(k_f r)}{(k_f r)} \left( \frac{v_{ex}^{\ l} + v_{ex}^{\ ul}}{2} \right) d^3r \right]$$

$$+ \left[ \rho \int \partial \left( v_d^{\ l} + v_d^{\ ul} \right)_{d^3r} + \rho \int 9j_1^2(k_f r) \partial \left( v_{ex}^{\ l} + v_{ex}^{\ ul} \right)_{d^3r} \right]$$

$$+\left[\frac{\rho}{2}\int\frac{\partial}{\partial\rho}\left(\frac{\mathbf{v}_{d}^{l}+\mathbf{v}_{d}^{ul}}{2}\right)d^{3}\mathbf{r}+\frac{\rho}{2}\int\frac{9j_{1}^{2}(k_{f}r)}{(k_{f}r)^{2}}\frac{\partial}{\partial\rho}\left(\frac{\mathbf{v}_{ex}^{l}+\mathbf{v}_{ex}^{ul}}{2}\right)d^{3}\mathbf{r}\right]$$

$$u(k,\rho) = u(k_{f},\rho) + [u(k,\rho) - u(k_{f},\rho)]$$
  
=  $e(\rho) + \rho \frac{d e(\rho)}{d \rho} - \frac{\hbar^{2} k_{f}^{2}}{2m} + u^{ex}(k,\rho)$ 

$$u^{ex}(k,\rho) = \frac{\rho}{2} \int \left[ j_0(kr) - j_0(k_f r) \right] \frac{3j_1(k_f r)}{(k_f r)} \left[ v_{ex}^l(r) + v_{ex}^{ul}(r) \right] d^3r$$

$$u_{\tau}(k,\rho) = 2E_{S}(\rho) - \frac{\hbar^{2}k_{f}^{2}}{3m^{*}(k=k_{f},\rho)} + u_{\tau}^{ex}(k,\rho)$$

$$u_{\tau}^{ex}(k,\rho) = \frac{\rho}{2} \int [j_0(kr) - j_0(k_f r)] [v_{ex}^l(r) - v_{ex}^{ul}(r)] j_0(k_f r) d^3r$$

$$\begin{aligned} V_{eff}(r) &= t_0 (1 + x_0 P_{\sigma}) \delta(r) + \frac{t_3}{6} (1 + x_3 P_{\sigma}) \left(\frac{\rho}{1 + b\rho}\right)^{\gamma} \delta(r) \\ &+ (W + BP_{\sigma} - HP_{\tau} - MP_{\sigma} P_{\tau}) f(r) \\ f(r) &= \frac{e^{-r/\alpha}}{r/\alpha}, Yukawa \\ e^{-r^{2}/\alpha^{2}}, Gaussian \\ e^{-r/\alpha}, \exp onential \end{aligned}$$

$$\begin{aligned} & SEI has total 11 - numbers of Interaction parameters \\ & b, \gamma, t_0, x_0, t_3, x_3, W, B, H, M and \alpha \end{aligned}$$

$$b &\geq \frac{1}{\rho_0} \left[ \left(\frac{mc^2}{\frac{T_{f_0}}{5} - e(\rho_0)}\right)^{\frac{1}{\gamma+1}} \right]^{-1} \\ & T_{f_0} &= \frac{\hbar^2 k_{f_0}^2}{2m}, e(\rho_0) = [H(\rho)/\rho]_{\rho=\rho_0}, k_{f_0} = \left(\frac{3\pi^2 \rho_0}{2}\right)^{1/3}, \end{aligned}$$

$$\rho \int j_0(kr) \frac{3j_1(k_f r)}{(k_f r)} \left( \frac{\mathbf{v}_{ex}^{-l} + \mathbf{v}_{ex}^{-ul}}{2} \right) d^3\mathbf{r} = \varepsilon_{ex} \frac{\rho}{\rho_0} I(k,\rho)$$

$$I_Y(k,\rho) = \frac{3\Lambda^2(\Lambda^2 + k_f^2 - k^2)}{8kk_f^3} \ln \left[ \frac{\Lambda^2 + (k+k_f)^2}{\Lambda^2 + (k-k_f)^2} \right]$$

$$+ \frac{3\Lambda^2}{2k_f^2} - \frac{3\Lambda^2}{2k_f^2} \left[ \tan^{-1} \left( \frac{k+k_f}{\Lambda} \right)^2 - \tan^{-1} \left( \frac{k-k_f}{\Lambda} \right)^2 \right] (Yukawa)$$

$$I_G(k,\rho) = \frac{3\Lambda^4}{8kk_f^3} \left[ e^{-\left( \frac{k+k_f}{\Lambda} \right)^2} - e^{-\left( \frac{k+k_f}{\Lambda} \right)^2} \right] + \frac{3\Lambda^3}{4k_f^3} \int_{(k-k_f)/\Lambda}^{(k+k_f)/\Lambda} e^{-t^2} dt$$
(Gaussian)
$$\varepsilon_{ex} = \frac{\varepsilon_{ex}^l + \varepsilon_{ex}^{ul}}{2} \quad and \quad \Lambda \quad range, \quad \alpha = 1/\Lambda (Yukawa),$$

 $\alpha = 2 / \Lambda (Gaussian)$ 

two parameters in the momentum dependent part of mean field



$$= e(\rho_0) - T_{f_0} + \varepsilon_{ex} S(\lambda), \qquad \Lambda = \lambda k_{f_0}$$
$$S(\lambda) = [I(k_{300}, \rho_0) - I(k_{f_0}, \rho_0)] = \frac{T_{f_0} - e(\rho_0)}{\varepsilon_{ex}},$$

 $\varepsilon_{\rm ex} = \frac{\varepsilon_{ex}^l + \varepsilon_{ex}^{ul}}{2}$ 

and  $\Lambda$  - determined simultaneously using an optimisation procedure

$$e(\rho_0) = -16 MeV, \quad T_{f_0} = 37 MeV (\rho_0 = 0.161 \, fm^{-3})$$



$$u^{ex}(k,\rho) = \frac{\rho}{2} \int \left[ j_0(kr) - j_0(k_f r) \right] \frac{3j_1(k_f r)}{(k_f r)} \left[ v_{ex}^l(r) + v_{ex}^{ul}(r) \right] d^3r$$

150

100

50





SEI UV14+UVII (a)

0.3

*R.Wiringa PRC* 38 2967(1988)



$$b, \gamma, \varepsilon_0^l, \varepsilon_0^{ul}, \varepsilon_{\gamma}^l, \varepsilon_{\gamma}^{ul}, \varepsilon_{ex}^l, \varepsilon_{ex}^{ul}$$
 and  $\alpha - 09$  parameters  
of ANM

$$\begin{split} \varepsilon_{0}^{l} &= \rho_{0} \bigg[ \frac{t_{0}}{2} (1 - x_{0}) + (W + B/2 - H - M/2) \int f(r) d^{3}r \bigg] \\ \varepsilon_{0}^{ul} &= \rho_{0} \bigg[ \frac{t_{0}}{2} (2 + x_{0}) + (W + B/2) \int f(r) d^{3}r \bigg] \\ \varepsilon_{1}^{1} &= \frac{t_{3}}{12} \rho_{0}^{\gamma + 1} (1 - x_{3}) \\ \varepsilon_{1}^{1} &= \frac{t_{3}}{12} \rho_{0}^{\gamma + 1} (2 + x_{3}) \\ \varepsilon_{ex}^{l} &= \rho_{0} (M - W/2 + H/2 - B) \int f(r) d^{3}r \\ \varepsilon_{ex}^{ul} &= \rho_{0} (M + H/2) \int f(r) d^{3}r \qquad \int f(r) d^{3}r = 4\pi \alpha^{3} (Yukawa) \\ &= \pi^{3/2} \alpha^{3} (Gaussian) \end{split}$$



Danielewicz P et al., Science 298 (2002) 1592

$$(\varepsilon_0^l + \varepsilon_0^{ul}) \qquad (\varepsilon_\gamma^l + \varepsilon_\gamma^{ul}) \qquad (\varepsilon_{ex}^l + \varepsilon_{ex}^{ul})$$

into two specific channels for interactions between like (I) and unlike (u) nucleons.

 $(\varepsilon_{ex}^{l} + \varepsilon_{ex}^{ul}) = 2\varepsilon_{ex}$ , The splitting can be arbitrary subject to this condition

The sign of  $(\epsilon_{ex}^{\ \ l} - \epsilon_{ex}^{\ \ ul}) \Rightarrow$  splitting of **n** and **p** effective masses.

$$u_{\tau}(k,\rho) = 2E_{s}(\rho) - \frac{\hbar^{2}k_{f}^{2}}{3m} \left\{ \left( \frac{m^{*}(k,\rho)}{m} \right) \right\}_{k=k_{f}}^{-1} + u_{\tau}^{ex}(k,\rho)$$

$$u_{\tau}^{ex}(k,\rho) = \frac{(\varepsilon_{ex}^{l} - \varepsilon_{ex}^{ul})}{\rho_{0} \int f(r) d^{3}r} \int [j_{0}(kr) - j_{0}(k_{f}r)] j_{0}(k_{f}r) f(r) d^{3}r$$

$$\left[\frac{\mathbf{m}^*}{\mathbf{m}}(\mathbf{k},\rho,Y_p)\right]_{n,p} = \left[1 + \frac{\mathbf{m}}{\hbar^2 \mathbf{k}} \frac{\partial \mathbf{u}^{n,p}(\mathbf{k},\rho,Y_p)}{\partial \mathbf{k}}\right]^{-1}$$

$$\varepsilon_{ex} < \varepsilon_{ex}^{l} \le 0 \ (m_{n}^{*} > m_{p}^{*})$$

 $2\varepsilon_{ex} \leq \varepsilon_{ex}^l < \varepsilon_{ex} (m_n^* > m_n^*)$ 



#### **Entropy Density**

$$S^{0,n}(\rho,T) = -\frac{\xi}{(2\pi)^3} \int \left[ n_T^{0,n}(k) \ln n_T^{0,n}(k) + \left(1 - n_T^{0,n}(k)\right) \ln \left(1 - n_T^{0,n}(k)\right) \right] d^3k,$$







beta – stable  $n + p + e + \mu$  matter (NSM)  $\mu_n(\rho, Y_p) - \mu_p(\rho, Y_p) = \mu_e(\rho, Y_e) = \mu_\mu(\rho, Y_\mu)$ 

$$Y_p(\rho) = Y_e(\rho) + Y_\mu(\rho)$$



 $H^{NSM}(\rho, Y_{p}) = H^{N}(\rho, Y_{p}) + H^{e}(\rho, Y_{e}) + H^{\mu}(\rho, Y_{\mu})$  $P^{NSM}(\rho, Y_{p}) = P^{N}(\rho, Y_{p}) + P^{e}(\rho, Y_{e}) + P^{\mu}(\rho, Y_{\mu})$ 

$$S^{NSM}(\rho, Y_p) = \begin{bmatrix} H^N(\rho, Y_p) - H(\rho, Y_p = 1/2) \\ = \begin{bmatrix} (1 - 2Y_p(\rho))^2 \rho E_s(\rho) \end{bmatrix}_{NSM} \end{bmatrix}$$



Behera et al., JPG 36 (2009) 125105

Same density dependence of the EOS in ANM for widely varying momentum dependence of the mean field and vice versa; Therefore, the simple effective interaction can be used in the transport model analysis of the flow data.

- 09-parameters of ANM  $\longrightarrow$  three standard values  $e(\rho_0)$ ,  $\rho_0$  and  $E_s(\rho_0)$  for a given  $\gamma$ .
- Out of 11-interaction parameters, two parameters open. We have taken

 $t_0$  and  $x_0$ 

as the open parameters.

We further constrain one more parameter, namely, x<sub>0</sub>

from spin polarization property in NM.

Spin polarized Neutron Matter

$$\varepsilon_{ex}^{l} = \frac{(\varepsilon_{ex}^{l,l} + \varepsilon_{ex}^{l,ul})}{2}, \quad \varepsilon_{\gamma}^{l} = \frac{(\varepsilon_{\gamma}^{l,l} + \varepsilon_{\gamma}^{l,ul})}{2}, \quad \varepsilon_{0}^{l} = \frac{(\varepsilon_{0}^{l,l} + \varepsilon_{0}^{l,ul})}{2}$$



$$\varepsilon_{ex}^{l,l} = -\rho_0 V_0^{TO} \int f(r) d^3r$$

$$x_0 = 1 - \frac{2}{t_0 \rho_0} \left( \varepsilon_0^l - \varepsilon_{ex}^l + \frac{3}{2} \varepsilon_{ex}^{l,l} \right)$$

Only one parameter  $t_0$  kept open which shall be determined from finite nucleus.

#### **FINITE NUCLEI**

#### The many-body Hamiltonian with an effective interaction



$$\mathcal{H}_{0} = \frac{\hbar^{2}}{2m} (\tau_{n} + \tau_{p}) + \mathcal{H}_{d}^{Nucl} + \mathcal{H}_{exch}^{Nucl} + \mathcal{H}^{SO}(\mathbf{r}) + \mathcal{H}_{d}^{Coul}(\mathbf{r}) + \mathcal{H}_{exch}^{Coul}(\mathbf{r}).$$

The total nuclear direct and exchange energy density coming from SEI. The non-local exchange part is made local by using semiclassical ħ-expansion of density matrix upto 2<sup>nd</sup> order. (X.Viñas et. al. PRC 67 (2003) 014324)

The Spin-orbit interaction taken similar to Skyrme and Gogny interaction.

 $t_0 \longrightarrow {}^{40}Ca$   $W_0$ (strength parameter of Spin-orbit interaction)  $\longrightarrow {}^{208}Pb$ BEs of 161 - and Radii of 86- spherical nuclei: A=16 to A=224









Behera et. al., J.Phys G (2013) 095105; (2015) 045103

HFB results with SEI for 579 even-even nuclei, deformed and spherical, over the nuclear chart

$$\frac{\gamma = 1/3}{t_0 = 201, W_0 = 115} \quad \Delta E_{rms} = 1.873 (\text{HFB})$$

$$t_0 = 214, W_0 = 115 \quad \Delta E_{rms} = 1.788 (\text{EROT})$$

$$\frac{\gamma = 1/2}{t_0 = 438, W_0 = 112} \quad \Delta E_{rms} = 1.958 (\text{HFB})$$

$$t_0 = 450, W_0 = 115 \quad \Delta E_{rms} = 1.843 (\text{EROT})$$



### **CONCLUSION:**

- By adjusting the parameters of SEI from considerations of momentum dependence of the mean fields in NM, the interaction could predict the finite nuclei properties similar in quality as that of any other conventional interaction.
- The procedure of parameter fixation provide the flexibility of systematically varying the momentum dependent properties, where the EOSs remains the same and vice-versa.
- The preliminary results of HFB with SEI shows that it is also competent to account for the deformation properties in finite nuclei similar in quality as any other successful effective interaction.

• Collaborators

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# THANK YOU