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High-spin Molecular Resonances in $^{12}C+^{12}C$

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- Introduction: Experiments and theoretical works on $^{12}C+^{12}C$ system
- Basic dynamical properties
- Molecular Normal Modes Analyses
- Coriolis coupling and resulting aligned state and new disaligned states
- Comparison with the Experimental Excitation functions
- Concluding remarks

INTRODUCTION on Exp.

$^{12}C+^{12}C$ shows many resonance levels
Spin assignments have been made

(1) below and near Coulomb barrier: spins J=0-8
W. Galster et al., PRC15(1977)950 : $^{12}C(^{12}C,\alpha)^{20}Ne$
H. Voit et al., Phys Lett 67B(1977)399 : $^{12}C(^{12}C,\alpha)^{20}Ne$

(2) above Coulomb barrier: spins J=8-12
E.R. Cosman et al., PRL35(1975)265: $^{12}C(^{12}C,p)^{23}Na$
N. Fletcher et al., PRC(1976)1173: $^{12}C(^{12}C,^{8}Be)^{16}O$

(3) well above the Coulomb barrier: High spins J=10, 12
L. Greenwood et al., PRC12(1975)156: $^{12}C(^{12}C,\alpha)^{20}Ne$
H. Fortune, et al., PRC14(1976)1271: $^{12}C(^{12}C,\alpha)^{20}Ne$

$^{12}C+^{12}C$ High spin resonances Excitation functions

T.M. Cormier et al.

PRL38(1977)940,40(1978)924

and

A spin is revised such as J=12 -> 14

R.J. Ledoux et al. PRC27(1983)1103, PRC30(1984)866

Results of phase shift analyses

R.J. Ledoux et al.

Ecm	J	Γ_{el}	Γ_{inel}	Γ_{tot}
18.4	12+	52	144	400
19.3	12+	80	125	400
20.4	14+	42	52	300

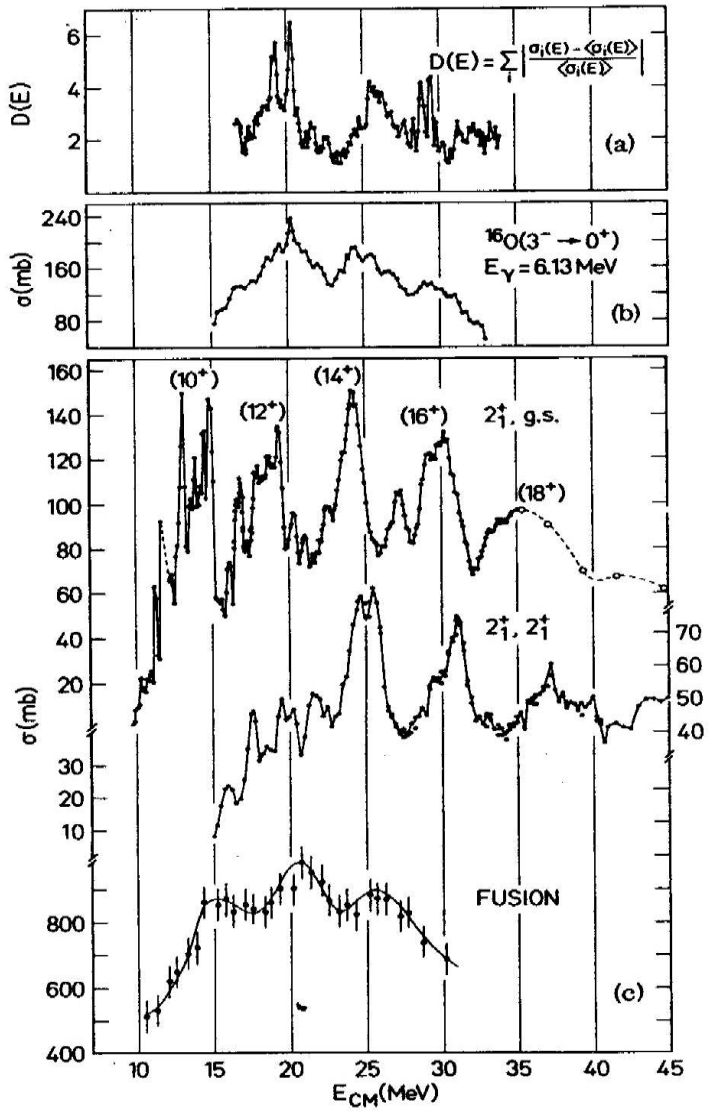


FIG. 1. (a) Deviation function, $D(E)$, calculated from sixteen elastic excitation functions using an averaging interval of 1.5 MeV. (b) Excitation function for the $^{16}O(3^- \rightarrow 0^+)$ γ ray (Ref. 11). (c) Total-cross-section excitation functions for single inelastic (2_1^+ + g.s.) scattering (present work), mutual inelastic ($2_1^+ + 2_1^+$) scattering (Ref. 4), and fusion (Ref. 2).

INTRODUCTION on theoretical works

Double resonance mechanism: near the Coulomb barrier

Nogami-Imanishi Model: B. Imanishi, Nucl. Phys. A125(1969)33

H. Fink, W. Scheid and W. Greiner, Nucl. Phys. A188(1972)259

Y. Kondo, T. Matsuse and Y. Abe, PTP 59(1978)465

Band Crossing Model (BCM): Well above the Coulomb barrier for High spins J~12

Y. Abe, Y. Kondo and T. Matsuse, PTP Suppl.68(1980)303

E(ΔL=2)～E(12C(2+)) causes Crossing of the elastic band and the inelastic bands.

Weak coupling picture reproduces small level splitting to obtain many levels.

We have developed

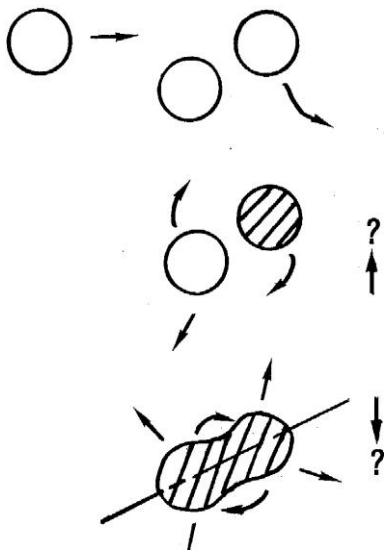
“Di-nuclear molecular model for Strong coupling regime”

applied to high spin resonances of the $^{24}Mg+^{24}Mg$ and $^{28}Si+^{28}Si$ systems with Geometrical Configurations.

We now apply the model to the high spin resonances of the $^{12}C+^{12}C$ system with respect to Coriolis Coupling.

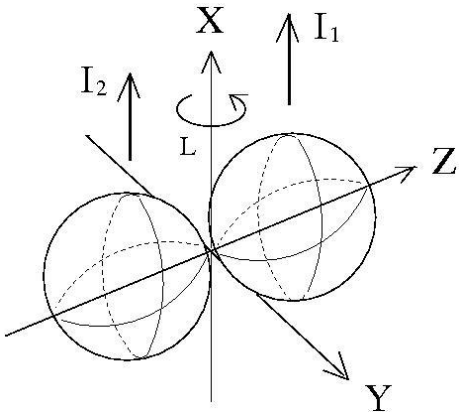
Consider $^{12}C+^{12}C$ aligned configuration by double resonance mechanism:

what connects the molecular model with coupled channel one?



Weak coupling regime

Double resonance mechanism



Aligned configuration
L = J—(I1+I2)

Strong coupling regime

a rotation-vibration type,

$$\Psi_\lambda \sim D^J_{MK}(\theta_i)\chi_K(R,\alpha,\beta_1,\beta_2).$$

Description of the Aligned Configuration in the molecular model is possible, with respect to the Coriolis coupling.
Centrifugal energy is optimized with Coriolis coupling, which brings alignments, like the Channel Coupling Framework.

We can translate rotational functions of the usual Laboratory frame into the molecular frame (M.F.) by the molecular rotation $\Omega(\theta_i)$ as follows.

Variables of molecular description $(q_i) = (\theta_1, \theta_2, \theta_3, R, \alpha, \beta_1, \beta_2),$

Lab. Frame to M.F. with Rotation $\Omega_i(\tilde{\alpha}_i, \tilde{\beta}_i) = \Omega_M(\theta_1, \theta_2, \theta_3)\Omega_i(\alpha_i, \beta_i).$

Rotational function in Lab. $\chi_{I_i M_i}(\tilde{\alpha}_i, \tilde{\beta}_i) = \sum_{\mu_i} D^{I_i}_{M_i \mu_i}(\theta_1, \theta_2, \theta_3)\chi_{I_i \mu_i}(\alpha_i, \beta_i).$

$$\sum_{M_1 M_2} (I_1 I_2 M_1 M_2 | I M_I) D^{I_1}_{M_1 \mu_1}(\theta_i) D^{I_2}_{M_2 \mu_2}(\theta_i) = (I_1 I_2 \mu_1 \mu_2 | I\, \mu_1 + \mu_2) D^{I}_{M_I, \mu_1 + \mu_2}(\theta_i),$$

(A·10)

$$\sum_{M_I, m} (I l M_I m | J M) D^{I}_{M_I, \mu_1 + \mu_2}(\theta_i) Y_{lm}(\theta_2, \theta_1)$$

$$= (I l\, \mu_1 + \mu_2\ 0 | J\ \mu_1 + \mu_2) \sqrt{(2l+1)/4\pi}\, D^{J}_{M, \mu_1 + \mu_2}(\theta_i), \qquad \text{(A·11)}$$

μ1= (K+nu)/2, μ2= (K-nu)/2 indicate quantum no. of each 12C's rotation around the molecular z'-axis,
which are Important to connect the model wave functions with the Channel description.

put K = μ1+μ2 ≤ l (total rotation around z'-axis), then

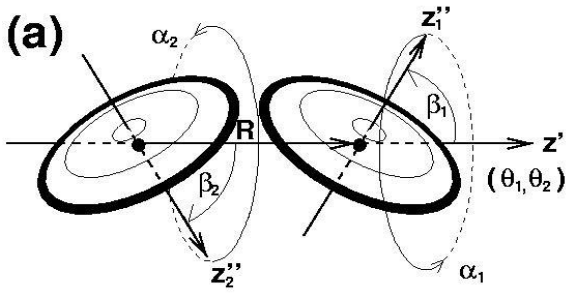
$$\mathcal{Y}_{(I_1I_2)Il;JM} = \sum_K (IlK0|JK)\sqrt{\frac{2l+1}{4\pi}}\mathcal{S}_{12}\left[D^J_{MK}(\theta_i)\Phi_{(I_1I_2)IK}(\alpha,\beta_1,\beta_2)\right], \qquad (\mathrm{A}\cdot 12)$$

$$\Phi_{(I_1I_2)IK}(\alpha,\beta_1,\beta_2) = \sum_{\mu_1\mu_2}(I_1I_2\mu_1\mu_2|IK)\chi_{I_1\mu_1}(\alpha,\beta_1)\chi_{I_2\mu_2}(-\alpha,\beta_2)$$

$$\sim \sum_{\mu_1\mu_2}(I_1I_2\mu_1\mu_2|IK)\, e^{i\nu\alpha} d^{I_1}_{\mu_1 0}(\beta_1) d^{I_2}_{\mu_2 0}(\beta_2),$$

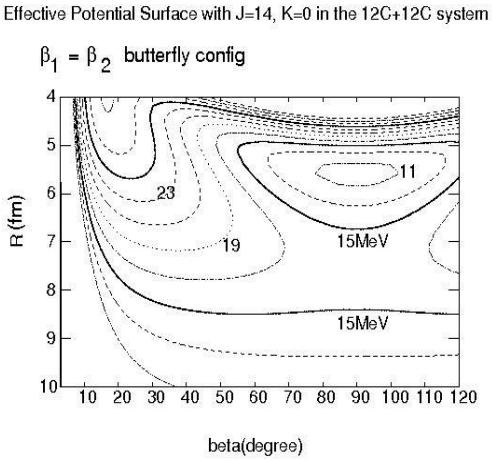
The above relations give the Channel Function in the Molecular frame, which is a good prescription to obtain channel probabilities with molecular wave functions.

- Dynamics of the Di-nuclear molecular model
  Nucleus-nucleus interaction is considered to be important.
  Thus interaction is taken into account firstly in the molecular model
- Internal degrees of freedom are orientations of symmetry axes of 12C nuclei and R described with Euler angles referring to molecular axis z' (Fig. (a))
- Nucleus-Nucleus interaction with folding of nucleon-nucleon interactions (DDM3Y), is presented directly with the internal degrees of freedom such as Fig. on the right-hand side.
- System have a strong geometrical confinement



$\theta_3 = (\alpha_1 + \alpha_2)/2$

$\alpha = (\alpha_1 - \alpha_2)/2$



Total wave function by

a rotation-vibration type,

$$\Psi_\lambda \sim D^J_{MK}(\theta_i)\chi_K(R, \alpha, \beta_1, \beta_2).$$

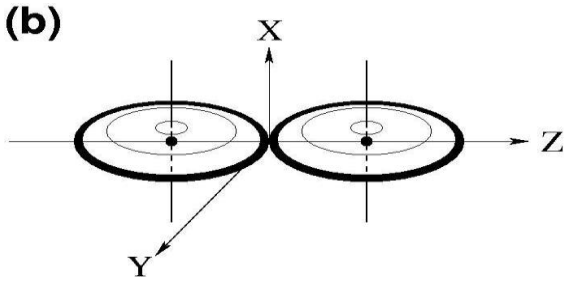
Formulations are given in
E. Uegaki and Y. Abe,PTP127(2012)831, ibid. 877 (^{28}Si+^{28}Si resonances)

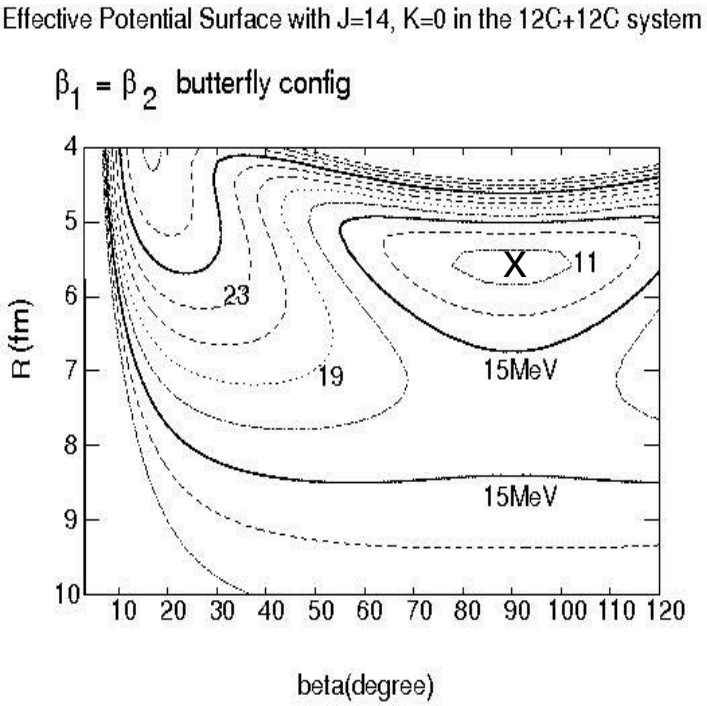
Kinetic energy operator is classified into 3 groups,
rotational, internal and Coriolis ones.

$$\hat{T} = \hat{T}' + \hat{T}'_{\rm C},$$
$$\hat{T}' = \hat{T}'_{\rm rot} + \hat{T}_{\rm int},$$

Effective potential specified with J, K is defined by
$V_{JK} = T'_{rot}(J,K) + V_{interaction}$ (Fig. in right-hand side)

Stable config. is obtained at x-mark,
i.e. at Equator-equator one (Fig.(b))





We solve normal mode
around this E-E configuration.

we expand V_{JK}
at the equilibrium E-E configuration,
into a quadratic form for R, β_1 and β_2,

Our description is basically Harmonic Oscillations for the internal degrees.
For example, vibrational mode for beta, hwβ∼5MeV(J=14)

Resulting Energy Spectrum with specified K

$$E^J(n,n_+,n_-,K) = E_0(R_e) + \frac{\hbar^2}{2}\left[\frac{J(J+1)-K^2-1}{\mu R_e^2} + \frac{K^2-2}{2I}\right] + \left(n+\frac{1}{2}\right)\hbar\omega_R + (n_1++n_2+1)\hbar\omega_\beta + E_\alpha + E_{\text{coupl}},$$

$$\hbar\omega_\beta = \sqrt{k_0\left(\frac{\hbar^2}{I} + \frac{\hbar^2}{\mu R_e^2}\right)}\,.$$

Selection rule for the states

$(-1)^{n_1} = (-1)^{(K+\nu)/2}$ and $(-1)^{n_2} = (-1)^{(K-\nu)/2}$, due to the symmetry

thus $(-1)^{n_1+n_2} = (-1)^K$.

Table I. Examples of the molecular states specified by the K-quantum number, the β-vibrational quanta (n_1, n_2) and ν for the α-motion allowed by the selection rule.

K	(n_1, n_2)	ν
0	(0,0), (2,0)	0, 4, 8,.. : $K \pm \nu = 4m$
0	(1,1)	2,6,.. : $K \pm \nu = 4m+2$
2	(0,0)	2, 6,10,...
4	(0,0)	0, 4, 8,...
1, 3	(1,0) + (0,1)	1, 3, 5,...

We solve Coriolis coupling T'c between different K's

Coriolis coupling operators between different K

lowering and rasing operators $\hat{J}'_{\pm} = \hat{J}'_1 \pm i\hat{J}'_2$

and for internal variables creation annihiration operators are used

$$-\frac{\partial}{\partial\beta_i} = -\frac{\partial}{\partial\Delta\beta_i} = \sqrt{\frac{m\omega}{2\hbar}}(b_i^* - b_i),$$

$$\Delta\beta_i = \sqrt{\frac{\hbar}{2m\omega}}(b_i^* + b_i),$$

- Selection of the basis states

With correspondence to the channel functions, $\mu_1=(K+nu)/2$, $\mu_2=(K-nu)/2$ specifies each ^{12}C rotation around z'-axis.

For $^{12}C(2+)$ excitation, $\mu_i+n_i<=l_i=2$ (n_i is β_i vibrational quanta)

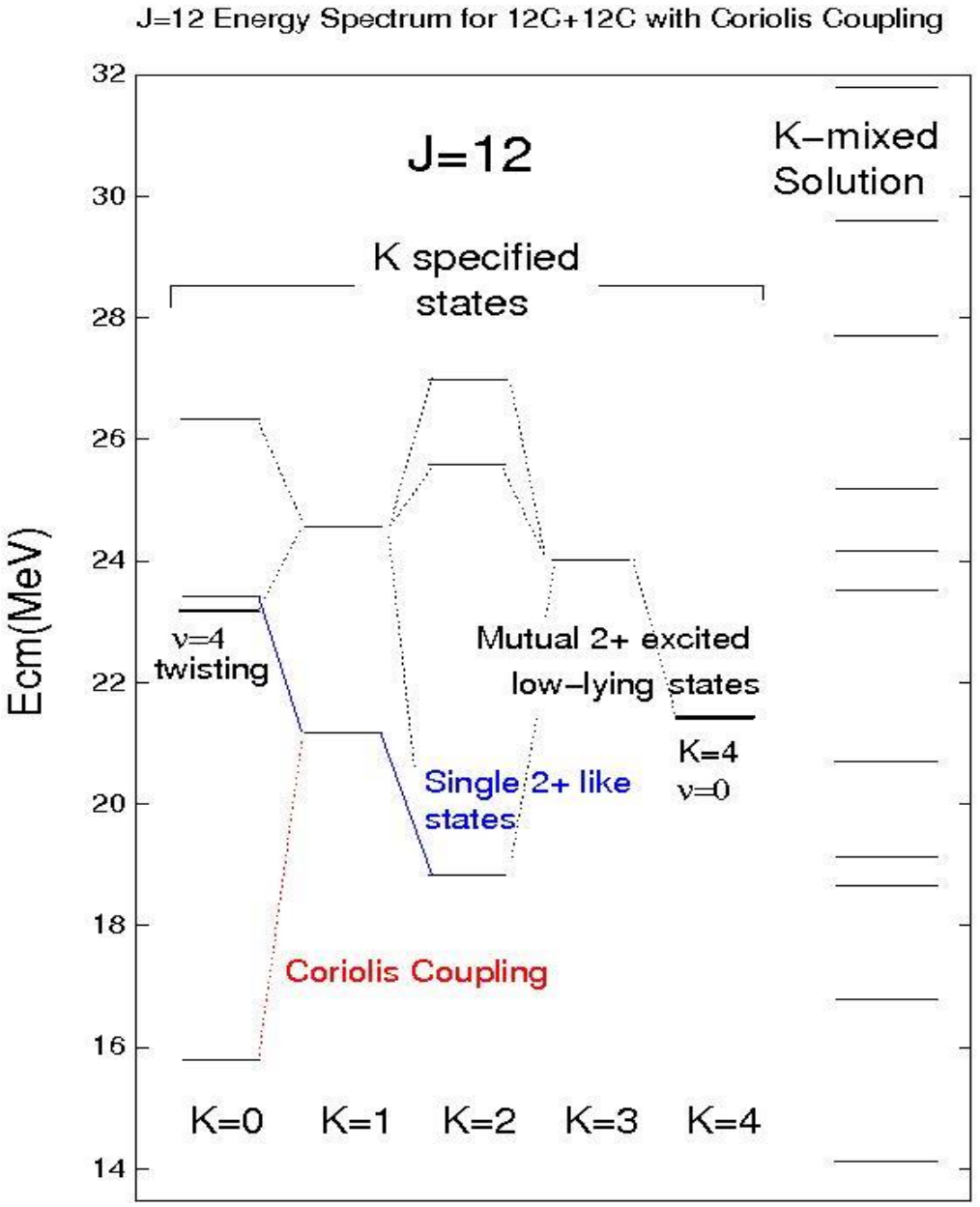
For example, for K=4, nu=0 state ($\mu_1=\mu_2=2$), $n_i=0$ should be imposed.

- We take states corresponding to the elastic, single 2+ and mutual 2+ excitations in the channel description.

We diagonalize 11 basis K-specified states, to obtain K-mixed solutions (levels on the right side)

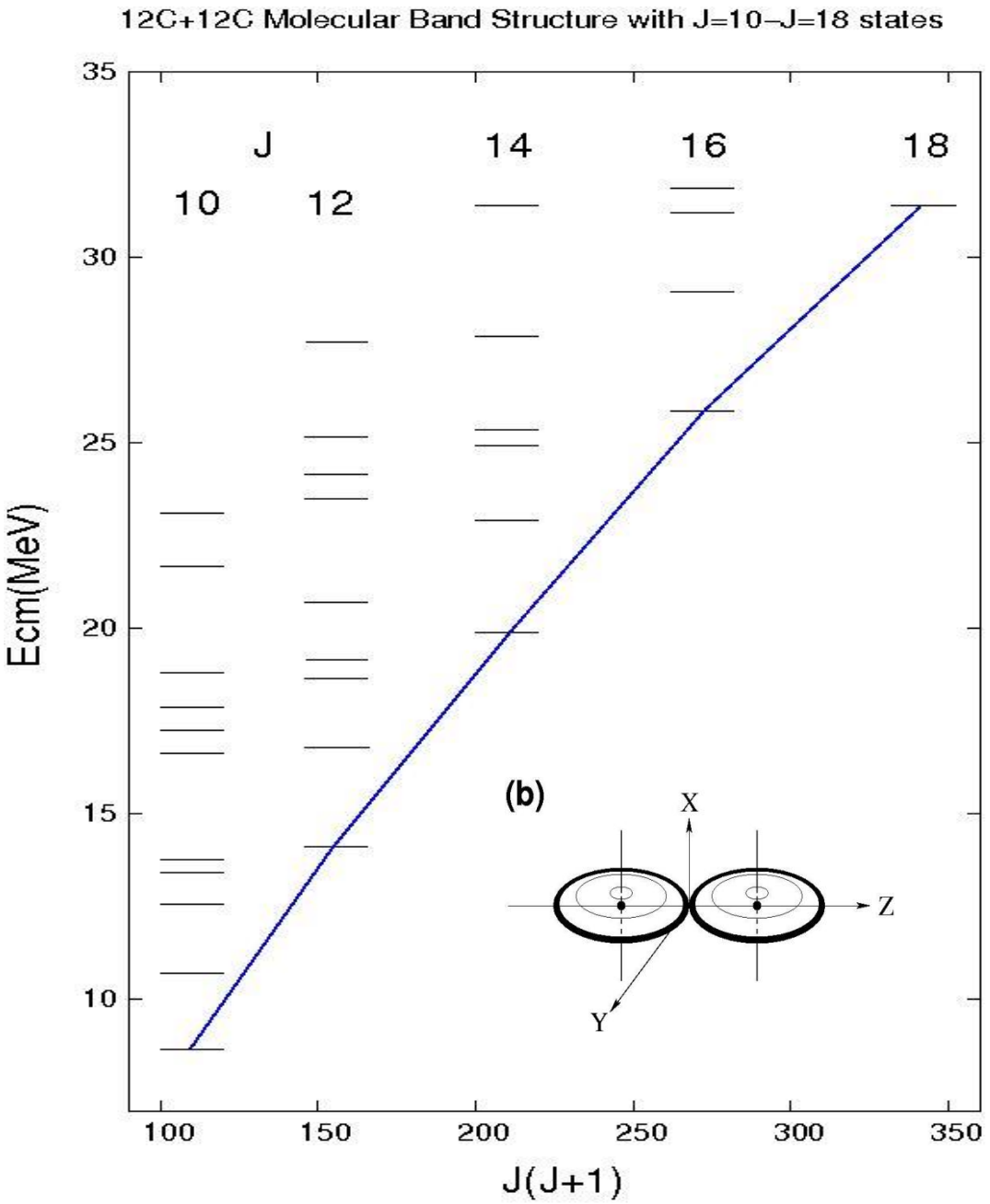
- Coriolis Coupling occurs between states with $\Delta K = 1$ & $\Delta nu = 1$.

For example, the molecular ground state couples with K=1, nu=1 state (indicated with dotted red line)



- There are also couplings between basis K-specified states, which are indicated with blue lines and dotted black lines.
- Low-lying states are n1=n2=0 Zero-point oscillation for β-motions, with μ1, μ2 excitation, such as K=4 state, or (K=0 nu=4) state.
- Thus, dynamically, Rotations around the Molecular z'-Axis (RAMA) is important!
- RAMA states are spin disaligned states, because each spin vector of the constituent 12C nuclei is along the molecular z'-axis
- They appear as second and third molecular excited states, with rather high-excitation energy, but their decay widths are expected to be small due to weak coupling to the elastic channel. (shown later by excitation functions)

- Figure shows molecular band structure btained, in which states with large widths over 4 MeV are omitted to have a correspondence with observation.
- J=10-18 are exhibited, which are dominated by the equator-equator config. (inserted fig. (b)).
- Below spin J=8, stable configurations turns to be different ones, which needs more careful investigation, as constituent ^{12}C nuclei touch closely and the system is expected to be more complex.



Excitation functions are compared with Exp.

Define the Total widths tentatively as $\Gamma_\lambda = (\sum_c \Gamma_{\lambda c}) \times 2$

$\sigma_{c'c} = 2\pi/k_c^2 \cdot (2J+1) \cdot |\sum_\lambda \sqrt{(\Gamma_{\lambda c} \Gamma_{\lambda c'})}/(E_\lambda - E - i\,\Gamma_\lambda/2)|^2$

note : for the elastic channel, δcc' term is omitted.

- Experimental data shows decays to 20Ne+α channel are comparable with those in 12C+12C channel, and thus total widths are assumed to be twice of the calculated widths in the 12C+12C decay.
- In the next two pages, the results are displayed.

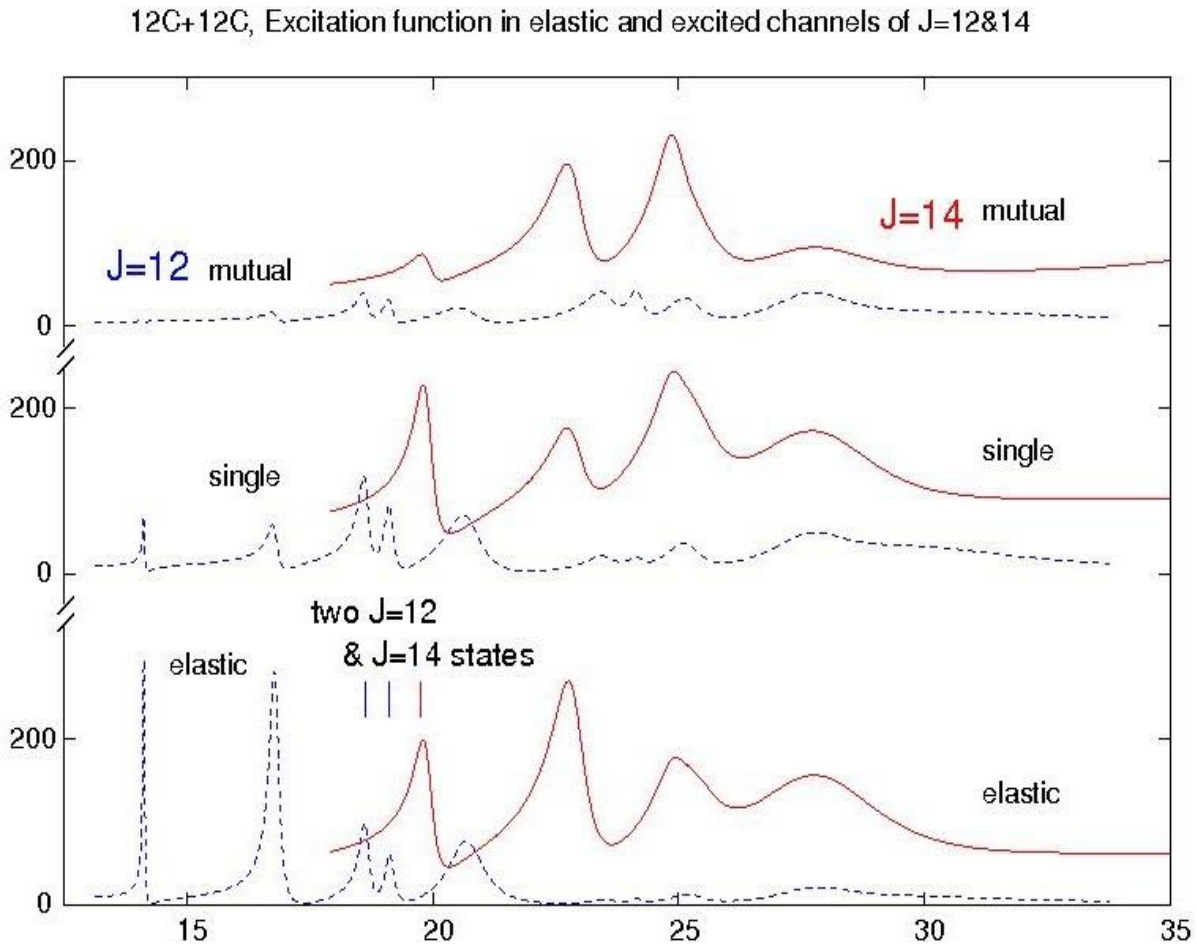
Page 16: Figure of excitation functions for the elastic, single 2+ and mutual 2+ decays of 12C+12C on J=12 and J=14.

Page 17: Fig. of total excitation functions (sum of J=10-18) in those channels, compared with Exp. Data obtained by T.M. Cormier et al. PRL38(1977)940,40(1978)924.

- Most of the elastic components appear in the molecular ground state and the 1st excited state.
- From the 2nd excited state to above, elastic contributions are weak.

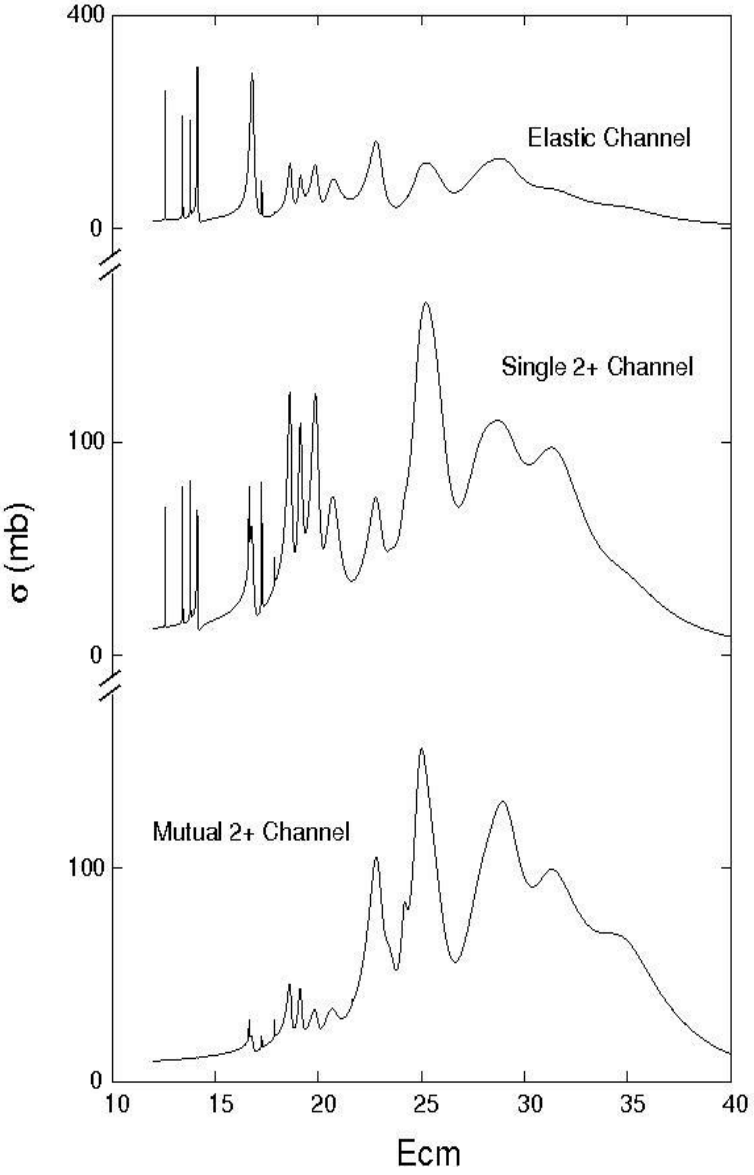
  => small width, narrow peaks.

- Dotted lines show excitation functions for J=12, and red lines show those for J=14 in comparison.
  Surprisingly, two J=12 states and one J=14 state appear very closely in energy, and thus they are obtained in good correspondence with experiment.



Theoretical Results

12C+12C, Excitation function in elastic and excited channels: Sum of J=10–18



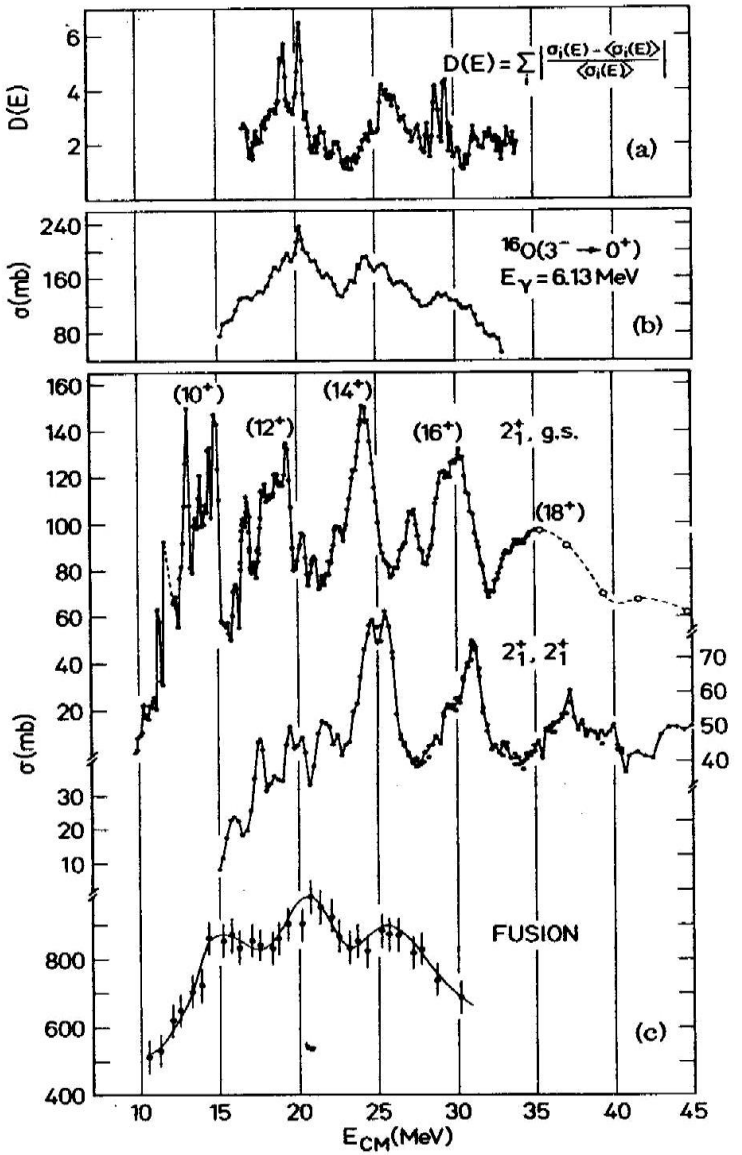


FIG. 1. (a) Deviation function, $D(E)$, calculated from sixteen elastic excitation functions using an averaging interval of 1.5 MeV. (b) Excitation function for the $^{16}O(3^- \rightarrow 0^+)$ γ ray (Ref. 11). (c) Total-cross-section excitation functions for single inelastic $(2_1^+ + g.s.)$ scattering (present work), mutual inelastic $(2_1^+ + 2_1^+)$ scattering (Ref. 4), and fusion (Ref. 2).

Concluding Remarks

- Molecular model has been applied to the high spin resonances of $^{12}C+^{12}C$ system. Stable configuration for high spins J=10-18 are found to be Equator-Equator one, and thus normal modes around this stable configuration are solved with each specified K-quantum number. Next, Coriolis coupling is solved in the space of those normal-mode states, which brings strong K-mixing to optimize centrifugal energy for L-base's one from J-base's.
- The resulting molecular ground state appears as a state in which ^{12}C spin is aligned with the orbital angular momentum. This is similar to BCM and is known as double resonance mechanism.
- We have found a new aspect of the dynamical property. The interaction of the oblate-oblate system ^{12}C-^{12}C gives low lying intrinsic excited states with high rotational motions around the molecular z'-axis, such as K=4 or (K=0, nu=4) state. Those multi-excited states of a nature of ^{12}C spin disaligned with the orbital angular momentum weakly couple to the elastic channel, and thus they are expected to be observed as resonances with small widths.
- Excitation functions of the single 2^+ and mutual 2^+ channels are calculated and are compared with experiment. The results reproduce characteristic features of the experimental data in a wide energy range very well.
- Molecular model gives both the intuitive understanding on dynamics of dinuclear structure and a tool of analyses for the reaction mechanism.