Fluctuations of conserved charges and freeze-out conditions in heavy ion collisions

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S. Borsanyi, Z. Fodor, S. Katz, S. Krieg, C. R., K. Szabo, PRL 2014

P. Alba, W. Alberico, R. Bellwied, M. Bluhm, V. Mantovani Sarti, M. Nahrgang, C. R., PLB 2014

Motivation

- Synergy between fundamental theory and experiment
- We can create the deconfined phase of QCD in the laboratory
- Lattice QCD simulations have reached unprecedented levels of accuracy
  - physical quark masses
  - several lattice spacings $\longrightarrow$ continuum limit
- Can we learn something about hadronization from the synergy between fundamental theory and experiment?

The observables: fluctuations of conserved charges

- They can be calculated on the lattice as combinations of quark number susceptibilities

- They can be compared to experimental measurements (with some caveats)

- The chemical potentials are related:

$$\begin{aligned}\mu_u &= \frac{1}{3}\mu_B + \frac{2}{3}\mu_Q;\\ \mu_d &= \frac{1}{3}\mu_B - \frac{1}{3}\mu_Q;\\ \mu_s &= \frac{1}{3}\mu_B - \frac{1}{3}\mu_Q - \mu_S.\end{aligned}$$

- susceptibilities are defined as follows:

$$\chi_{lmn}^{BSQ} = \frac{\partial^{\,l+m+n} p/T^4}{\partial(\mu_B/T)^l \partial(\mu_S/T)^m \partial(\mu_Q/T)^n}.$$

Relating susceptibilities to moments

In a thermally equilibrated system we can define susceptibilities χ as 2nd derivative of pressure with respect to chemical potential (1st derivative of ρ).

Starting from a given partition function we define the fluctuations of a set of conserved charges as:

$$\frac{p}{T^4} = \frac{\ln \mathcal{Z}}{VT^3} \qquad \chi_{lmn}^{BSQ} = \frac{\partial^{l+m+n}\left(p/T^4\right)}{\partial\left(\mu_B/T\right)^l \partial\left(\mu_S/T\right)^m \partial\left(\mu_Q/T\right)^n}$$

The fluctuations of conserved charges are related to the moments of the multiplicity distributions of the same charge measured in HIC.

$$\delta N = N - \langle N \rangle$$

mean: $M = \langle N \rangle = VT^3\chi_1,$

variance: $\sigma^2 = \langle (\delta N)^2 \rangle = VT^3\chi_2,$

skewness: $S = \frac{\langle (\delta N)^3 \rangle}{\sigma^3} = \frac{VT^3\chi_3}{(VT^3\chi_2)^{3/2}},$

kurtosis: $k = \frac{\langle (\delta N)^4 \rangle}{\sigma^4} - 3 = \frac{VT^3\chi_4}{(VT^3\chi_2)^2};$

Measurable ratios:

$$R_{32} = S\sigma = \frac{\chi_3^{(B,S,Q)}}{\chi_2^{(B,S,Q)}}$$

$$R_{42} = K\sigma^2 = \frac{\chi_4^{(B,S,Q)}}{\chi_2^{(B,S,Q)}}$$

To measure μ_B:

$$R_{12} = \frac{M}{\sigma^2} = \frac{\chi_1^{(B,S,Q)}}{\chi_2^{(B,S,Q)}}$$

To measure T:

$$R_{31} = \frac{S\sigma^3}{M} = \frac{\chi_3^{(B,S,Q)}}{\chi_1^{(B,S,Q)}}$$

Relating lattice results to experimental measurement

- we can relate susceptibilities to moments of multiplicity distributions:

$$\mathrm{mean}:\ M = \chi_1 \qquad \mathrm{variance}:\ \sigma^2 = \chi_2$$

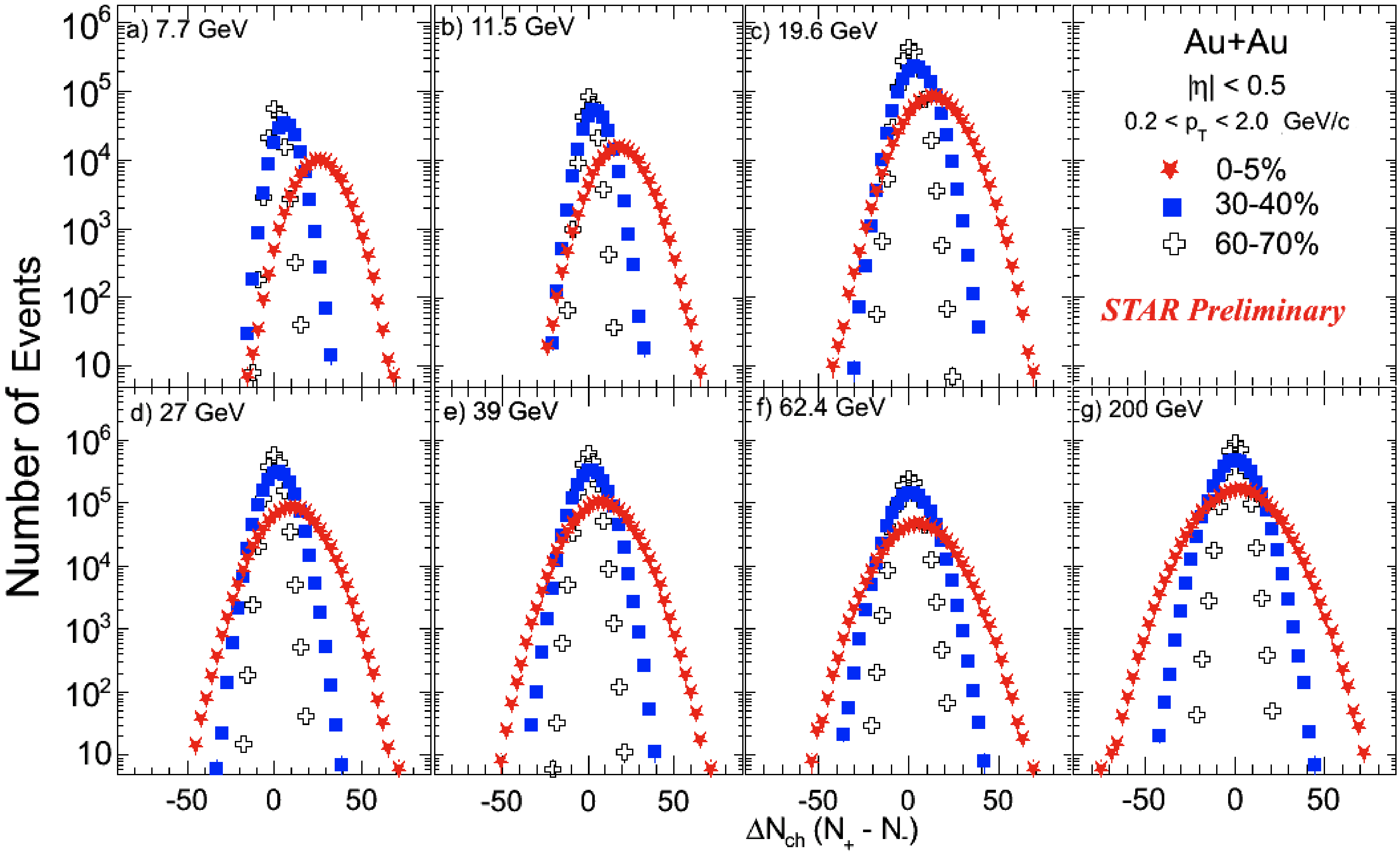
$$\mathrm{skewness}:\ S = \chi_3/\chi_2^{3/2} \qquad \mathrm{kurtosis}:\ \kappa = \chi_4/\chi_2^2$$

$$S\sigma = \chi_3/\chi_2 \qquad \kappa\sigma^2 = \chi_4/\chi_2$$

$$M/\sigma^2 = \chi_1/\chi_2 \qquad S\sigma^3/M = \chi_3/\chi_1$$

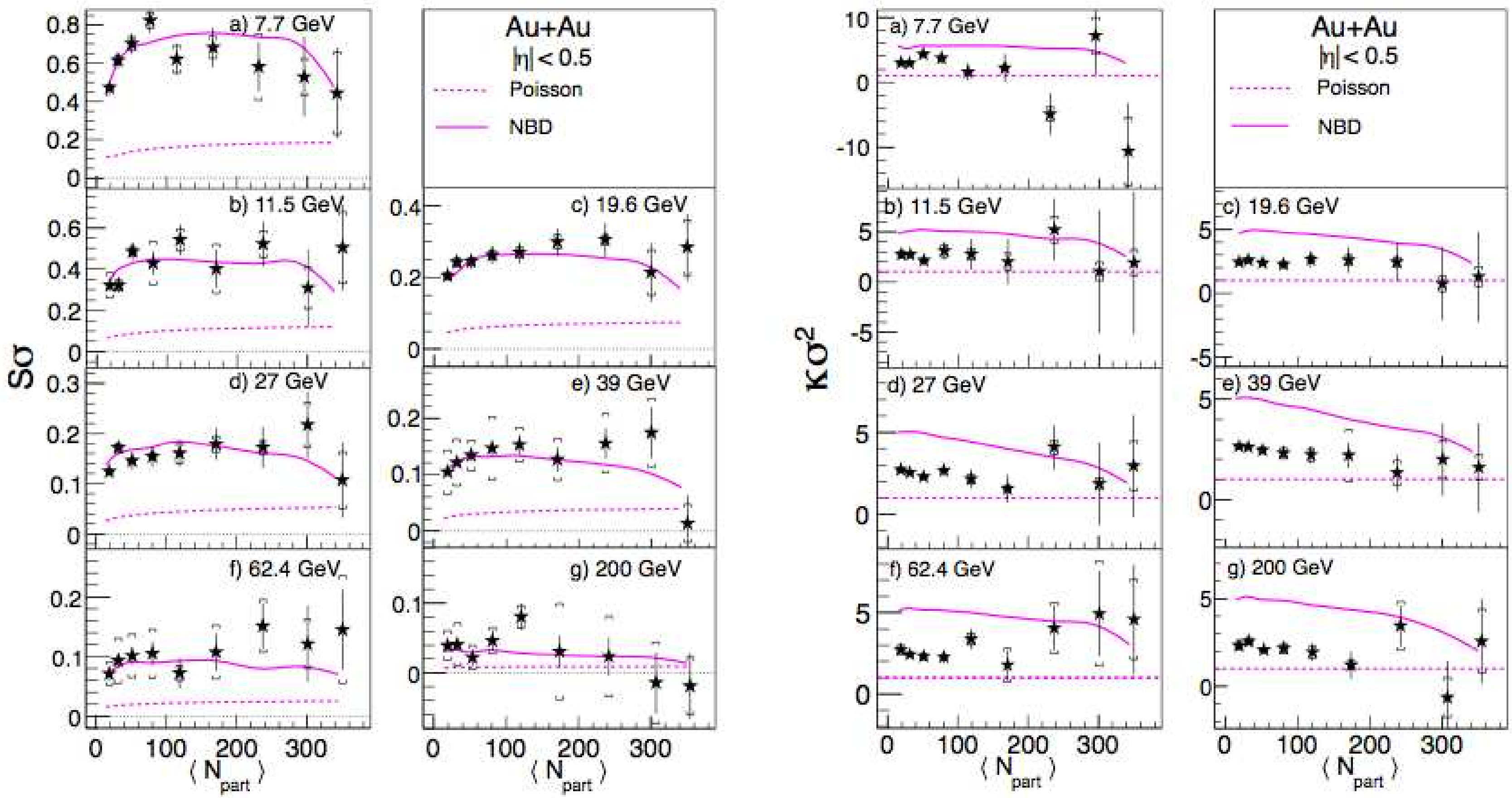
F. Karsch (2012)

Experimental measurement I



Star Collaboration: arXiv 1212.3892

Experimental measurement II



Star Collaboration: PRL 2014

Caveats

- Effects due to volume variation because of finite centrality bin width V. Skokov, B. Friman, K. Redlich, PRC (2013)
  - Experimentally corrected by centrality-bin-width correction method
- Finite reconstruction efficiency
  - Experimentally corrected based on binomial distribution A. Bzdak, V. Koch, PRC (2012)
- Spallation protons
  - Experimentally removed with proper cuts in p_T
- Canonical vs Gran Canonical ensemble
  - Experimental cuts in the kinematics and acceptance V. Koch, S. Jeon, PRL (2000)
- Proton multiplicity distributions vs baryon number fluctuations
  - Numerically very similar once protons are properly treated M. Asakawa and M. Kitazawa, PRC (2012), M. Nahrgang *et al.*, 1402.1238
- Final-state interactions in the hadronic phase J.Steinheimer *et al.*, PRL (2013)
  - Consistency between different charges = fundamental test

Thermometer and Baryometer

- R^B_{31}: thermometer

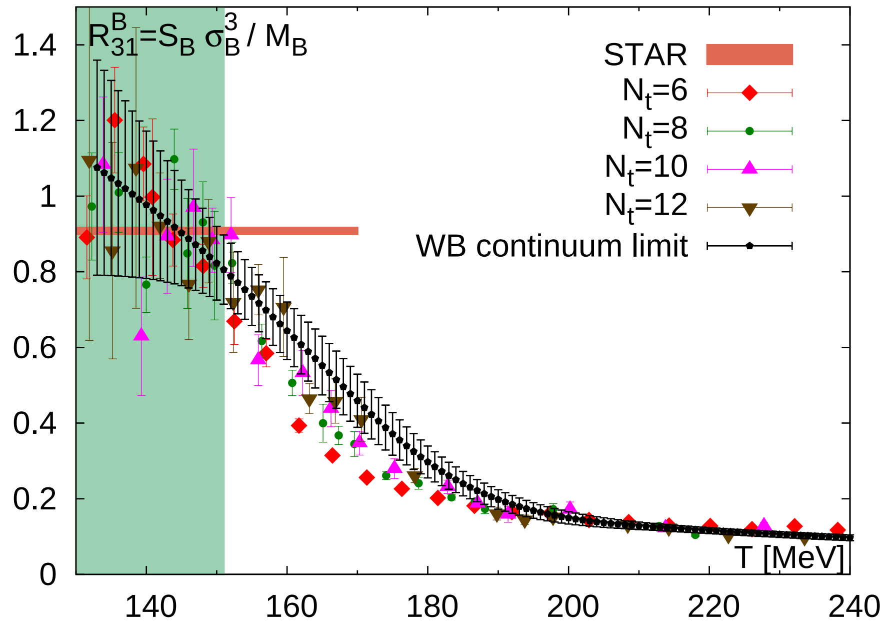
$$R^B_{31}(T,\mu_B) = \frac{\chi^B_3(T,\mu_B)}{\chi^B_1(T,\mu_B)} = \frac{\chi^B_4(T,0) + \chi^{BQ}_{31}(T,0)q_1(T) + \chi^{BS}_{31}(T,0)s_1(T)}{\chi^B_2(T,0) + \chi^{BQ}_{11}(T,0)q_1(T) + \chi^{BS}_{11}(T,0)s_1(T)} + \mathcal{O}(\mu_B^2)$$

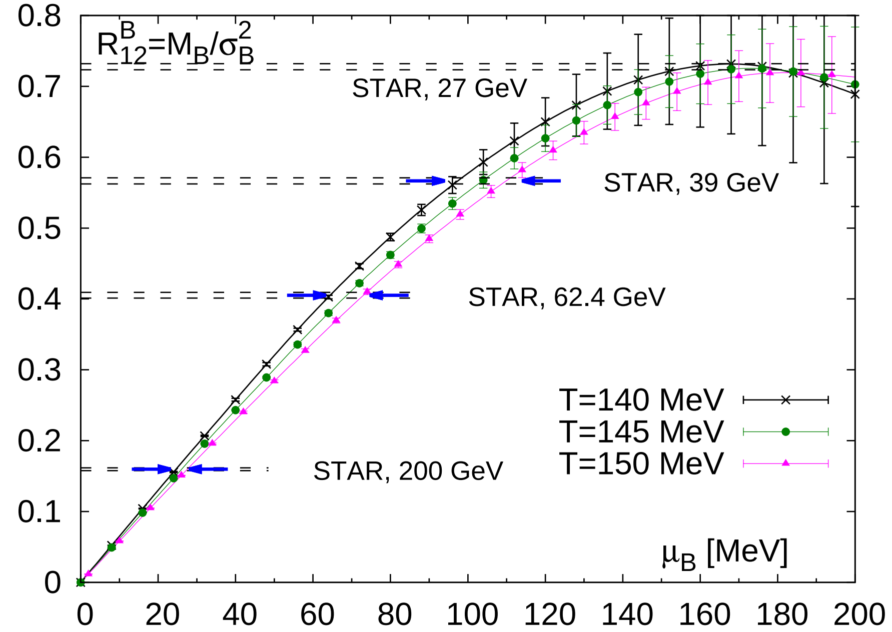
- Expand numerator and denominator around $\mu_B = 0$: ratio is independent of μ_B
- R^B_{12}: baryometer

$$R^B_{12}(T,\mu_B) = \frac{\chi^B_1(T,\mu_B)}{\chi^B_2(T,\mu_B)} = \frac{\chi^B_2(T,0) + \chi^{BQ}_{11}(T,0)q_1(T) + \chi^{BS}_{11}(T,0)s_1(T)}{\chi^B_2(T,0)} \frac{\mu_B}{T} + \mathcal{O}(\mu_B^3)$$

- Expand numerator and denominator around $\mu_B = 0$: ratio is proportional to μ_B

Extracting freeze-out parameters from baryon number



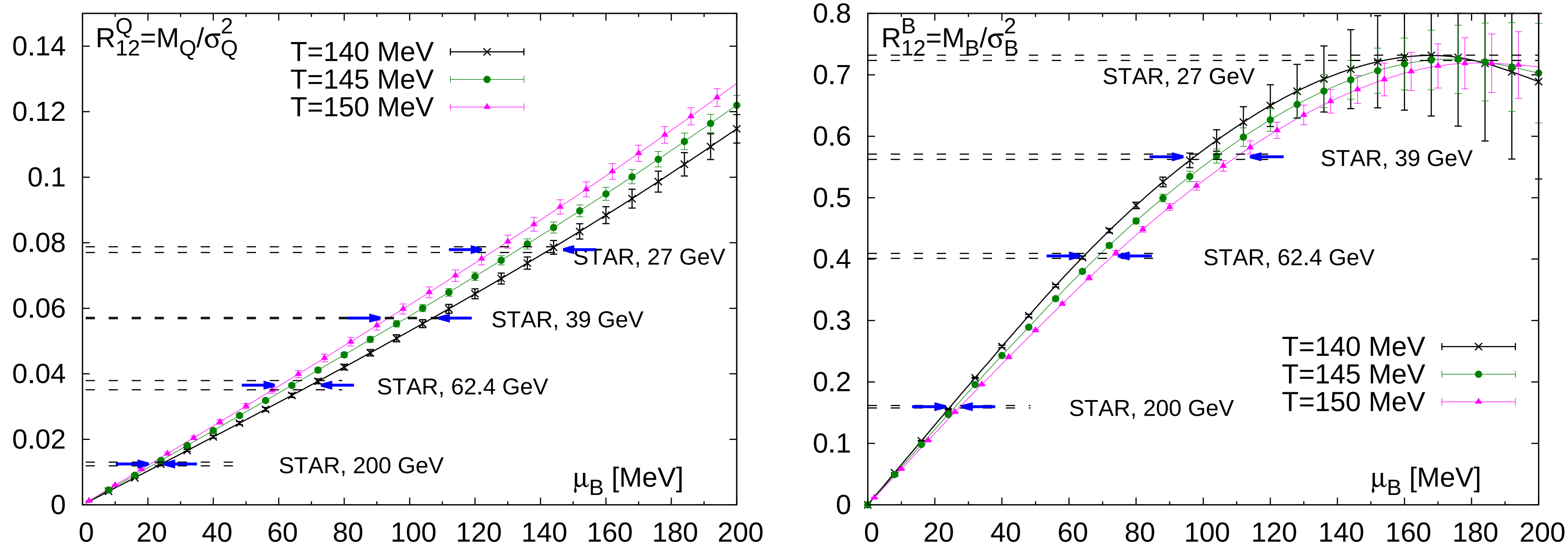


WB Collaboration: PRL (2014); STAR data from 1309.5681

- Upper limit: $T_f \leq 151 \pm 4$ MeV

$\sqrt{s}[GeV]$	μ_B^f [MeV]
200	25.8$\pm$2.7
62.4	69.7$\pm$6.4
39	105$\pm$11
27	-

Extracting freeze-out μ_B from electric charge



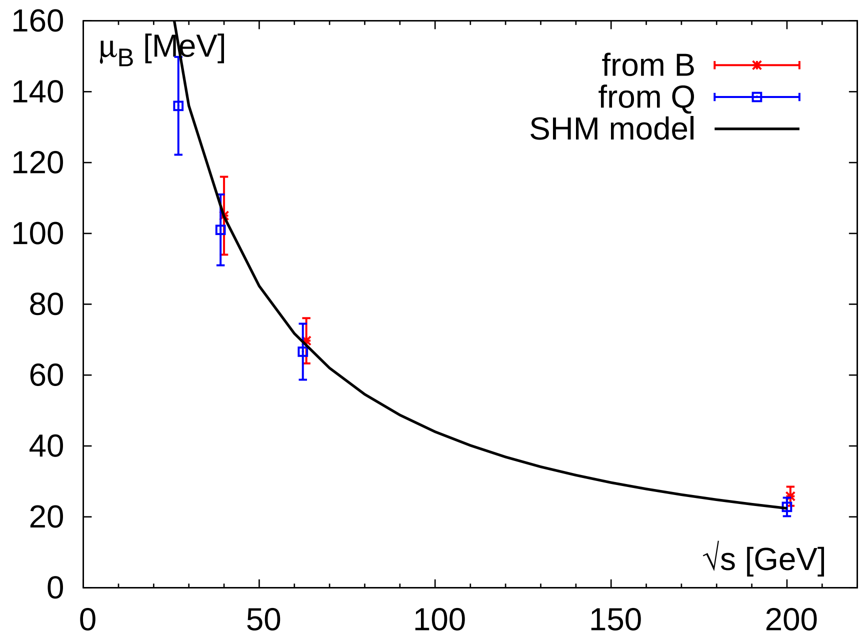
WB Collaboration: PRL (2014); STAR data from 1309.5681 and 1402.1558

- It is of fundamental importance to test the consistency between the freeze-out parameters obtained with different conserved charges

- This consistency check validates the method and shows equilibration of the medium

Consistency is found!

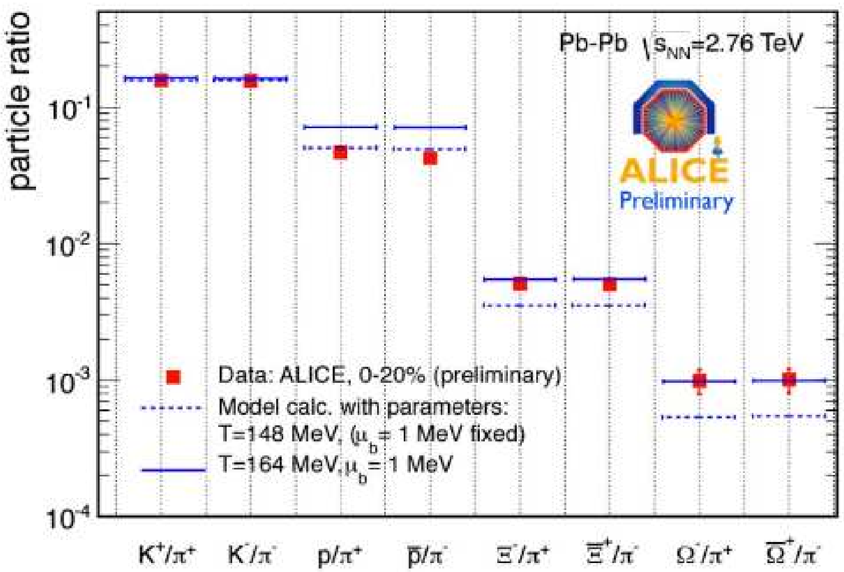
$\sqrt{s}[GeV]$	μ_B^f [MeV] (from B)	μ_B^f [MeV] (from Q)
200	25.8$\pm$2.7	22.8$\pm$2.6
62.4	69.7$\pm$6.4	66.6$\pm$7.9
39	105$\pm$11	101$\pm$10
27	-	136$\pm$13.8

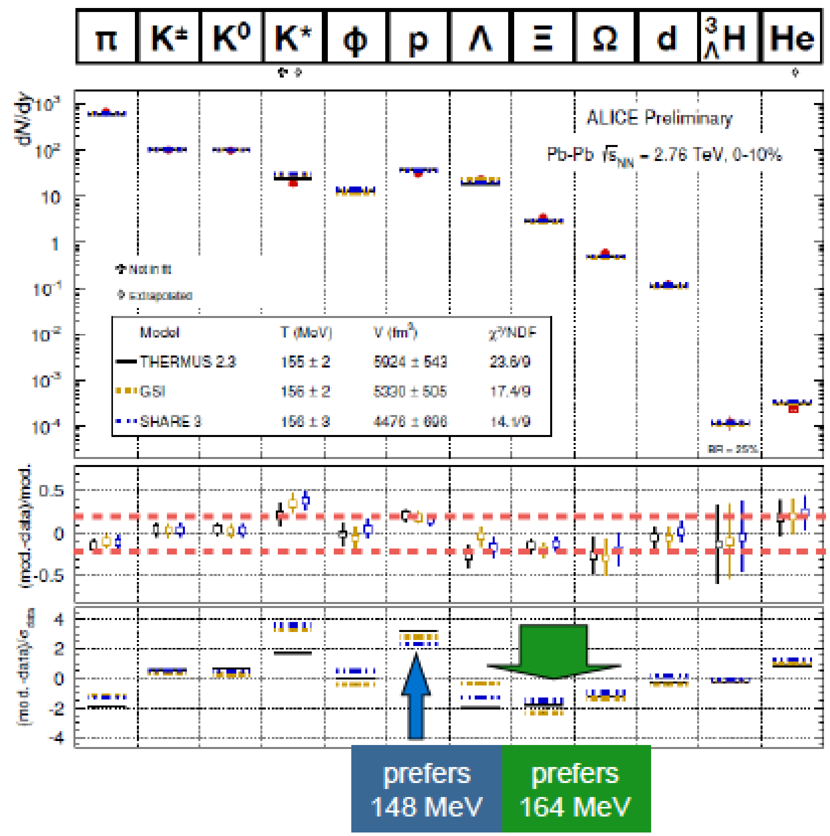


Lattice: WB Collaboration: PRL (2014); SHM: Andronic *et al.*, NPA (2006)

Freeze-out temperature from yields

- Fit to yields of identified particles: Statistical Hadronization Model (SHM)
- Model-dependent. Parameters: freeze-out temperature and chemical potential



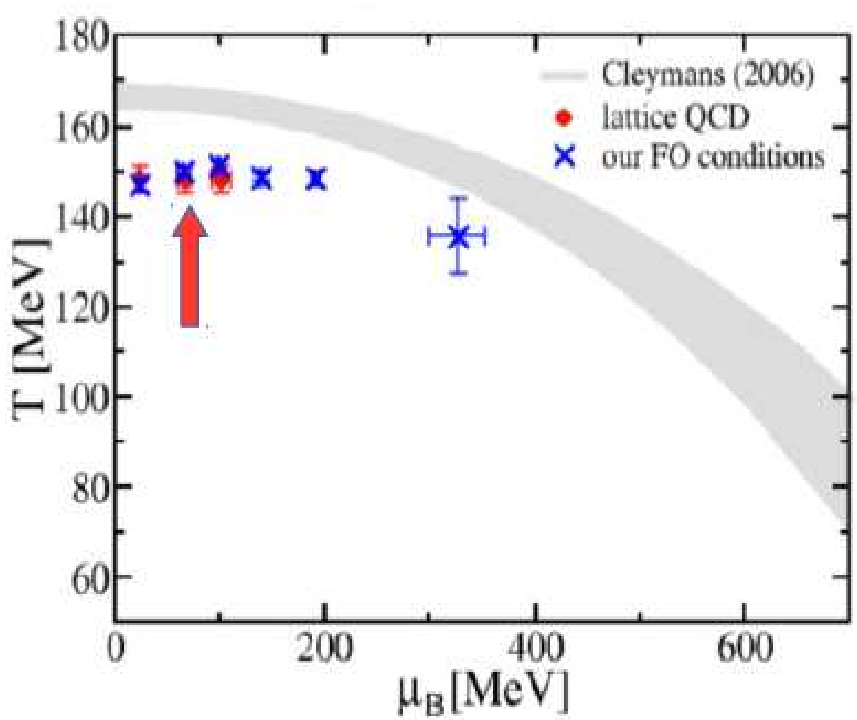


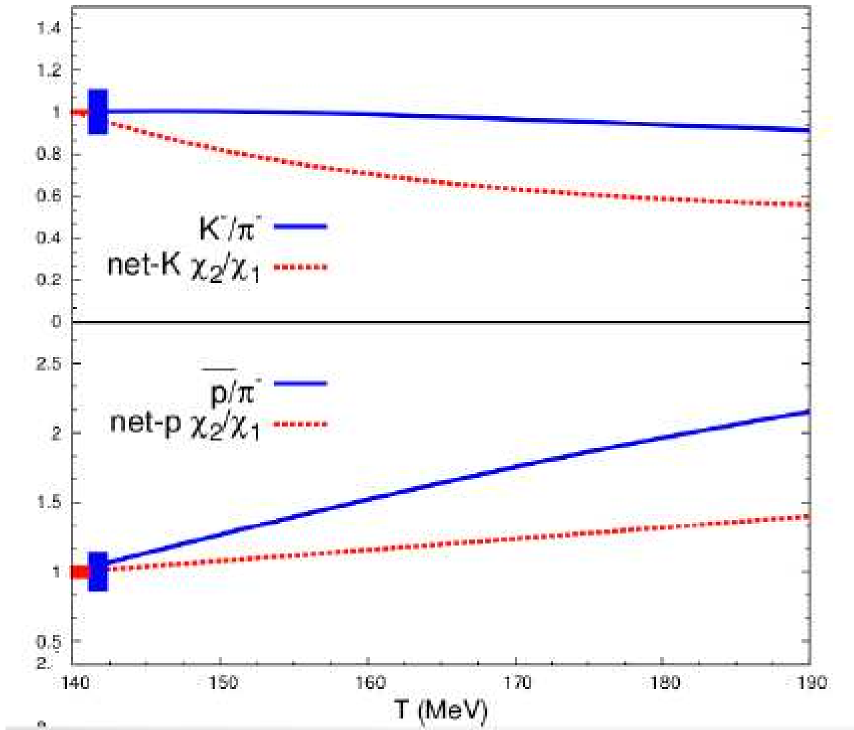
R. Preghenella for ALICE, SQM 2012

M. Floris, QM 2014.

HRG model analysis

- Experimental cuts in acceptance and momentum
- Resonance decay and regeneration

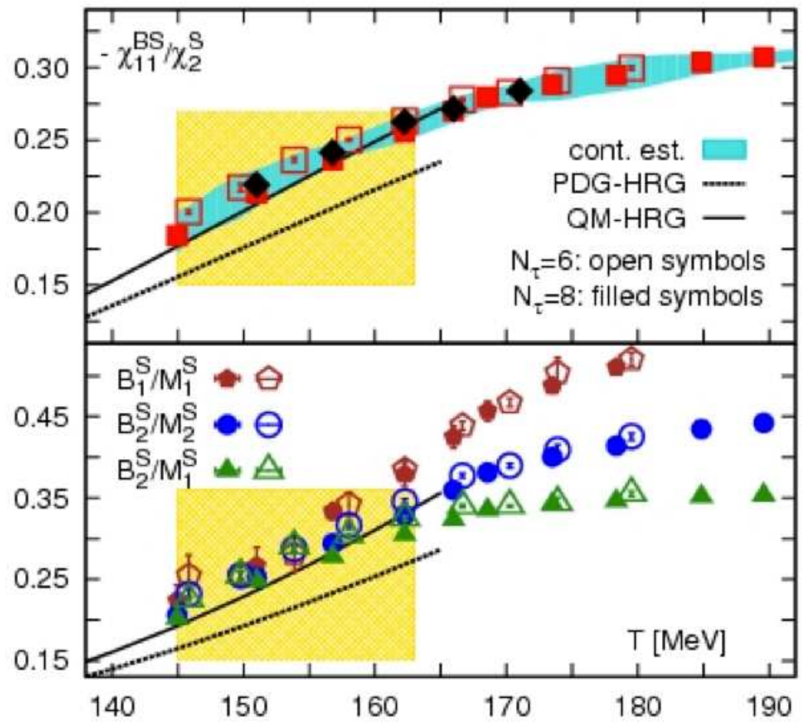




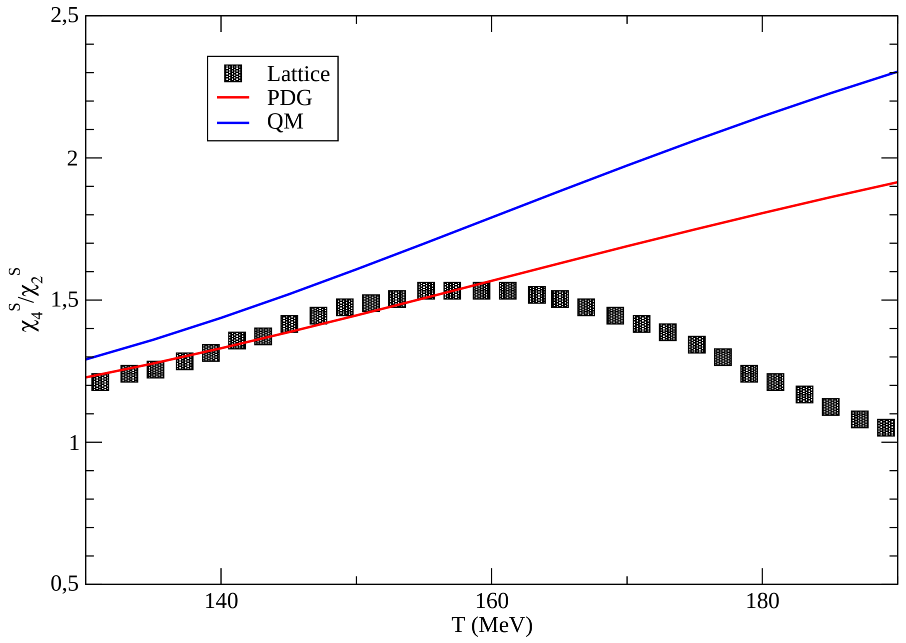
P. Alba *et al.*, PLB 2014

P. Alba *et al.*, arXiv:1504.03262

Quark model strange states



A. Bazavov *et al.*: PRL (2014)



R. Bellwied *et al.*: in preparation

- Not-yet discovered strange states improve the BS correlator but χ_4^S/χ_2^S gets worse

Fluctuations from yields

- Net baryon number susceptibility

$$\frac{\chi_B}{T^2} \approx \frac{1}{VT^3}(\langle p\rangle + \langle N\rangle + \langle \Lambda + \Sigma_0\rangle + \langle \Sigma^+\rangle + \langle \Sigma^-\rangle + \langle \Xi^-\rangle + \langle \Xi^0\rangle + \langle \Omega^-\rangle + \overline{par}\ )$$

- Net strangeness

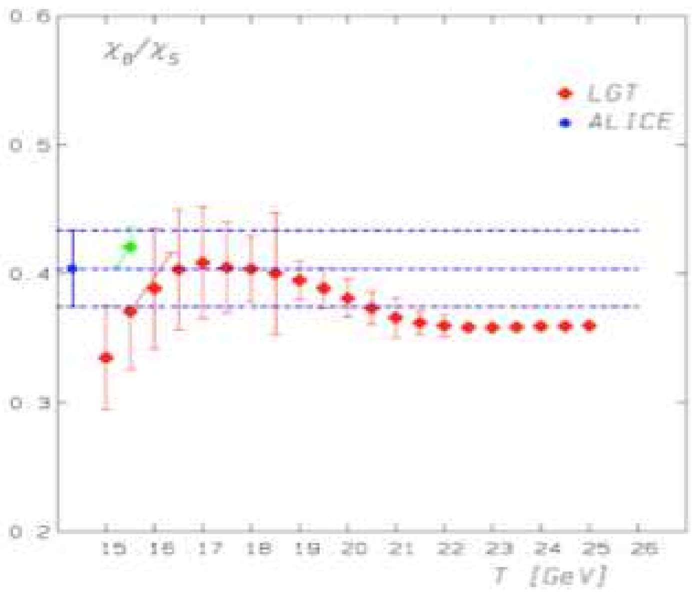
$$\frac{\chi_S}{T^2} \approx \frac{1}{VT^3}(\langle K^+\rangle + \langle K_S^0\rangle + \langle \Lambda + \Sigma_0\rangle + \langle \Sigma^+\rangle + \langle \Sigma^-\rangle + 4\langle \Xi^-\rangle + 4\langle \Xi^0\rangle + 9\langle \Omega^-\rangle + \overline{par}$$
$$-(\Gamma_{\varphi\to K^+} + \Gamma_{\varphi\to K^-} + \Gamma_{\varphi\to K_S^0} + \Gamma_{\varphi\to K_L^0})\langle \varphi\rangle\ )$$

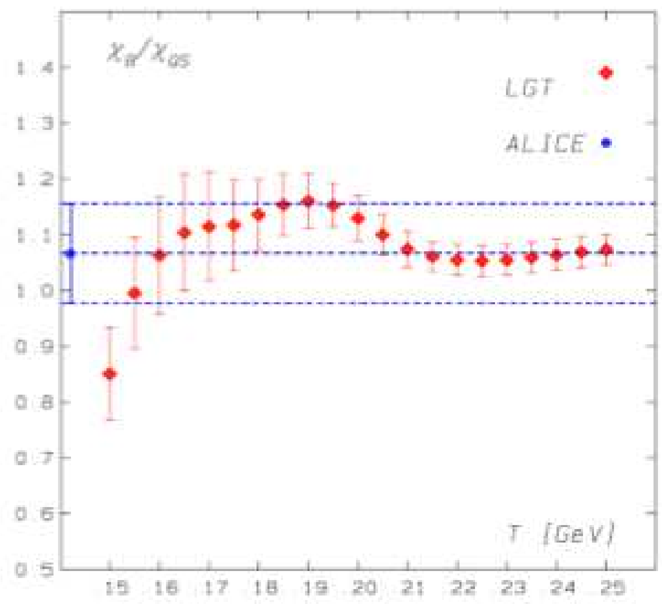
- Charge-strangeness correlation

$$\frac{\chi_{QS}}{T^2} \approx \frac{1}{VT^3}(\langle K^+\rangle + 2\langle \Xi^-\rangle + 3\langle \Omega^-\rangle + \overline{par}$$
$$-(\Gamma_{\varphi\to K^+} + \Gamma_{\varphi\to K^-})\langle \varphi\rangle - (\Gamma_{K_0^*\to K^+} + \Gamma_{K_0^*\to K^-})\langle K_0^*\rangle\ )$$

K. Redlich *et al.*, 2014

Fluctuations from yields





K. Redlich *et al.*, 2014

Conclusions

- It is possible to extract freeze-out parameters from first principles
- Higher order fluctuations of baryon number:
  - $R_{31}^B(T,\mu_B)=\frac{\chi_3^B(T,\mu_B)}{\chi_1^B(T,\mu_B)}$: Thermometer
  - $R_{12}^B(T,\mu_B)=\frac{\chi_1^B(T,\mu_B)}{\chi_2^B(T,\mu_B)}$: Baryometer
- Higher order fluctuations of electric charge:
  - independent measurement
  - $R_{12}^Q(T,\mu_B)=\frac{\chi_1^Q(T,\mu_B)}{\chi_2^Q(T,\mu_B)}$: Baryometer
- The freeze-out parameter sets obtained from B and Q are consistent with each other
- Looking forward to strangeness fluctuation data!