Toward a solution of the $R_{AA}$ and $v_2$ puzzle for heavy quarks

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The puzzling Relation between $R_{AA}$ and $v_2$ for Heavy Flavors

- T-dependence of the interaction
- Boltzmann vs Fokker-Planck approach

Boltzmann vs Fokker-Planck: $c\bar{c}$ and $b\bar{b}$ angular correlation
Introduction Heavy Quarks

\[ M_{HQ} \gg gT \approx K \left( M_{\text{Charm}} \approx 1.3 \text{ GeV}; \ M_{\text{Bottom}} \approx 4.2 \text{GeV} \right) \]

HQ propagation in the QGP is described by the Fokker-Planck eq.

\[ \frac{\partial f}{\partial t} = \frac{\partial}{\partial p_i} \left[ A_i(p)f + \frac{\partial}{\partial p_j} \left[ B_{ij}(p)f \right] \right] \]

The interaction is encoded in the drag and diffusion coefficients

\[ A = \int d^3k |M(k, p)|^2 p \]
\[ B = \frac{1}{2} \int d^3k |M(k, p)|^2 p^2 \]

Evaluated from scattering matrix \(|M|^2\)

From experiments and theoretical simulations we know that drag from \(p \overline{q} \text{cd} \rightarrow R_{AA}\) larger than exp. data

\(R_{AA}\) gives information on the average strength of the interactions between HQ and the bulk

The relation between \(R_{AA}\) and \(v_2\) can give further informations on the interaction
Various model at work for RHICs

Simultaneous description of $R_{AA}$ and $v_2$ is a tough challenge for all models.
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$R_{AA}$ and $v_2$ correlation

Larger interaction $\to$ smaller $R_{AA}$ $\to$ larger the $v_2$

The correlation between $R_{AA}$ and $v_2$ is related with the time-dependence of the interaction (for an expanding medium) $\leftrightarrow$ Temperature-dependence

This is general, seen also for light quarks
[Scardina, Di Toro, Greco, PRC82(2010)] [J.Liao and E. Shuryak PRL 102 (2009)]
T- dependence of the Drag Coefficient

Drag Coefficient

- **pQCD** (*Combridge cross-section*)
  \[ \alpha_{pQCD} = \frac{4\pi}{11\ln(2\pi T \Lambda^{-1})} \]

- **AdS/CFT**
  \[ \text{AdS/CFT} = k \frac{T^2}{M} \]
  [Akamatsu et al. PRC79 (09) 054907]
  [S. K. Das PRC89 (2014) 054912]

- **Quasi-Particle-Model** (*fit lQCD e,P*)
  \[ m_q^2 = \frac{1}{6} \left( N_c + \frac{1}{2} N_f \right) g^2 T^2 \]
  \[ m_g^2 = \frac{N_c - 1}{8N_c} g^2 T^2 \]
  \[ g_{QPM}(T) = \frac{4\pi^2}{(11N_c - 2N_f) \ln \left[ \lambda \left( \frac{T_c}{T} - \frac{T}{T_c} \right) \right]^2} \]
  [S. Plumari et al PRD 84 094004 (2011)]

- **\( \alpha_{QPM}(T), \ m_{q,g} = 0 \)**
  we mean simply the coupling of the QPM, but with a bulk of massless q and g

T- dependence of the Drag Coefficient

Drag Coefficient

- **pQCD** (*Combridge cross-section*)
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  \[ \frac{T^2}{M} = k \]
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- **Quasi-Particle-Model** (*fit to QCD e,P*)
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- **aQPM(T) , m_{q,g}=0**
  we mean simply the coupling of the QPM, but with a bulk of massless q and g

Model independent discussion within a Fokker-Planck dynamics

It is only a way to have different Temperature Dependence of the drag coefficient
Impact of T-dependence of the Drag

Au+Au@200AGeV, b=8 fm

Interaction rescaled to have very similar $R_{AA}$ for all the cases

- $R_{AA}(p_T)$ well reproduced whatever is the T-dependence
- At fixed $R_{AA}(p_T)$ -> $v_2(p_T)$ is larger if $\gamma$ is larger at low $T$

Impact of T-dependence of the Drag

LHC - Pb+Pb@2.76ATeV

- Similar trends as for RHIC case

**Boltzmann approach**

\[ p^\mu \partial_\mu f(\mathbf{x}, \mathbf{p}) + M(X) \partial_\mu M(X) \partial^\mu f(X, \mathbf{p}) = C_{22} \]

Free-streaming  
Mean Field  
Collisions

Describes the evolution of the one-body distribution function \( f(\mathbf{x}, \mathbf{p}) \)

It is valid to study the evolution of both bulk and Heavy quarks

To solve numerically the B-E we divide the space into a 3-D lattice and we use the standard test particle method to sample \( f(\mathbf{x}, \mathbf{p}) \)

\[ C_{22} = \int d^3k [\omega(p + k, k) f(p + k) - \omega(p, k) f(p)] \]

The Collision integral is solved by means of a stochastic algorithm

\[ \omega(p, k) = g \int \frac{d^3q}{(2\pi)^3} f'(q) v_{rel} \sigma_{p,q \rightarrow p-k,q+k} \]
Boltzmann vs Fokker Planck approach

The Fokker Planck eq can be derived from the B-E making an expansion of the collision integral in terms of the transferred momentum $k$

B-E

\[
\left( \frac{\partial}{\partial t} + \frac{p}{E} \frac{\partial}{\partial x} \right) f(x, p, t) = C_{22}
\]

\[
C_{22} = \int d^3k \left[ \omega(p + k, k)f(p + k) - \omega(p, k)f(p) \right]
\]

If $|k| \ll |P|

\[
\omega(p + k, k)f(p + k) \approx \omega(p, k)f(p) + k \frac{\partial}{\partial p} (\omega f) + \frac{1}{2} k_i k_j \frac{\partial^2}{\partial p_i \partial p_j} (\omega f)
\]

F-P

\[
\frac{\partial f}{\partial t} = \frac{\partial}{\partial p_i} \left[ A_i(p) f + \frac{\partial}{\partial p_j} \left[ B_{ij}(p) f \right] \right]
\]

The Fokker-Planck equation is equivalent to an ordinary stochastic differential equation

\[
dx_i = \frac{p_j}{E} dt
\]

\[
dp_j = -\Gamma p_j dt + \sqrt{dt} C_{jk}(t, p + \xi dp) \rho_k
\]
Evaluation of Drag and Diffusion

Common approach between LV and BM

The infrared singularity is regularized introducing a Debye-screaning-mass $m_D$

\[
\frac{1}{t} \rightarrow \frac{1}{t - m_D}
\]

Langevin approach

For Collision Process the $A_i$ and $B_{ij}$ can be calculated as following:

\[
A_i = \frac{1}{2E_p} \int \frac{d^3 q}{(2\pi)^3 2E_q} \int \frac{d^3 q'}{(2\pi)^3 2E_q'} \int \frac{d^3 p'}{(2\pi)^3 2E_p'} \frac{1}{\gamma_c} \sum |M|^2 (2\pi)^4 \delta^4(p + q - p' - q') f(q)[(p - p')]_i = \langle \langle (p - p')_i \rangle \rangle
\]

\[
B_{ij} = \frac{1}{2} \langle \langle (p - p')_i (p' - p)_j \rangle \rangle
\]

Boltzmann approach

\[
M \rightarrow \sigma
\]

\[
\sigma_{gc \rightarrow gc} = \frac{1}{16\pi(s - M_c^2)^2} \int_0^0 \int_{(s - M_c^2)^2/s} dt \sum |M|^2
\]
Boltzmann vs Langevin

static medium

T=400 MeV

We have considered different average momentum transferred $\leftrightarrow$ different $m_D$
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We have plotted the results as a ratio between:

$$\frac{\frac{dN}{d^3p}}{\text{Langevin}} \div \frac{\frac{dN}{d^3p}}{\text{Boltzmann}}$$

at different time

to quantify how much the ratio differs from 1
The differences between BM and LV depends on:

- **Average momentum transferred**
- **Mass**

[S. K. Das, F. Scardina, V. Greco PRC90 044901 (2014)]
$R_{AA}$ and $v_2$ Boltzmann vs Langevin

Au+Au@200AGeV, $b=8$ fm

✓ Fixed same $R_{AA}(p_T)$ [reduce the drag by 40%]

Same $R_{AA}$ but different $v_2$

The differences between F-P and BM are larger for a more differential observable like the $v_2$

[S. K. Das, F. Scardina, V. Greco PRC90 044901 (2014)]
Impact of hadronization mechanism

Impact of hadronization

Coalescence increase
both $R_{AA}$ and $v_2$

reverse the correlation
toward agreement with data

Hees-Mannarelli-Greco-Rapp, PRL100 (2008)

$$f_q \text{ from } \pi, K$$
Greco,Ko,Levai - PRL90
Summary on the build-up of $v_2$ at fixed $R_{AA}$

$R_{AA}$ and $V_2$ are correlated but still one can have $R_{AA}$ about the same while $V_2$ can change up to a factor 3:

$\gamma(T)$ + Boltzmann dynamics + hadronization
LV vs BM approach: Energy loss of a single HQ

Langevin vs Boltzmann approach with $T=400$ MeV.

$M_c/T \approx 3$
$M_b/T \approx 10$

Langevin vs Boltzmann angular correlation

Initially the $c\bar{c}$ and $b\bar{b}$ are distributed back to back (LO)

We have fixed the RAA on exp. data for both the two approaches

A difference of an order of magnitude
Langevin vs Boltzmann angular correlation

Initially the $c\bar{c}$ and $b\bar{b}$ are distributed back to back (LO).

There are no differences at RHIC.

Significant differences at LHC.
Summary

✓ The exp. data for $R_{AA}$ and $v_2$ seem to indicate an interaction about constant in $T$

✓ The more one looks at differential observables $R_{AA} \rightarrow V_2 \rightarrow dN_{cc}/d\Delta \phi$ the more the differences between the BM and F-P approach increases

✓ We can realize that charm in hot QGP is not that heavy and the motion not really Brownian

✓ Very similar dynamics between F-P and BM for Bottom at least for $R_{AA}$ and $V_2$, not negligible differences instead for azimuthal correlations especially at LHC