Shear Viscosity to Electric Conductivity ratio of the QGP

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arxiv:1407.2559, Phys. Rev. D90 (2014) 11, 114009
Phys. Rev. C86 (2012) 054902

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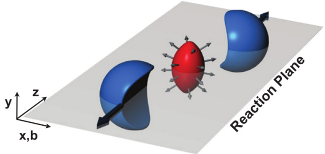
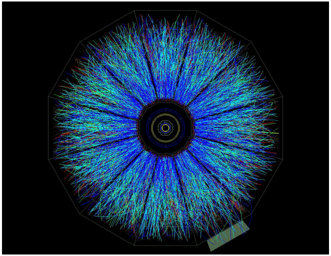
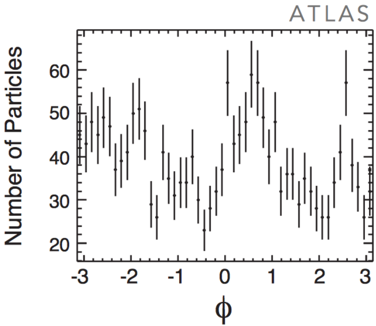
Collective Flows





First Au-Au @ 100 GeV, RHIC, STAR



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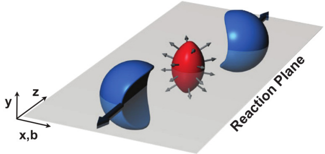
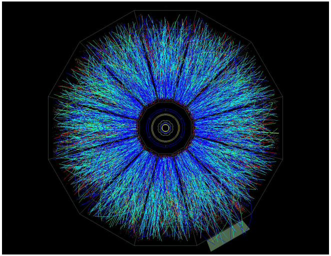




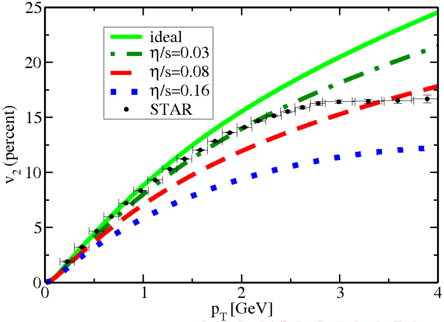
First Au-Au @ 100 GeV, RHIC, STAR

$$\frac{1}{p_T}\frac{d^3N}{dp_T\,dy\,d\phi} = \frac{1}{2\pi p_T}\frac{d^2N}{dp_T\,dy}\left(1 + 2\sum_{n=1}^{\infty} v_n(p_T, y)\cos[n(\phi - \psi_R)]\right)$$

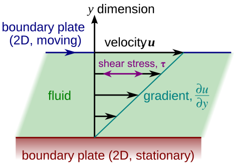
Elliptic Flow: $n = 2$

$$v_2(p_T, b) = \left\langle \frac{p_x^2 - p_y^2}{p_x^2 + p_y^2} \right\rangle$$

P. Romatschke and U. Romatschke, Phys. Rev. Lett 99, 172301 (2007)



Shear Viscosity



Shear Stress

$$\tau = \frac{F_x}{A_{yz}} = -\eta \frac{\partial u_x}{\partial y}$$

fluid	$P\,[Pa]$	$T\,[K]$	$\eta\,[Pa \cdot s]$	$\eta/s\,[\hbar/k_B]$
H_2O	$0.1 \cdot 10^6$	370	$2.9 \cdot 10^{-4}$	8.2
^4He	$0.1 \cdot 10^6$	2.0	$1.2 \cdot 10^{-6}$	1.9
H_2O	$22.6 \cdot 10^6$	650	$6.0 \cdot 10^{-5}$	2.0
^4He	$0.22 \cdot 10^6$	5.1	$1.7 \cdot 10^{-6}$	0.7
QGP	$88 \cdot 10^{33}$	$2 \cdot 10^{12}$	$\leq 5 \cdot 10^{11}$	≤ 0.4

T.Schäfer, D. Teaney, Rep. Prog. Phys **72** (2009) 126001

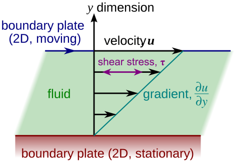
Viscosity ⟷ microscopic details ?

$$\eta \rightarrow \lambda, \sigma, \langle p \rangle \ldots$$

Pitch drop experiment

- started in 1927
- 8th drop on 28th November 2000
- $\eta_{pitch} = 2.3 \cdot 10^{11}\, \eta_{H_2O} \sim 6.67 \cdot 10^7\, Pa\, s$
- $\eta_{pitch} \ll \eta_{QGP}$
- η/s

Shear Viscosity



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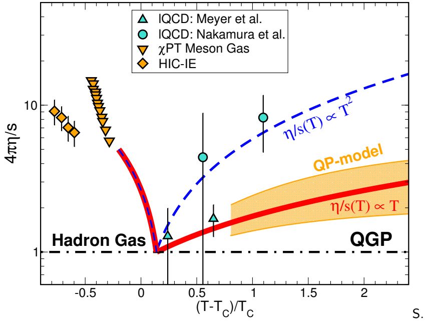
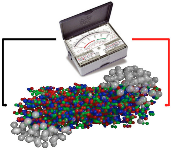


Plumari et al: arXiv:1304.6566v1

- QM: $\eta/s \simeq \frac{4}{15}\langle p \rangle \tau \Rightarrow \eta/s > \frac{1}{15}$
- AdS/CFT: $\eta/s = \frac{1}{4\pi}$

Electric Conductivity



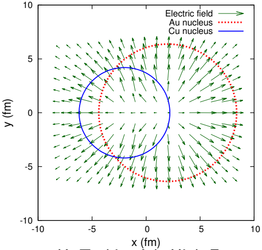
Ohm's Law

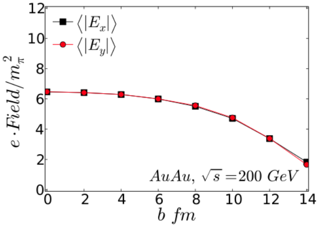
$$I = \frac{V}{R} \Rightarrow j = \sigma_{el} E$$

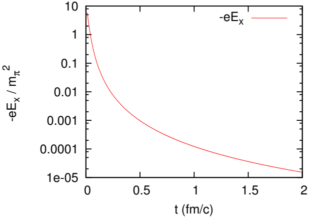
$$eE \simeq (m_\pi)^2 \to E \sim 10^{21}\, V/cm$$

Old unit: Mho

℧







K. Tuchin, Adv.High Energy Phys. 2013 (2013) 490495   Y. Hirono, M. Hongo, T. Hirano, arXiv:1211.1114 (2012)

Direct Flow

$$v_1 = \left\langle \frac{p_x}{p_T} \right\rangle$$

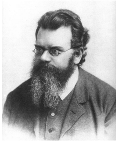
Photon rate

$$\omega \frac{d\Gamma_\gamma}{d^3p} = \frac{\alpha_{EM}}{\pi^2 e^2} \frac{\sigma_{el}\,\omega}{e^{\omega/T} - 1}$$

Lattice QCD

$$G_{\mu\nu}(\tau, T) = \int d^3x \, \langle J_\mu(\tau, \mathbf{x}) J_\nu(0, \mathbf{0})^\dagger \rangle$$

Relativistic Transport Equation



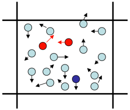
Relativistic Boltzmann Transport Equation

$$p^{\mu}\partial_{\mu} f(x,p) = \mathcal{C}_{22}[f]$$

$$\mathcal{C}_{22} = \frac{1}{2E_1}\int \frac{d^3p_2}{(2\pi)^3 2E_2}\frac{1}{\nu}\int \frac{d^3p_1'}{(2\pi)^3 2E_1'}\frac{d^3p_2'}{(2\pi)^3 2E_2'} f_1' f_2' \times$$
$$|\mathcal{M}_{1'2'\to 12}|^2 (2\pi)^4 \delta^{(4)}(p_1'+p_2'-p_1-p_2)$$
$$-\frac{1}{2E_1}\times\int \frac{d^3p_2}{(2\pi)^3 2E_2}\frac{1}{\nu}\int \frac{d^3p_1'}{(2\pi)^3 2E_1'}\frac{d^3p_2'}{(2\pi)^3 2E_2'} f_1 f_2 \times$$
$$|\mathcal{M}_{12\to 1'2'}|^2 (2\pi)^4 \delta^{(4)}(p_1+p_2-p_1'-p_2')$$

gain

loss



Test-particle method

$f(x,p) = \sum_{i=1}^{N} \delta^4(x_i(t)-x)\delta^4(p_i(t)-p)$

$N = N_{real} \times N_{test}$, $\sigma \to \sigma/N_{test}$

Stochastic method

$$P_{22} = \frac{\Delta N_{coll}^{2\to 2}}{\Delta N_1 \Delta N_2} = v_{rel}\frac{\sigma_{22}}{N_{test}}\frac{\Delta t}{\Delta^3 x}$$

if $P_{22} > rand()$ collision takes place

$v_{rel} = \sqrt{(p_1 \cdot p_2)^2 - m_1^2 m_2^2}/E_1 E_2$

Z. Xu and C. Greiner, Phys.Rev. C71 (2005) 064901

Transport Coefficients: Green-Kubo

Transport coefficients: $\eta, \zeta, k, D, \sigma_{el}$ characterize the non-equilibrium behaviour of a system.

Fluctuation-Dissipation Theorem

It establishes a relation between equilibrium fluctuations of a physical observable and a dissipative process that takes place when the system is perturbed from equilibrium.

Green-Kubo Relation

$$A = \frac{V}{T}\int_0^\infty dt\, \langle J(t)J(0)\rangle$$

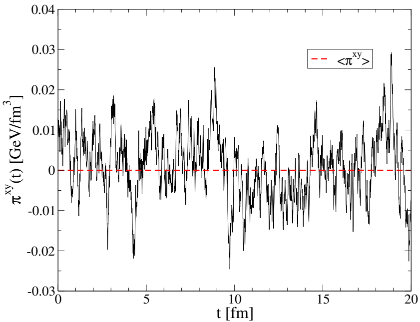
$$\eta \Rightarrow J = \pi^{xy} = -\eta\frac{\partial u_x}{\partial y}$$

$$\sigma_{el} \Rightarrow J = \sigma_{el}E$$

$$D \Rightarrow J = -D\nabla\rho$$

$$\zeta \Rightarrow J = P$$

$$k \Rightarrow J = Q = -k\nabla T$$

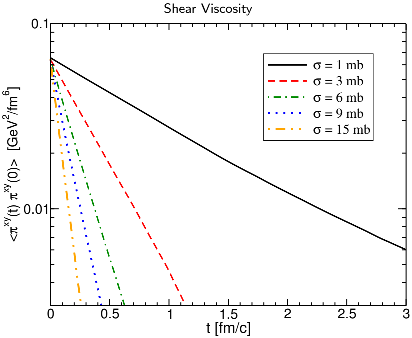


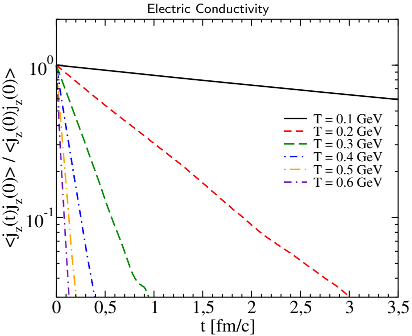
Transport Coefficients: Green-Kubo

Correlation Functions

$$\langle \pi^{xy}(t)\pi^{xy}(0)\rangle = \langle \pi^{xy}(0)^2\rangle e^{-t/\tau} \Longrightarrow \eta = \frac{V}{T}\int_0^\infty dt\, \langle \pi^{xy}(t)\pi^{xy}(0)\rangle = \frac{V}{T}\langle \pi^{xy}(0)^2\rangle \tau$$

$$\langle J_z(t)J_z(0)\rangle = \langle J_z(0)^2\rangle e^{-t/\tau} \Longrightarrow \sigma_{el} = \frac{V}{T}\int_0^\infty dt\, \langle J_z(t)J_z(0)\rangle = \frac{V}{T}\langle J_z(0)^2\rangle \tau$$





C. Wesp et al, Phys.Rev. C84 (2011) 054911, S. Plumari, A. Puglisi et al., Phys.Rev. C86 (2012) 054902, A. Puglisi et al, arXiv: 1408.7043, M. Greif et al., arXiv:1408:7049

Fixing the Thermodynamics

General Formulas from Transport Theory

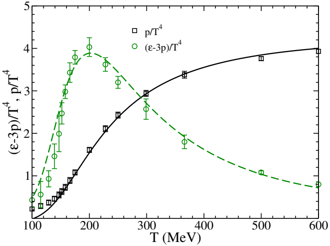
$$\eta/s = \frac{1}{15\,T\,s}\left(\sum_q \int \frac{d^3p}{(2\pi)^3}\frac{p^4}{E^2}\tau_q f(p) + \int \frac{d^3p}{(2\pi)^3}\frac{p^4}{E^2}\tau_g f(p)\right) = \underbrace{\frac{1}{15\,Ts}\left\langle \frac{p^4}{E^2}\right\rangle}_{\text{Thermodynamics}} \overbrace{\left(\tau_q \rho_q^{tot} + \tau_g \rho_g\right)}^{\text{Dynamics}}$$

$$\frac{\sigma_{el}}{T} = \frac{e^2}{3\,T^2}\sum_{j=q,\bar{q}} q_j^2 \int \frac{d^3p}{(2\pi)^3}\frac{\vec{p}^2}{E^2}\tau_i f(p) = \underbrace{\frac{e_\star^2}{3T^2}\left\langle \frac{p^2}{E^2}\right\rangle}_{\text{Thermodynamics}} \overbrace{\tau_q}^{\text{Dynamics}} \rho_q$$

Thermodynamics from Lattice QCD, S. Plumari et al. PRD 84, 094004 (2011)

$$P(T) = \sum_{i=q,g} d_i \int \frac{d^3p}{(2\pi)^3}\frac{\vec{p}^2}{3E_i(p)} f_i(p) - B(T)$$

$$\epsilon(T) = \sum_i d_i \int \frac{d^3p}{(2\pi)^3} E_i f(p) + B(m_i(T))$$



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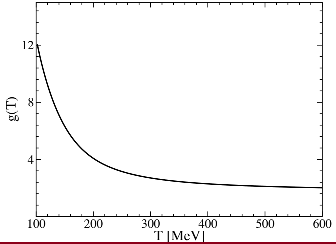
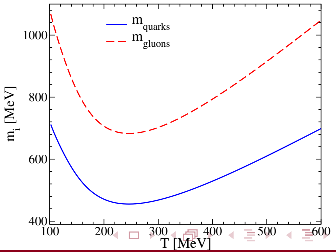
$$g^2(T) = \frac{48\pi^2}{(11N_c - 2N_f)\log\left[\lambda\left(\frac{T}{T_c} - \frac{T}{T_s}\right)\right]^2}$$

$$m_g^2(T) = \frac{1}{6}g^2\left(N_c + \frac{1}{2}N_f\right)T^2, \quad m_q^2(T) = \frac{N_c^2-1}{8N_c}g^2T^2$$





Shear Viscosity

Fixing the Dynamics

$$\tau_{q,tr}{}^{-1} = \langle\sigma(s)_{tr} v_{rel}\rangle \left(\rho_q \sum_{i=u,d,s}^{\bar{u},\bar{d},\bar{s}} \beta^{qi} + \rho_g \beta^{qg}\right)$$

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$$\tau \sim \frac{1}{\rho\sigma}$$

$$\sigma_{tot}^{ij}(s) \sim \beta^{ij} \frac{\alpha_s^2}{m_D^2(T)} \frac{s}{(s + m_D^2(T))}$$

ij	β
$qq \to qq$	$2\,\frac{8\pi}{9}$
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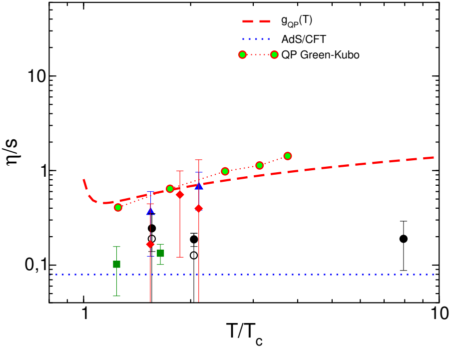

$$g_{QP}^2(T) = \frac{48\pi^2}{(11N_c - 2N_f)\log\left[\lambda\left(\frac{T}{T_c} - \frac{T}{T_s}\right)\right]^2}$$

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- GK vs RTA
- QP: $\eta/s \sim 6 \times (1/4\pi)$

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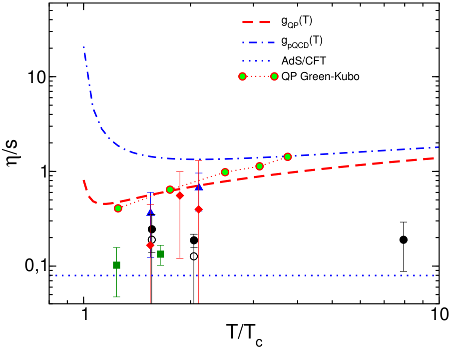

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$$g_{pQCD}^2 = \frac{16\pi^2}{\left(11 - \frac{2}{3}N_f\right)\log\left[\frac{2\pi T}{\Lambda_{QCD}}\right]^2}$$

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Electric Conductivity

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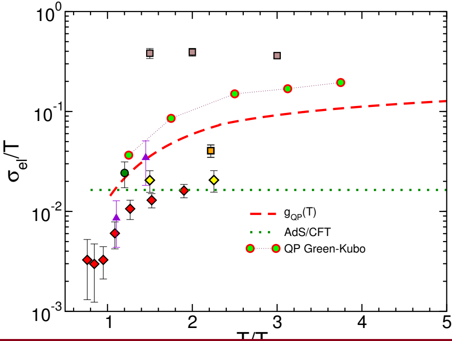
A. Puglisi et al., PRD 90(2014)11,114009, M. Greif et al., PRD 90(2014)9,094014



- RTA understimates Green-Kubo results: $\sim$ 1.7 factor
- $\sigma_{el}^{QP} \sim 4 \times \sigma_{el}^{Lattice}$

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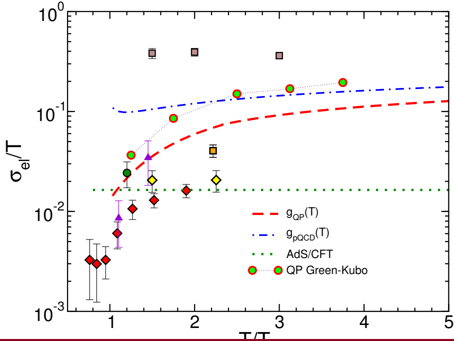

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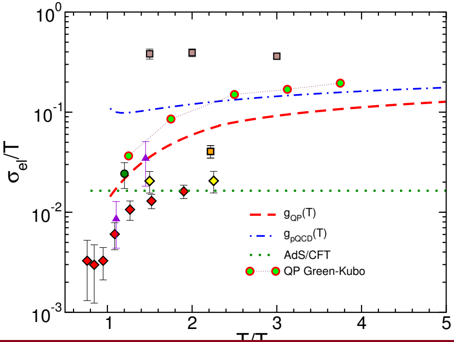


T-dependence

$$\langle p^4/E^2 \rangle \simeq \epsilon T/\rho \Rightarrow \quad \eta/s \simeq \tau\rho/T^2$$

$$\langle p^2/E^2 \rangle \simeq T/m(T) \Rightarrow \quad \sigma_{el}/T \simeq T/m(T)\eta/s$$

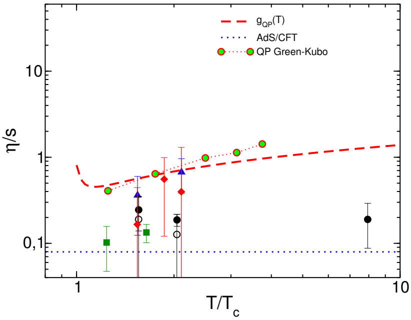
- extra T-dependence: $m(T)$ increases as $T \to T_c$ (from Lattice)
- Conformal Theory (or massless particles): $T^\mu_\mu = \epsilon - 3P = 0 \Rightarrow \sigma_{el}/T \sim \eta/s$
- $\epsilon - 3P > 0$ as the origin of extra T-dependence

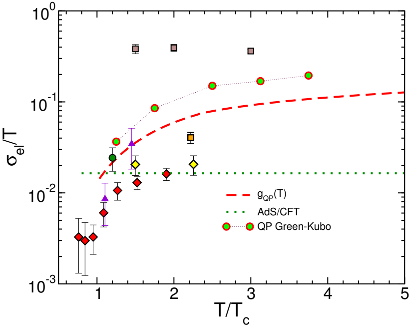
Setting relaxation times to reproduce the minimum of $\eta/s = 1/4\pi$: K factor

$$\sigma^{ij}_{tot}(s) \sim K\beta^{ij} \frac{\alpha_s^2}{m_D^2(T)} \frac{s}{(s+m_D^2(T))}$$

$$\eta/s = \frac{1}{15Ts} \left\langle \frac{p^4}{E^2} \right\rangle (\tau_q \rho_q^{tot} + \tau_g \rho_g)$$

$$\frac{\sigma_{el}}{T} = \frac{e_\star^2}{3T^2} \left\langle \frac{p^2}{E^2} \right\rangle \tau_q \rho_q$$





Lattice: A. Amato et al. 2013 *Phys. Rev. Lett.* **111** 172001

Setting relaxation times to reproduce the minimum of $\eta/s = 1/4\pi$: K factor ~ 6.36

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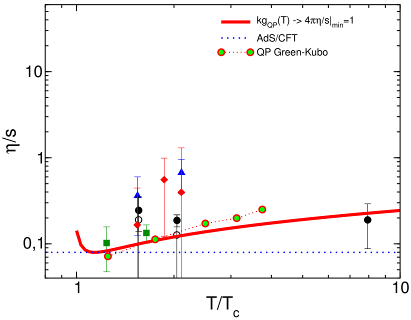
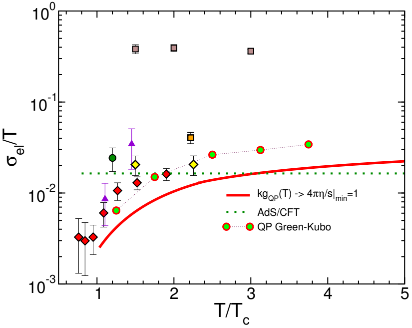




σ_{el}/T consistent with $\eta/s = 1/4\pi$

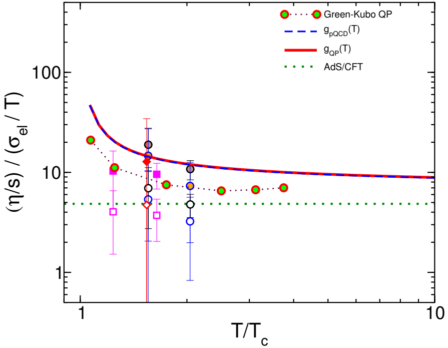
Lattice: A. Amato et al. 2013 *Phys. Rev. Lett.* **111** 172001

Shear Viscosity η to Electric Conductivity σ_{el} ratio

Taking the Ratio

$$\frac{\eta/s}{\sigma_{el}/T} = \frac{6}{5}\,\frac{T\langle p^2/E^2\rangle^{-1}}{s e_\star^2}\left\langle \frac{p^4}{E^2}\right\rangle\left(1+\frac{\tau_g}{\tau_q}\,\frac{\rho_g}{\rho_q^{tot}}\right),$$

- $C^q = (\beta^{qq} + \beta^{q\bar q} + 2\beta^{q\bar q'} + 2\beta^{qq'})/\beta^{qg}$
- $C^q|_{pQCD} = \frac{28}{9} \simeq 3.1$
- $C^g = \beta^{gg}/\beta^{qg}$
- $C^g|_{pQCD} = \frac{9}{2}$



Ratio

- independent on K factor
- independent on $\alpha_s(T)$
- not sensitive to gluon scatterings (C^g)
- sensitive only on quarks scatterings (C^q)
- increases near T_c
- constant value for $T \gg T_c$
- Conformal Theory: flat behaviour
- σ_{el}/T extra T-dependence

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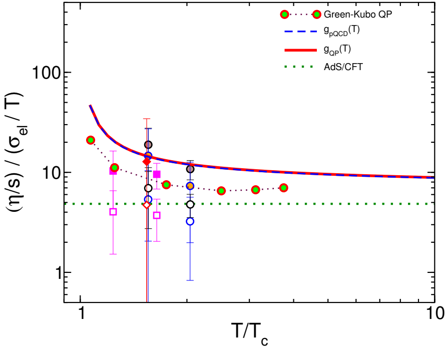

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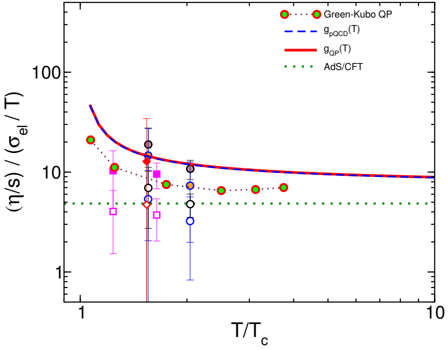

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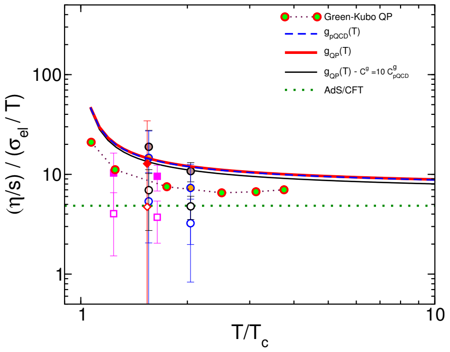

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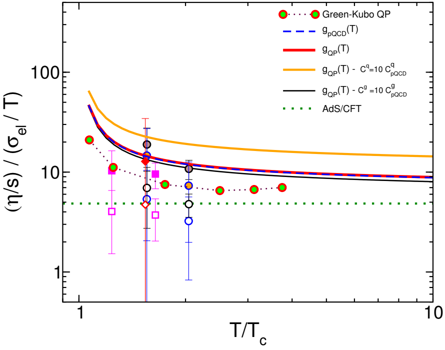

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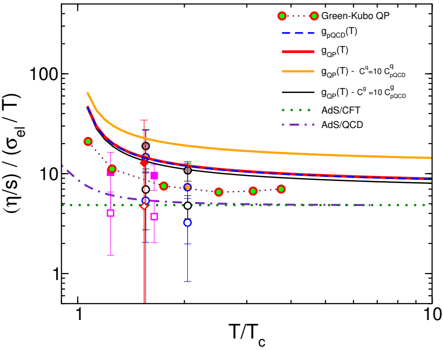

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AdS/QCD: J. Noronha et al, Phys. Rev. D 89

Conclusions

Shear Viscosity and Electric Conductivity results

- η/s: Relaxation Time Approximation in agreement with Green-Kubo results
- σ_{el}/T: Relaxation Time Approximation underestimates Green-Kubo results
- extra T-dependence for σ_{el}
- Fixing the minimum of $\eta/s = 1/4\pi \Longrightarrow \sigma_{el}/T$ in agreement with recent Lattice QCD data.

Ratio $(\eta/s)/(\sigma_{el}/T)$

- depends only on quark scatterings sector
- increases near T_c contrary to the constant behaviour of AdS/CFT
- independent on $\alpha_s(T)$
- supplies a measure of the quarks to gluons scattering rates

Thank you!