Shape Phase Transitions Far From Stability

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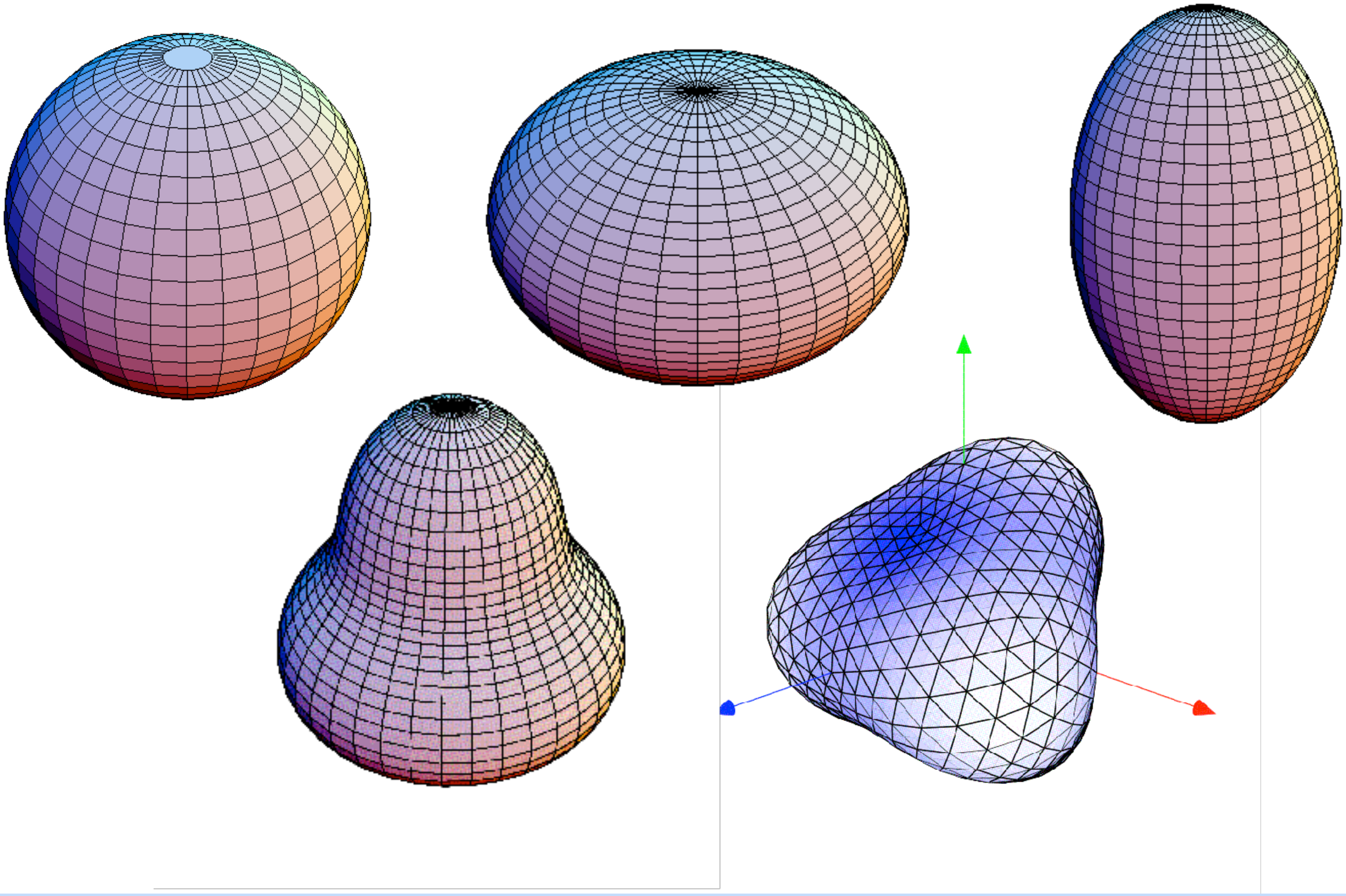
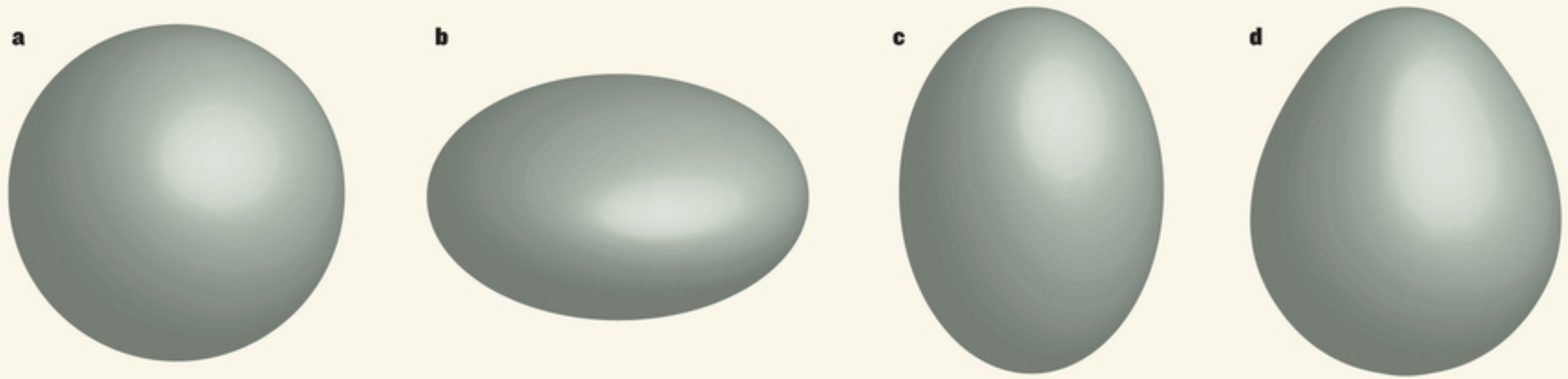
How can nucleonic matter support a diversity of shapes in finite systems?



Evolution of shell structures with nucleon number and/or angular momentum.

Stabilizing effect of closed shells → spherical shape

Deformation-driving part of the effective interaction → $T=0$ $Q_p \cdot Q_n$ force



$^{20}\mathrm{O} \Longrightarrow \nu(1d5/2)^4$

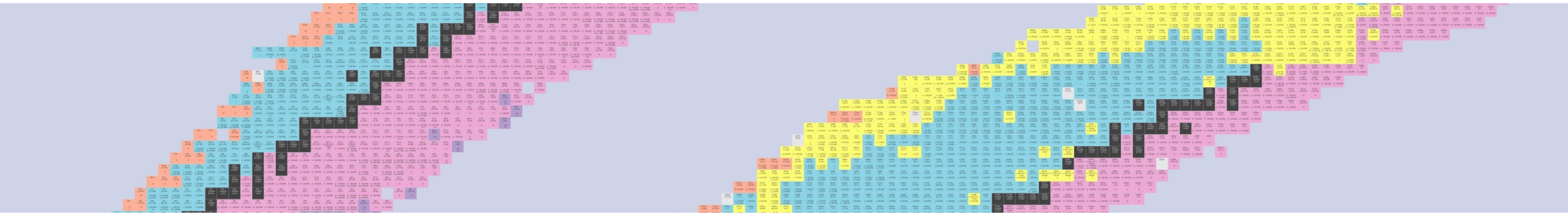
$^{20}\mathrm{Ne} \Longrightarrow \nu(1d5/2)^2\ \pi(1d5/2)^2$

Shape transitions within a single nucleus (shape coexistence) or as a function of nucleon number (shape evolution), present universal phenomena that occur in light, medium-heavy, heavy and superheavy nuclei.

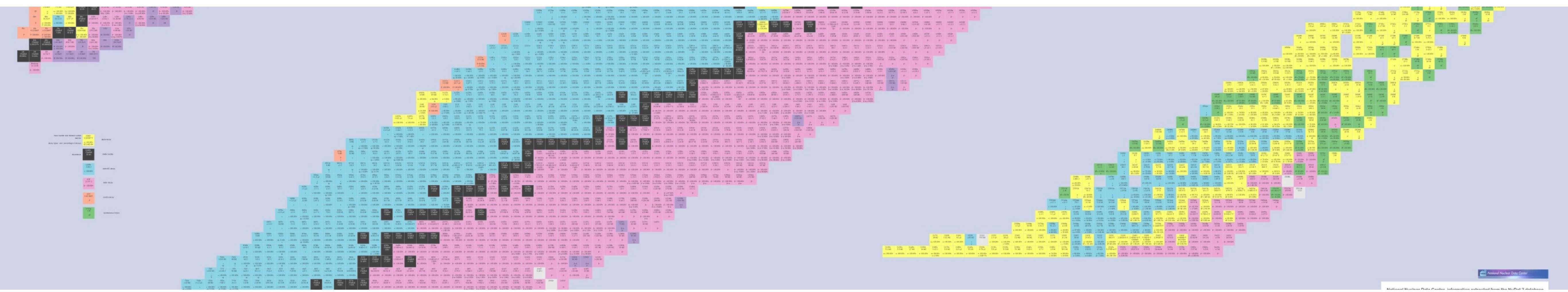
Universal theory framework: Nuclear Energy Density Functionals

Energy Density Functionals

✔ the nuclear many-body problem is effectively mapped onto a ***one-body problem*** without explicitly involving inter-nucleon interactions!



✔ the exact density functional is approximated with ***powers and gradients of ground-state densities and currents.***

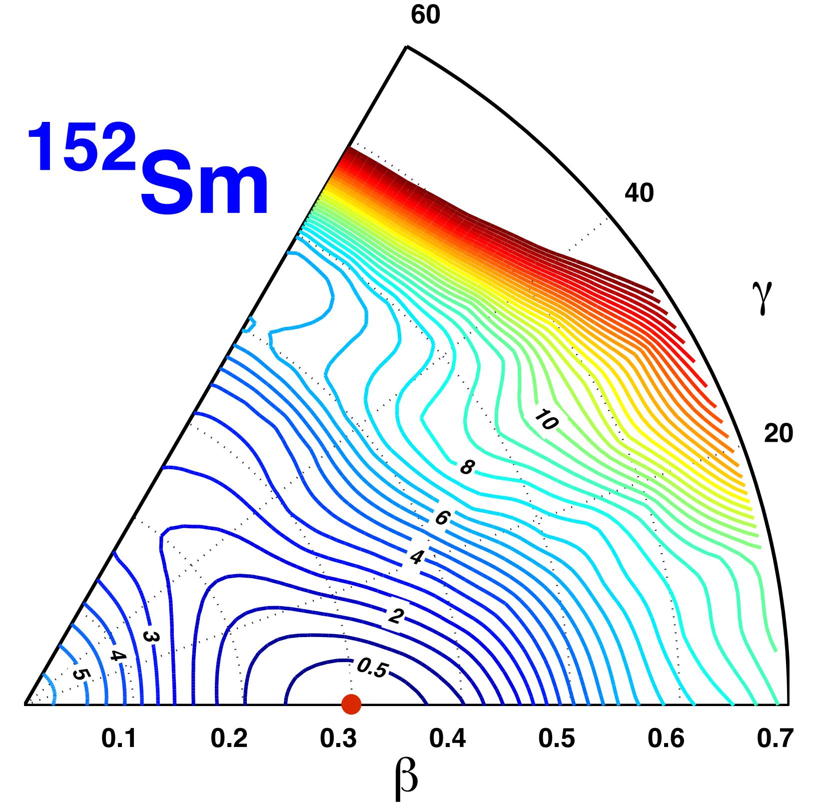


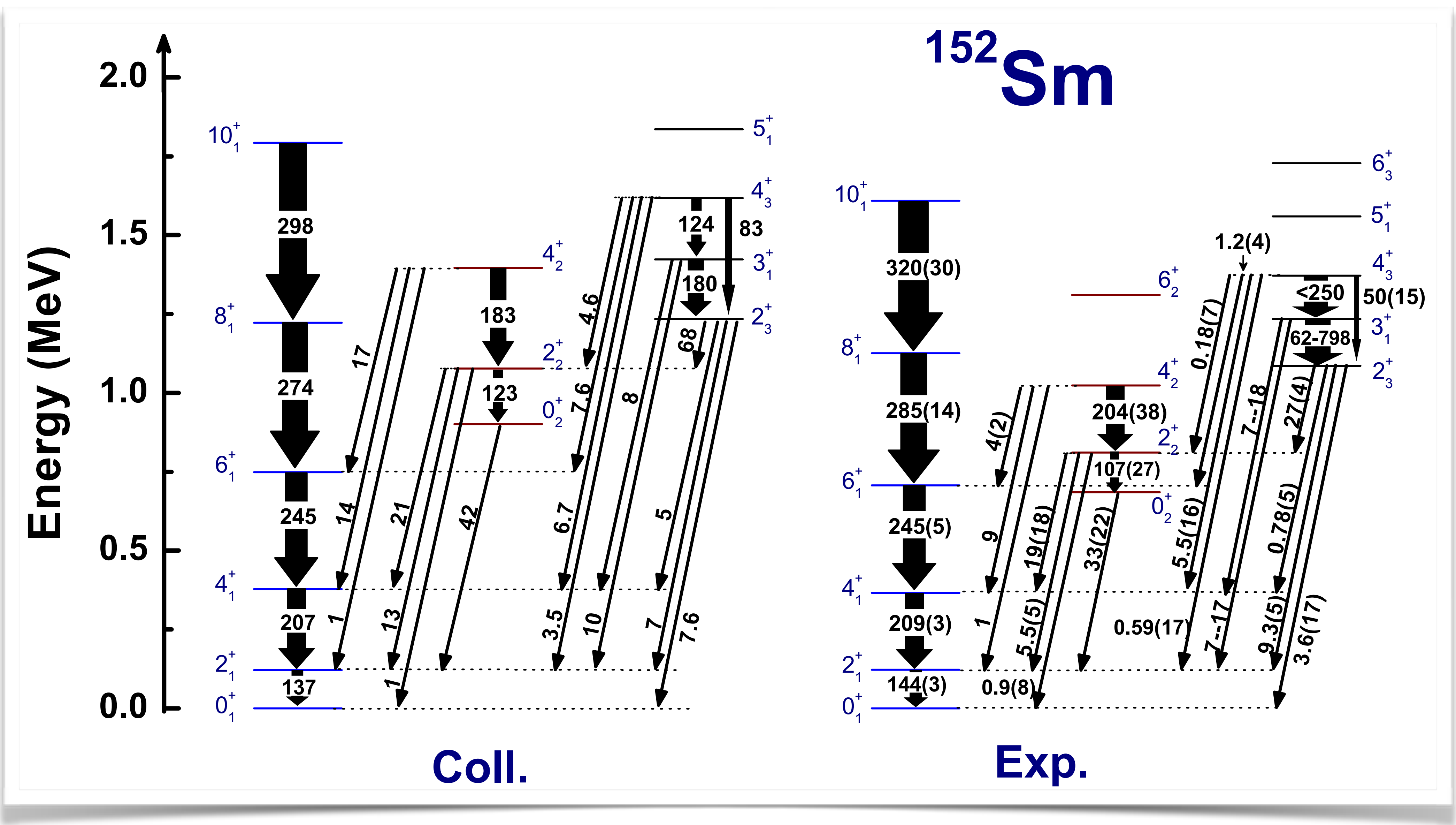
✔ ***universal density functionals*** can be applied to all nuclei throughout the chart of nuclides.

Important for extrapolations to regions far from stability!

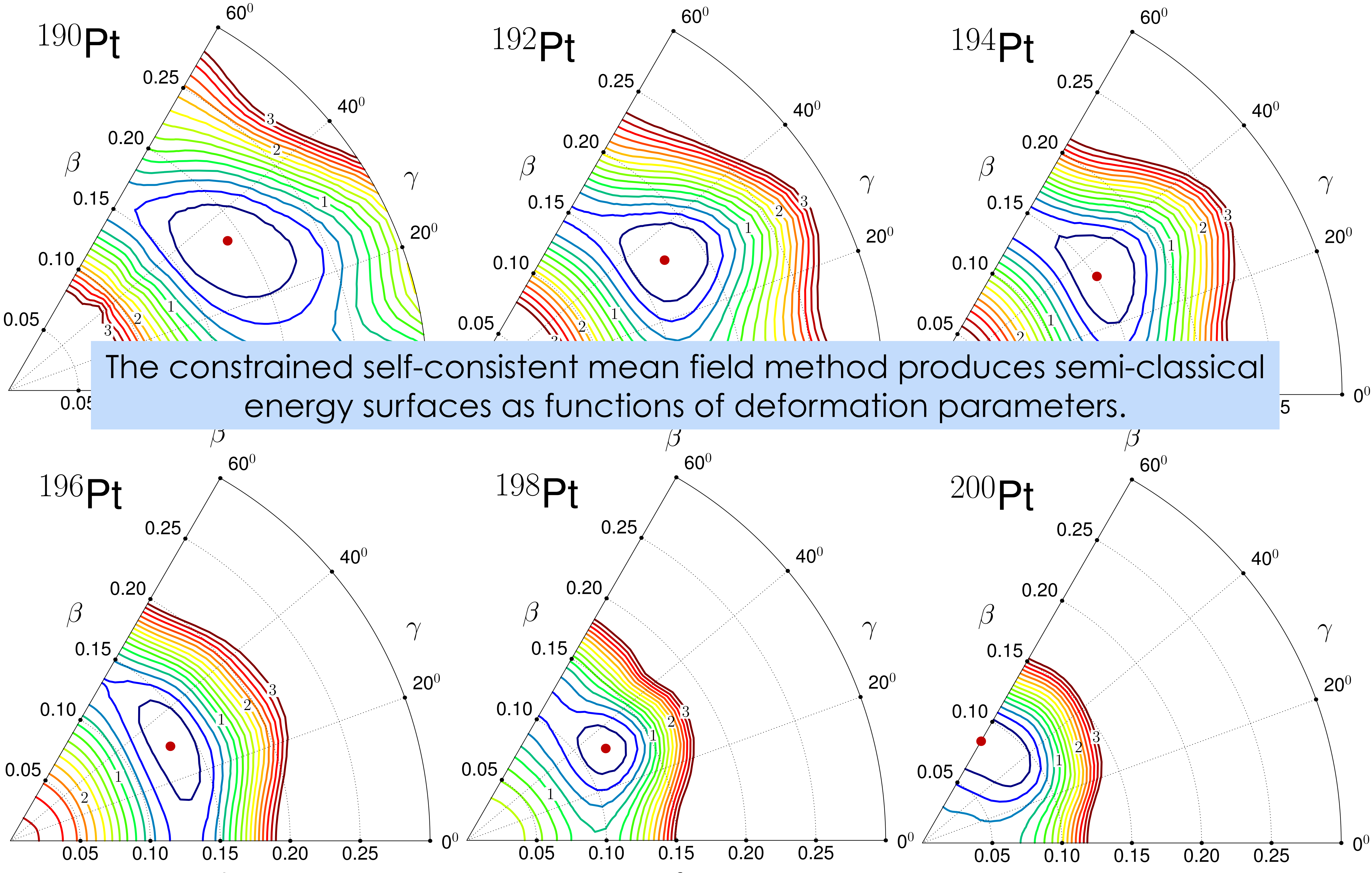
✔ an intuitive interpretation of mean-field results in terms of ***intrinsic shapes*** and ***single-particle states.***

✔ the ***full model space*** of occupied states can be used; no distinction between core and valence nucleons, ***no need for effective charges!***





Evolution of nucleonic shells ⇒ gradual transition between equilibrium shapes



The constrained self-consistent mean field method produces semi-classical energy surfaces as functions of deformation parameters.

Nuclear Many-Body Correlations

short-range
(hard repulsive core of the NN-interaction)

long-range
nuclear resonance modes (giant resonances)

collective correlations
large-amplitude soft modes:
(center of mass motion, rotation, low-energy quadrupole vibrations)

...vary smoothly with nucleon number! Implicitly included in an Energy Density Functional.

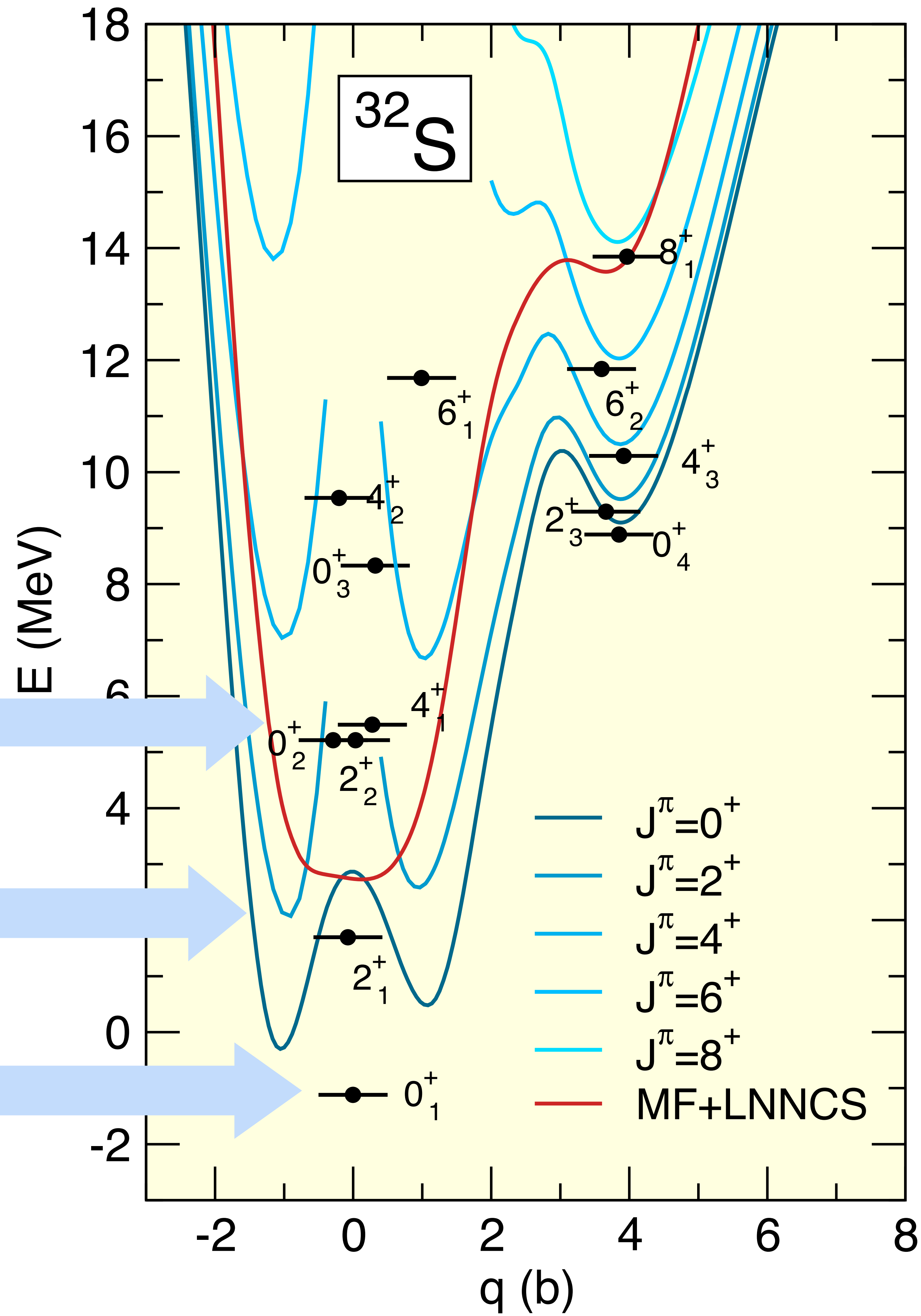
...sensitive to shell-effects and strong variations with nucleon number! Cannot be included in a simple EDF framework.

Restoration of broken symmetries (rotational, particle number) and fluctuations of collective variables (quadrupole deformation).

1. Mean-field calculations, with a constraint on the quadrupole moment.

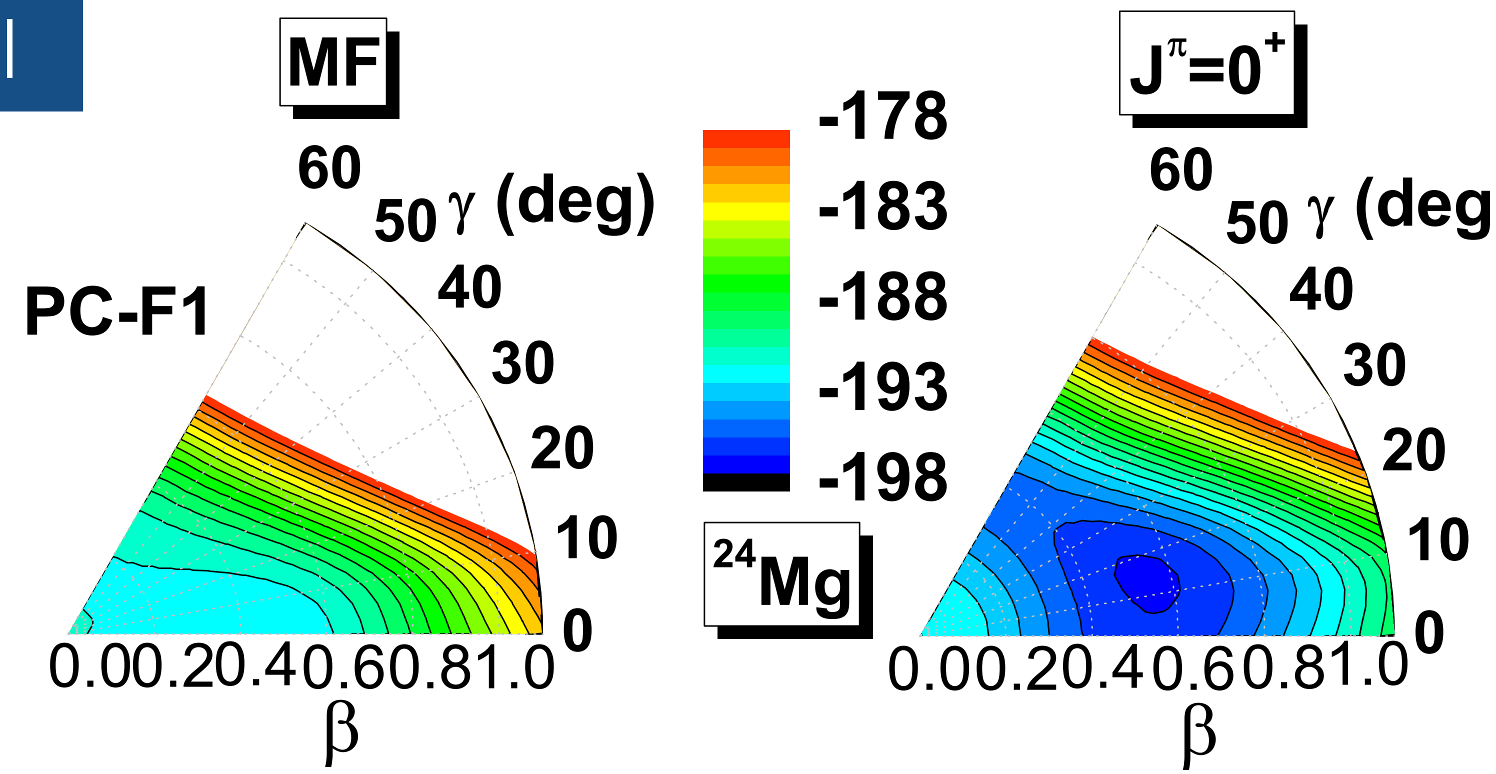
2. Angular-momentum and particle-number projection.

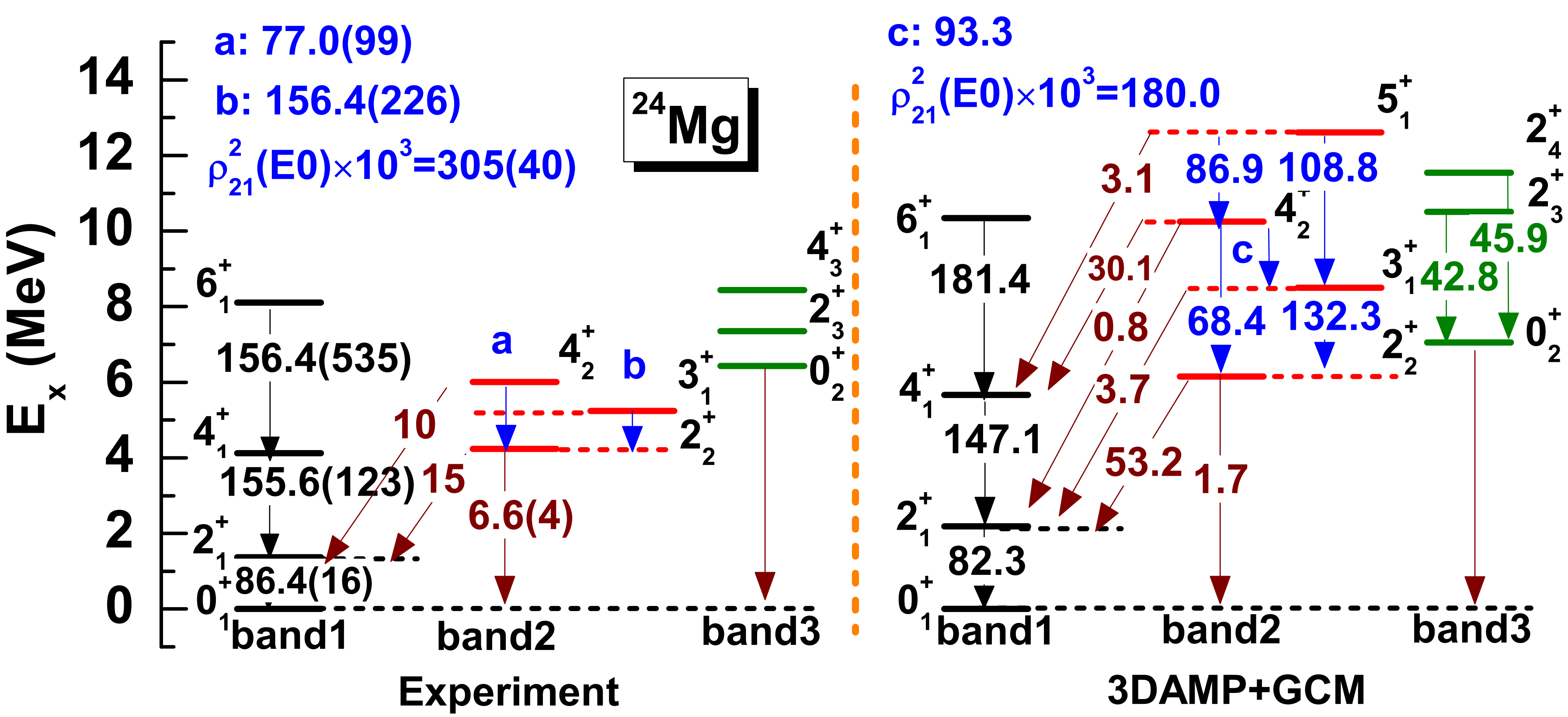
3. Generator Coordinate Method ⇒ configuration mixing



... larger variational space for projected GCM calculations!

3D AMP + GCM model





Yao, Meng, Ring, Vretenar, Phys. Rev. C **81**, 044311 (2010)

Five-dimensional collective Hamiltonian

Nikšić, Li, Vretenar, Prochniak, Meng, Ring, Phys. Rev. C **79**, 034303 (2009).
Nikšić, Vretenar, Ring, Prog. Part. Nucl. Phys. **66**, 519 (2011).

... nuclear excitations determined by quadrupole vibrational and rotational degrees of freedom

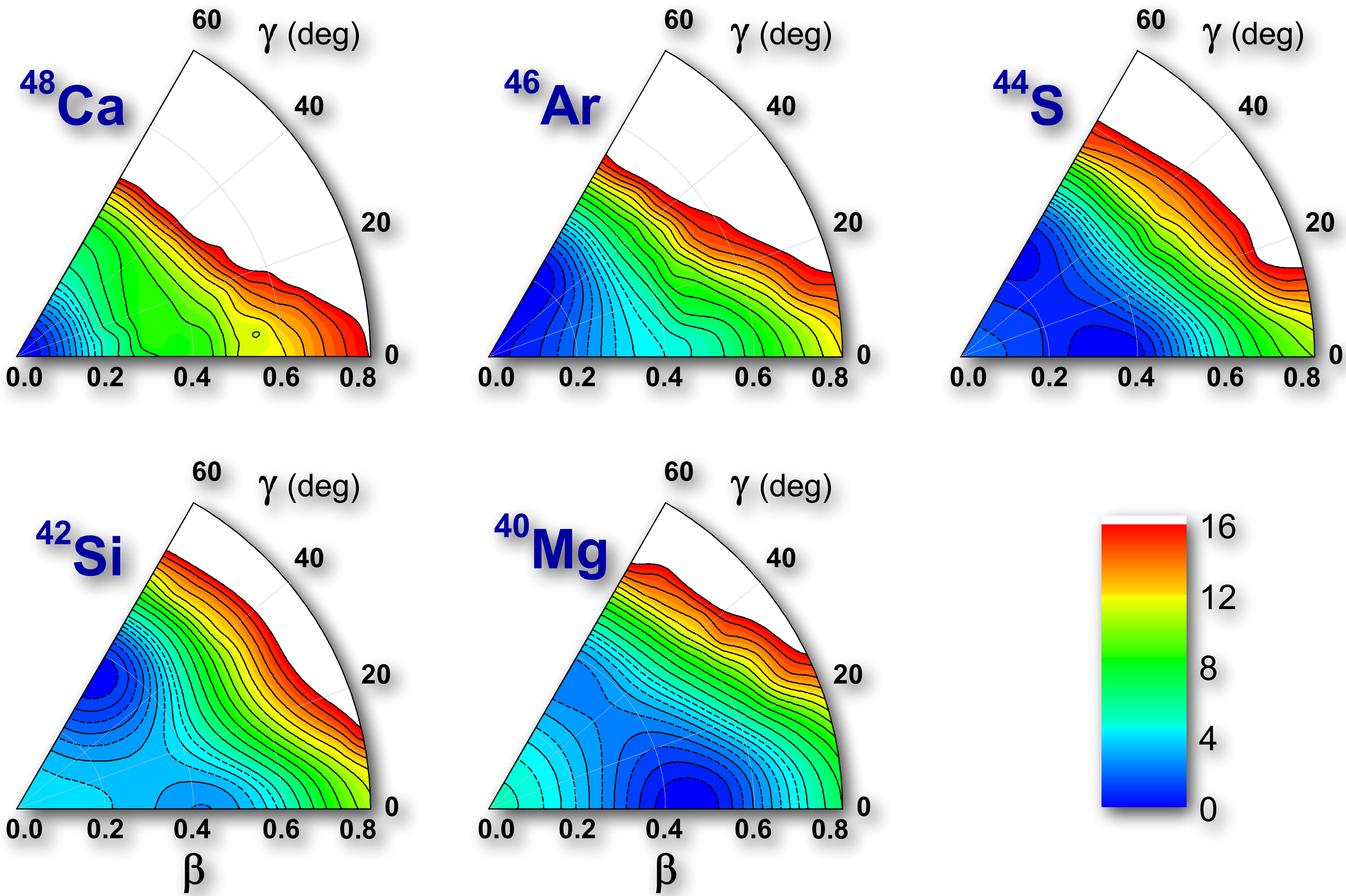
$$H_{\mathrm{coll}} = \mathcal{T}_{\mathrm{vib}}(\beta,\gamma) + \mathcal{T}_{\mathrm{rot}}(\beta,\gamma,\Omega) + \mathcal{V}_{\mathrm{coll}}(\beta,\gamma)$$

$$\mathcal{T}_{\mathrm{vib}} = \frac{1}{2}B_{\beta\beta}\dot{\beta}^2 + \beta B_{\beta\gamma}\dot{\beta}\dot{\gamma} + \frac{1}{2}\beta^2 B_{\gamma\gamma}\dot{\gamma}^2$$

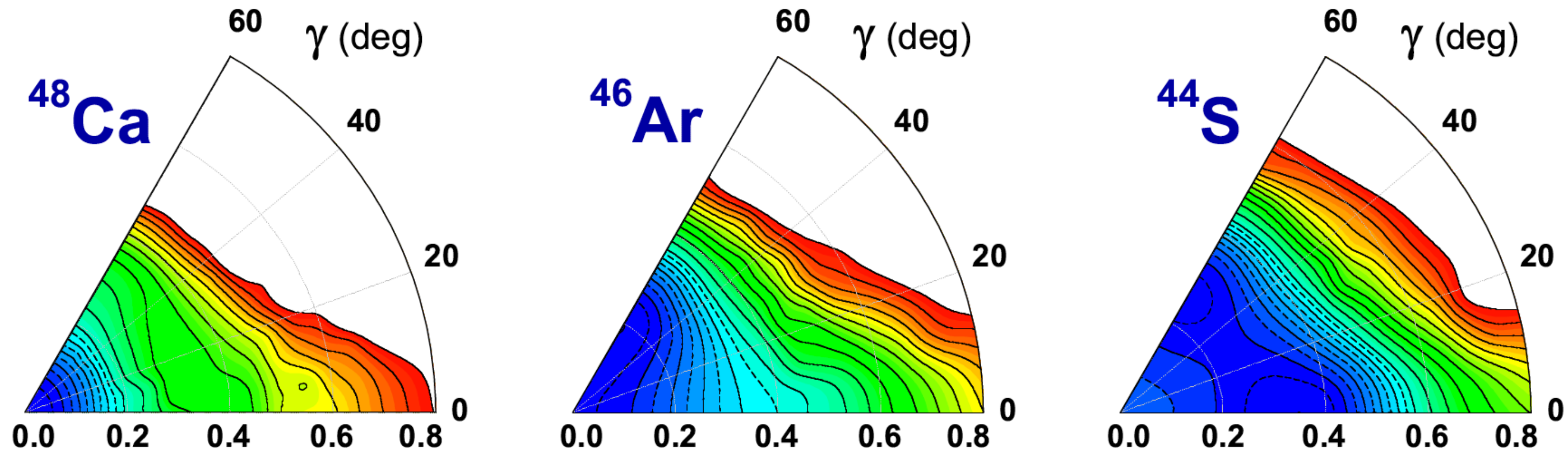
$$\mathcal{T}_{\mathrm{rot}} = \frac{1}{2}\sum_{k=1}^{3}\mathcal{I}_k\omega_k^2$$

The entire dynamics of the collective Hamiltonian is governed by the seven functions of the intrinsic deformations β and γ: the collective potential, the three mass parameters: $B_{\beta\beta}$, $B_{\beta\gamma}$, $B_{\gamma\gamma}$, and the three moments of inertia I_k.

Coexisting shapes in N=28 isotones



Z. P. Li, J. M. Yao, D. Vretenar, T. Nikšić, H. Chen, and J. Meng
Phys. Rev. C 84, 054304 (2011)



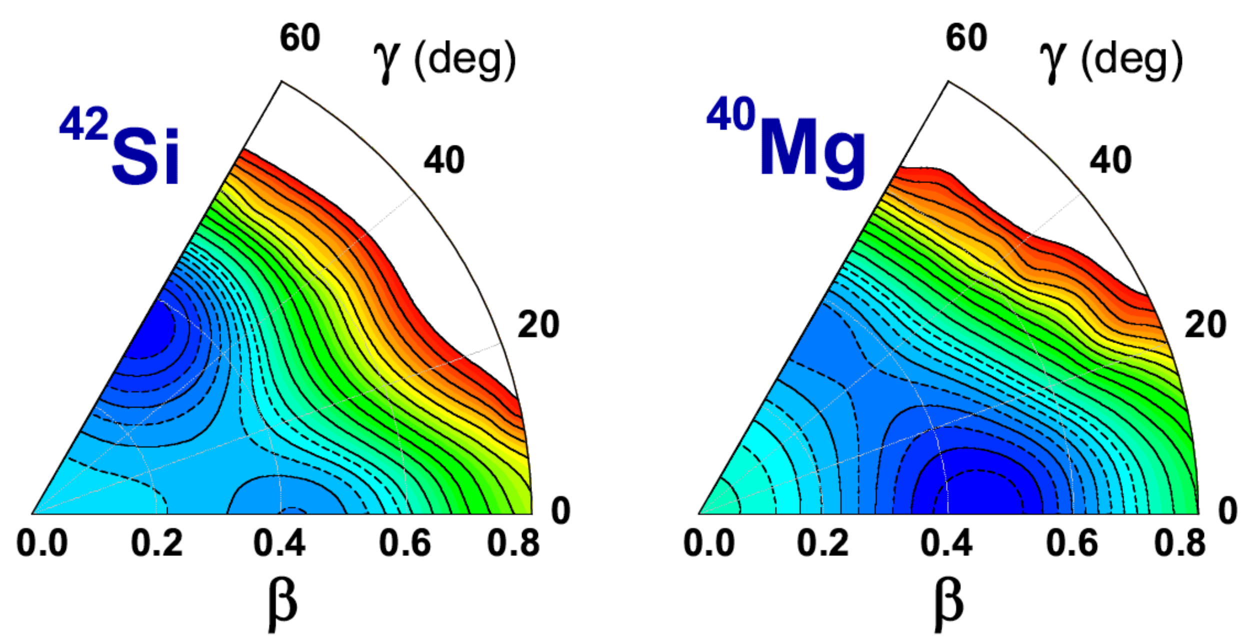
Neutron N=28 spherical energy gaps

	$\Delta^{\text{sph.}}_{N=28}$	$\beta_{\min}$
^{48}Ca	4.73	0.00
^{46}Ar	4.48	-0.19
^{44}S	3.86	0.34
^{42}Si	3.13	-0.35
^{40}Mg	2.03	0.45

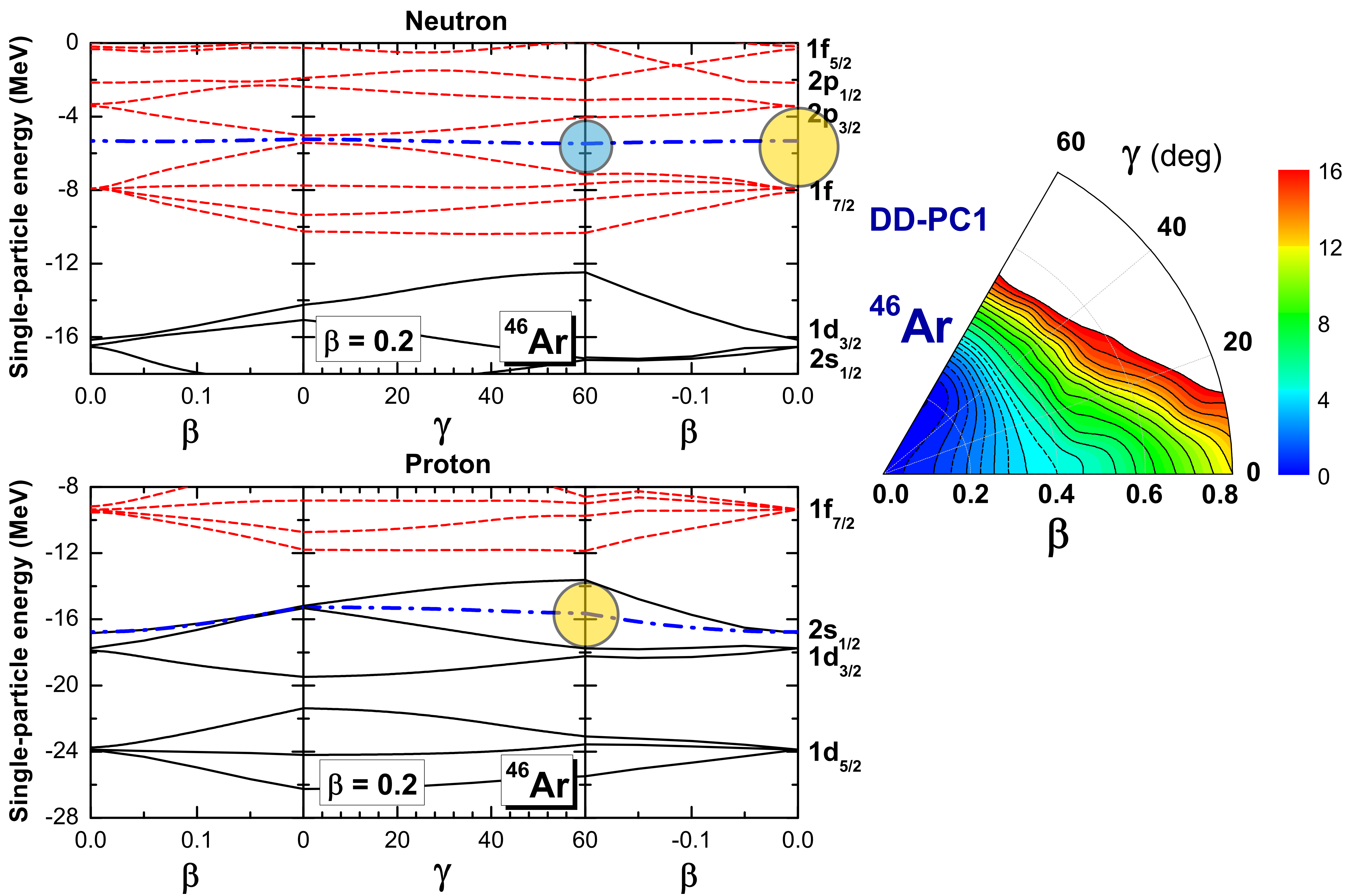
<u>Experimental values:</u>

4.80 MeV

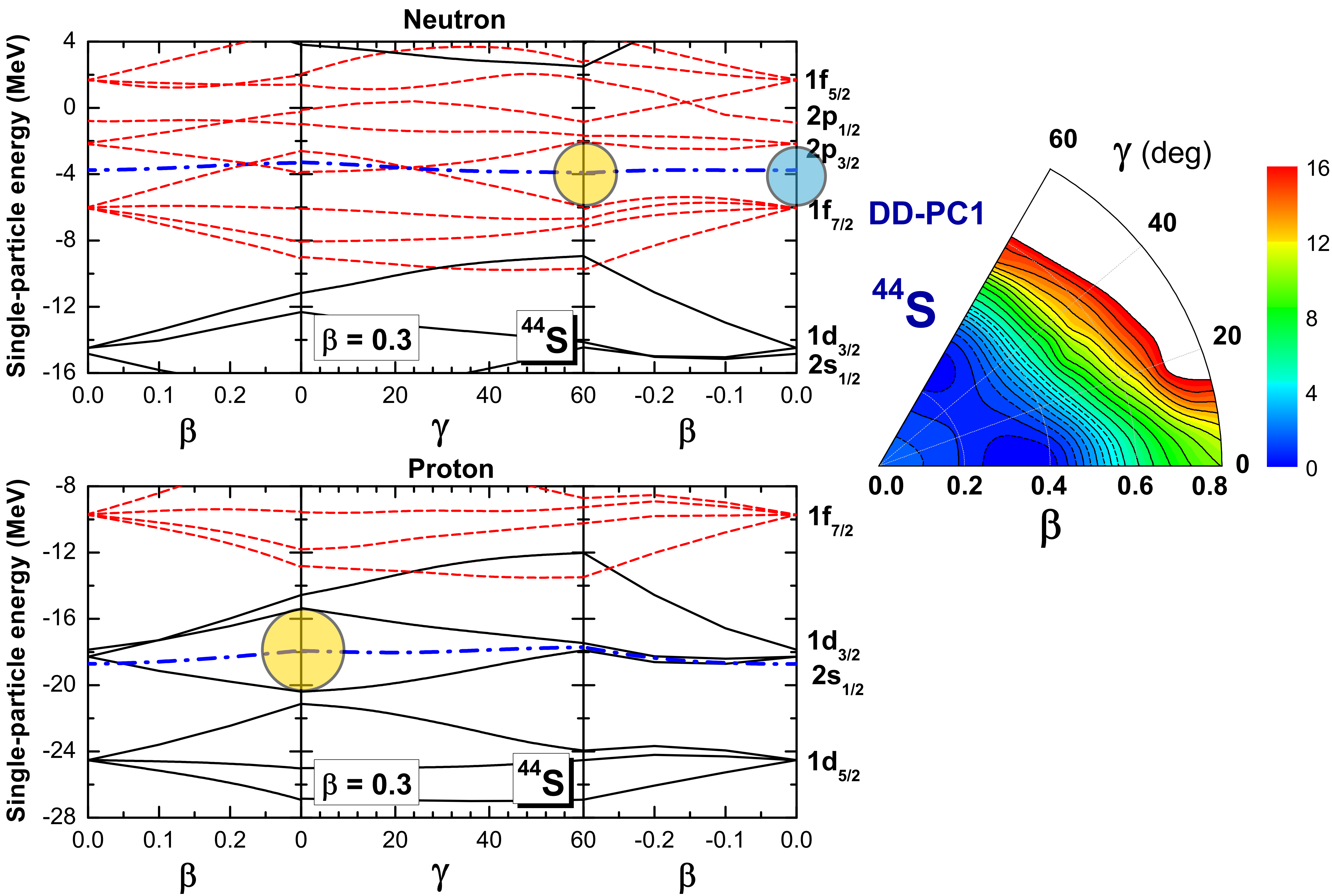
4.47 MeV



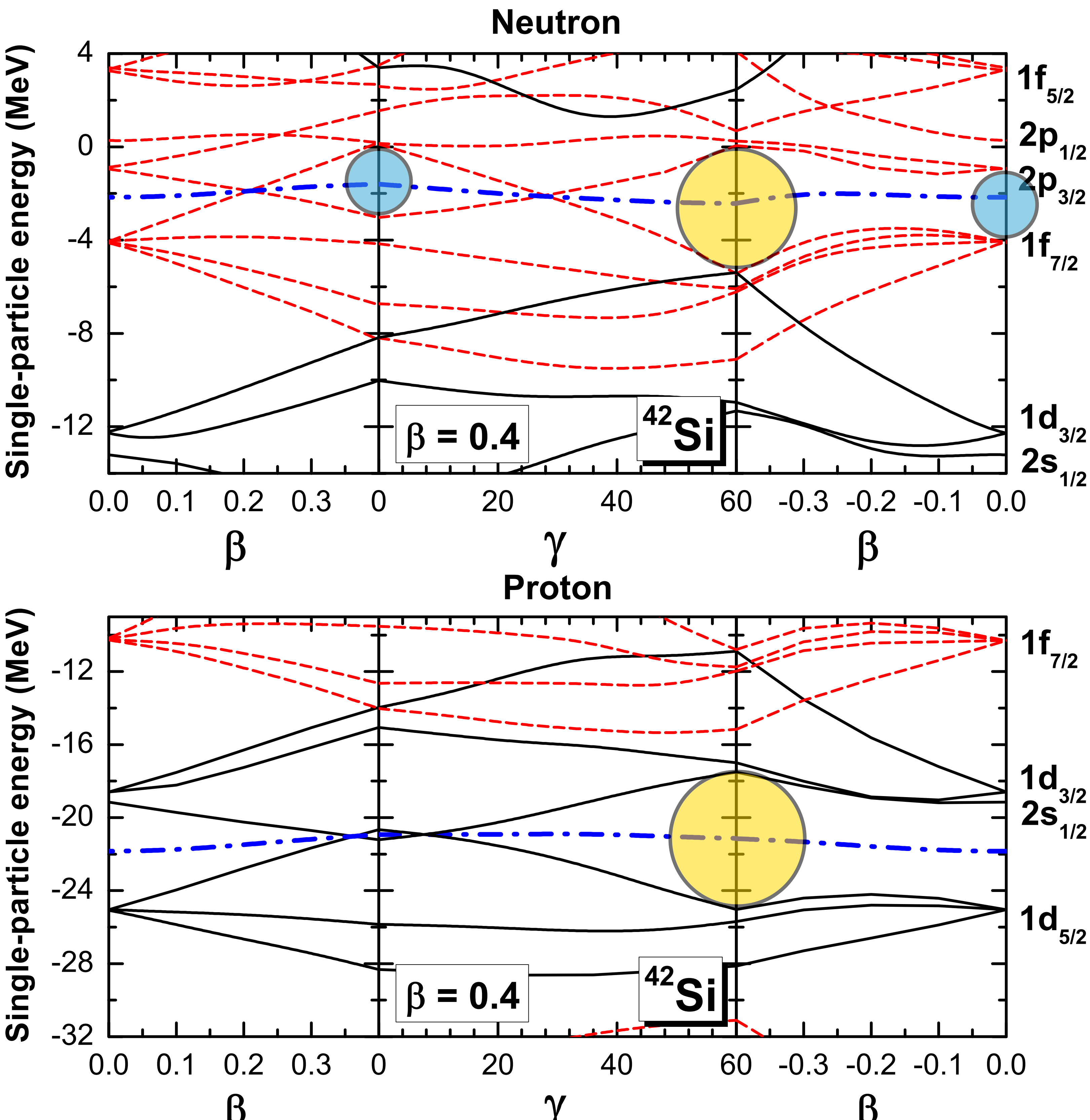
^{46}Ar: single-particle levels

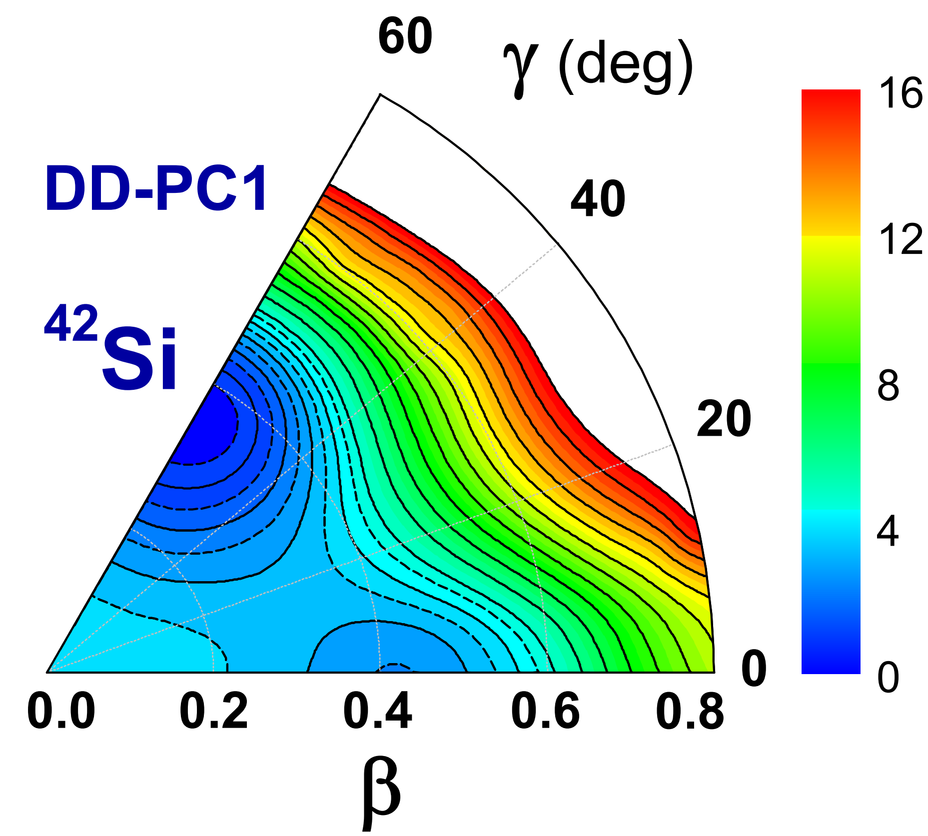


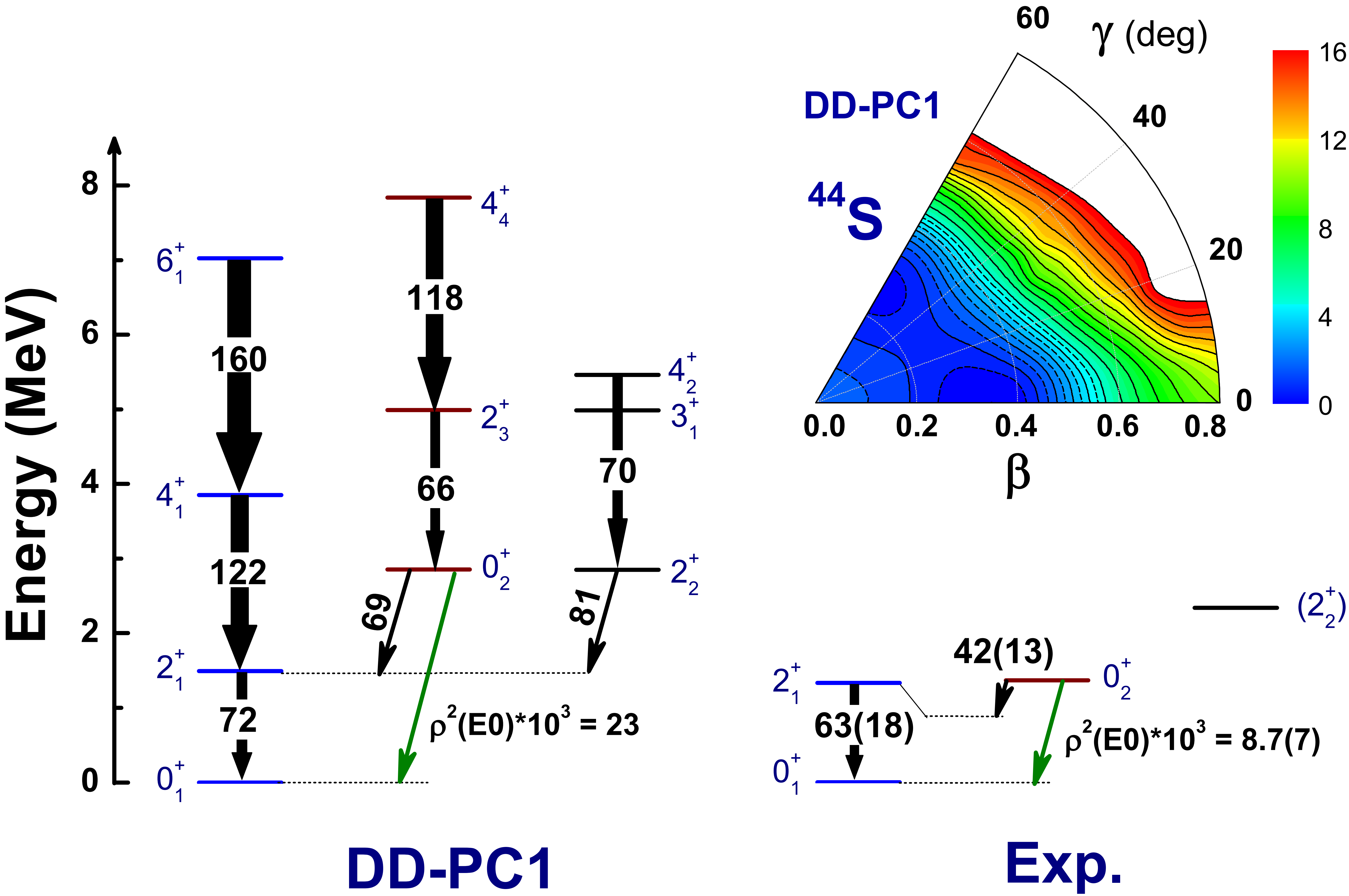
^{44}S: single-particle levels



^{42}Si: single-particle levels



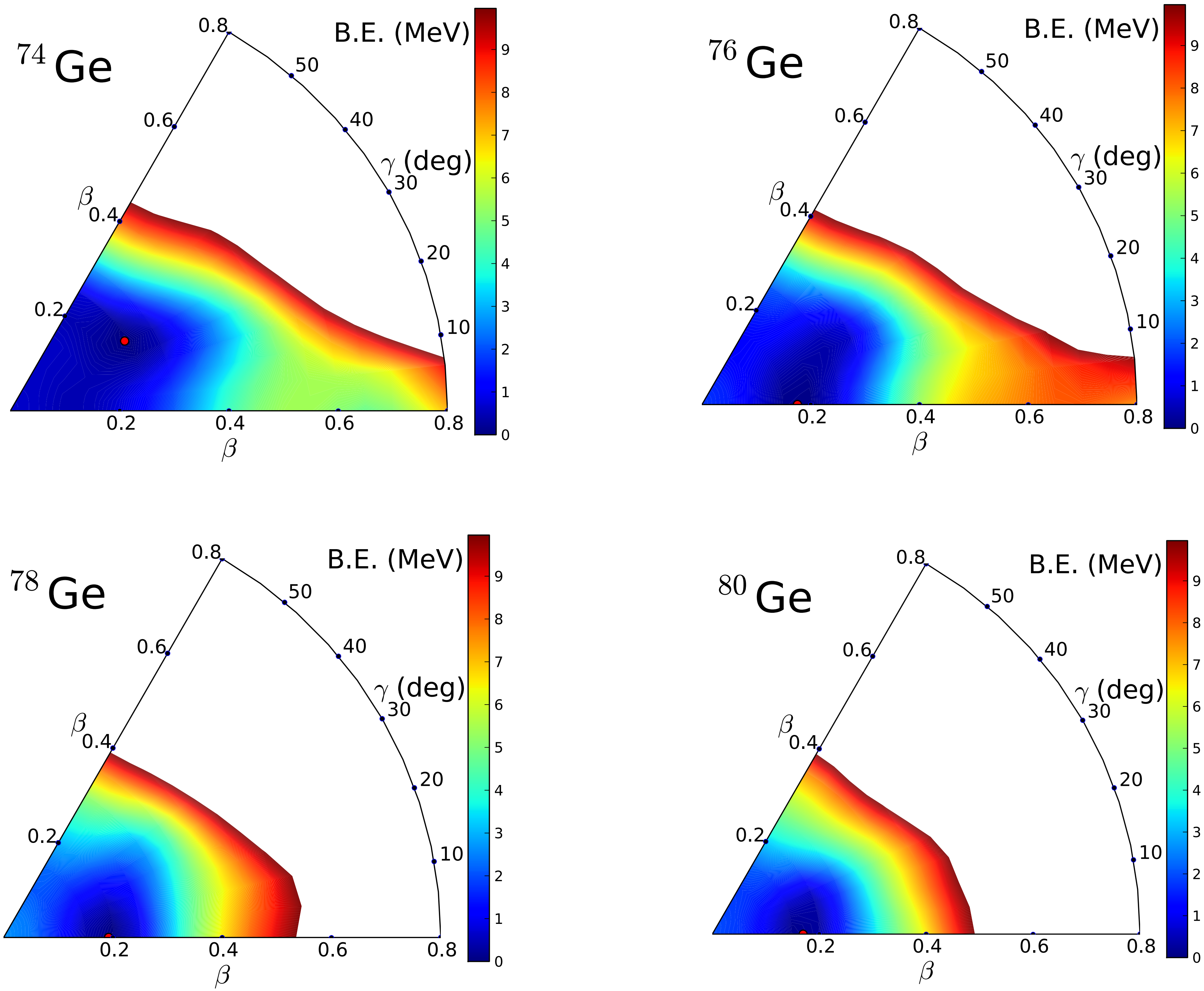




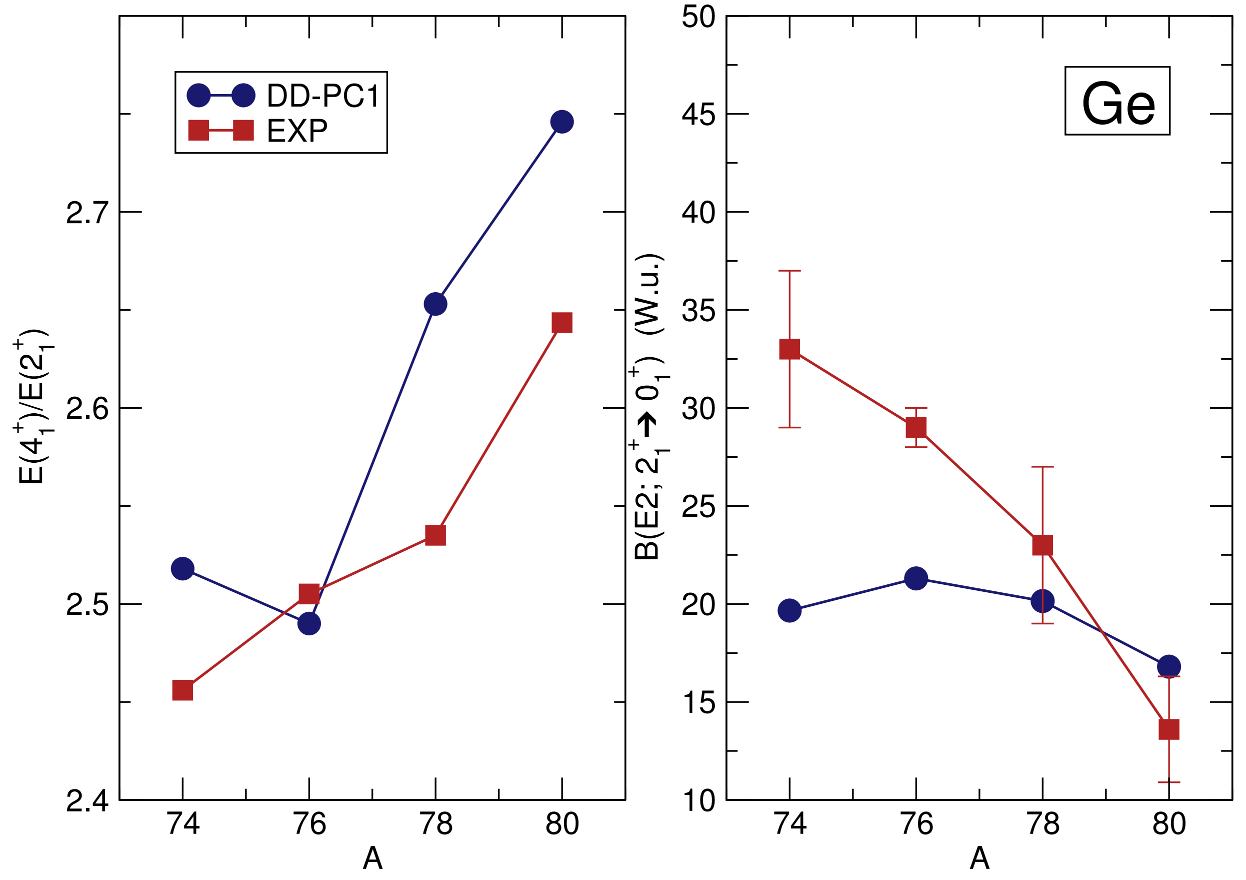
Energy (MeV)
8
6
4
2
0
6^+_1
4^+_1
2^+_1
0^+_1
160
122
72
4^+_4
2^+_3
0^+_2
118
66
69
4^+_2
3^+_1
2^+_2
70
81
$\rho^2(E0)*10^3 = 23$
DD-PC1
DD-PC1
^{44}S
γ (deg)
60
40
20
0
0.0
0.2
0.4
0.6
0.8
β
16
12
8
4
0
(2^+_2)
42(13)
2^+_1
0^+_2
63(18)
0^+_1
$\rho^2(E0)*10^3 = 8.7(7)$
Exp.

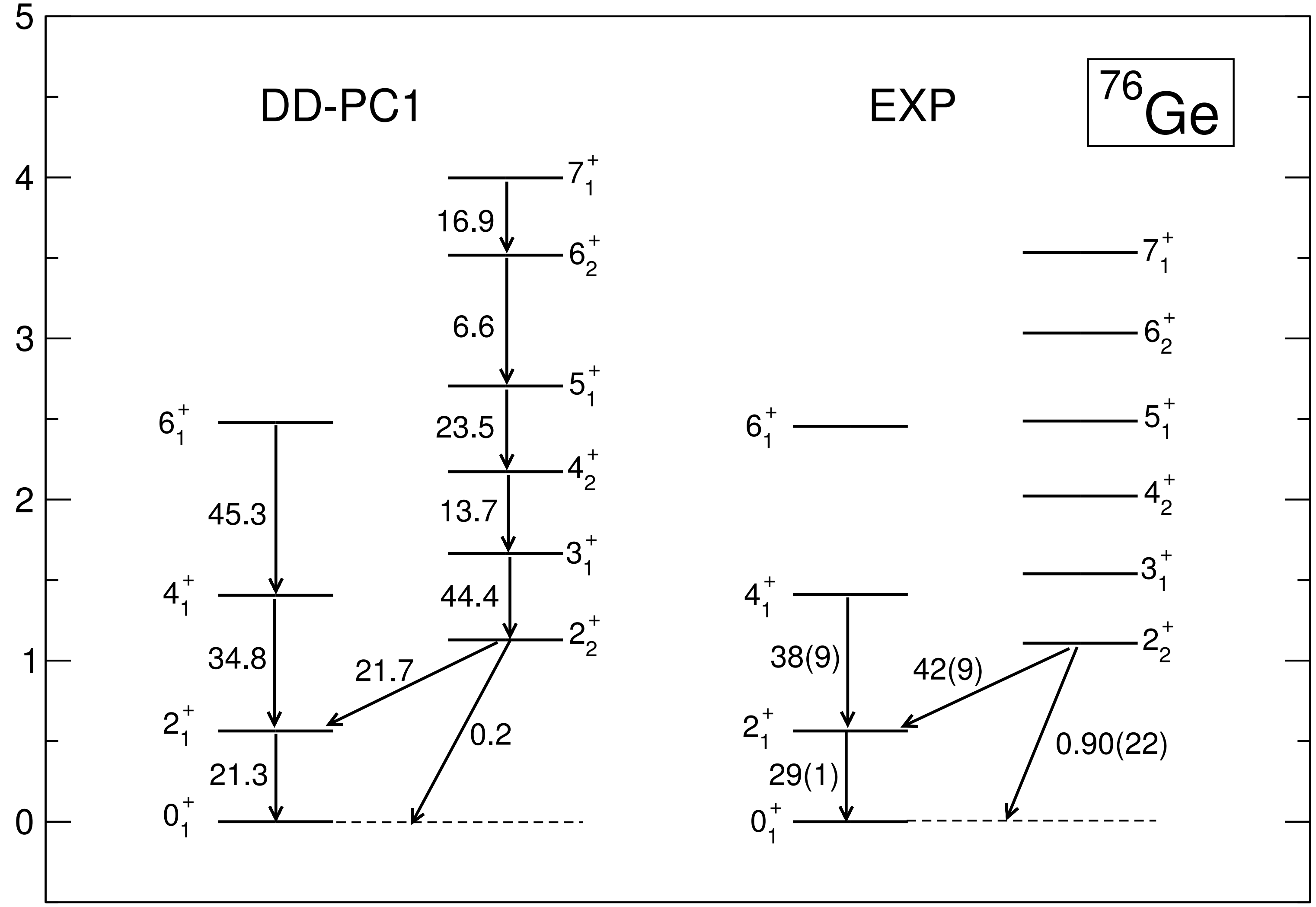
Shape evolution and triaxiality in germanium isotopes

T. Nikšić, P. Marević, and D. Vretenar, Phys. Rev. C 89, 044325 (2014)



Quadrupole collective Hamiltonian based on the functional DD-PC1



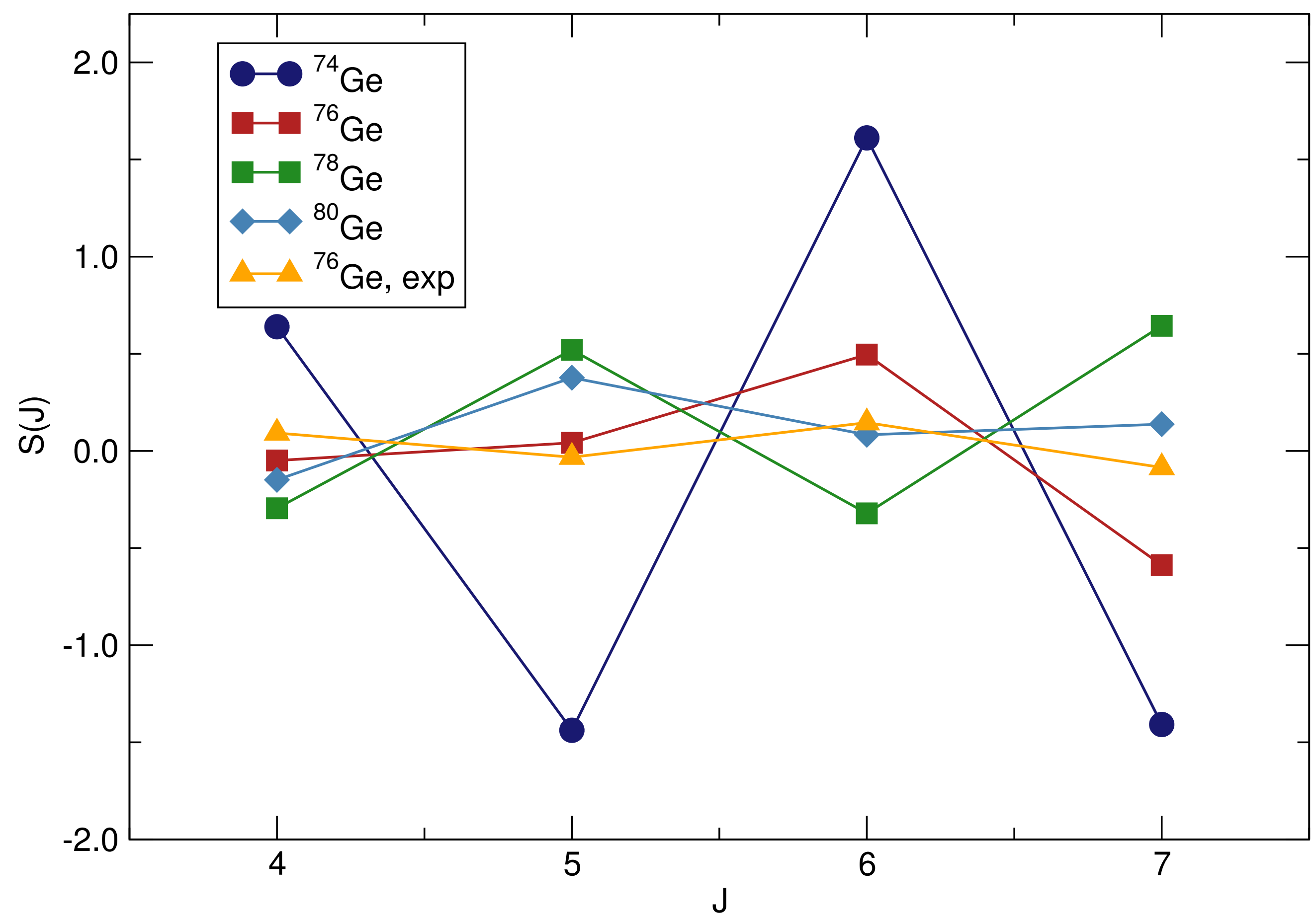


The level of K-mixing is reflected in the staggering in energy between odd- and even-spin states in the γ band:

$$S(J) = \frac{E[J_\gamma^+] - 2E[(J-1)_\gamma^+] + E[(J-2)_\gamma^+]}{E[2_1^+]}$$

Deformed γ-soft potential $\Rightarrow$ S(J) oscillates between negative values for even-spin states and positive values for odd-spin states.

γ-rigid triaxial potential $\Rightarrow$ S(J) oscillates between positive values for even-spin states and negative values for odd-spin states.



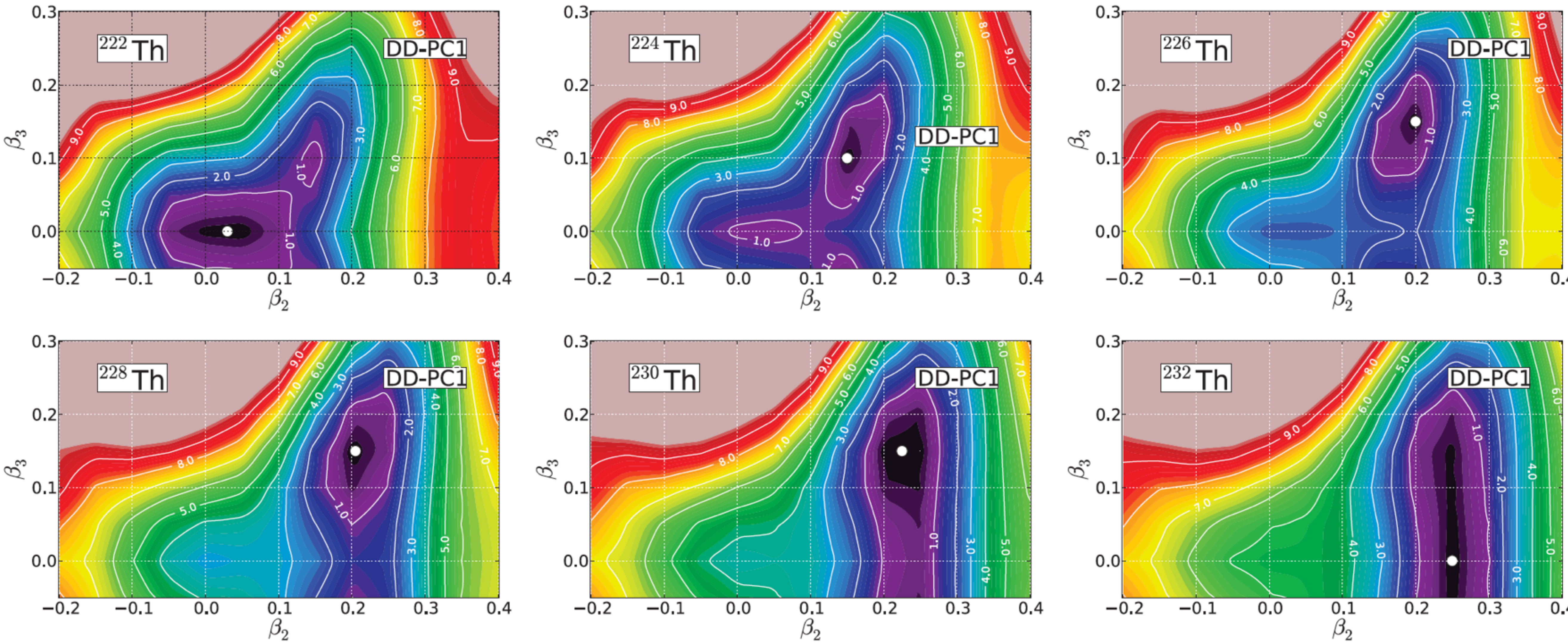
The mean-field potential of ^{76}Ge is γ soft. The inclusion of collective correlations (symmetry restoration and quantum fluctuations) drives the nucleus toward triaxiality, but not strong enough to stabilize a $\gamma \approx 30^\circ$ shape.

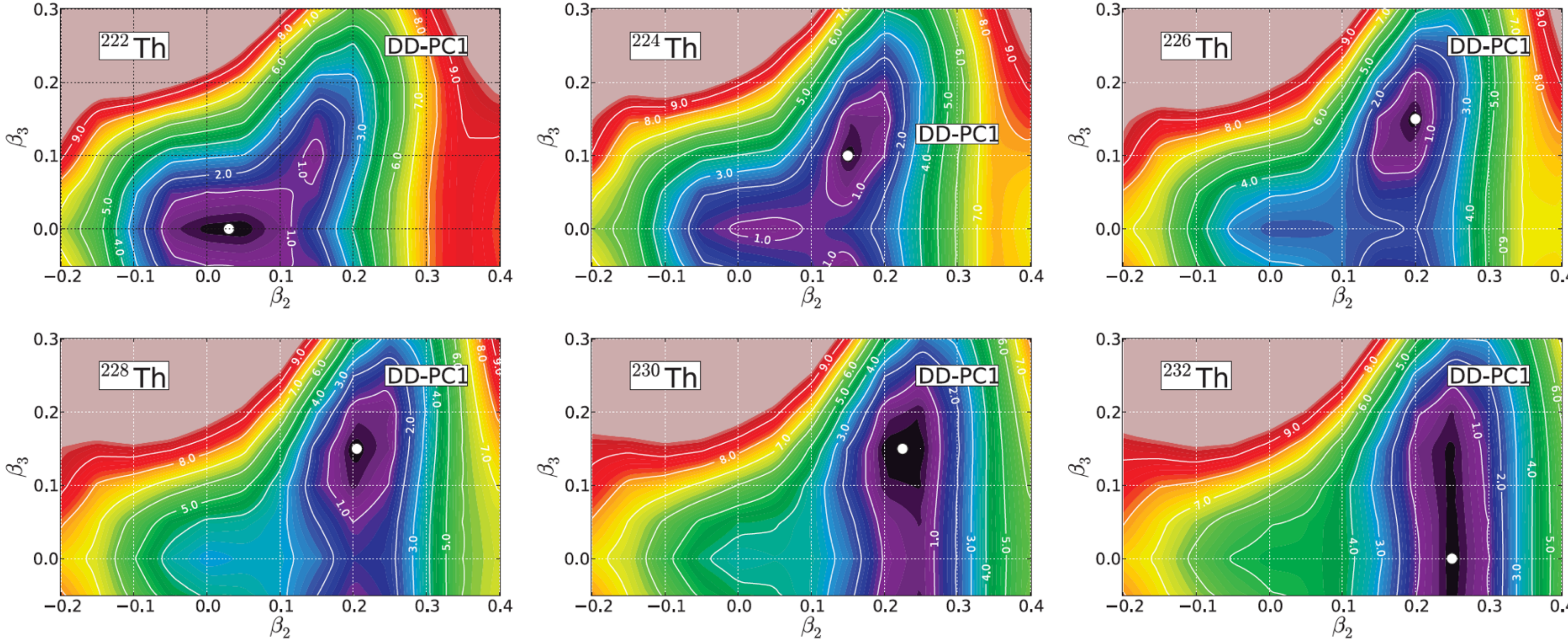
Octupole shape-phase transitions in light actinide and rare-earth nuclei

K. Nomura, D. Vretenar, and B.-N. Lu, Phys. Rev. C 88, 021303(R) (2013)

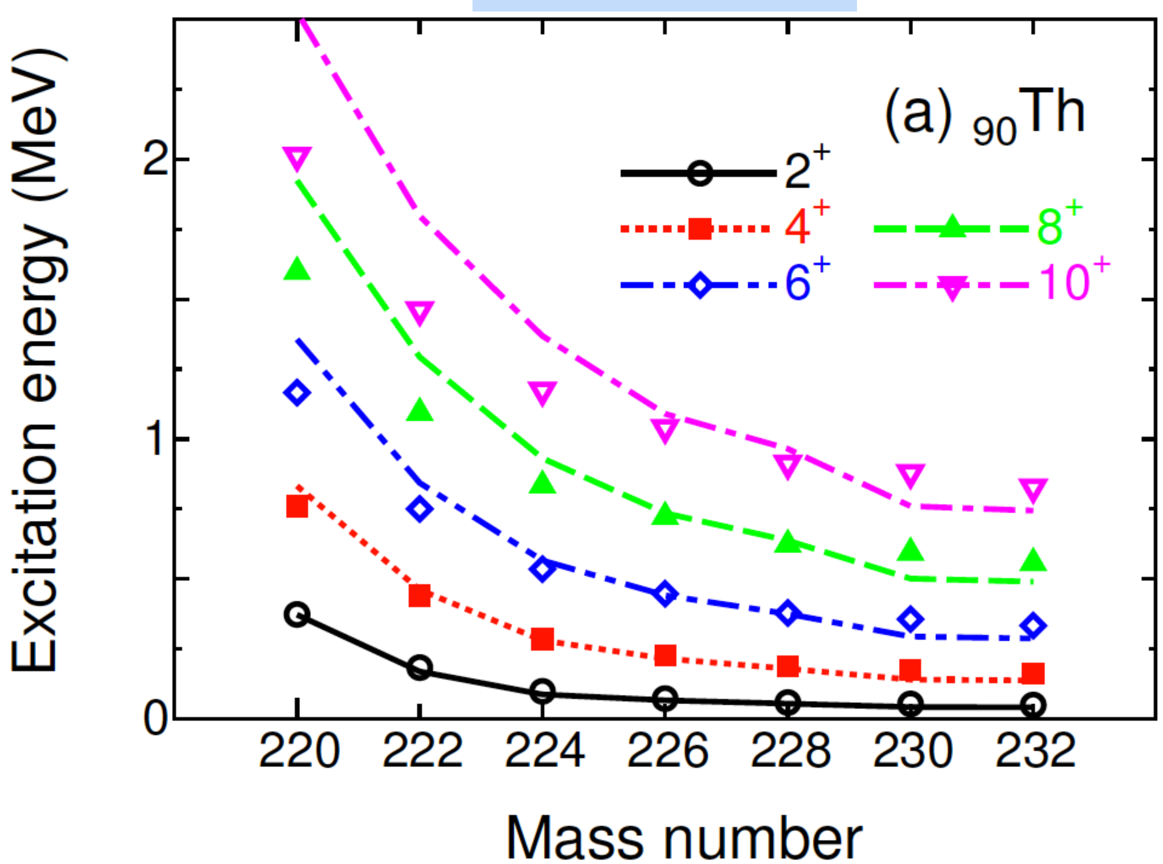
K. Nomura, D. Vretenar, T. Nikšić, and Bing-Nan Lu, Phys. Rev. C 89, 024312 (2014).

Axially symmetric deformation energy surfaces of $^{222-232}$Th in the (β_2,β_3) plane:

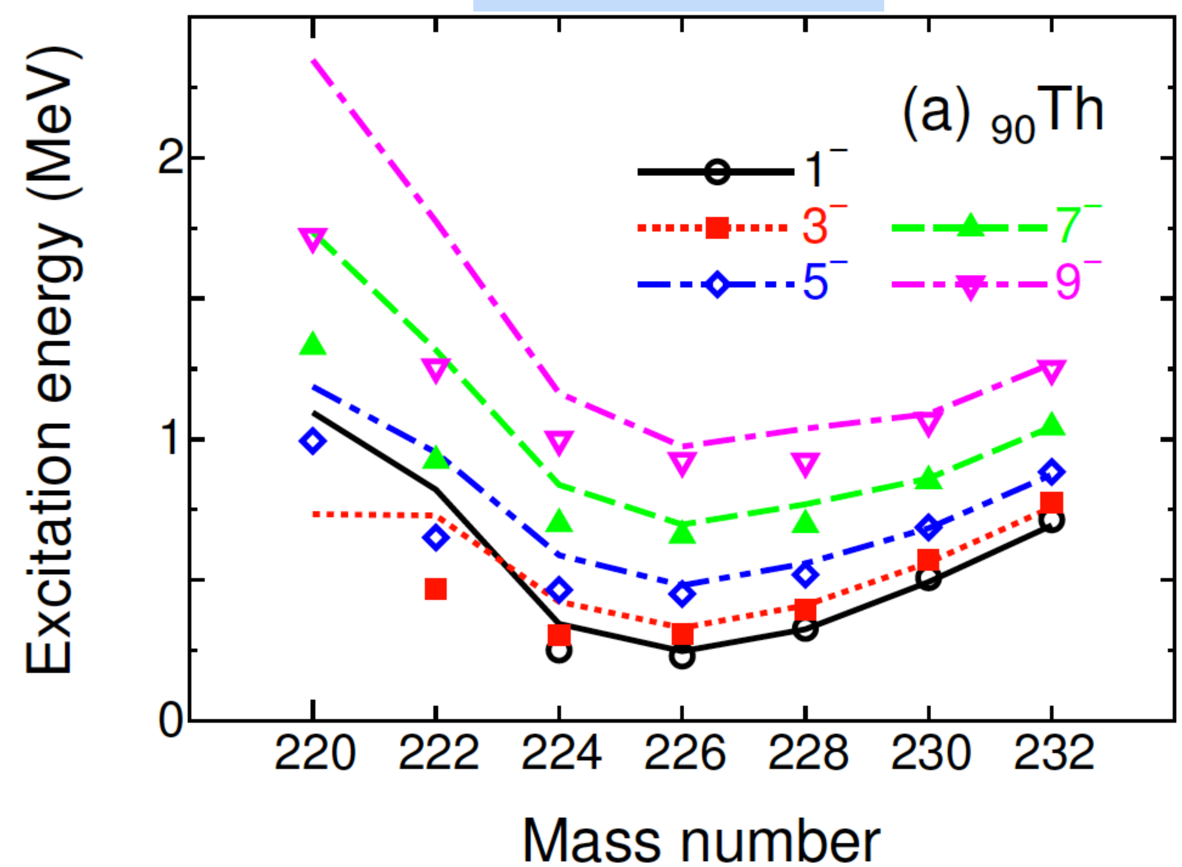


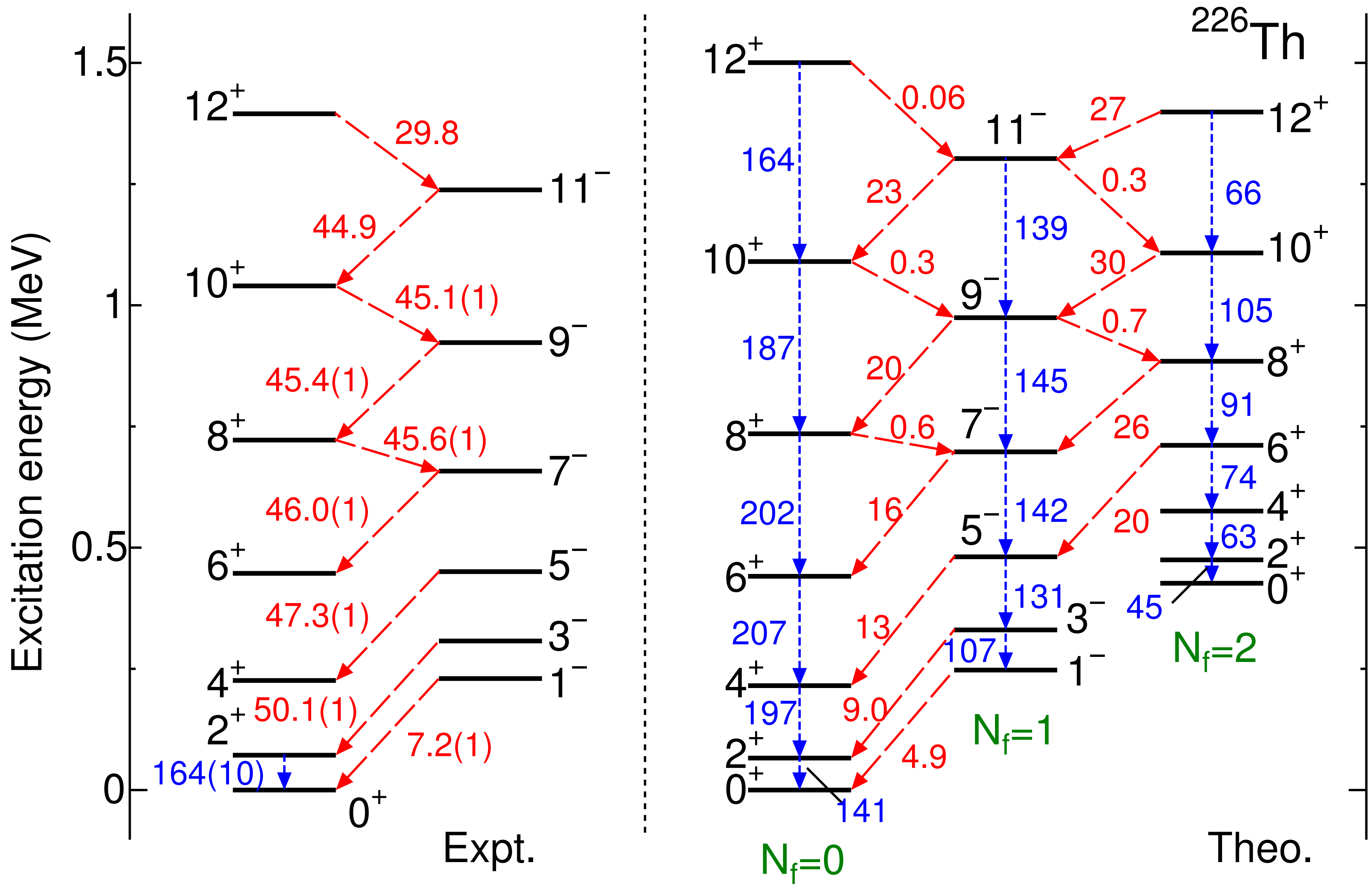


POSITIVE PARITY



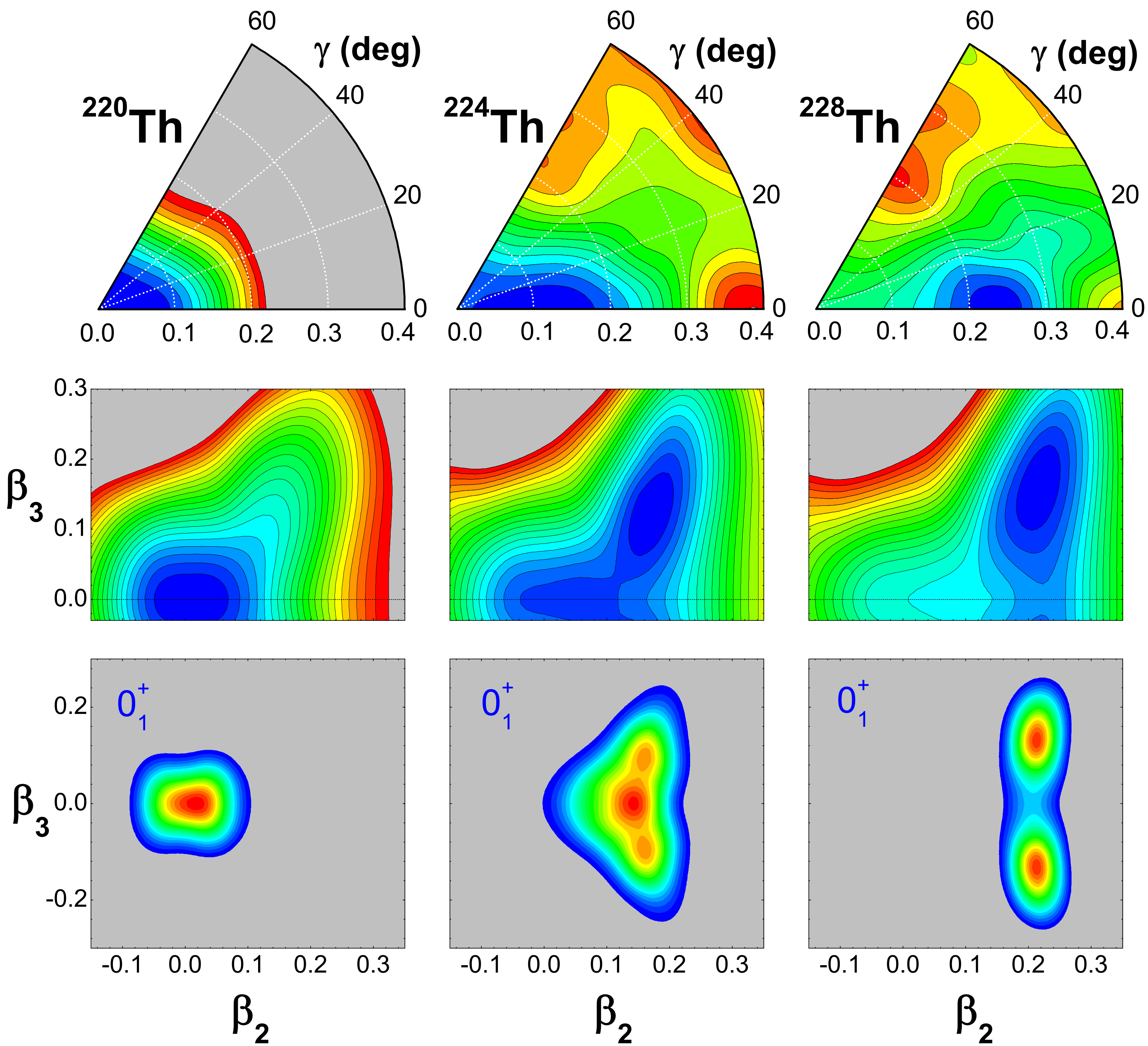
NEGATIVE PARITY

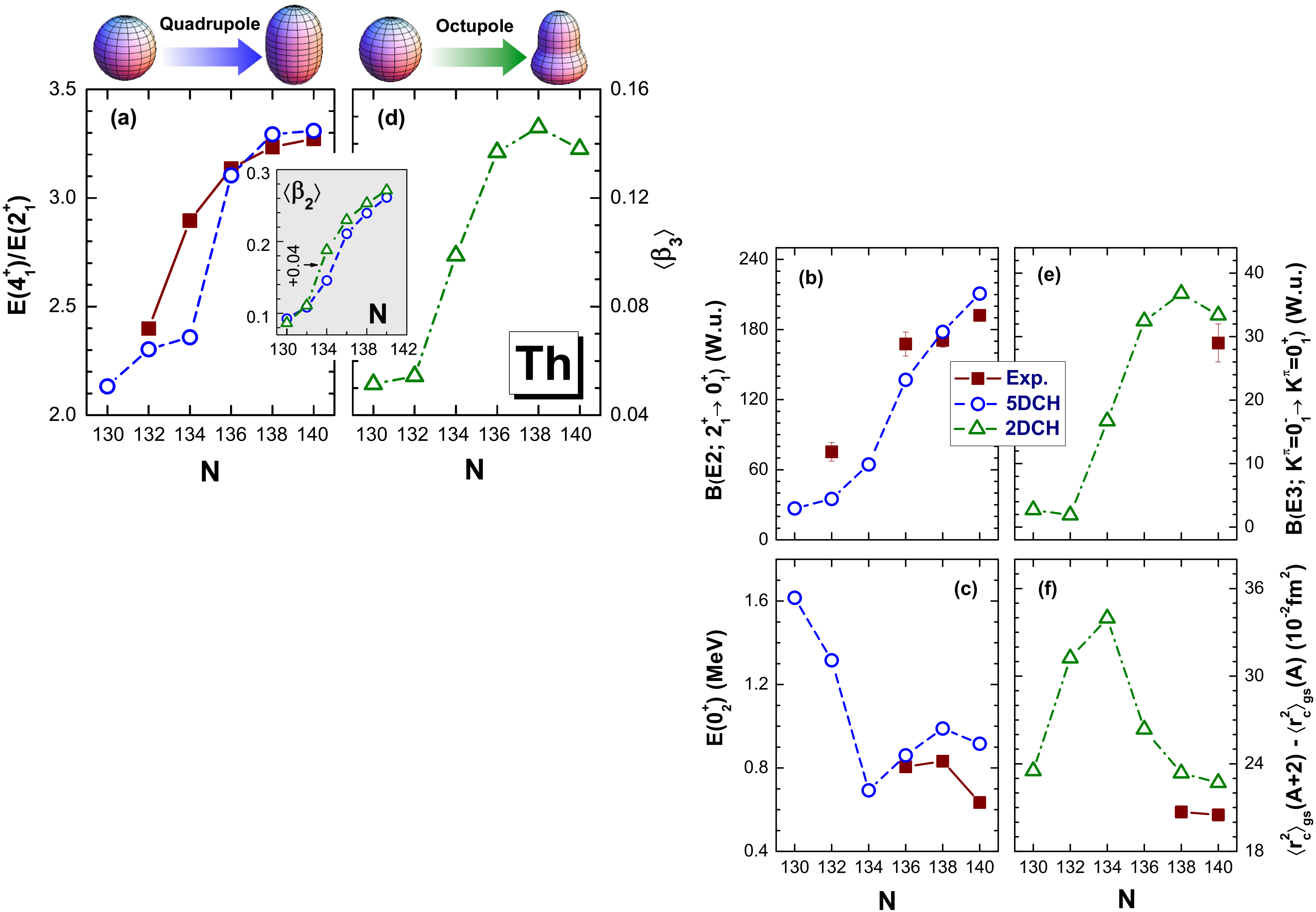



Excitation energy (MeV)
0
0.5
1
1.5
Expt.
Theo.
^{226}Th
N_f=0
N_f=1
N_f=2
164(10)
50.1(1)
7.2(1)
47.3(1)
46.0(1)
45.6(1)
45.4(1)
45.1(1)
44.9
29.8

Simultaneous quadrupole and octupole shape phase transitions in Thorium

Z.P. Li, B.Y. Song, J.M. Yao D. Vretenar, and J. Meng, Phys. Lett. B 726 (2013) 866





Quadrupole
Octupole
(a)
(d)
E(4+1)/E(2+1)
⟨β3⟩
⟨β2⟩
+0.04
N
Th
(b)
(e)
(c)
(f)
B(E2; 2+1→ 0+1) (W.u.)
B(E3; Kπ=0-1→ Kπ=0+1) (W.u.)
E(0+2) (MeV)
⟨r2c⟩gs(A+2) - ⟨r2c⟩gs(A) (10-2fm2)
Exp.
5DCH
2DCH

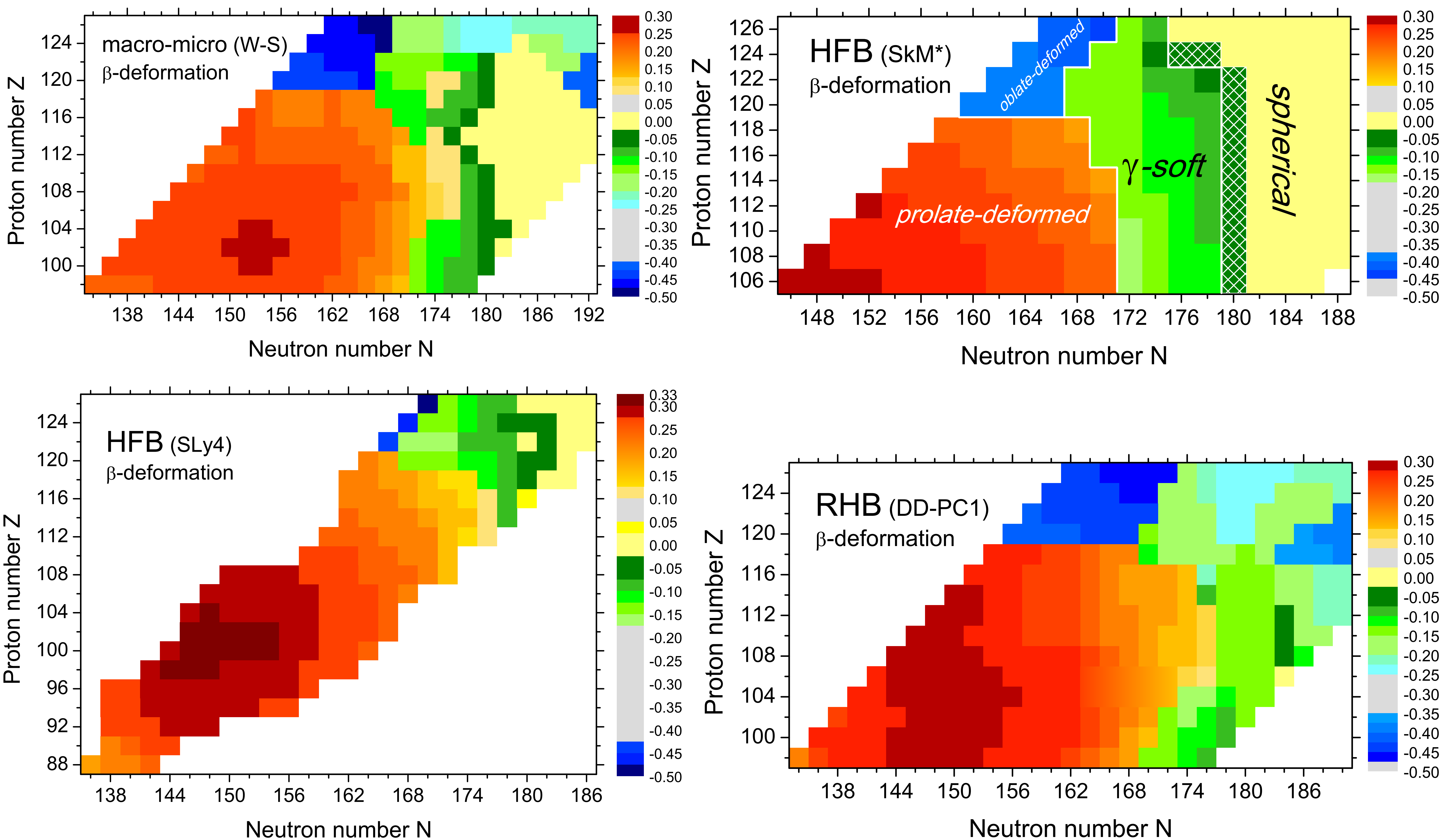
Extrapolation to Super-heavy Nuclei

EDFs and the corresponding structure models are applied to a region far from those in which their parameters are determined by data ➠ uncertainty of model predictions?

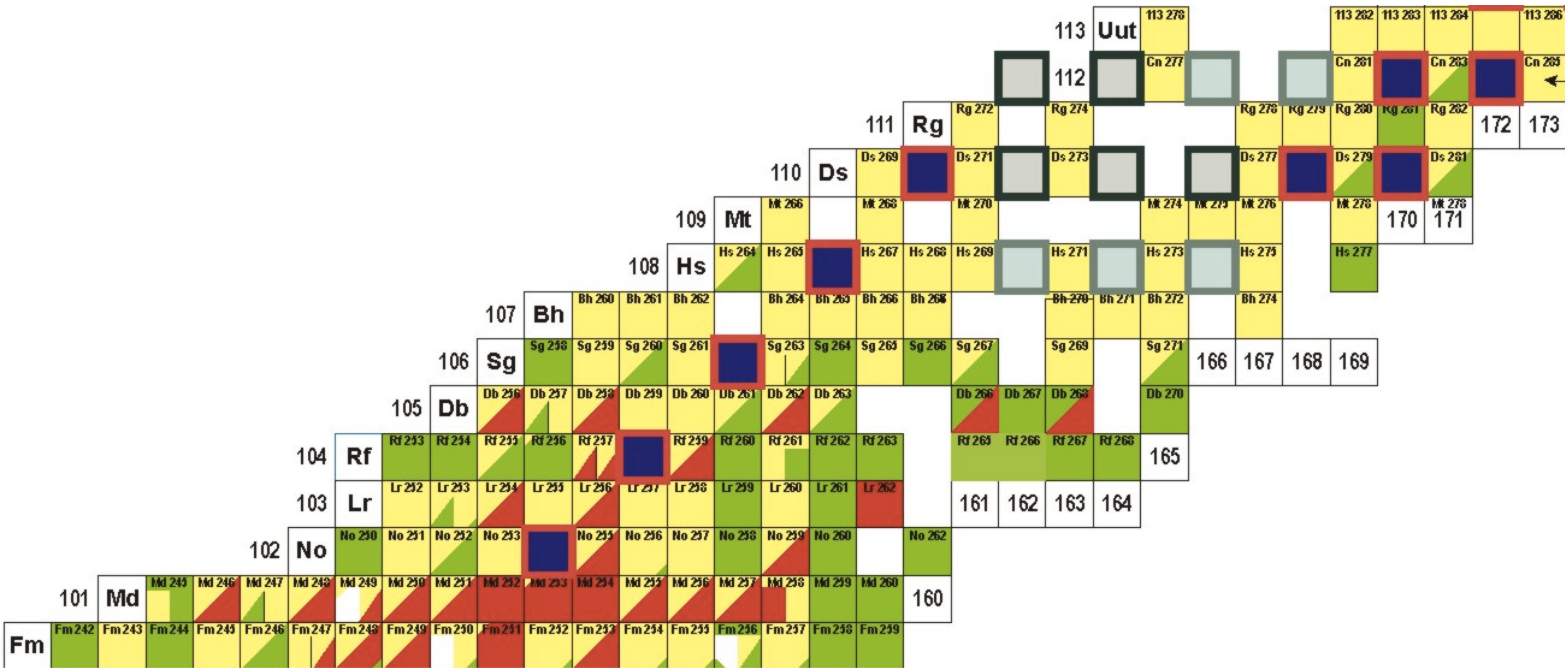
Much higher density of single-particle states close to the Fermi energy ➠ details of the evolution of deformed shells with nucleon number will have more pronounced effects on energy gaps, separation energies, Q_α-values, band-heads in odd-A nuclei, K-isomers …

Much stronger competition between the attractive short-range nuclear interaction and the long-range electrostatic repulsion ➠ pronounced effects on the Coulomb, surface and isovector energies! Fast shape transitions! Exotic shapes!

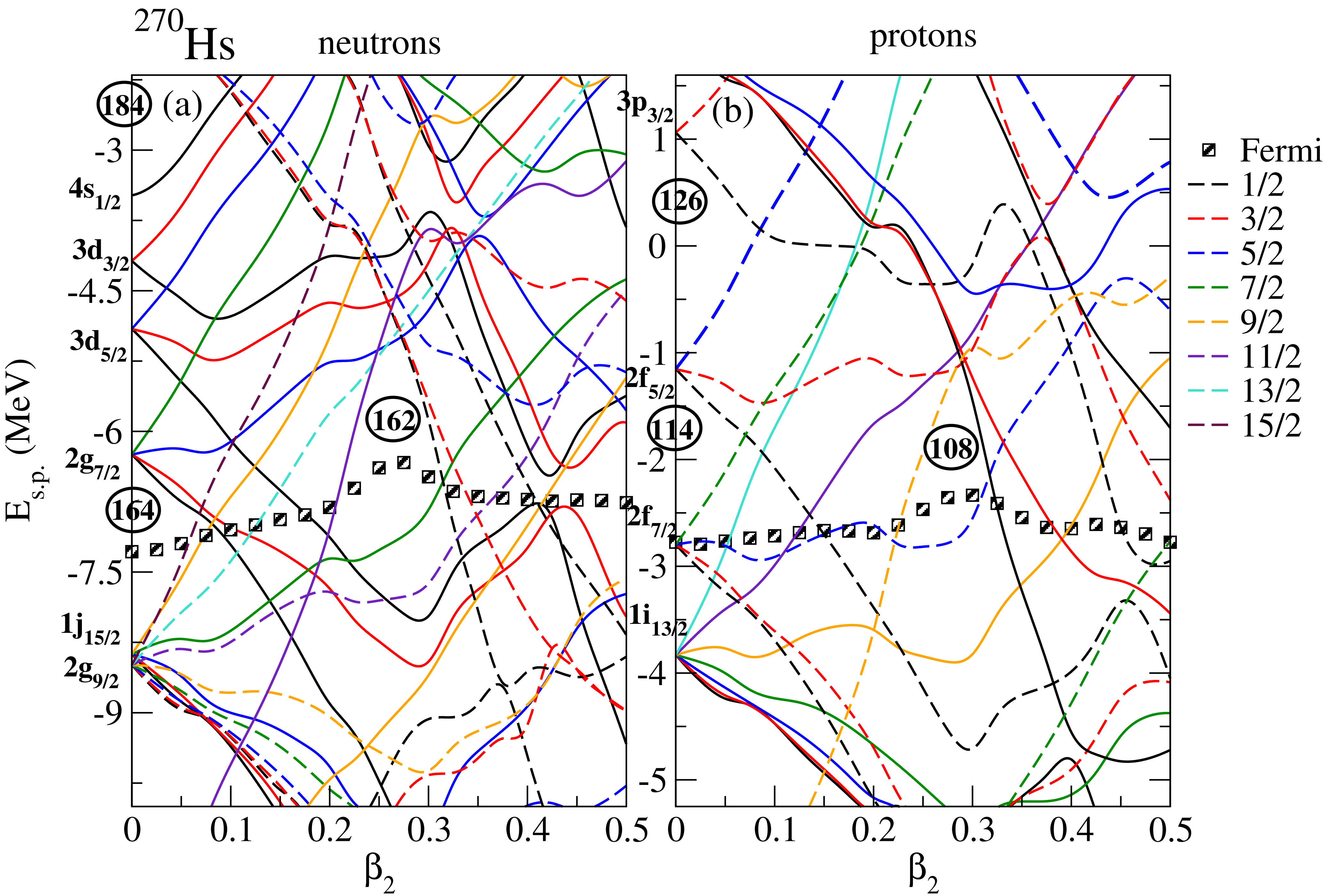
Shape transitions in super-heavy nuclei



Transactinides

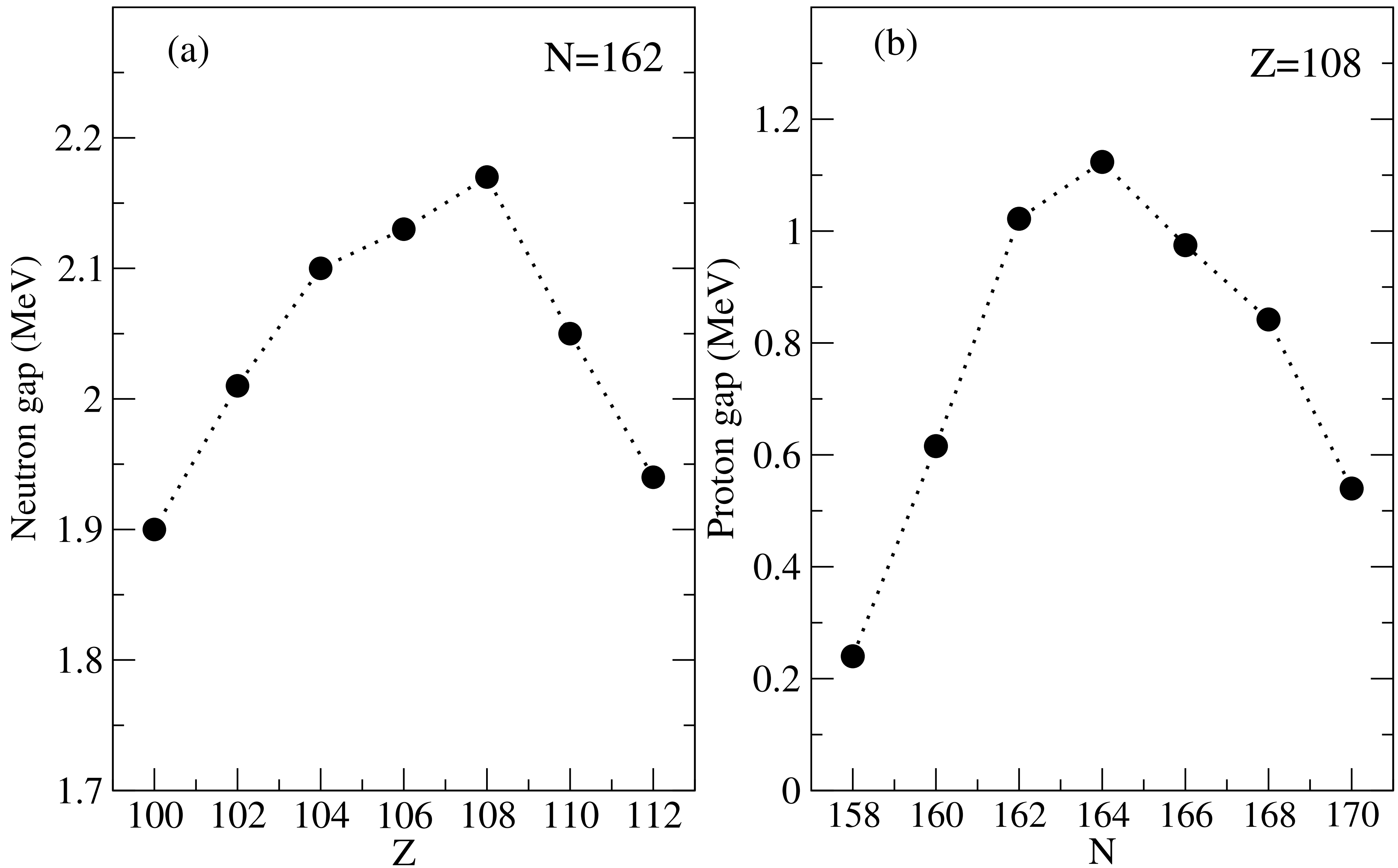


Energy gaps are small. Shape stabilisation depends on how fast shell structures vary with deformation!

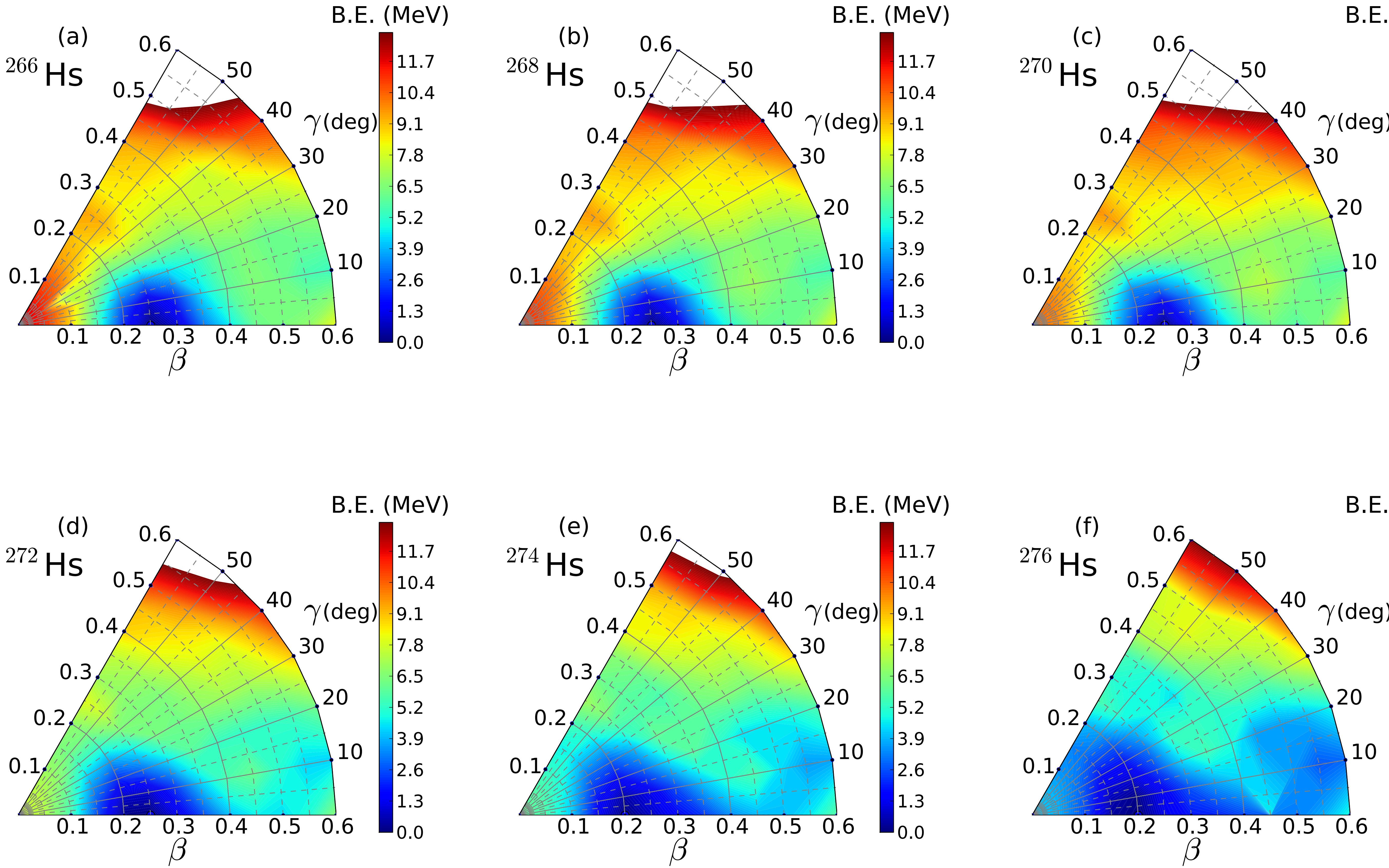


V. Prassa, T. Nikšić, and D. Vretenar
Phys. Rev. C 88, 044324 (2013)

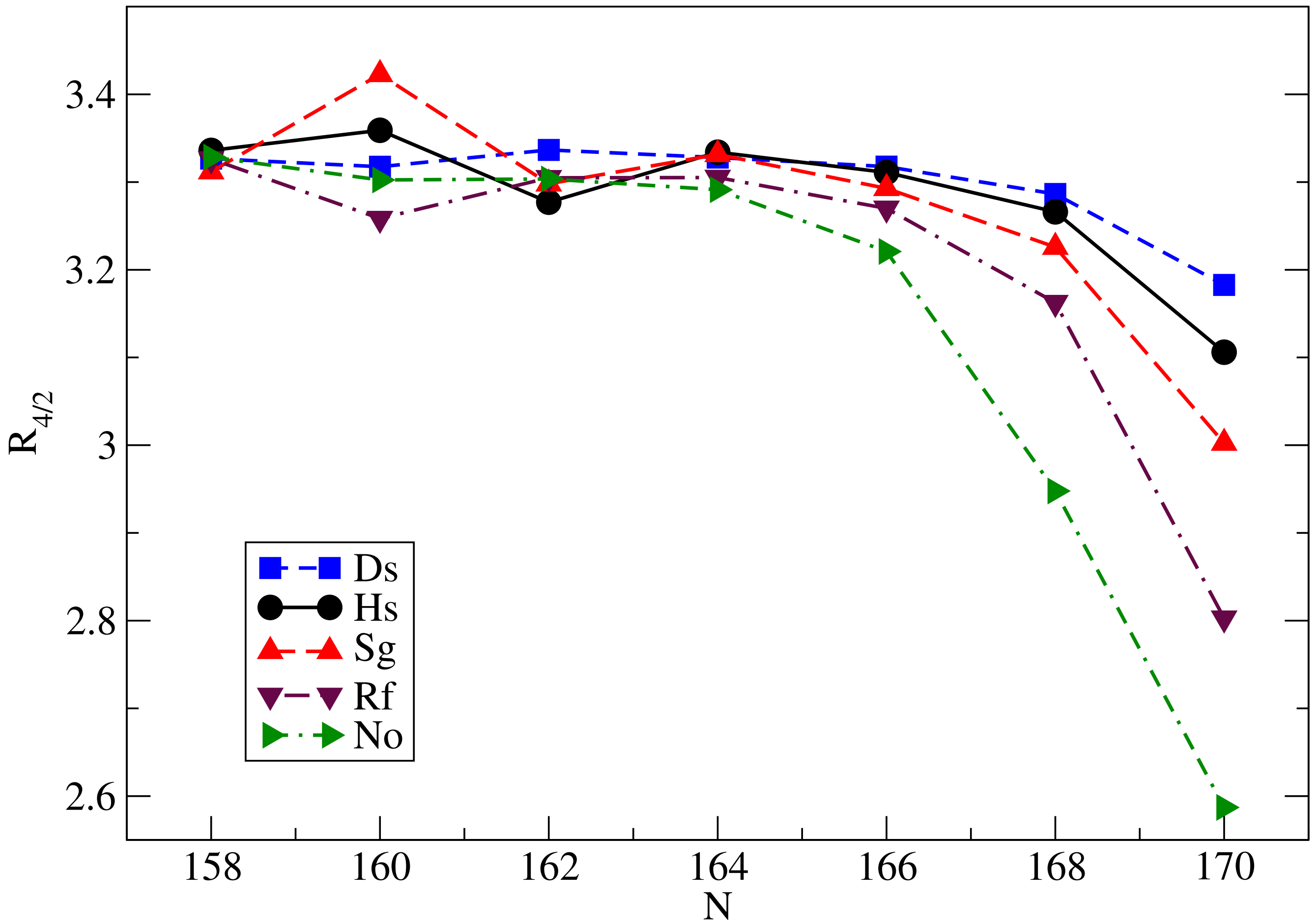
Neutron and proton shell gaps



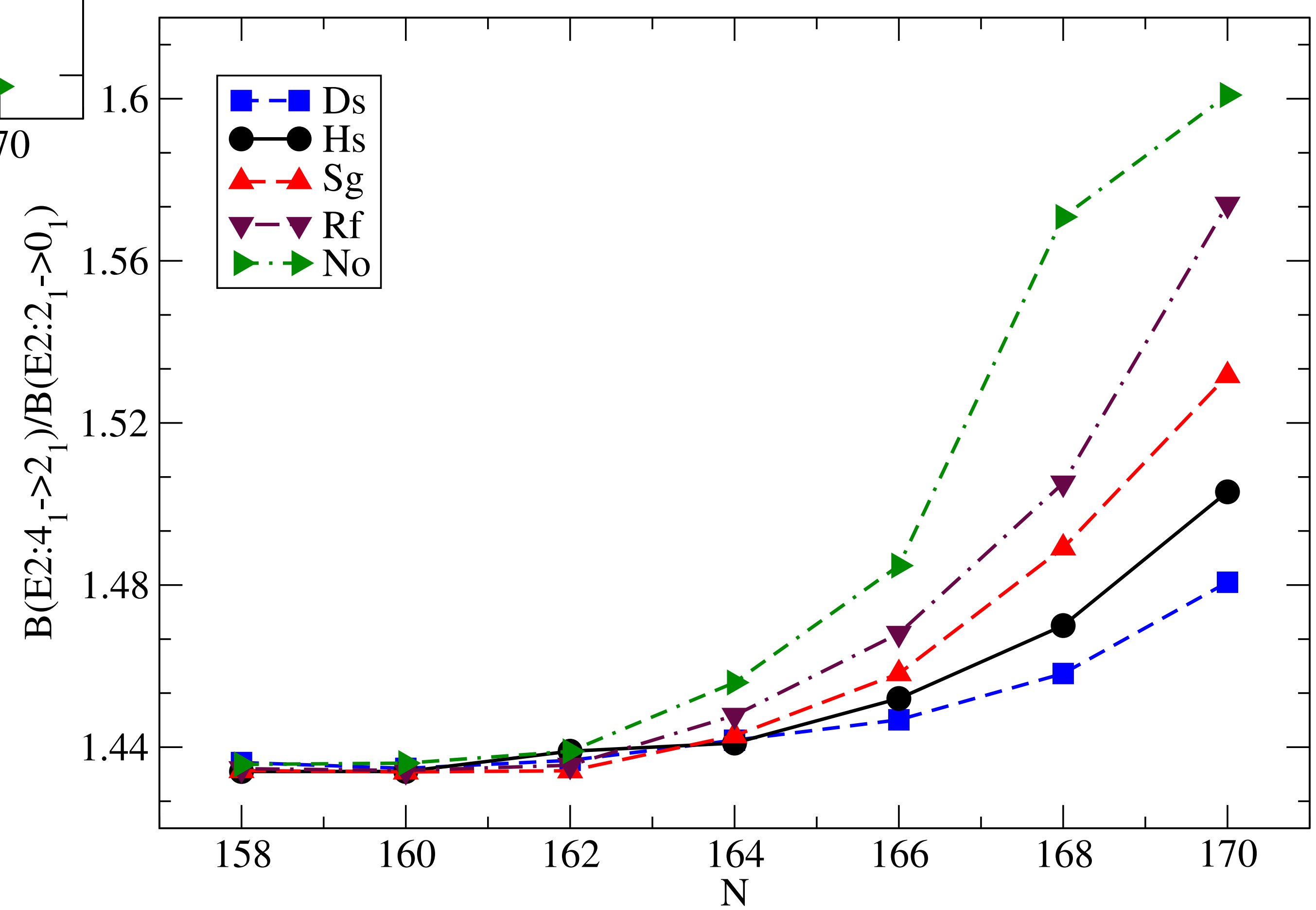
^{270}Hs ➠ deformed “doubly magic” nucleus


(a)
^{266}Hs
(b)
^{268}Hs
(c)
^{270}Hs
(d)
^{272}Hs
(e)
^{274}Hs
(f)
^{276}Hs
B.E. (MeV)
γ(deg)
β
0.1 0.2 0.3 0.4 0.5 0.6
10 20 30 40 50
0.0 1.3 2.6 3.9 5.2 6.5 7.8 9.1 10.4 11.7

Collective states



The ratio R4/2 of excitation energies of the yrast states $4^+{}_1$ and $2^+{}_1$ as a function of the neutron number for the isotopic chains of No, Rf, Sg, Hs and Ds.



The ratio of reduced transition probabilities $B(E2;\, 4^+{}_1 \rightarrow 2^+{}_1\,)/\, B(E2;\, 2^+{}_1 \rightarrow 0^+{}_1\,)$ as a function of the neutron number.

Nuclear Energy Density Functional Framework

✔ The evolution of shapes, shape coexistence and shape (phase) transitions present universal phenomena that reflect the organisation of nucleonic matter in finite nuclei, and occur in every region of the chart of nuclides.

✔ NEDFs enable a unified microscopic description of the structure of stable and nuclei far from stability, and reliable extrapolations toward the drip lines.

✔ when extended to take into account collective correlations, universal EDFs describe deformations, shape-coexistence and shape-phase transition phenomena associated with shell evolution.