Spatial properties of pairing and quarteting correlations in nuclear systems

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1 Pairing Correlations
   Pairing Tensor
   Pairing Coherence Length
   Results, Conclusions

2 Quarteting Correlations
   Quartet and $\alpha$ tensors
   Quartet and $\alpha$ Coherence Lengths
   More Results, More Conclusions
Outline

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• This presentation is largely based on [Delion & Baran 2015].
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   - Pairing Tensor
   - Pairing Coherence Length
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   - Quartet and $\alpha$ tensors
   - Quartet and $\alpha$ Coherence Lengths
   - More Results, More Conclusions
The hypothesis of correlated pairs and superfluidity in nuclei is supported by a wealth of arguments [Ring & Schuck 1980, Pillet et al. 2010]:

- energy gap;
- th.-exp. discrepancy in level density and moments of inertia;
- odd-even staggering;
- sudden onset of deformation away from shell closure;
- large cross section of two-particle transfer.
Pairing Tensor

Note:

- **pair transfer amplitude** $\approx$ **pairing tensor** [Pillet et. al. 2010];

- The pairing tensor $\kappa$ captures the nontrivial correlations.

Paired systems present two types of densities [Ring & Schuck 1980]:

- normal: $\rho_{ab} = \langle c_a^\dagger c_b \rangle$
- abnormal: $\kappa_{ab} = \langle c_a c_b \rangle$

Pairing tensor [Pillet et. al. 2007, Pillet et. al. 2010]:

$$\kappa(\vec{r}_1, \vec{r}_2) = \langle BCS| \hat{\psi}(\vec{r}_1) \hat{\psi}(\vec{r}_2) |BCS\rangle$$
Nonlocal part of $\kappa$: The Coherence Length

$$\xi(R) = \sqrt{\langle r^2 \rangle_{\kappa(r,R)}}$$
The Coherence Length: $\xi_{HFB} \sim \xi_{BCS}$
Coherence Length Systematics

(a) protons

(b) neutrons

\[ \langle \xi \rangle / R_N \]
Coherence Length Scaling

(a) protons
\[ \langle \xi \rangle \approx 2.4 \, \text{A}^{1/5} \]

(b) neutrons
\[ \langle \xi \rangle \approx 2.0 \, \text{A}^{1/4} \]
Temperature dependence of $\xi$

What we expect:
Temperature dependence of $\xi$

What we expect:

$T$ increases

↓

pairing correlations fade

↓

$\xi$ should vary significantly
Temperature dependence of $\xi$

- **Almost no variation!**
- Mostly affected by the mixing between its parts $\kappa_{odd}$ and $\kappa_{even}$ [Pillet et al. 2010].
The Coherence Length vs $\lambda$

Keep $N$ constant!
The Coherence Length vs $\lambda$

(a) protons

(b) neutrons

- $\lambda = 0.96$ MeV
- $\lambda = 0$ MeV
- $\lambda = -2.96$ MeV

- $\lambda = 0.88$ MeV
- $\lambda = 0$ MeV
- $\lambda = -13.75$ MeV
As far as pairing effects are concerned:

- $\xi$ has similar properties for all considered interactions.

- $\xi_{HFB} \sim \xi_{BCS}$.

- Nice scaling behavior of $\langle \xi \rangle$, with some shell effects.

- $\xi$ insensitive to variations of the intensity of pairing correlations due to thermal pair breaking.
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Quartet Correlations Density

Simplest way to build a quartet: proton and neutron pairs independent from each other [Mang 1960, Sandulescu 1962].

This allows us to define the quarteting density as:

\[
\kappa_q(R_\pi, R_\nu) = \langle \phi_0^{(\beta_\alpha/2)}(r_\pi) | \kappa_\pi(r_1, r_2) \rangle \cdot \langle \phi_0^{(\beta_\alpha/2)}(r_\nu) | \kappa_\nu(r_3, r_4) \rangle
\]

\(\alpha\)-particle internal wavefunction

\[
\psi_\alpha = \phi_0^{(\beta_\alpha/2)}(r_\pi) \cdot \phi_0^{(\beta_\alpha/2)}(r_\nu) \cdot \phi_0^{(\beta_\alpha)}(r_\alpha)
\]
Quartet Coherence Length

$$\xi_q(R_\alpha) = \sqrt{\langle r^2_\alpha \rangle} \kappa_q$$
p-n correlations are described by the term $\phi_{00}^{(\beta_\alpha)}(r_\alpha)$ of $\psi_\alpha$.

The $\alpha$ tensor

$$\kappa_\alpha(r_\alpha, R_\alpha) = \kappa_q(r_\alpha, R_\alpha) \cdot \phi_{00}^{(\beta_\alpha)}(r_\alpha)$$
p-n correlations are described by the term $\phi_{00}(r_\alpha)$ of $\psi_\alpha$.

The $\alpha$ tensor

$$\kappa_\alpha(r_\alpha, R_\alpha) = \kappa_q(r_\alpha, R_\alpha) \cdot \phi_{00}(r_\alpha)$$

Formation Amplitude

The amplitude $\langle \psi_\alpha | \text{quartet} \rangle$ is [Mang 1960]:

$$\mathcal{F}(R_\alpha) = \int_0^\infty \kappa_\alpha(r_\alpha, R_\alpha) r_\alpha^2 \, dr_\alpha$$
Quartet and $\alpha$ Coherence Lengths
Coherence Length

\[ \xi_{\alpha} (\text{fm}) \]

\[ A \]
\[ \xi(R)^2 = \frac{\int r^2dr \; r^2 \kappa(r,R)^2}{\int r^2dr \; \kappa(r,R)^2} = \frac{I^{(2)}}{I^{(1)}} \]
Coherence Length

For $^{220}\text{Ra}$:

- (a) $\kappa_q(\alpha, R_\alpha)^2$
- (b) $\kappa_\alpha(\alpha, R_\alpha)^2$
- (c) $w_q(\alpha, R_\alpha)$
- (d) $w_\alpha(\alpha, R_\alpha)$

where

$$\xi(R)^2 = \int dr \ r^2 \ w(r, R)$$
Conclusions - 2nd part

• Our simple treatment evidences the surface nature of $\alpha$ condensation: the formation amplitude $\mathcal{F}(R_N) = \text{max.}$

• The quartet CL is somewhat similar to the pairing CL, but with larger values on the nuclear surface.

• The p-n correlations play an important role, as the $\alpha$-CL has a quasiconstant value $\xi_\alpha \sim 1.7\text{fm} \leq r_\alpha = 1.9\text{fm}.$
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Thank you!

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References

Pairing:


Quartentning and $\alpha$: