

# Nuclear symmetry energy in density-dependent relativistic Hartree-Fock theory: the role of Fock terms and tensor force

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# Outline

1. Introduction
2. Theoretical Framework of DDRHF Theory in Nuclear Matter
3. Results and Discussion
  - Symmetry Energy Properties in Nuclear Matter
  - Self-Consistent Tensor Effects on Nuclear Matter Systems
  - Possible Ways to Improve DDRHF EDF
4. Summary and Outlook

# Symmetry Energy in Nuclear Matter

- Equation of state isospin asymmetric nuclear matter

$$E_b(\rho_b, \delta) = E_0(\rho_b) + E_S(\rho_b)\delta^2 + \mathcal{O}(4), \quad \delta = (\rho_n - \rho_p)/(\rho_n + \rho_p)$$

- Empirical parabolic law:

*\* I. Bombaci and U. Lombardo, Phys. Rev. C 44, 1892 (1991).*

$$E_S(\rho_b) = \frac{1}{2} \frac{\partial^2 E_b(\rho_b, \delta)}{\partial \delta^2} \Big|_{\delta=0}, \quad L = 3\rho_0 \left. \frac{\partial E_S(\rho_b)}{\partial \rho_b} \right|_{\rho_b=\rho_0}, \quad K_{\text{sym}} = 9\rho_0^2 \left. \frac{\partial^2 E_S(\rho_b)}{\partial \rho_b^2} \right|_{\rho_b=\rho_0}.$$

- Important to understand

- Information about nuclear structure:  
**fission properties, density distribution, collective excitation, etc.**
- Information about neutron star:  
**mass-radius relation, crust-core transition density, cooling rate, etc.**
- Information about heavy ion reaction mechanism: **isospin diffusion, DR(n/p), etc.**

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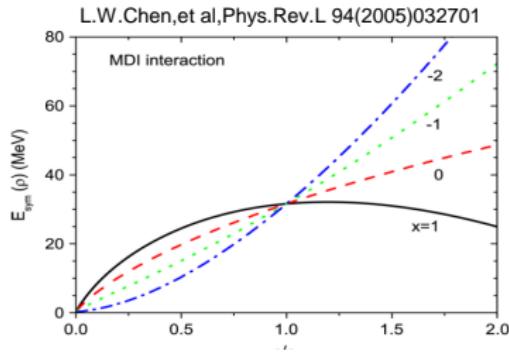
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- Density-dependent uncertainty, esp., at supra-saturation densities.

Ingredients of effective nuclear interaction:  
**momentum-dependence, exchange term, tensor force, SRC, ...**



# The Relativistic Hartree-Fock (RHF) Theory

## Including the exchange terms and the $\pi$ -meson

- ① In-medium effects not considered explicitly:  
⌚ Failed on incompressibility of nuclear matter      ☀ A. Bouyssy: PRL 1985, PRC 1987
- ② With the nonlinear self-couplings of the  $\sigma$ -field:  
⌚ Inconsistency, Broken chiral symmetry      ☀ P. Bernardos: PRC 1993
- ③ Nonlinear self-interaction of scalar field with zero-range limit:  
⌚ Complicated exchange contributions, Violating Pauli Principle      ☀ S. Marcos: JPG 2004
- ④ Density-dependent meson-nucleon couplings:  
⌚ Lacking rearrangement effect      ☀ H. L. Shi: PRC 1995

# The Relativistic Hartree-Fock (RHF) Theory

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⌚ Lacking rearrangement effect      ☀ H. L. Shi:PRC1995
  - ⑤ Density-dependent relativistic Hartree-Fock (DDRHF) theory:  
⌚ Quantitative descriptions comparable with RMF without dropping the exchange terms      ☀ W. H. Long:PLB2006
  - ⑥ Inclusion of  $\rho$ -tensor couplings in DDRHF theory:  
⌚ Enhanced  $np$  correlations, Better conserved relativistic symmetry      ☀ W. H. Long:PRC2007, Long:PRC2010, Liang:EPJA2010
- DDRHF Effective Interactions : PKO1, PKO2, PKO3, PKA1

# Improved Isospin Related Nuclear Structure Descriptions

## Fock terms :

→ Improved  $\beta$  and  $E$  dependence for the effective mass

✳ W. H. Long et al., PLB 640, 150 (2006).

## Exchange diagrams in isoscalar channels :

→ Self-consistent description of spin-isospin excitation

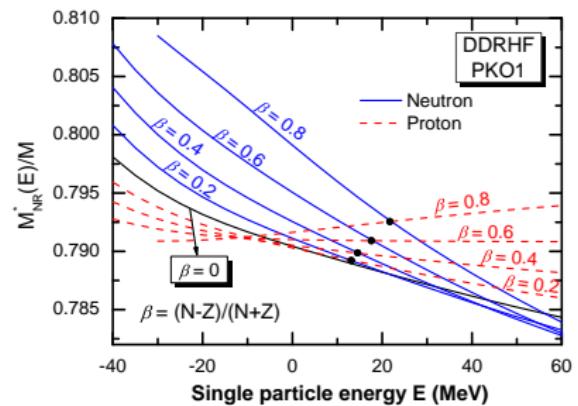
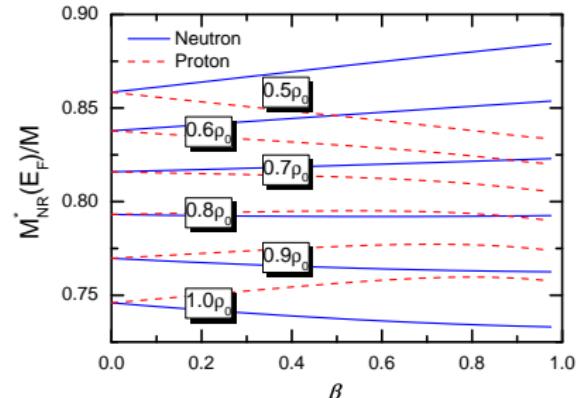
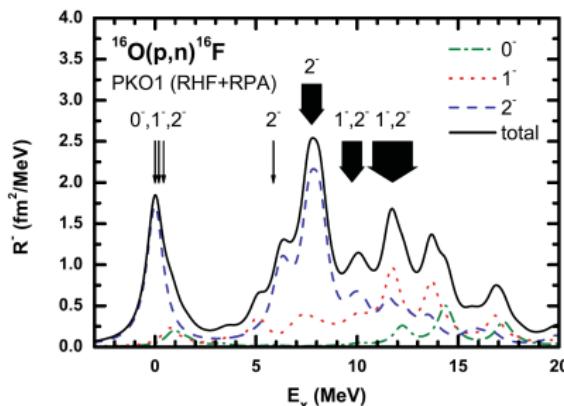
✳ H. Z. Liang, N. Van Giai, J. Meng, PRL 101, 122502 (2008).

✳ H. Z. Liang, P. W. Zhao, J. Meng, PRC 85, 064302 (2012).

→ Significant contributions in the symmetry energy

✳ B. Y. Sun et al., PRC 78, 065805 (2008).

✳ L. J. Jiang et al., PRC 91, 025802 (2015).



## Scalar and Vector effective masses in DBHF:

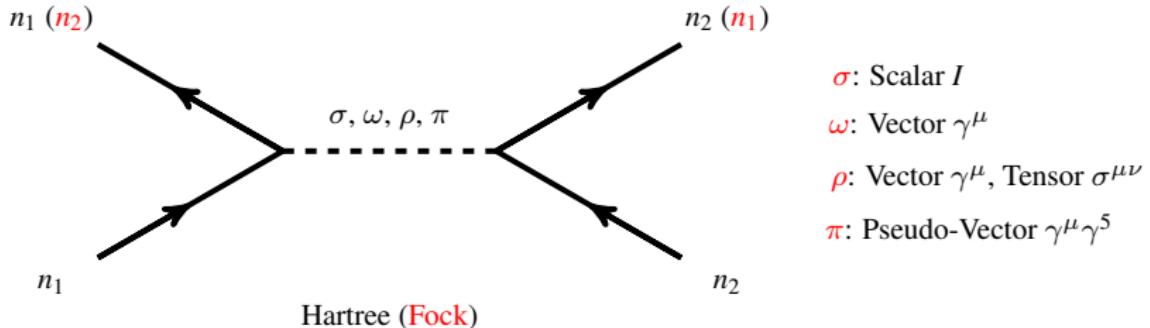
✳ Z. Y. Ma et al., Phys. Lett. B 604, 170 (2004).

# Motivations: Why Symmetry Energies in DDRHF Theory

- 1. What is the behavior of the symmetry energy at high densities?**
- 2. What is the role of the Fock terms in symmetry energy of nuclear matter?**
- 3. What is the role of the tensor force in symmetry energy of nuclear matter?**
- 4. Can we find a way to improve RHF EDF from constraints of symmetry energy?**

# RHF Lagrangian Density

- Relativistic Hartree & Fock (RHF): meson & photon exchanges



- RHF Lagrangian density: Nucleon ( $\psi$ ), Hyperon  $\Lambda$  ( $\psi_\Lambda$ ), Mesons ( $\sigma, \omega, \rho, \pi$ )

$$\begin{aligned}
 \mathcal{L} &= \mathcal{L}_M + \mathcal{L}_\Lambda + \mathcal{L}_I + \mathcal{L}_\sigma + \mathcal{L}_\omega + \mathcal{L}_\rho + \mathcal{L}_\pi \\
 &= \bar{\psi} [i\gamma^\mu \partial_\mu - M] \psi + \bar{\psi}_\Lambda (i\gamma^\mu \partial_\mu - M_\Lambda - g_{\sigma-\Lambda}\sigma - g_{\omega-\Lambda}\gamma^\mu\omega_\mu) \psi_\Lambda \\
 &\quad + \bar{\psi} \left[ -g_\sigma\sigma - g_\omega\gamma^\mu\omega_\mu - g_\rho\gamma^\mu\vec{\rho}_\mu + \frac{f_\rho}{2M}\sigma_{\mu\nu}\partial^\nu\vec{\rho}^\mu \cdot \vec{\tau} - \frac{f_\pi}{m_\pi}\gamma_5\gamma^\mu\partial_\mu\vec{\pi} \cdot \vec{\tau} \right] \psi \\
 &\quad + \frac{1}{2}\partial^\mu\sigma\partial_\mu\sigma - \frac{1}{2}m_\sigma^2\sigma^2 - \frac{1}{4}\Omega^{\mu\nu}\Omega_{\mu\nu} + \frac{1}{2}m_\omega^2\omega^\mu\omega_\mu \\
 &\quad - \frac{1}{4}\vec{R}^{\mu\nu}\cdot\vec{R}_{\mu\nu} + \frac{1}{2}m_\rho^2\vec{\rho}^\mu\cdot\vec{\rho}_\mu + \frac{1}{2}\partial^\mu\vec{\pi}\cdot\partial_\mu\vec{\pi} - \frac{1}{2}m_\pi^2\vec{\pi}\cdot\vec{\pi},
 \end{aligned}$$

with  $\Omega^{\mu\nu} \equiv \partial^\mu\omega^\nu - \partial^\nu\omega^\mu$ ,  $\vec{R}^{\mu\nu} \equiv \partial^\mu\vec{\rho}^\nu - \partial^\nu\vec{\rho}^\mu$ .

$\Lambda$  hyperon participates only in the interactions propagated by the isoscalar mesons.

# RHF Energy Functional in Momentum Representation

- Energy functional in momentum representation: energy density in nuclear matter

$$\varepsilon = \frac{1}{\Omega} \langle \Phi_0 | H | \Phi_0 \rangle = \varepsilon_k + \sum_{\phi} \left( \varepsilon_{\phi}^D + \varepsilon_{\phi}^E \right),$$

with kinetic energy density  $\varepsilon_k$ , direct ( $\varepsilon_{\phi}^D$ ) and exchange ( $\varepsilon_{\phi}^E$ ) terms of the potential energy density,

$$\varepsilon_k = \sum_{p,s,\tau} \bar{u}(p,s,\tau) (\gamma \cdot p + M_{\tau}) u(p,s,\tau), \quad \text{with} \quad \tau_n = \frac{1}{2}, \tau_p = -\frac{1}{2}, \tau_{\Lambda} = 0,$$

$$\varepsilon_{\phi}^D = + \frac{1}{2} \sum_{p_1,s_1,\tau_1} \sum_{p_2,s_2,\tau_2} \bar{u}(p_1,s_1,\tau_1) \Gamma_{\phi} u(p_1,s_1,\tau_1) \frac{1}{m_{\phi}^2} \bar{u}(p_2,s_2,\tau_2) \Gamma^{\phi} u(p_2,s_2,\tau_2),$$

$$\varepsilon_{\phi}^E = - \frac{1}{2} \sum_{p_1,s_1,\tau_1} \sum_{p_2,s_2,\tau_2} \bar{u}(p_1,s_1,\tau_1) \Gamma_{\phi} u(p_2,s_2,\tau_2) \frac{1}{m_{\phi}^2 + \mathbf{q}^2} \bar{u}(p_2,s_2,\tau_2) \Gamma^{\phi} u(p_1,s_1,\tau_1),$$

where  $\phi$  represents  $\sigma$ -S,  $\omega$ -V,  $\rho$ -V,  $\rho$ -T,  $\rho$ -VT, and  $\pi$ -PV couplings,

$$\Gamma_{\sigma\text{-S}} = ig_{\sigma} \text{ or } ig_{\sigma\text{-}\Lambda}, \quad \Gamma_{\omega\text{-V}} = g_{\omega} \gamma_{\mu} \text{ or } g_{\omega\text{-}\Lambda} \gamma_{\mu}, \quad \Gamma_{\rho\text{-V}} = g_{\rho} \gamma_{\mu} \vec{\tau},$$

$$\Gamma_{\rho\text{-T}} = \frac{f_{\rho}}{2M} q^{\nu} \sigma_{\mu\nu} \vec{\tau}, \quad \Gamma_{\rho\text{-VT}} = \Gamma_{\rho\text{-V}} \text{ or } \Gamma_{\rho\text{-T}}, \quad \Gamma_{\pi\text{-PV}} = \frac{f_{\pi}}{m_{\pi}} \mathbf{q} \cdot \boldsymbol{\gamma} \gamma_5 \vec{\tau}.$$

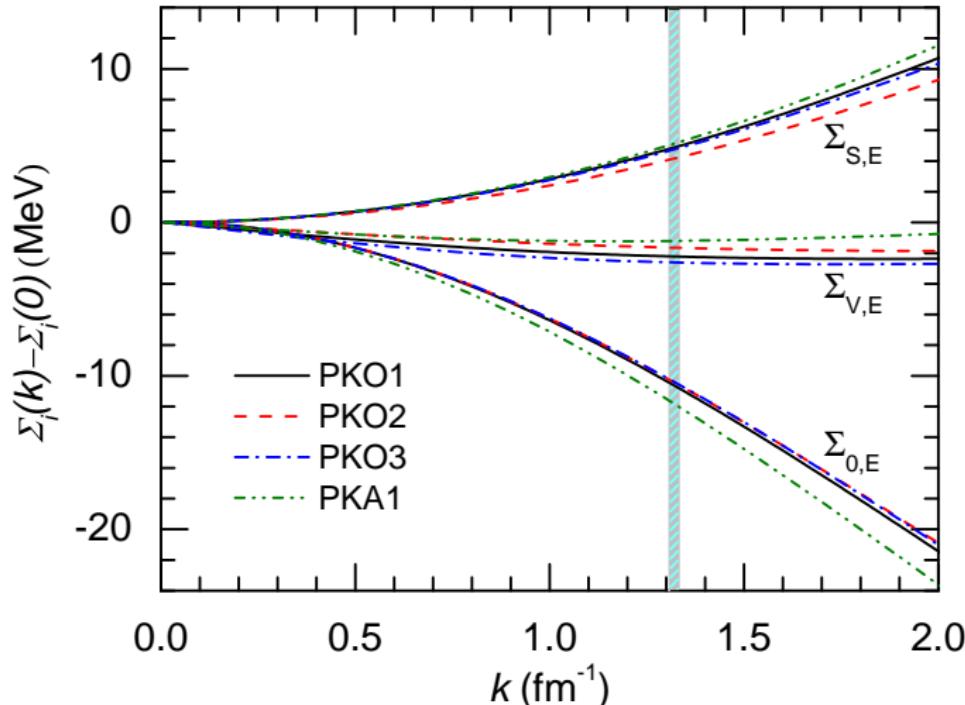
- Self-energies in nuclear matter from variation:  $\Sigma(p) = \Sigma_S(p) + \gamma_0 \Sigma_0(p) + \boldsymbol{\gamma} \cdot \hat{\mathbf{p}} \Sigma_V(p)$

$$\boxed{\Sigma(p)u(p,s,\tau) = \frac{\delta}{\delta \bar{u}(p,s,\tau)} \sum_{\sigma,\omega,\rho,\pi} \left[ \varepsilon_{\phi}^D + \varepsilon_{\phi}^E \right].}$$

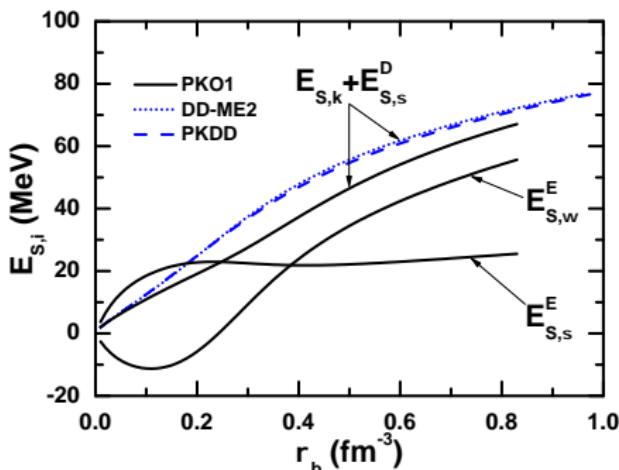
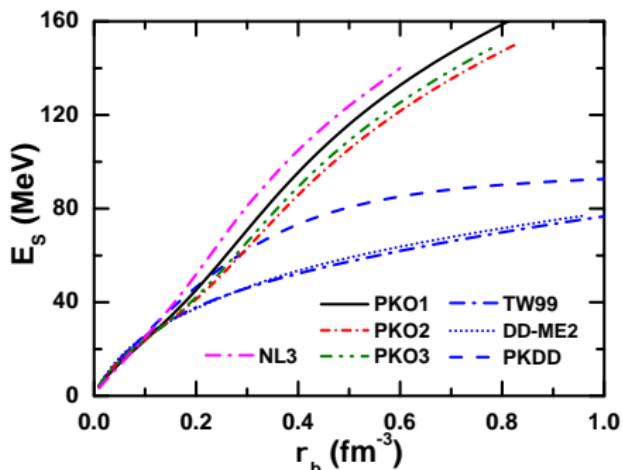
# Momentum Dependence of Nucleon Self-Energy

Where does momentum dependence of the potential energy come from?

- ① Range of  $NN$  force
- ② Intrinsic  $k$ -dependence of  $NN$  interaction
- ③ Fock term



# Symmetry Energy — Correlation with Neutron Star Radius



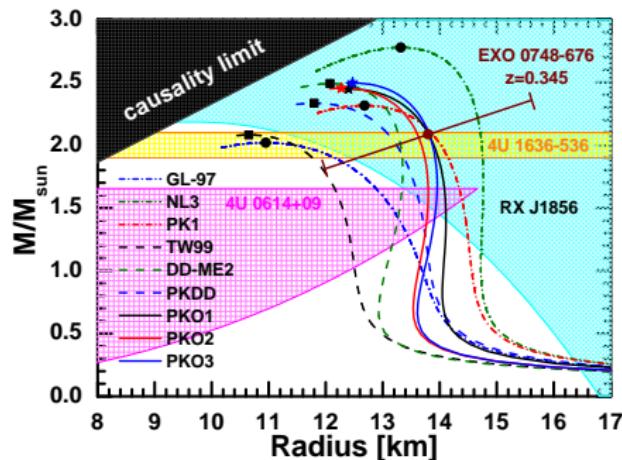
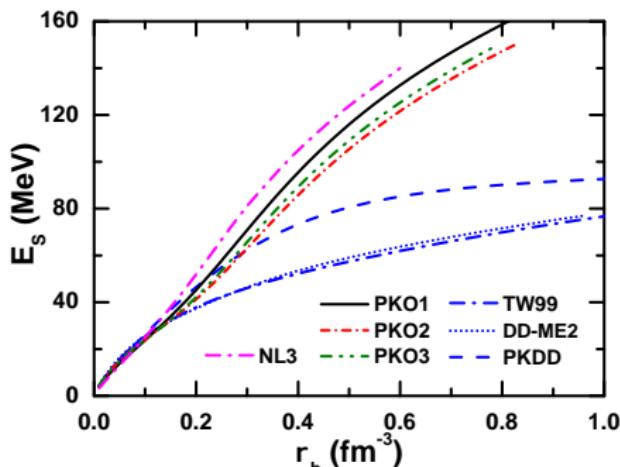
✿ B. Y. Sun, W. H. Long, J. Meng, and U. Lombardo, Phys. Rev. C **78**, 065805 (2008).

- Not only the  $\rho$  meson but all the mesons take part in the isospin properties in the DDRHF theory
  - In charge of producing the symmetry energy via the Fock channel
  - Significant contributions from isoscalar  $\sigma$  and  $\omega$  exchange diagram in the symmetry energy
- Observation limit to neutron star radius imposes a strong constraint on the symmetry energy

Correlation with neutron star radius:

✿ C. J. Horowitz and J. Piekarewicz, Phys. Rev. Lett. **86**, 5647 (2001).

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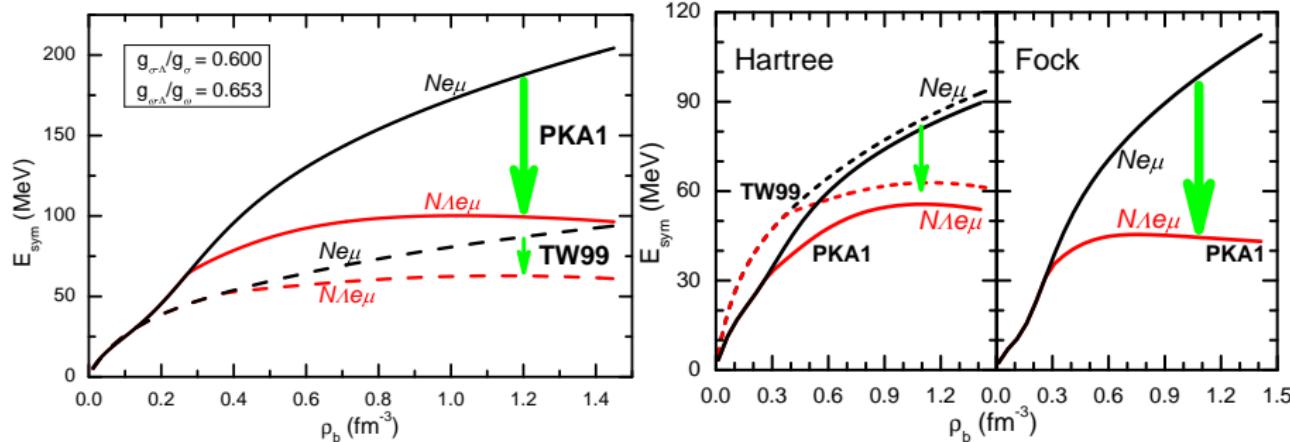
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# Symmetry Energy — Hyperon effects

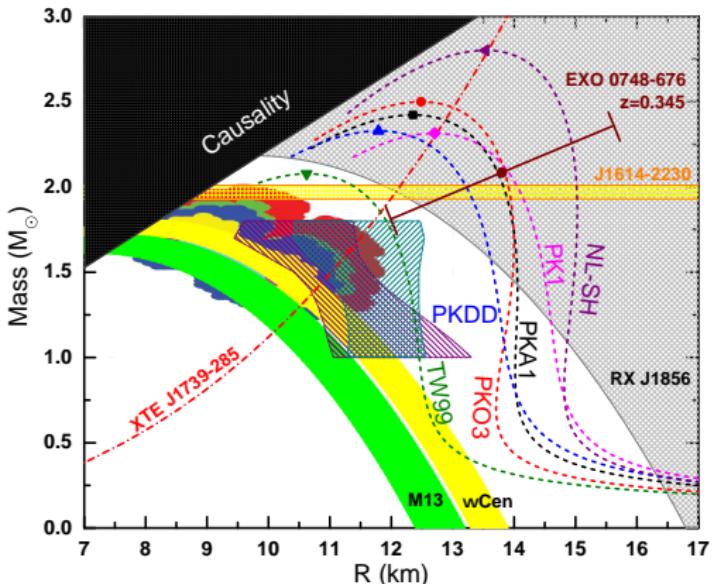


- Softened  $E_{\text{sym}}$  with hyperons → Reduced  $R_{\text{max}}$
- Softened EoSs: The stiffer the EoS is, the greater the effort by  $\Lambda$  to soften the EoS is.
- The  $\Lambda\omega$  couplings in the Fock channel give an attraction even at high densities  
→ Extra mass reduction due to the Fock terms

✿ W. H. Long, B. Y. Sun, K. Hagino, and H. Sagawa, Phys. Rev. C 85, 025806 (2012).

Extra  $E_{\text{sym}}$  softening due to Fock terms: 2-3 times reduction by Fock terms as Hartree ones do  
Fock terms play more significant role in determining the symmetry energy.

# Mass-Radius Relations of Neutron Stars

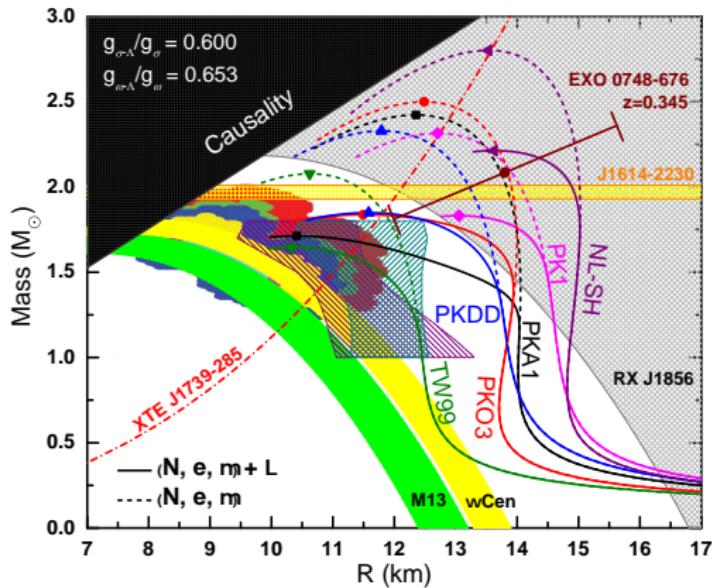


✳ W. H. Long, B. Y. Sun, K. Hagino, and H. Sagawa, Phys. Rev. C 85, 025806 (2012).

	PKA1	PKO3	PKDD	TW99	PK1	NL-SH
$M_{\max}$	1.713	1.837	1.849	1.647	1.832	2.213
$R_{\max}$	10.425	11.495	11.583	10.333	13.048	13.633
$\Delta M_{\max}$	0.710	0.663	0.480	0.431	0.483	0.589
$\Delta R_{\max}$	1.929	0.992	0.215	0.299	-0.343	-0.099

- Reduced deviations on EoSs → Vicinal  $M_{\max}$
- Softened  $E_{\text{sym}}$  with hyperons → Reduced  $R_{\max}$
- Softened EoSs: The  $\Lambda-\omega$  couplings in the Fock channel give an attraction even at high densities → Extra mass reduction due to the Fock terms

# Mass-Radius Relations of Neutron Stars — Hyperon effects



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# Nuclear Tensor Interaction

Nuclear tensor interaction is identified by the form:

$$S_{12} = 3(\boldsymbol{\sigma}_1 \cdot \mathbf{q})(\boldsymbol{\sigma}_2 \cdot \mathbf{q}) - \boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2 \mathbf{q}^2$$

Usually thought it is from (isovector)  $\pi$  and  $\rho$ - meson exchange

⌘ *T. Otsuka, T. Suzuki et al., Phys. Rev. Lett. 95, 232502 (2005).*

- Nuclear structure of ground state: Shell model, SHF+Tensor
  - ⌘ *T. Otsuka, T. Suzuki et al., Phys. Rev. Lett. 95, 232502 (2005).*
  - ⌘ *Colò, H. Sagawa, et al., Phys. Lett.B 646 227–231 (2007).*
- Excitation and decay modes: GT、 SD,  $\beta$ -decay SHF+Tensor+RPA
  - ⌘ *C.L. Bai, H. Q. Zhang, H. Sagawa, et al., Phys. Rev. Lett. 105 072501 (2010).*
  - ⌘ *F. Minato and C.L. Bai, Phys. Rev. Lett. 110 122501 (2013).*
- Density dependence of the symmetry energy
  - ⌘ *C. Xu and B. A. Li, Phys. Rev. C 81, 064612 (2010).*
  - ⌘ *I. Vidaña, A. Polls, and C. Providência, Phys. Rev. C 84, 062801(R) (2011).*

→ GT and SD in RHF+RPA: significant contribution from  $\sigma^E + \omega^E$ :

central+tensor?

⌘ *H.Z. Liang, N.V. Giai, and J. Meng, PRL 101(2008)122502; H.Z. Liang, P.W. Zhao, and J. Meng, PRC 85(2012)064302.*

→  $\beta$ -decay in RHF+QRPA:

⌘ *Z.M. Niu, Y.F. Niu, H.Z.Liang, et al., Phys. Lett. B 723, 172-176, (2013).*

# Relativistic Formalism of Tensors

Relativistic formalism to quantify tensors in Fock diagrams of  $\pi$ -PV,  $\sigma$ -S,  $\omega$ -V,  $\rho$ -T couplings:

✉ L. J. Jiang, S. Yang, B. Y. Sun, W. H. Long, and H. Q. Gu, Phys. Rev. C **91**, 034326 (2015).

$$\mathcal{H}_{\pi\text{-PV}}^T = -\frac{1}{2} \left[ \frac{f_\pi}{m_\pi} \bar{\psi} \gamma_0 \Sigma_\mu \vec{\tau} \psi \right]_1 \cdot \left[ \frac{f_\pi}{m_\pi} \bar{\psi} \gamma_0 \Sigma_\nu \vec{\tau} \psi \right]_2 D_{\pi\text{-PV}}^{T,\mu\nu}(1,2), \quad (1)$$

$$\mathcal{H}_{\sigma\text{-S}}^T = -\frac{1}{4} \left[ \frac{g_\sigma}{m_\sigma} \bar{\psi} \gamma_0 \Sigma_\mu \psi \right]_1 \left[ \frac{g_\sigma}{m_\sigma} \bar{\psi} \gamma_0 \Sigma_\nu \psi \right]_2 D_{\sigma\text{-S}}^{T,\mu\nu}(1,2), \quad (2)$$

$$\mathcal{H}_{\omega\text{-V}}^T = +\frac{1}{4} \left[ \frac{g_\omega}{m_\omega} \bar{\psi} \gamma_\lambda \gamma_0 \Sigma_\mu \psi \right]_1 \left[ \frac{g_\omega}{m_\omega} \bar{\psi} \gamma_\delta \gamma_0 \Sigma_\nu \psi \right]_2 D_{\omega\text{-V}}^{T,\mu\nu\lambda\delta}(1,2), \quad (3)$$

$$\mathcal{H}_{\rho\text{-T}}^T = +\frac{1}{2} \left[ \frac{f_\rho}{2M} \bar{\psi} \sigma_{\lambda\mu} \vec{\tau} \psi \right]_1 \cdot \left[ \frac{f_\rho}{2M} \bar{\psi} \sigma_{\delta\nu} \vec{\tau} \psi \right]_2 D_{\rho\text{-T}}^{T,\mu\nu\lambda\delta}(1,2), \quad (4)$$

where  $\Sigma^\mu = (\gamma^5, \Sigma)$ , and  $D^T$  ( $\phi$  for  $\sigma$  and  $\pi$ ,  $\phi'$  for  $\omega$  and  $\rho$ ) read as,  $G^{\mu\nu\lambda\delta} \equiv \left( g^{\mu\nu} g^{\lambda\delta} - \frac{1}{3} g^{\mu\lambda} g^{\nu\delta} \right)$

$$D_\phi^{T,\mu\nu}(1,2) = \left[ \partial^\mu(1) \partial^\nu(2) - \frac{1}{3} g^{\mu\nu} m_\phi^2 \right] D_\phi(1,2) + \frac{1}{3} g^{\mu\nu} \delta(x_1 - x_2),$$

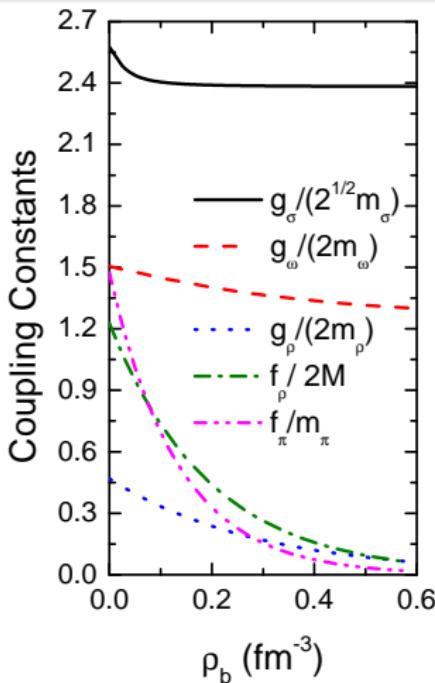
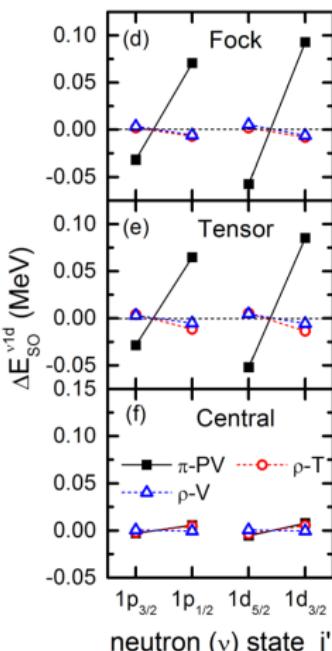
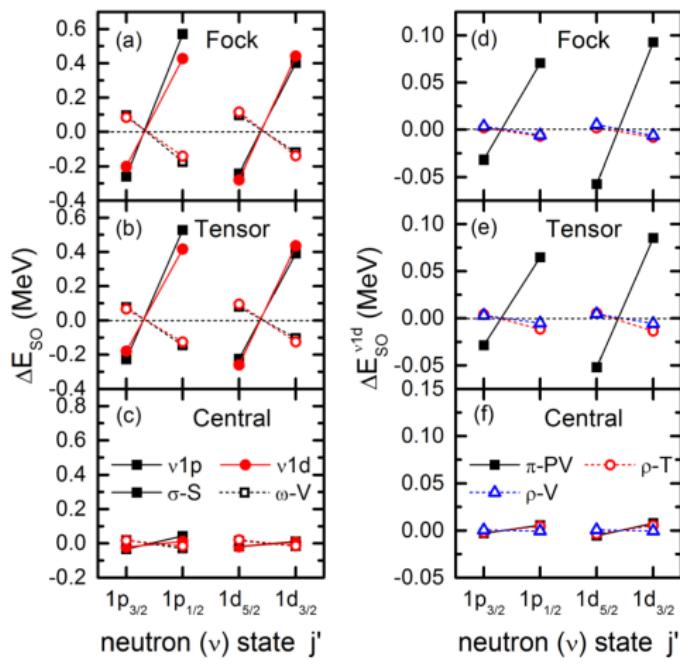
$$D_{\phi'}^{T,\mu\nu\lambda\delta}(1,2) = \left[ \partial^\mu(1) \partial^\nu(2) g^{\lambda\delta} - \frac{1}{3} G^{\mu\nu\lambda\delta} m_{\phi'}^2 \right] D_{\phi'}(1,2) + \frac{1}{3} G^{\mu\nu\lambda\delta} \delta(x_1 - x_2).$$

Relativistic Formalism of Second-Order Irreducible Tensor  $S_{12}$ :

For  $\pi$ -PV,  $\sigma$ -S

$$S_{12} = 3(\gamma_0 \Sigma_1 \cdot \mathbf{q})(\gamma_0 \Sigma_2 \cdot \mathbf{q}) - (\gamma_0 \Sigma_1) \cdot (\gamma_0 \Sigma_2) \mathbf{q}^2$$

# Nature of Tensor Force: Spin Dependence



- Tensor feature involved by Fock diagram is evaluated almost completely by the proposed formalism.
- Much distinct tensor effects are found in the isoscalar rather than the isovector.

✉ L. J. Jiang, S. Yang, B. Y. Sun, W. H. Long, and H. Q. Gu, Phys. Rev. C 91, 034326 (2015).

# Nature of Tensor Force: Tensor Sum Rule

- Tensor sum rule:

⌘ T. Otsuka, T. Suzuki et al., Phys. Rev. Lett. 95, 232502 (2005).

$$(2j_> + 1) V_{j>j'}^T + (2j_< + 1) V_{j<j'}^T = 0$$

$V_{j \geq j'}^T$ : Interaction matrix elements of the tensor force components

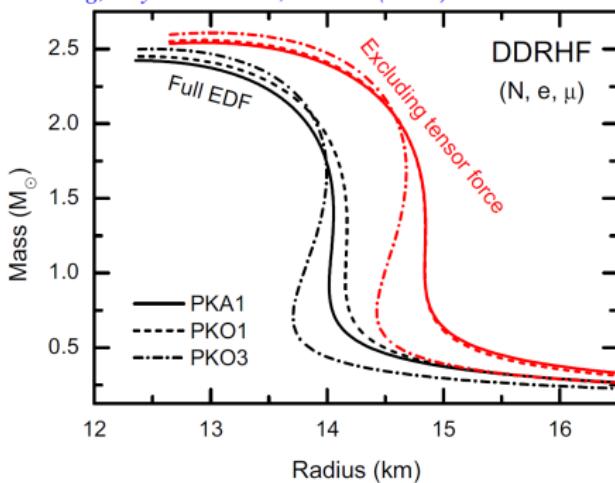
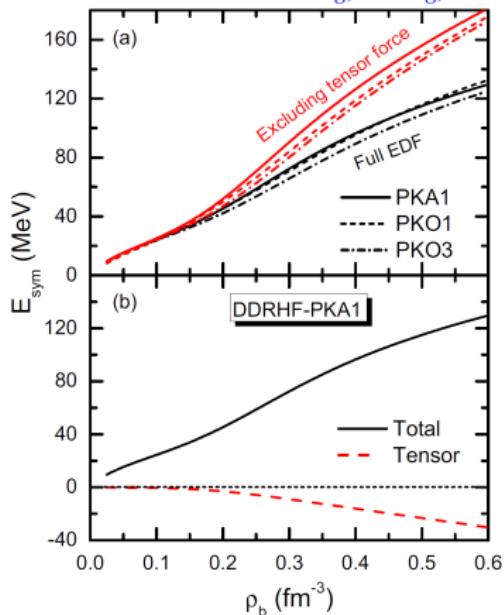
$^{48}\text{Ca}$	$V_{j \geq j'}^{T,\sigma} (10^{-1}\text{MeV})$				$V_{j \geq j'}^{T,\omega} (10^{-1}\text{MeV})$			
	$\nu 1p_{1/2}$	$\nu 1d_{5/2}$	$\nu 1d_{3/2}$	$\nu 1f_{7/2}$	$\nu 1p_{1/2}$	$\nu 1d_{5/2}$	$\nu 1d_{3/2}$	$\nu 1f_{7/2}$
$\nu 1p_{3/2}$	-1.72	+0.80	-1.24	+0.56	+0.27	-0.13	+0.21	-0.10
$\nu 1p_{1/2}$	+3.43	-1.60	+2.48	-1.11	-0.54	+0.26	-0.41	+0.19
$\nu 1d_{5/2}$	-1.62	+1.13	-1.66	+1.02	+0.27	-0.19	+0.28	-0.18
$\nu 1d_{3/2}$	+2.44	-1.69	+2.50	-1.53	-0.40	+0.29	-0.42	+0.27

⌘ L. J. Jiang, S. Yang, B. Y. Sun, W. H. Long, and H. Q. Gu, Phys. Rev. C 91, 034326 (2015).

- Combined with the contributions in spin-orbit splittings, the relativistic formalism are then confirmed to be of the nature of tensor force.
- The tensors are involved naturally by the Fock diagrams and quantified by the relativistic formalism **without introducing any additional free parameters**.

# Tensor Effects — Nuclear Matter

✉ L. J. Jiang, S. Yang, J. M. Dong, and W. H. Long, Phys. Rev. C **91**, 025802 (2015).



- Smaller proton fraction and larger  $\rho^{\text{DU}}$
- Smaller NS radius due to tensor force

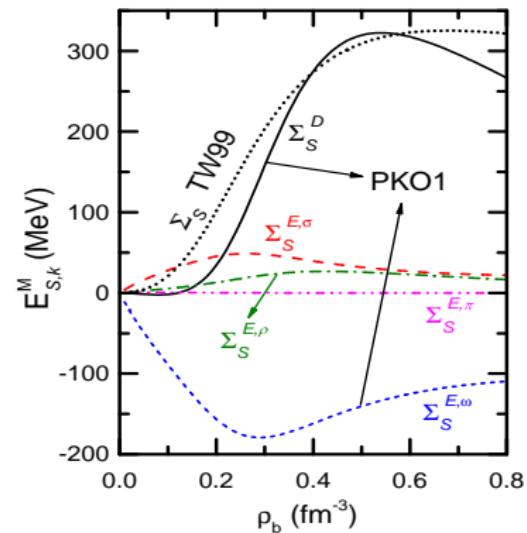
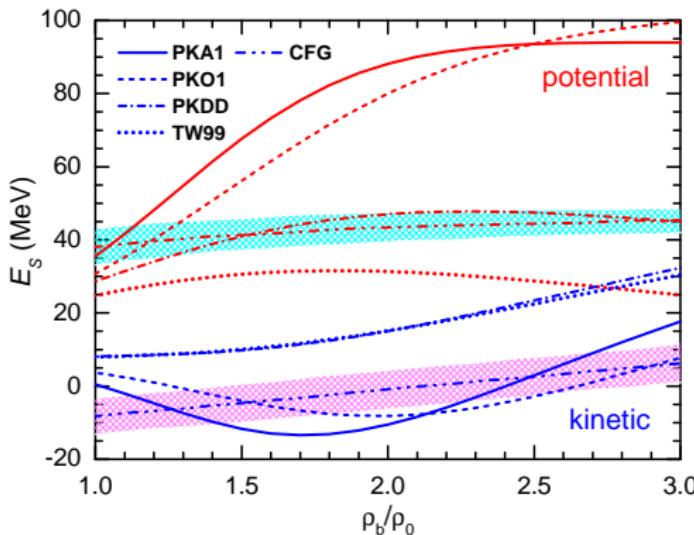
- Softer  $E_S$  due to tensor effects
- Tensor effects enhance with increasing density

Tensor Effects: Responsible for the uncertainty of  $E_S$  at supranuclear densities

✉ C. Xu and B. A. Li, Phys. Rev. C **81**, 064612 (2010); I. Vidaña et al., Phys. Rev. C **84**, 062801(R) (2011).

# Tensor Effects — Kinetic Symmetry Energy

✉ Qian Zhao, Bao Yuan Sun, Wen Hui Long, arXiv:1411.6274, accepted by J. Phys. G.



	TW99	PKDD	PKO1	PKA1	BHF
kin	8.0	8.1	3.7	0.5	-1.0
J	$T = 0$	51.0	50.8	38.8	42.4
	$T = 1$	-26.2	-22.1	-8.1	-5.7
					-9.0

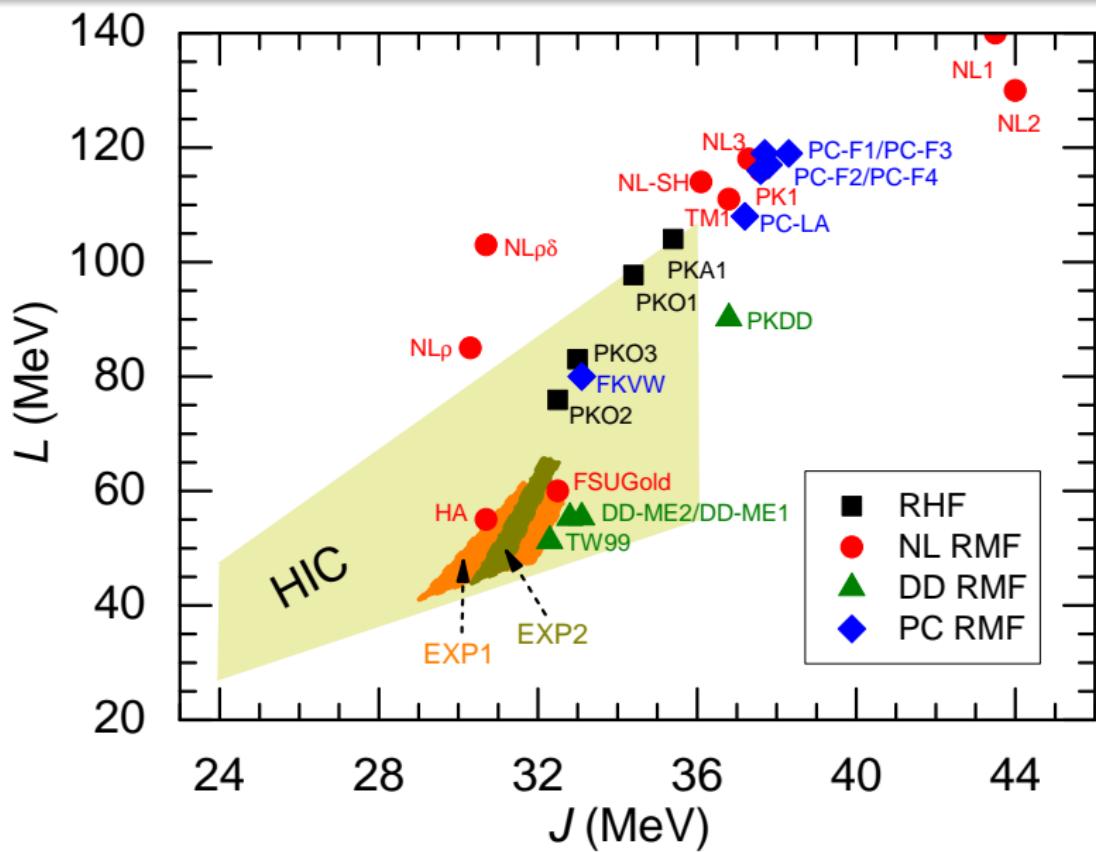
✉ Or Hen et al., Phys. Rev. C **91**, 025803 (2015).  
 ✉ I. Vidaña et al., PRC **84**, 062801(R) (2011).

Short Range Correlation?

$$E_\omega^T \leftrightarrow \int p_1 dp_1 p_2 dp_2 \left\{ \left[ \left( P_1^2 + p_2^2 + \frac{1}{6} m_\omega^2 \right) \Phi_\omega - p_1 p_2 \Theta_\omega \right] \hat{P}_1 \hat{P}_2 + \left( \frac{1}{4} m_\omega^2 \Theta_\omega - p_1 p_2 \right) (\hat{M}_1 \hat{M}_2 - 1) \right\}$$

Difference of  $E_{S,k}$  between RMF and RHF: mainly due to the exchange term of  $\omega$ -coupling

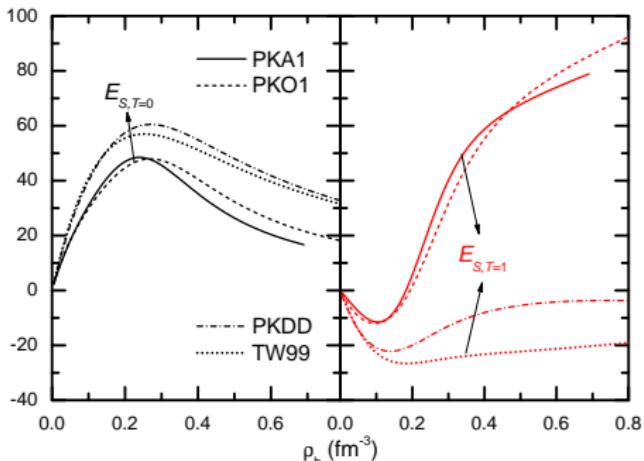
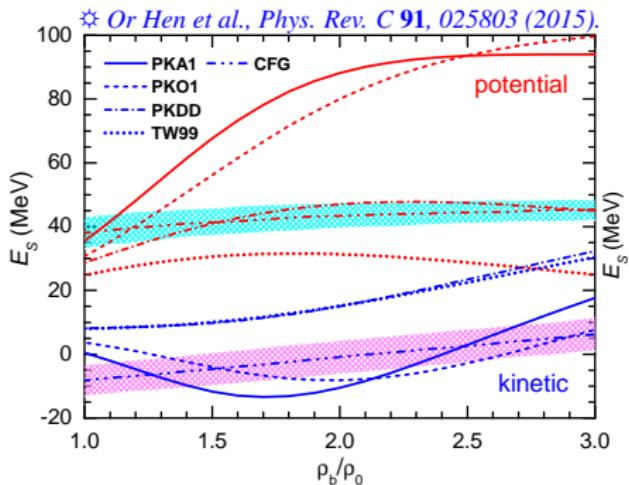
# Properties of $E_S(\rho_b)$ at saturation density: $J$ and $L$



✿ EXP1: J. Lattimer, *Astrophys. J.* 771, 51 (2013); EXP2: J. Lattimer, *Eur. Phys. J. A* 50, 40 (2014).

# Potential Symmetry Energy

$$E_S(\rho_b) = E_{S,k} + E_{S,T=0}^D + E_{S,T=0}^E + E_{S,T=1}^D + E_{S,T=1}^E$$



⌘ Qian Zhao, Bao Yuan Sun, Wen Hui Long, arXiv:1411.6274, accepted by J. Phys. G.

	TW99	PKDD	PKO1	PKA1	BHF
kin	5.9	5.0	-34.5	-69.6	14.9
L	$T = 0$	62.2	78.2	67.5	71.3
	$T = 1$	-12.8	7.0	64.8	103.2
					-17.5

Large model dependence in kinetic and  $T=1$  potential parts: Significant  $E_S^{E,\sigma+\omega}$

Stiff  $E_S$  in RHF: Too strong  $T=1$  ☺

⌘ I. Vidaña et al., PRC 84, 062801(R) (2011).

# Self-energy decomposition of $E_S(\rho_b)$

- Symmetry Energy from HVH theorem

✿ B. J. Cai, L. W. Chen, Phys.Lett. B 711 (2012) 104 – 108.

$$E_b + \rho_b \frac{\partial E_b}{\partial \rho_b} = \varepsilon_F \xrightarrow[\text{to asymmetry}]{\text{expansion}} E_S(\rho_b) = \frac{1}{4} \frac{d}{d\delta} \left[ \sum_{\tau} \tau \varepsilon_F^{\tau}(\rho_b, \delta, k_F^{\tau}) \right] \Big|_{\delta=0}$$

$$\frac{d\varepsilon_F^{\tau}}{d\delta} \Big|_{\delta=0} = \frac{\tau k_F}{3} \left( \frac{\partial \varepsilon}{\partial k} \right) \Big|_{k=k_F} + \frac{\partial \varepsilon_F^{\tau}}{\partial \delta} \Big|_{\delta=0}, \quad \Rightarrow E_S(\rho_b) = \boxed{E_S^{\text{kin}}(\rho_b) + E_S^{\text{mon}}(\rho_b)} + \boxed{E_S^{\text{1st}}(\rho_b)}$$

- Kinetic part,  $k$ -dependence of self-energy,  $\delta$ -dependence of self-energy:

$$E_S^{\text{kin}}(\rho_b) = \frac{k_F k_F^*}{6E_F^*}, \quad E_S^{\text{1st}}(\rho_b) = \frac{1}{2} \frac{g_{\rho}^2}{m_{\rho}^2} \rho_b + E_S^{\text{1st}, E}$$

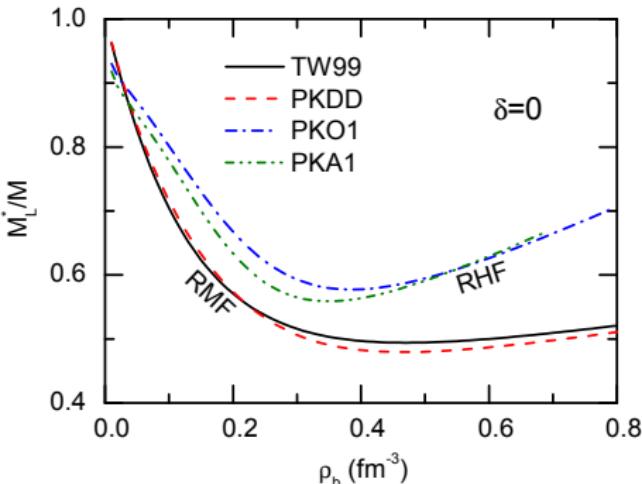
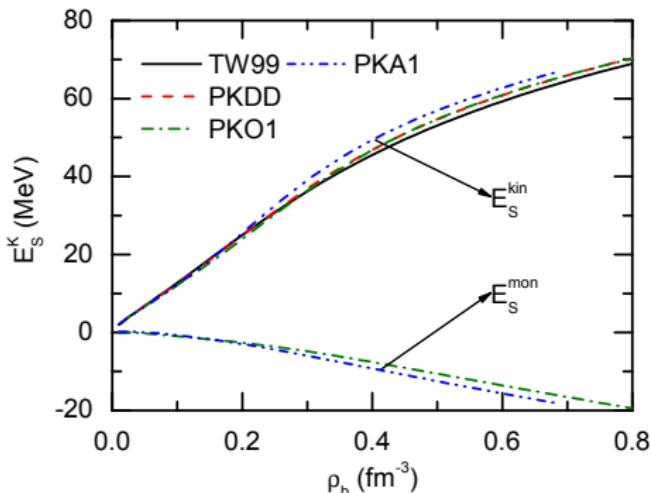
$$E_S^{\text{mon}}(\rho_b) = \frac{k_F k_F^*}{6E_F^*} \frac{\partial \Sigma_V}{\partial k} \Big|_{k=k_F} + \frac{k_F M_F^*}{6E_F^*} \frac{\partial \Sigma_S}{\partial k} \Big|_{k=k_F} + \frac{k_F}{6} \frac{\partial \Sigma_0}{\partial k} \Big|_{k=k_F},$$

- $k$ -dependent contribution and Landau mass:

$$E_S^K = E_S^{\text{kin}} + E_S^{\text{mon}} = \frac{k_F^2}{6M_L^*}$$

- The negative  $E_S^{\text{mon}}$  due to the  $k$ -dependence of self-energies in RHF, lead to larger  $M_L^*$

# Impact of Landau Mass



More reasonable  $M_L^*$  at  $\rho_0$  in RHF compared to empirical data, thus more reasonable  $E_S^K$  in RHF

✳ GQR: P.-G. Reinhard, *Nucl. Phys. A649*, 305c (1999).

✳  $\beta$ -decay: T. Marketin et al., *Phys. Rev. C* 75, 024304(2007).

Neutrino emission from neutron stars:

- Non-Relativistic: **Landau Mass**
- Relativistic: **Dirac Mass**

$$M_R^* = M_{NR}^* + \epsilon = M_g^* \neq M_L^*$$

✳ D.G. Yakovlev et al., *Phys. Rep.* **354**, 1 (2001).

✳ L.B. Leinson, A. Pérez, *Phys. Lett. B* **518**, 15 (2001).

# Possible Ways to Improve RHF EDF

- Inclusion of  $\delta$ -meson coupling channel
- $E_S, L, E_{S,k}$  and  $E_{S,pot}$  constraints in fitting process
- New density dependence of meson-nucleon coupling constants
- Correction of interaction vertex with the form factor
- SRC: a high-momentum tail for the nuclear momentum distribution

Meson-Nucleon Coupling Constants  $g_i^\circ$   
medium effects

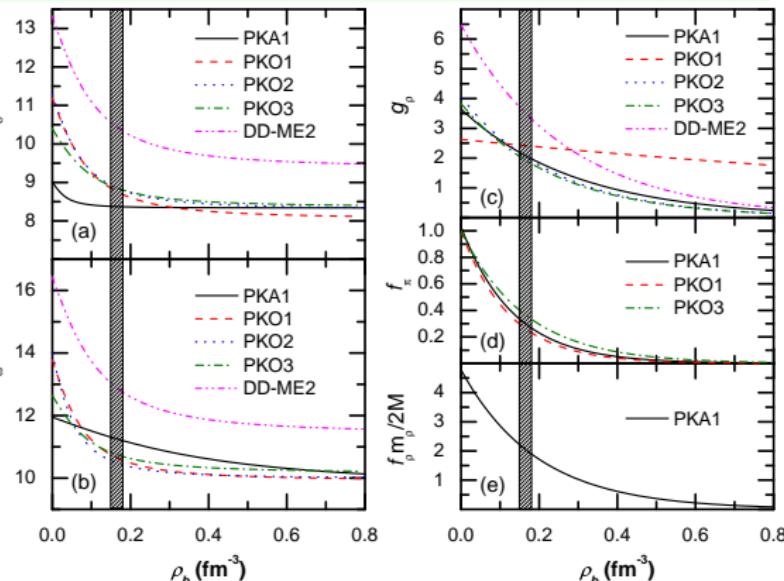
✿ R. Brockmann & H. Toki: PRL1992

$$g_i(\rho_b) = g_i(0)e^{-a_i \xi}, \quad i = \rho, \pi;$$

$$g_i(\rho_b) = g_i(\rho_0)f_i(\xi), \quad i = \sigma, \omega,$$

where  $\xi = \rho_b/\rho_0$  with  $\rho_b = \sqrt{\mu_j j_\mu}$ , and

$$f_i(\xi) = a_i \frac{1 + b_i(\xi + d_i)^2}{1 + c_i(\xi + d_i)^2}$$



# Summary and Outlook

- The effects of exchange terms: isoscalar channel  $\sigma$  and  $\omega$  Fock terms  
    ✿ *B. Y. Sun et al., PRC 78(2008)065805; W. H. Long et al., PRC 85(2012)025806.*
- Without introducing any additional free parameters, the DDRHF approach is a natural way to reveal the tensor effects on the nuclear matter system.  
    ✿ *L. J. Jiang et al., PRC 91(2015)034326; L. J. Jiang et al., PRC 91(2015)025802.*
- The inclusion of the Fock terms in the CDF theory reduces the kinetic part of the symmetry energy and enhances the Landau mass.  
    ✿ *Q. Zhao, B. Y. Sun, W. H. Long, arXiv:1411.6274, accepted by J. Phys. G.*
- Isospin related properties in RHF EDFs could be improved by constraining precisely each components of the symmetry energy.

# Collaborators

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Prof. Kouichi Hagino	Tohoku University, Japan
Prof. Hiroyuki Sagawa	University of Aizu, Japan
Dr. Jian Min Dong	IMPCAS, China
Dr. Li Juan Jiang	Lanzhou University, China
Mr. Shen Yang & Mr. Qian Zhao	Lanzhou University, China



*Thank you for your attention!*

# Quantization of RHF Hamiltonian

- System Hamiltonian ( $\phi = \sigma^S, \omega^V, \rho^V, \rho^{VT}, \rho^T, \pi^{PV}$ ):

$$H = \int d\mathbf{x} \bar{\psi} (-i\gamma \cdot \nabla + M) \psi + \frac{1}{2} \int d\mathbf{x} d\mathbf{x}' \bar{\psi}(\mathbf{x}) \bar{\psi}(\mathbf{x}') \Gamma_\phi D^\phi \psi(\mathbf{x}') \psi(\mathbf{x}),$$

with interaction vertices  $\Gamma_\phi(x, x')$  and meson propagators  $D_\phi(\mathbf{x}, \mathbf{x}')$  Retardation effects neglected

$$D_\phi(\mathbf{x}, \mathbf{x}') = \frac{1}{4\pi} \frac{e^{-m_\phi |\mathbf{x} - \mathbf{x}'|}}{|\mathbf{x} - \mathbf{x}'|}, \quad D_\phi(1, 2) = \frac{1}{m_\phi^2 + \mathbf{q}^2} \quad \text{where} \quad \mathbf{q} = \mathbf{p}_2 - \mathbf{p}_1.$$

- Self-energies  $\Sigma$  in nuclear matter:

Four-momentum of nucleon:  $p = (E(p), \mathbf{p})$

$$\Sigma(p) = \Sigma_S(p) + \gamma_0 \Sigma_0(p) + \boldsymbol{\gamma} \cdot \hat{\mathbf{p}} \Sigma_V(p).$$

- Dirac equation in nuclear matter:

$$(\boldsymbol{\gamma} \cdot \mathbf{p}^* + M^*) u(p, s, \tau) = \gamma_0 E^* u(p, s, \tau),$$

with starred quantities:

Relativistic mass-energy relation:  $E^{*2} = \mathbf{p}^{*2} + M^{*2}$

$$\mathbf{p}^* = \mathbf{p} + \hat{\mathbf{p}} \Sigma_V(p), \quad M^* = M + \Sigma_S(p), \quad E^* = E(p) - \Sigma_0(p).$$

- Quantization of the nucleon field:

No-sea approximation

$$\psi(x) = \sum_{p, s, \tau} u(p, s, \tau) e^{-ipx} c_{p, s, \tau}.$$

- RHF ground state:  $|\Phi_0\rangle = \prod_{p, s, \tau} c_{p, s, \tau}^\dagger |0\rangle$ , where  $|0\rangle$  is physical vacuum state.

# Selected CDF Effective Lagrangians

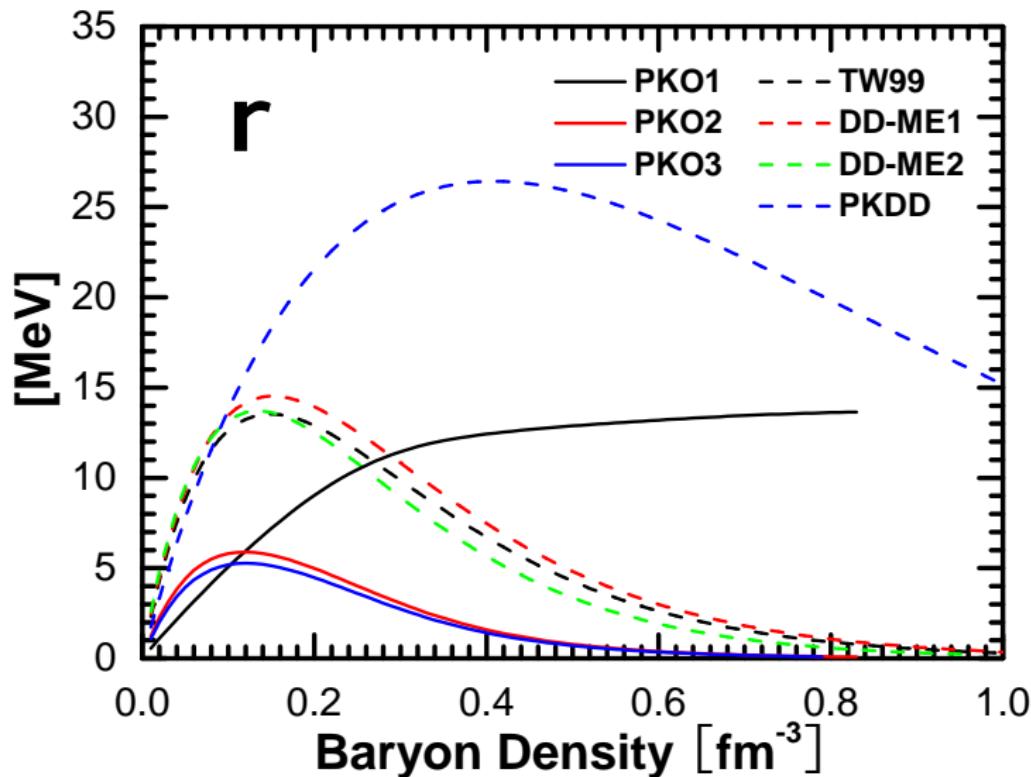
Table: Bulk properties of symmetric nuclear matter at saturation point

	Fock	$\sigma$ -NL	$\omega$ -NL	DD	$\pi$ -PV	$\rho$ -T	$\rho_0$ (fm $^{-3}$ )	$E_B/A$ (MeV)	$K$ (MeV)	$J$ (MeV)	$L$ (MeV)	Reference
PKA1	✓	✗	✗	✓	✓	✓	0.160	-15.83	230.0	36.0	104	<a href="#">Long:2007</a>
PKO1	✓	✗	✗	✓	✓	✗	0.152	-16.00	250.2	34.4	98	<a href="#">Long:2006</a>
PKO2	✓	✗	✗	✓	✗	✗	0.151	-16.03	249.6	32.5	76	<a href="#">Long:2008</a>
PKO3	✓	✗	✗	✓	✓	✗	0.153	-16.04	262.5	33.0	83	<a href="#">Long:2008</a>
NL1	✗	✓	✗	✗	✗	✗	0.152	-16.43	211.2	43.5	140	<a href="#">Reinhard:1986</a>
NL3	✗	✓	✗	✗	✗	✗	0.148	-16.25	271.7	37.4	118	<a href="#">Lalazissis:1997</a>
NL-SH	✗	✓	✗	✗	✗	✗	0.146	-16.33	354.9	36.1	114	<a href="#">Sharma:1993</a>
TM1	✗	✓	✓	✗	✗	✗	0.145	-16.26	281.2	36.9	111	<a href="#">Sugahara:1994</a>
PK1	✗	✓	✓	✗	✗	✗	0.148	-16.27	282.7	37.6	116	<a href="#">Long:2004</a>
TW99	✗	✗	✗	✓	✗	✗	0.153	-16.25	240.3	32.8	55	<a href="#">Typel:1999</a>
DD-ME1	✗	✗	✗	✓	✗	✗	0.152	-16.20	244.7	33.1	56	<a href="#">Nikšić:2002</a>
DD-ME2	✗	✗	✗	✓	✗	✗	0.152	-16.11	250.3	32.3	51	<a href="#">Lalazissis:2005</a>
PKDD	✗	✗	✗	✓	✗	✗	0.150	-16.27	262.2	36.8	90	<a href="#">Long:2004</a>

Relatively large values of  $K$  and  $J$  systematically in RMF with nonlinear self-coupling of mesons (**NLRMF**)

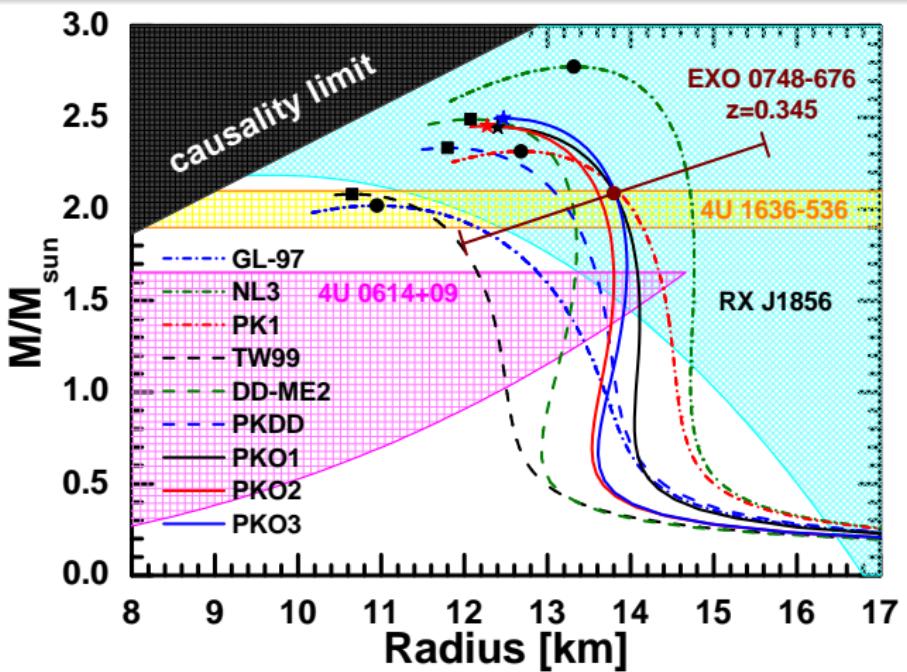
✿ [B. Y. Sun et al., PRC 78\(2008\)065805](#); [W. H. Long et al., PRC 85\(2012\)025806](#); [L. J. Jiang et al., PRC 91\(2015\)025802](#).

# Symmetry Energy — Contribution from $\rho$ Meson



The Fock terms play an important role in EoSs of asymmetric nuclear matter at high densities.

# Mass-Radius Relations of Neutron Stars

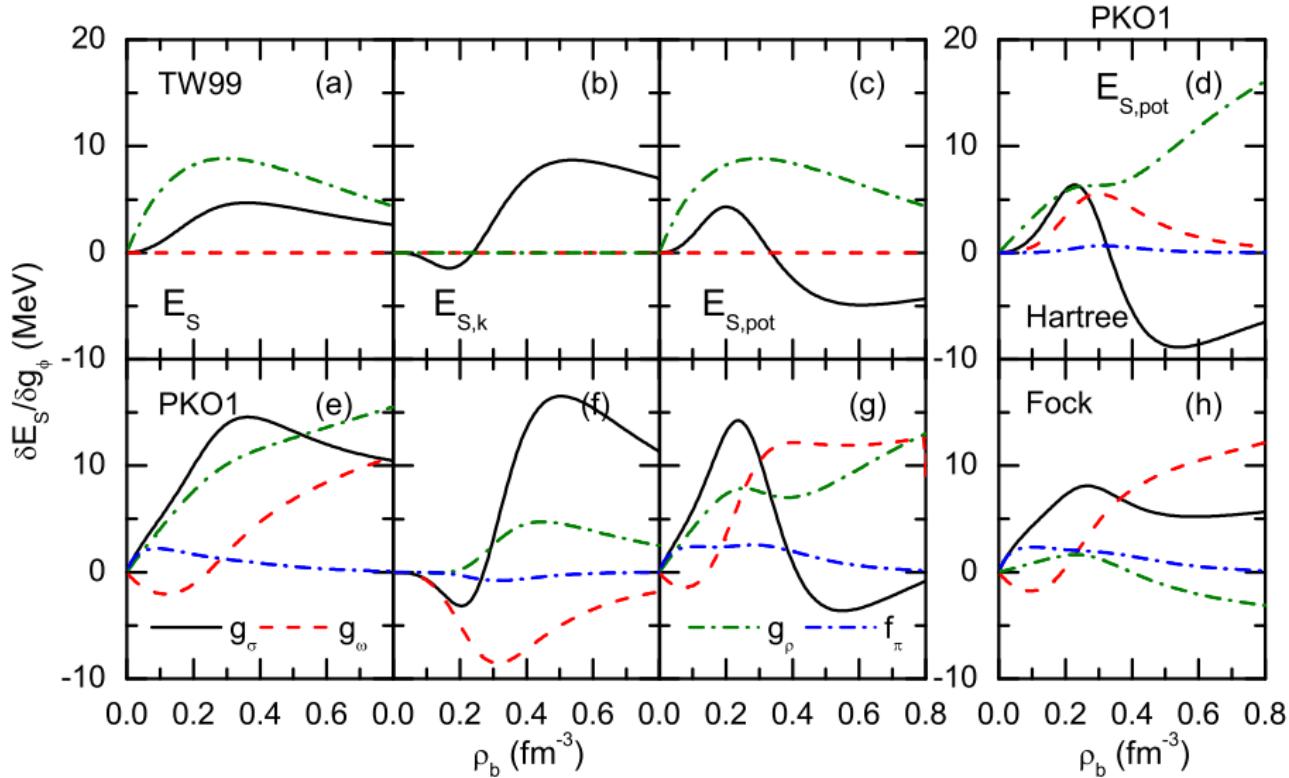


✿ B. Y. Sun, W. H. Long, J. Meng, and U. Lombardo, Phys. Rev. C 78, 065805 (2008).

	GL-97	NL1	NL3	NL-SH	TM1	PK1	TW99	DD-ME1	DD-ME2	PKDD	PKO1	PKO2	PKO3	$E_0$
$M_{\text{max}} (M_{\odot})$	2.02	2.81	2.78	2.80	2.18	2.32	2.08	2.45	2.49	2.33	2.45	2.45	2.49	$E_0$
$\rho_{\text{max}}(0) (\text{fm}^{-3})$	1.09	0.66	0.67	0.65	0.85	0.80	1.10	0.84	0.82	0.89	0.80	0.81	0.78	
$R(M_{\text{max}}) (\text{km})$	10.9	13.4	13.3	13.5	12.4	12.7	10.7	11.9	12.1	11.8	12.4	12.3	12.5	
$R(1.4 M_{\odot}) (\text{km})$	13.3	14.7	14.7	14.9	14.4	14.5	12.4	13.2	13.3	13.7	14.1	13.8	13.9	$E_S$

# Influence of theoretical model uncertainties

✉ Qian Zhao, Bao Yuan Sun, Wen Hui Long, arXiv:1411.6274, accepted by J. Phys. G.



- Zero-Range Tensor Terms in Skyrme EDF

$$V^T = \frac{T}{2} \left\{ \left[ (\boldsymbol{\sigma}_1 \cdot \mathbf{k}') (\boldsymbol{\sigma}_2 \cdot \mathbf{k}') - \frac{1}{3} (\boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2) \mathbf{k}'^2 \right] \delta(\mathbf{r}_1 - \mathbf{r}_2) \right. \\ \left. + \left[ (\boldsymbol{\sigma}_1 \cdot \mathbf{k}) (\boldsymbol{\sigma}_2 \cdot \mathbf{k}) - \frac{1}{3} (\boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2) \mathbf{k}^2 \right] \delta(\mathbf{r}_1 - \mathbf{r}_2) \right\} \\ + U \left\{ (\boldsymbol{\sigma}_1 \cdot \mathbf{k}') \delta(\mathbf{r}_1 - \mathbf{r}_2) (\boldsymbol{\sigma}_2 \cdot \mathbf{k}) - \frac{1}{3} (\boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2) [\mathbf{k}' \cdot \delta(\mathbf{r}_1 - \mathbf{r}_2) \mathbf{k}] \right\}$$

$T$ : triplet-even;  $U$ : triplet-odd

- Tensor Strength Factors:  $\alpha = \alpha_T + \alpha_C$  and  $\beta = \beta_T + \beta_C$

$$\alpha_C = \frac{1}{8} (t_1 - t_2) - \frac{1}{8} (t_1 x_1 + t_2 x_2), \quad \beta_C = -\frac{1}{8} (t_1 x_1 + t_2 x_2), \quad (5)$$

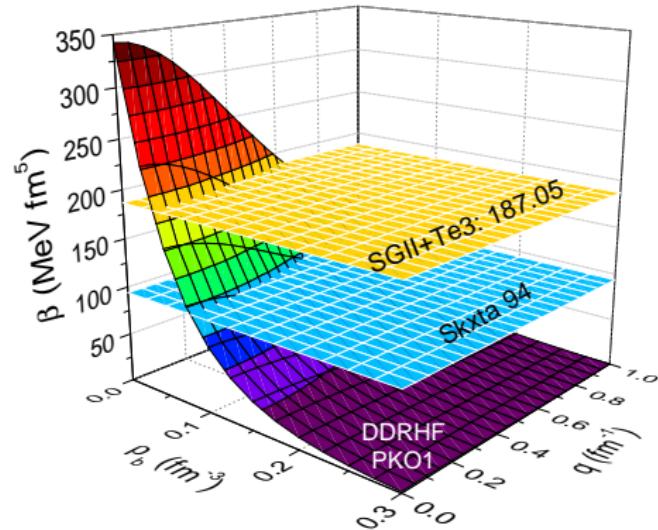
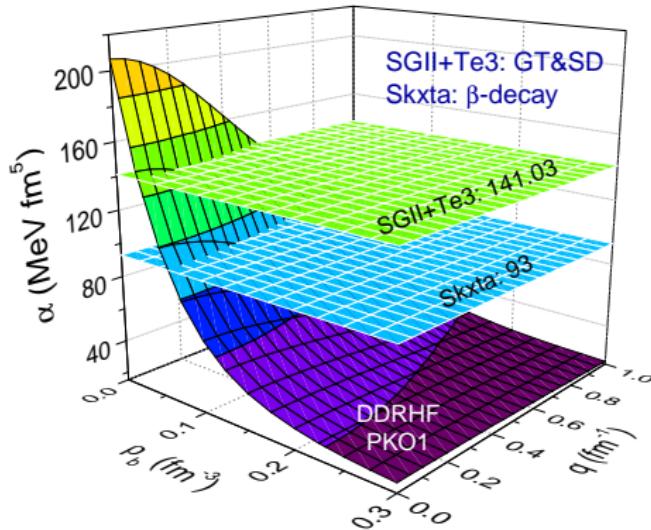
$$\alpha_T = \frac{5}{12} U, \quad \beta_T = \frac{5}{24} (T + U). \quad (6)$$

# Tensor Strength Factors

Tensor Strength Factors:  $\alpha$  and  $\beta$

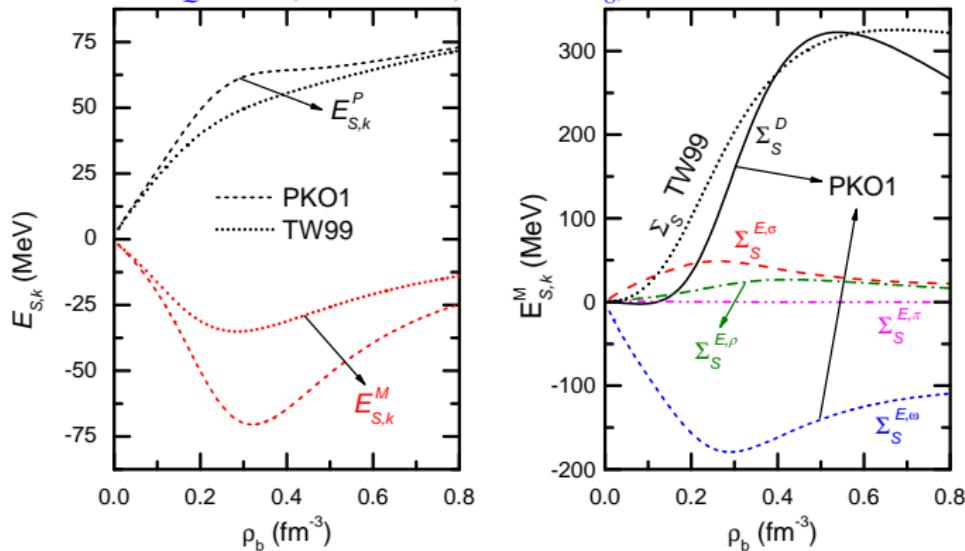
$$\alpha = \frac{5}{12} \left\{ \frac{1}{4} \frac{g_\sigma^2}{m_\sigma^2} \frac{1}{m_\sigma^2 + \mathbf{q}^2} - \frac{1}{8} \frac{g_\omega^2}{m_\omega^2} \frac{1}{m_\omega^2 + \mathbf{q}^2} + \frac{1}{2} \frac{f_\pi^2}{m_\pi^2} \frac{1}{m_\pi^2 + \mathbf{q}^2} - \left[ \frac{1}{8} \frac{g_\rho^2}{m_\rho^2} - \frac{1}{2} \frac{f_\rho^2}{4M^2} \right] \frac{1}{m_\rho^2 + \mathbf{q}^2} \right\},$$
$$\beta = \frac{5}{6} \left[ \frac{1}{2} \frac{f_\pi^2}{m_\pi^2} \frac{1}{m_\pi^2 + \mathbf{q}^2} - \left( \frac{1}{8} \frac{g_\rho^2}{m_\rho^2} - \frac{1}{2} \frac{f_\rho^2}{4M^2} \right) \frac{1}{m_\rho^2 + \mathbf{q}^2} \right]. \quad (7)$$

✉ L. J. Jiang, S. Yang, B. Y. Sun, W. H. Long, and H. Q. Gu, Phys. Rev. C 91, 034326 (2015).



# Kinetic Symmetry Energy in DDRHF Theory

Qian Zhao, Bao Yuan Sun, Wen Hui Long, arXiv:1411.6274.



- Kinetic energy density:

$$\mathcal{E}_k = \sum_{i=n,p} \frac{1}{\pi^2} \int_0^{k_F,i} p^2 dp \left( M \hat{M} + p \hat{P} \right) \equiv \mathcal{E}_k^M + \mathcal{E}_k^P,$$

$\hat{M} = \frac{M}{E^*} + \frac{\Sigma_S^D}{E^*} + \frac{\Sigma_S^{E,\phi}}{E^*}$  with direct term  $\Sigma_S^D$  and exchange term  $\Sigma_S^E$  of the scalar self-energy.

- Difference of  $E_{S,k}$  between RMF and RHF: mainly due to the exchange term of  $\omega$ -coupling