

# Present status of coupled-channels calculations for heavy-ion sub-barrier fusion reactions

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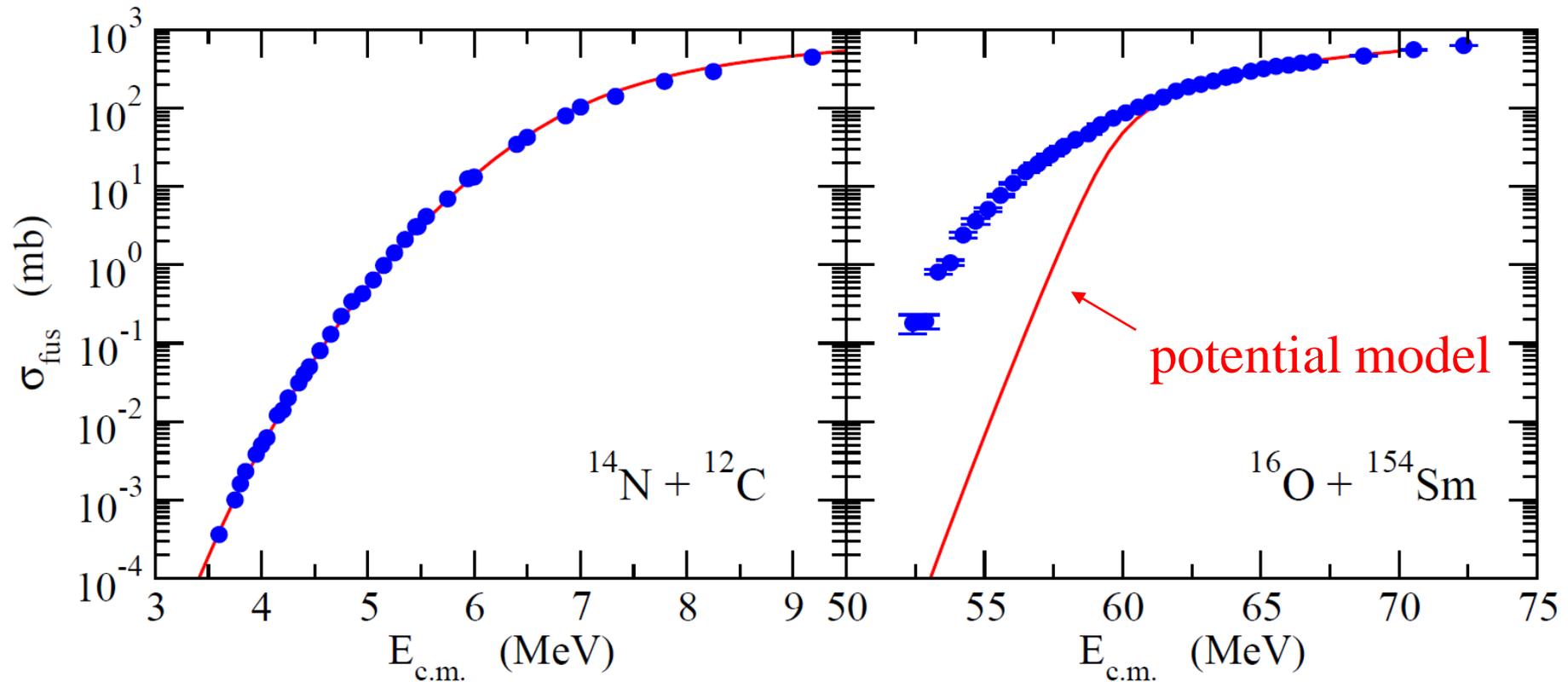


- 1. Introduction: H.I. sub-barrier fusion reactions*
- 2. Coupled-channels approach*
- 3. Coupling in the overlapping region: fusion hindrance*
- 4. C.C. calculations with “beyond-mean-field” method*
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# Introduction: heavy-ion sub-barrier fusion reactions

Discovery of large sub-barrier enhancement of  $\sigma_{\text{fus}}$  (~ the late 70's)

potential model:  $V(r) + \text{absorption}$

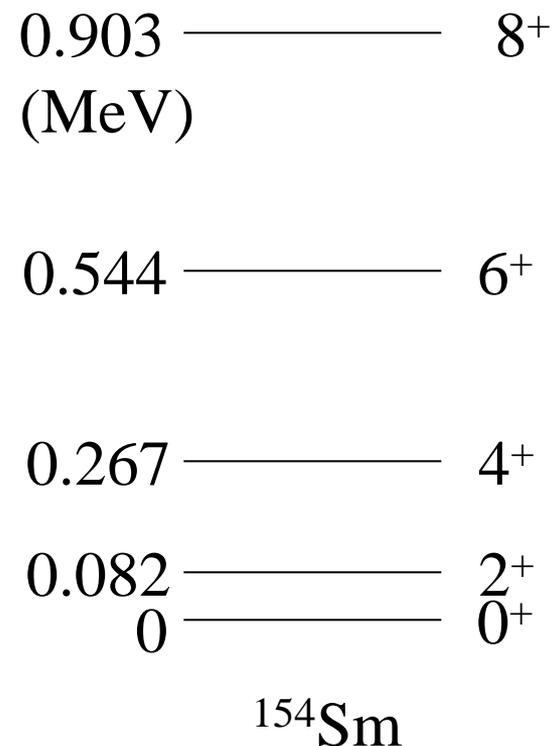
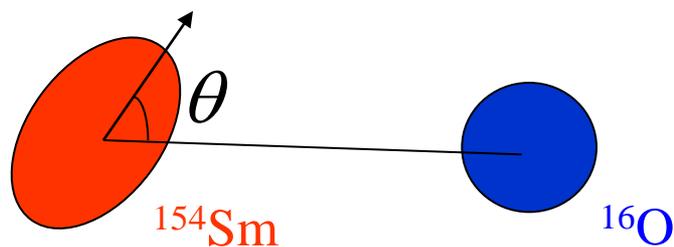
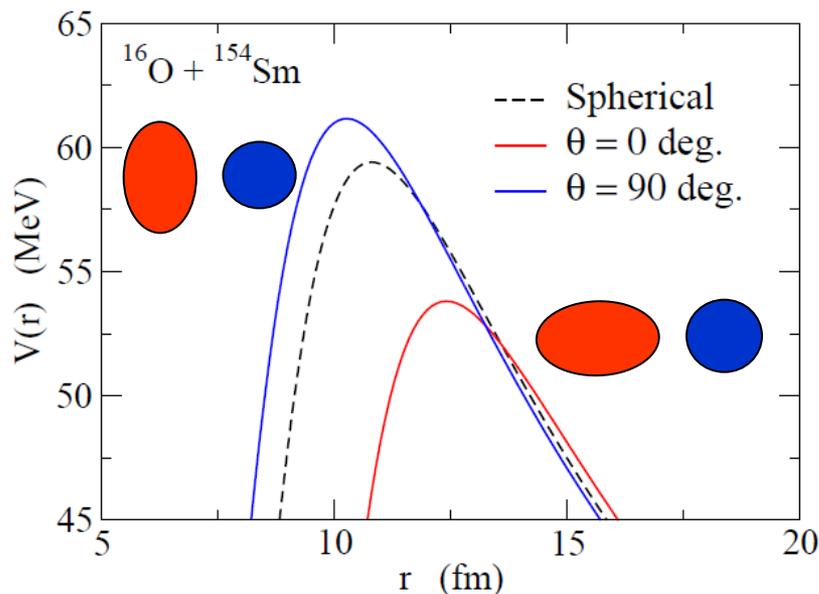


cf. seminal work:

R.G. Stokstad et al., PRL41('78) 465

# Effect of nuclear deformation

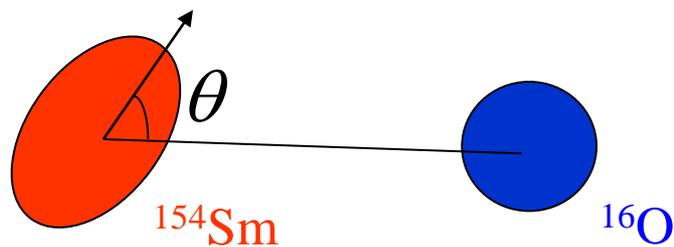
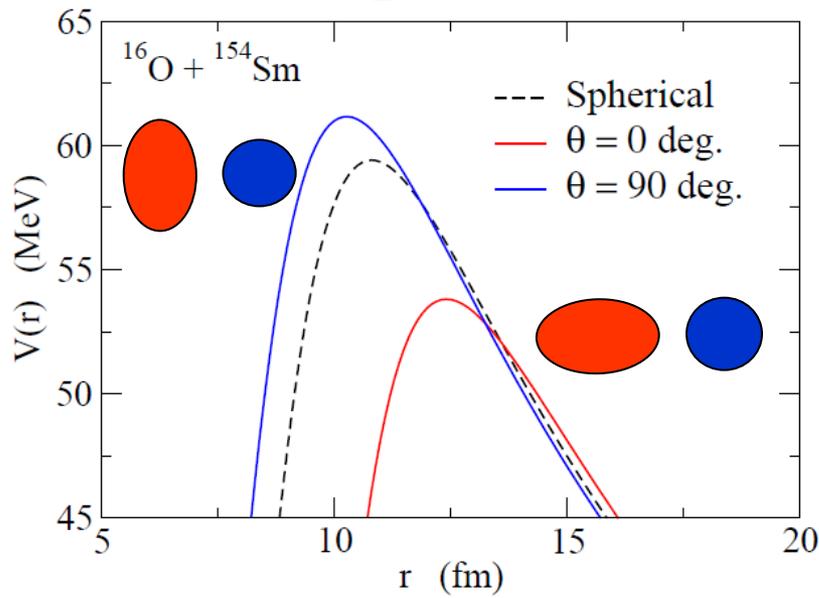
$^{154}\text{Sm}$  : a deformed nucleus with  $\beta_2 \sim 0.3$



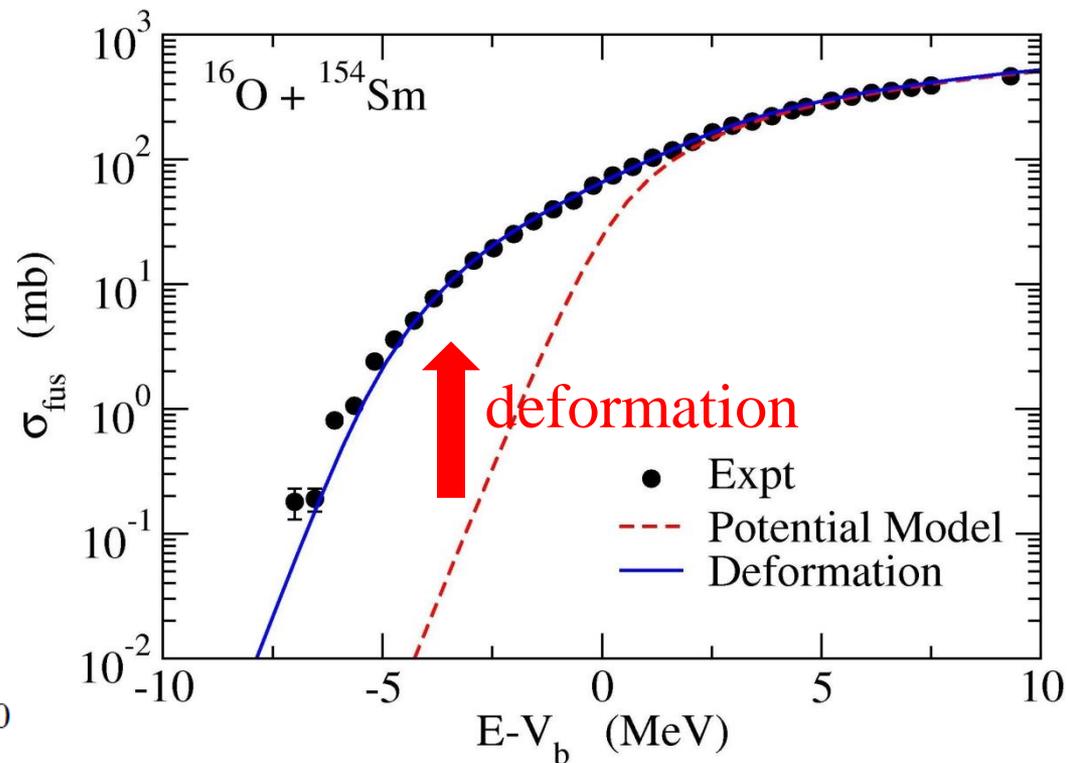
$$\sigma_{\text{fus}}(E) = \int_0^1 d(\cos \theta) \sigma_{\text{fus}}(E; \theta)$$

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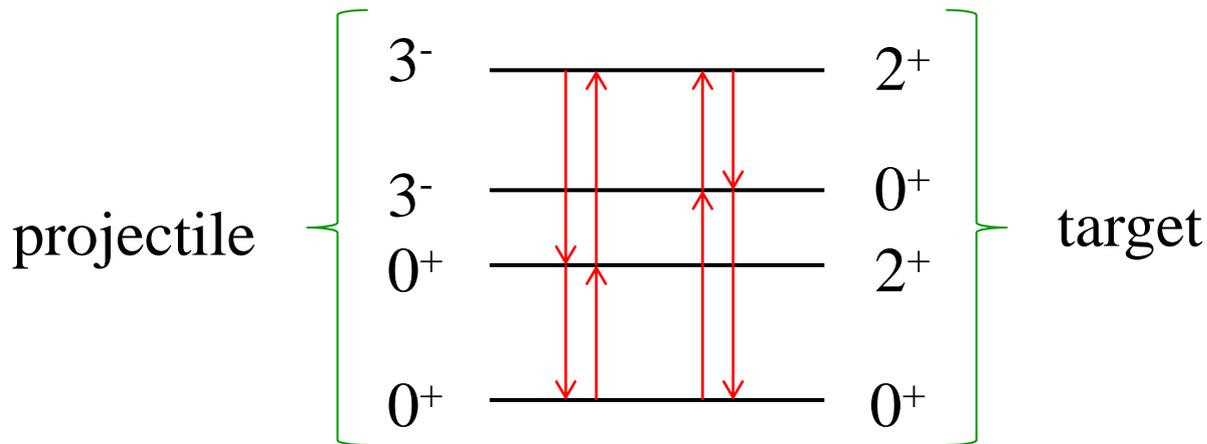
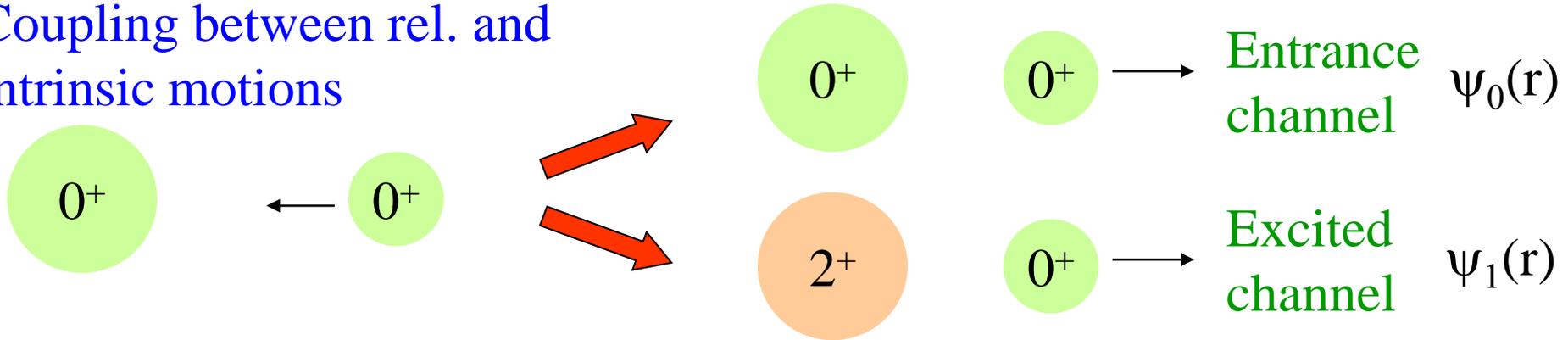


Fusion: strong interplay between  
nuclear structure and  
nuclear reaction

Quantum tunneling with many  
degrees of freedom

# Coupled-Channels method

Coupling between rel. and intrinsic motions



$$\Psi(\mathbf{r}, \xi) = \sum_k \psi_k(\mathbf{r}) \phi_k(\xi)$$



coupled Schroedinger equations for  $\psi_k(\mathbf{r})$

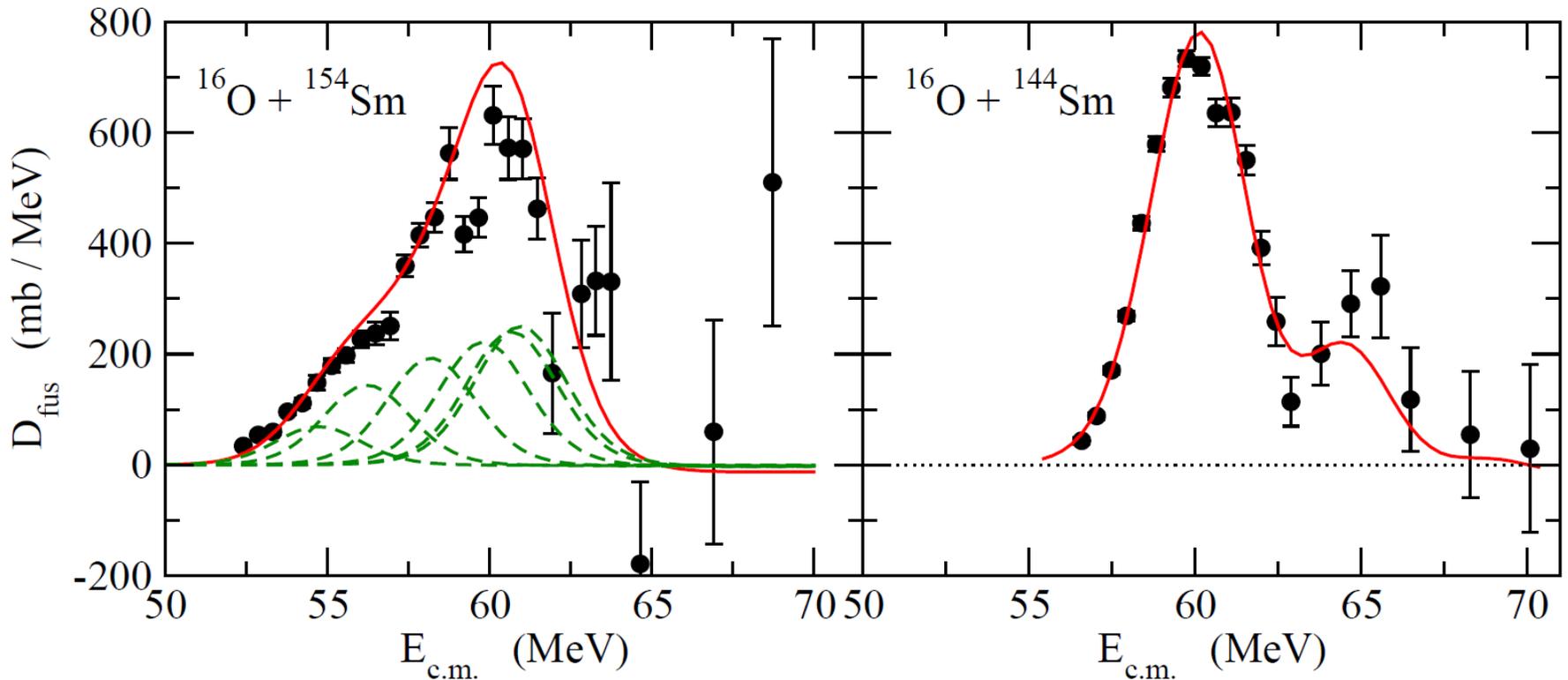
## C.C. approach: a standard tool for sub-barrier fusion reactions

cf. CCFULL (K.H., N. Rowley, A.T. Kruppa, CPC123 ('99) 143)

✓ Fusion barrier distribution (Rowley, Satchler, Stelson, PLB254('91))

$$D_{\text{fus}}(E) = \frac{d^2(E\sigma_{\text{fus}})}{dE^2}$$

— c.c. calculations



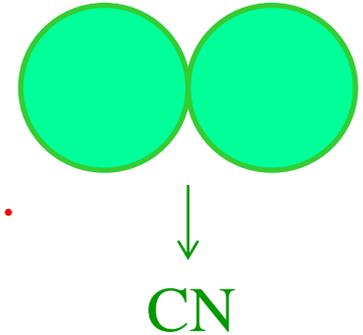
Expt. ANU ('95)

K.H., N. Takigawa, PTP128 ('12) 1061

# Coupled-channels calculations for sub-barrier fusion

## ➤ Low-lying collective excitations only

- low-lying collective excitations
  - strong coupling to the g.s., strong isotope dep.
- non-collective excitations
- giant resonances → high  $E_x$ , smooth isotope dep.



## ➤ Coupling strengths ( $B(E\lambda)$ ) and excitation energies ( $E_x$ )

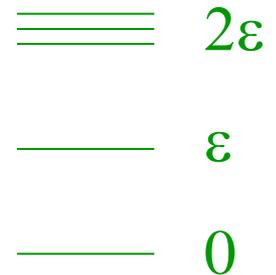
identical to those in isolated nuclei

← colliding nuclei: retain their identity during fusion

## ➤ Multiple excitations to higher collective states

- multi-phonon excitations
- higher members in the g.s. rotational band

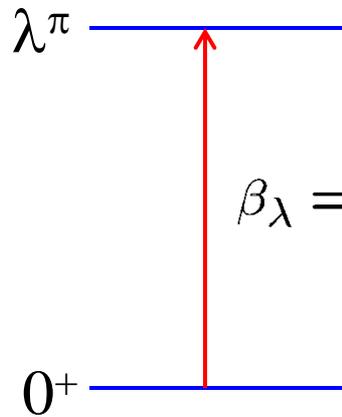
← simple harmonic oscillator/ rigid rotor



this talk:

**The validity of each of these assumptions?**

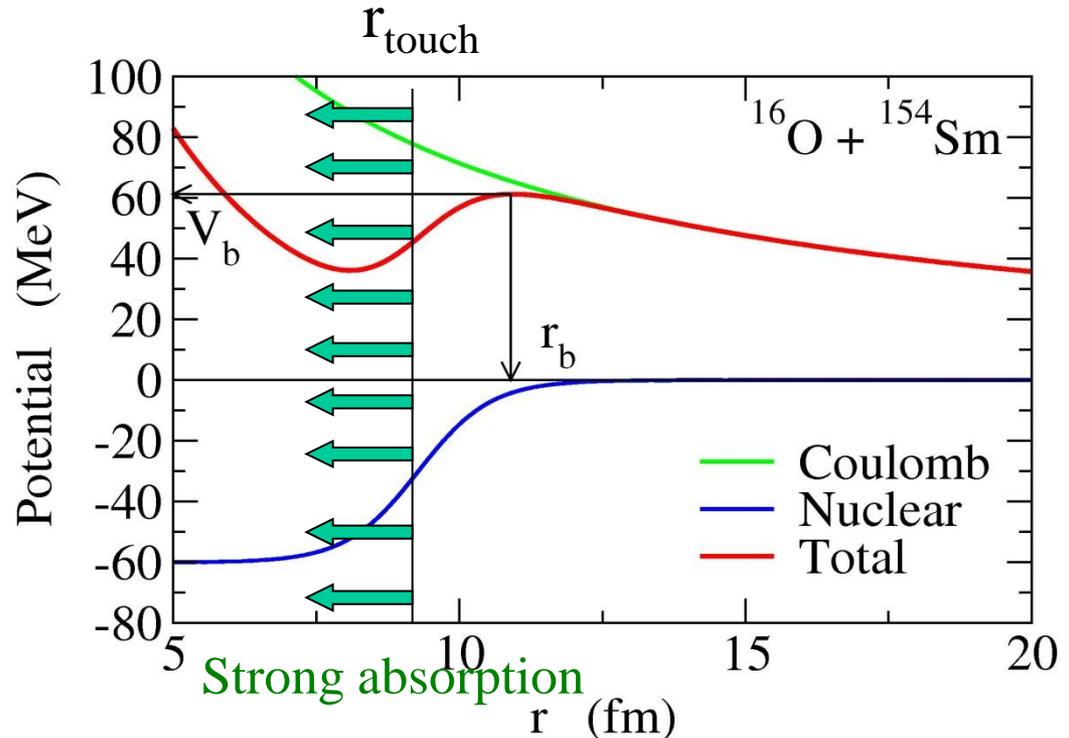
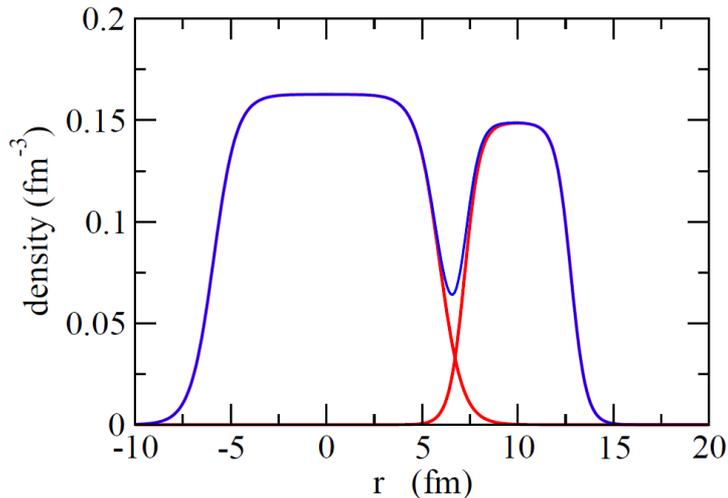
# Damping of collective motions?



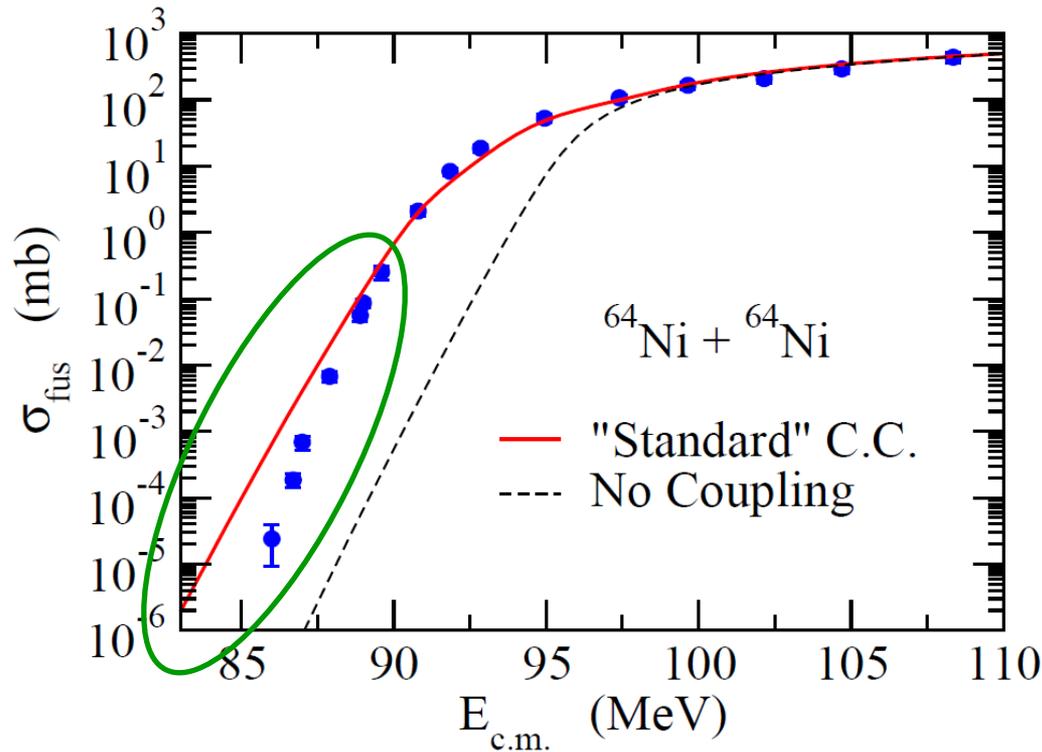
$$\beta_\lambda = \frac{4\pi}{3ZR^\lambda} \sqrt{\frac{B(E\lambda) \uparrow}{e^2}}$$

$$V_{\text{coup}}(r) \sim -\frac{\beta_\lambda}{\sqrt{4\pi}} \cdot R \frac{dV_N}{dr}$$

a level scheme in an isolated nucleus



# Fusion “hindrance” at deep sub-barrier energies



C.L. Jiang et al., PRL89('02)052701;  
PRL93('04)012701

## Theoretical models:

### ➤ Sudden model

S. Misicu and H. Esbensen,  
PRL96('06)112701

- ✓ frozen density
- ✓ repulsive inner core

→ shallow potential

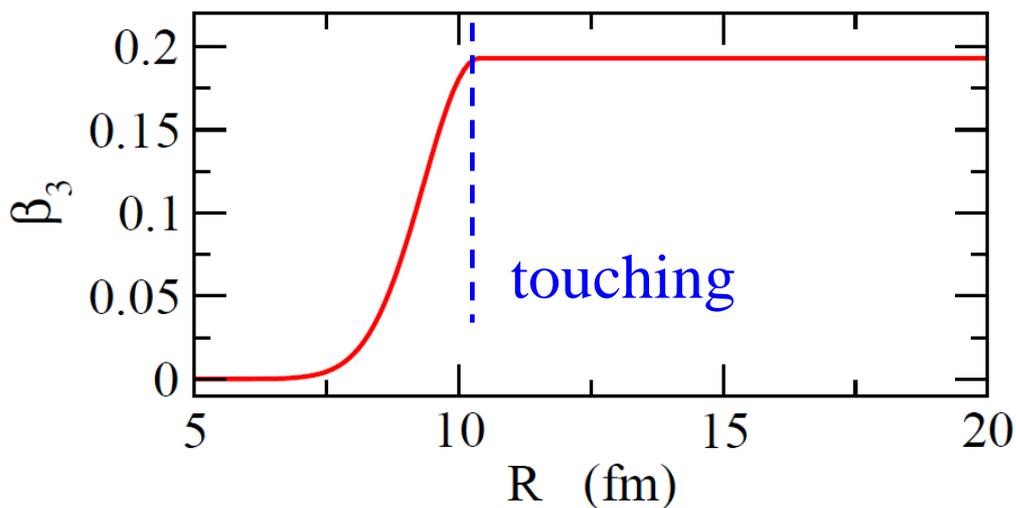
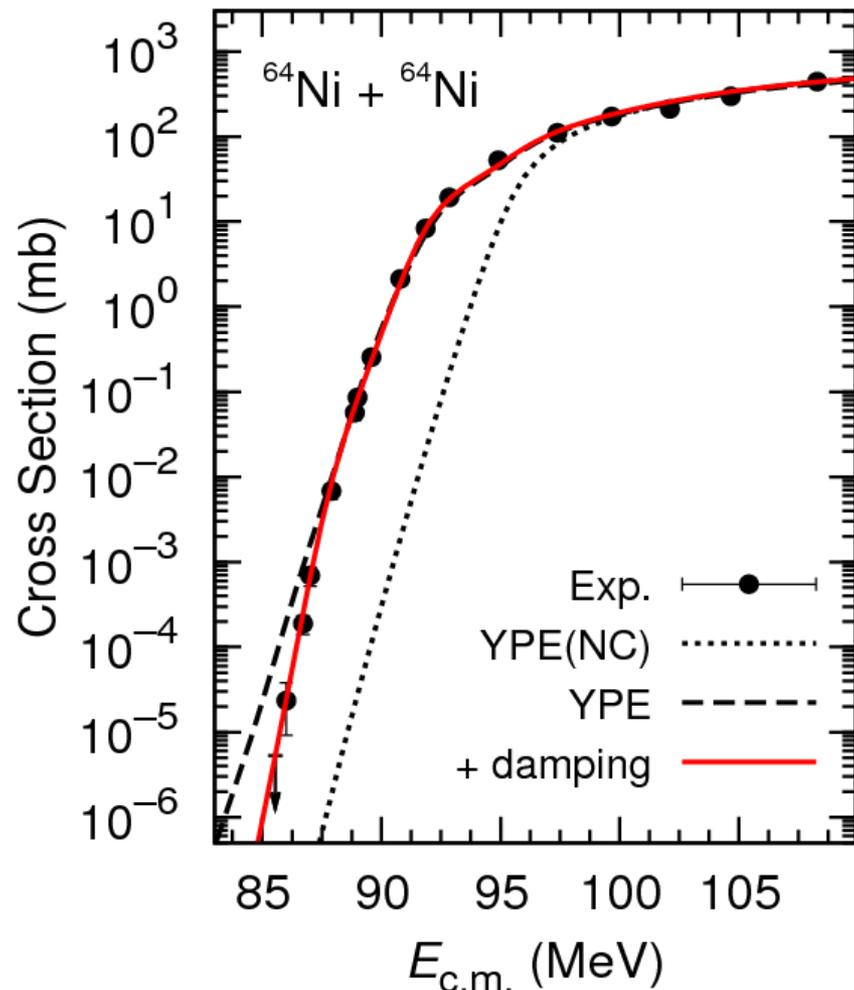
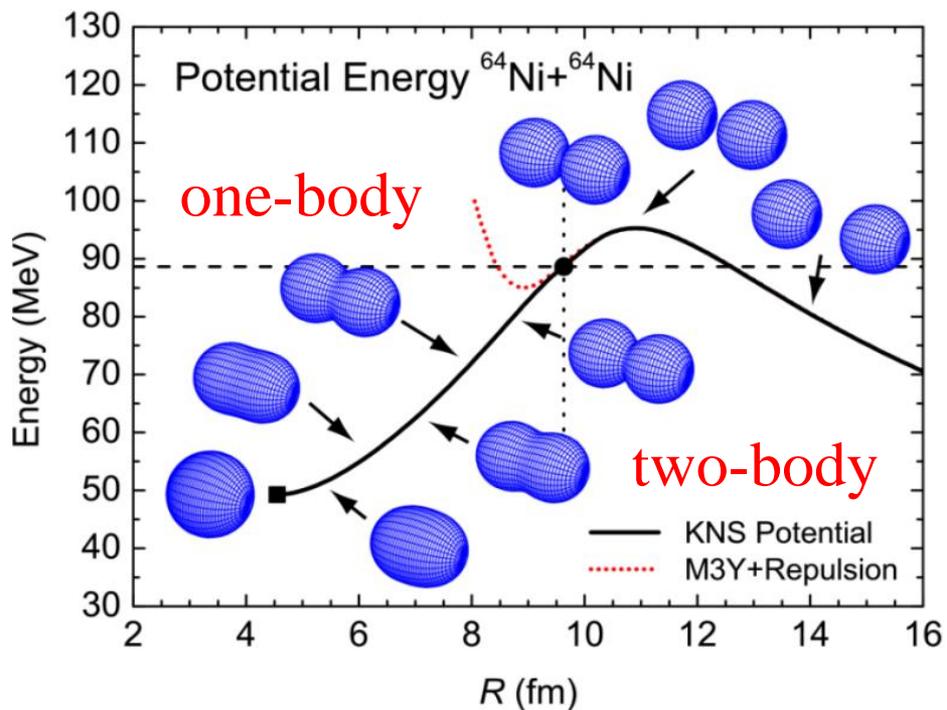
### ➤ Adiabatic model

T. Ichikawa, K.H., and  
A. Iwamoto,  
PRL103('09)202701

- ✓ density change after the touching
- ✓ neck formation

→ deep and thick potential

# Adiabatic model for fusion hindrance (Ichikawa, Hagino, Iwamoto)

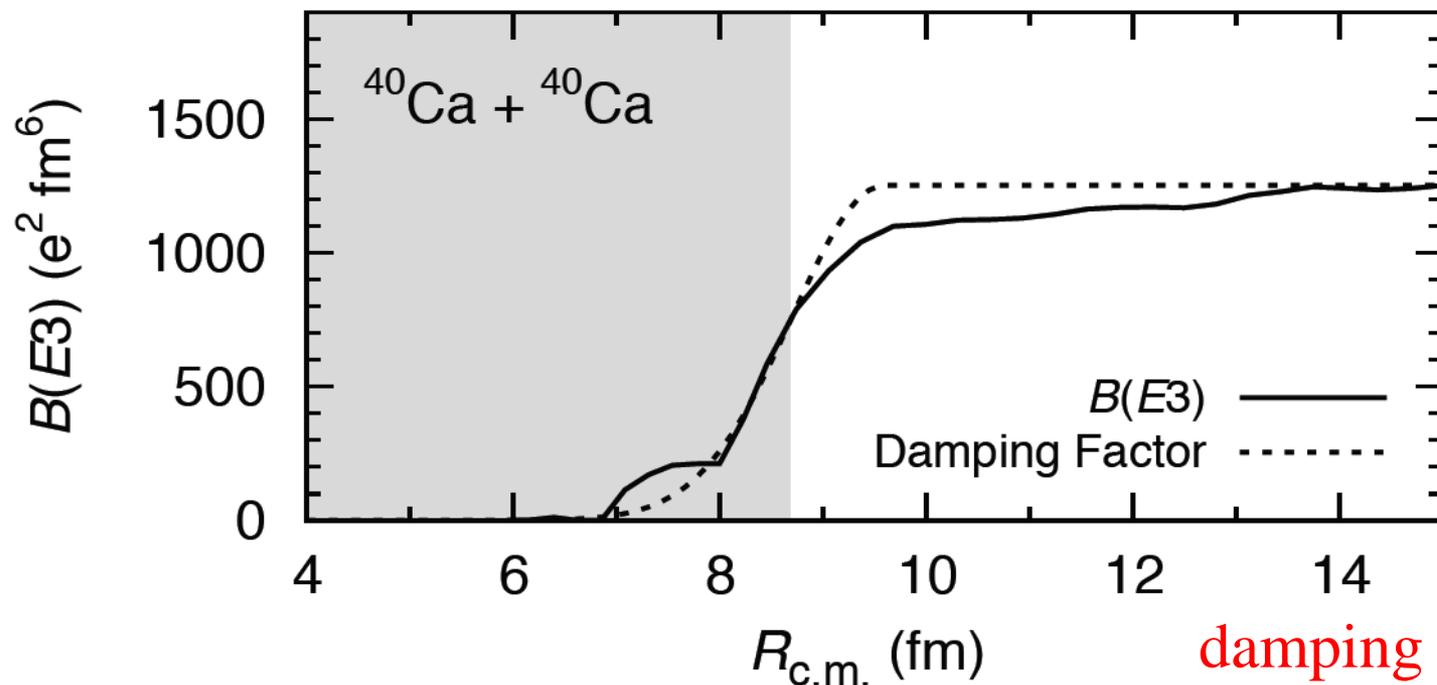
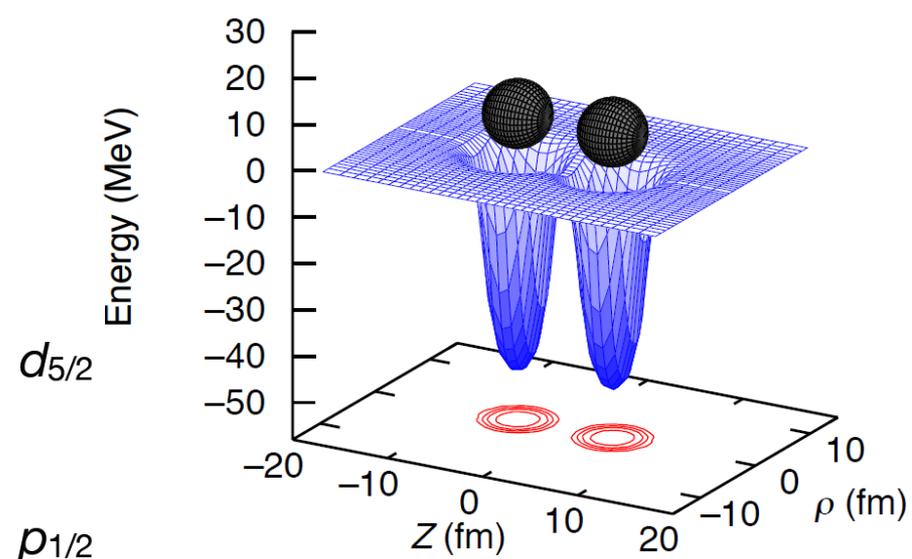
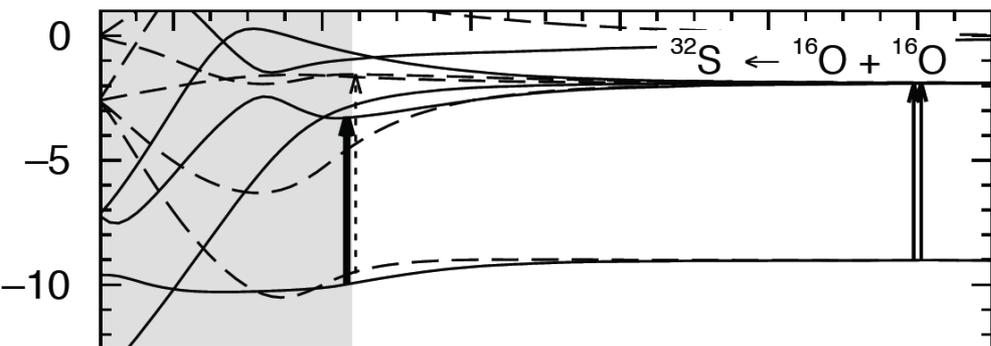


T. Ichikawa, K.H., and A. Iwamoto,  
PRL103('09)20270

# RPA calculation at each separation

T. Ichikawa and K. Matsuyanagi

PRC88('13) 011602(R)

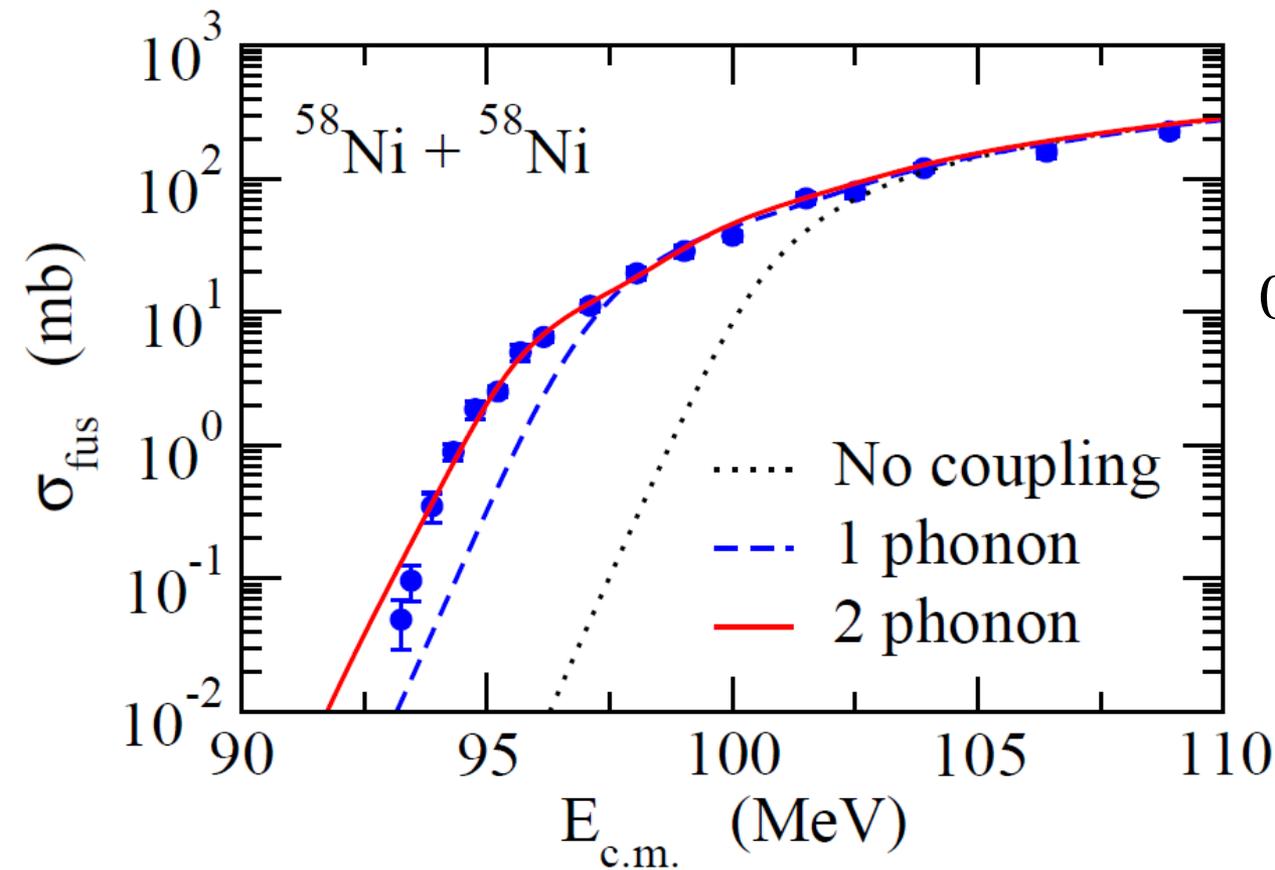


damping after the touching

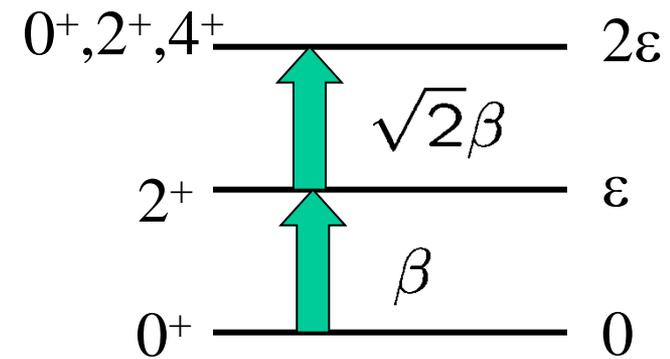
# Semi-microscopic modeling of sub-barrier fusion

K.H. and J.M. Yao, PRC91('15)064606

multi-phonon excitations

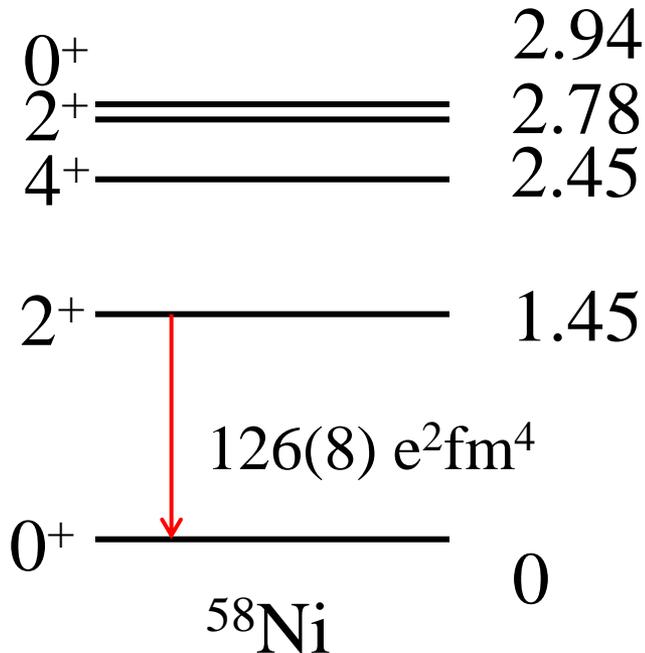


simple harmonic oscillator



## Anharmonic vibrations

- Boson expansion
- Quasi-particle phonon model
- Shell model
- Interacting boson model
- **Beyond-mean-field method**



$$Q(2_1^+) = -10 \pm 6 e\text{fm}^2$$

$$|JM\rangle = \int d\beta f_J(\beta) \hat{P}_{M0}^J |\Phi(\beta)\rangle$$

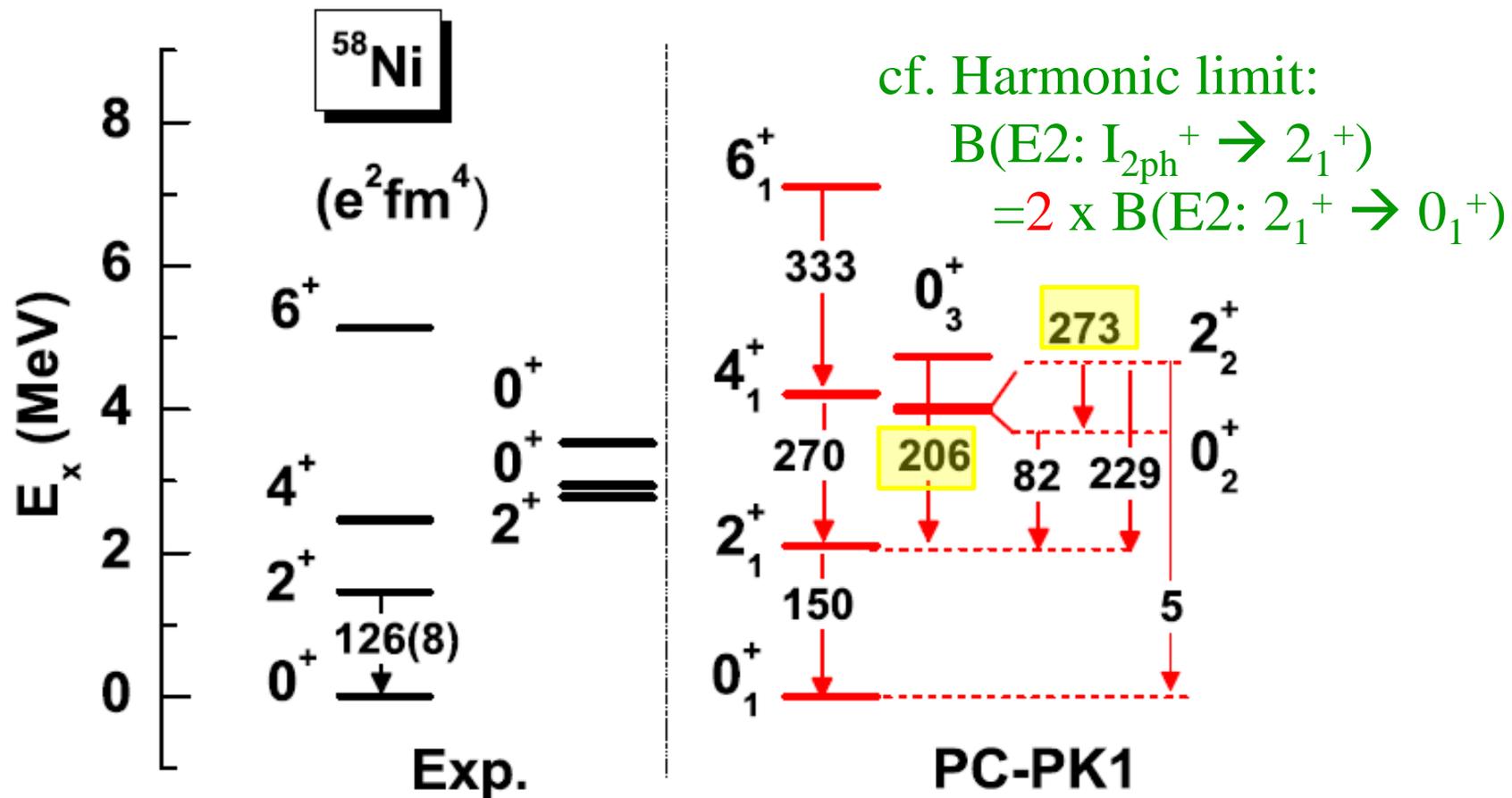
- ✓ **MF + ang. mom. projection**
- + particle number projection
- + **generator coordinate method (GCM)**

M. Bender, P.H. Heenen, P.-G. Reinhard,  
 Rev. Mod. Phys. 75 ('03) 121  
 J.M. Yao et al., PRC89 ('14) 054306

# Recent beyond-MF (MR-DFT) calculations for $^{58}\text{Ni}$

K.H. and J.M. Yao, PRC91 ('15) 064606

J.M. Yao, M. Bender, and P.-H. Heenen, PRC91 ('15) 024301



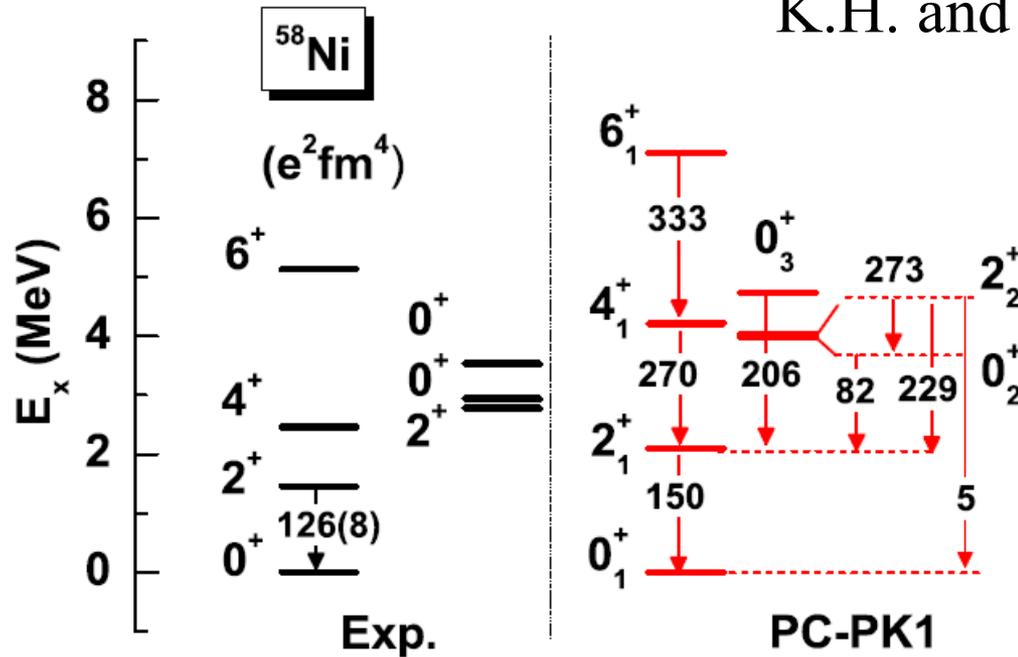
- ✓ A large fragmentation of  $(2^+ \times 2^+)_{J=0}$
- ✓ A strong transition from  $2_2^+$  to  $0_2^+$



effects on sub-barrier fusion?

# Semi-microscopic coupled-channels model for sub-barrier fusion

K.H. and J.M. Yao, PRC91 ('15) 064606



microscopic  
multi-pole operator

✓  $V_{\text{coup}}(r, \alpha_{\lambda 0}) \rightarrow V_{\text{coup}}(r, \hat{Q}_{\lambda 0})$

✓  $M(E2)$  from MR-DFT calculation ← among higher members of phonon states

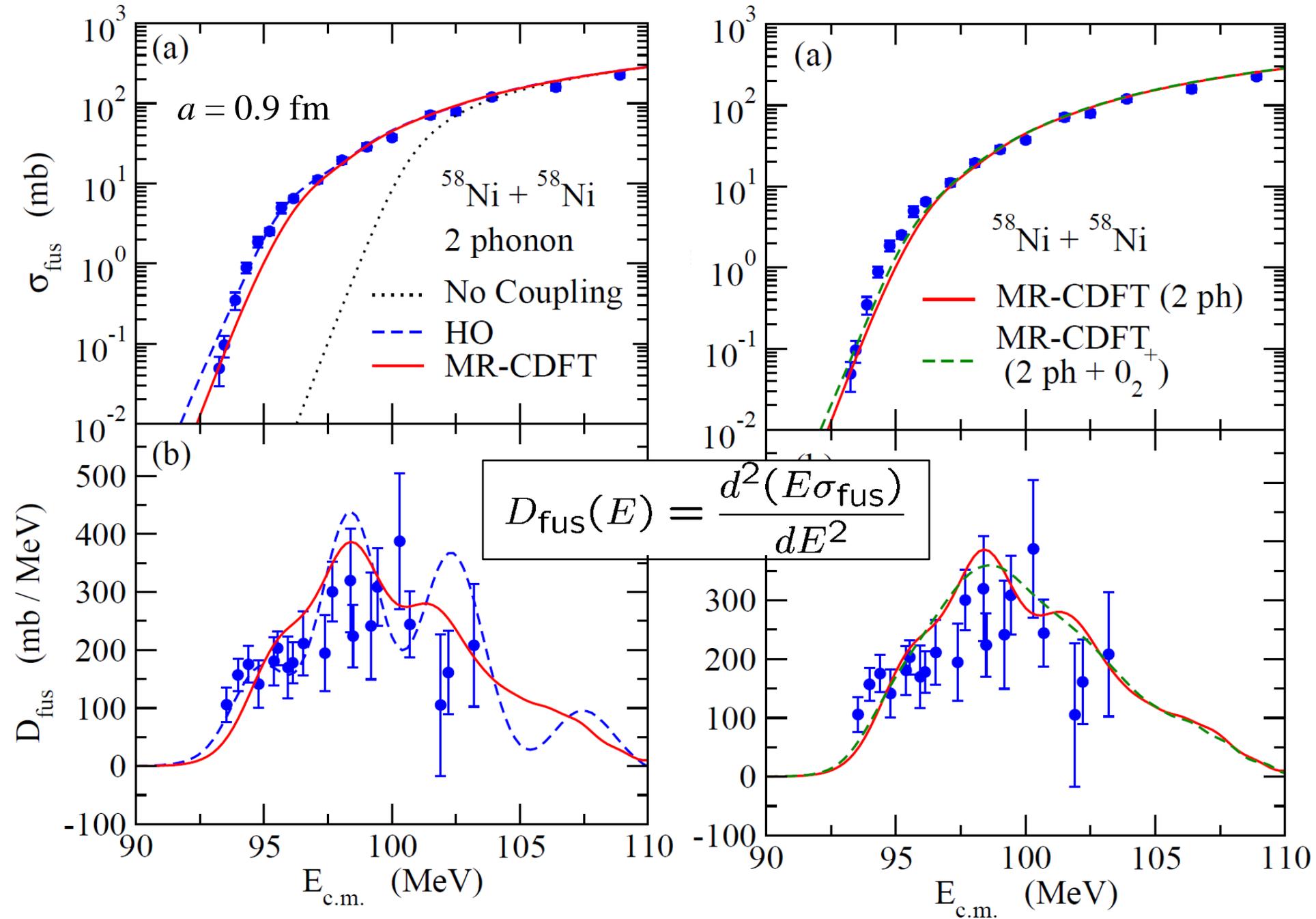
✓ scale to the empirical  $B(E2; 2_1^+ \rightarrow 0_1^+)$

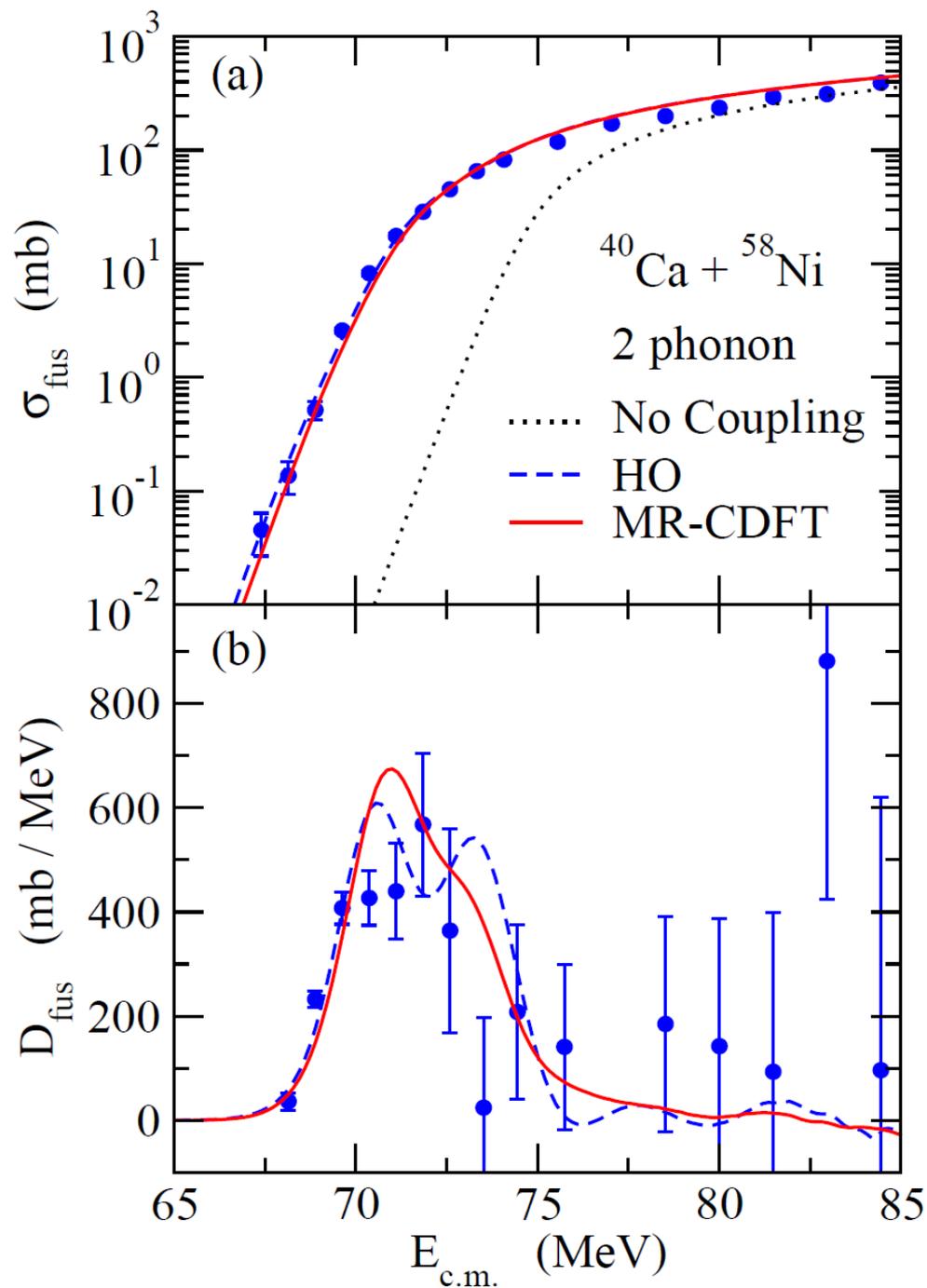
✓ still use a phenomenological potential

✓ use the experimental values for  $E_x$

✓  $\beta_N$  and  $\beta_C$  from  $M_n/M_p$  for each transition

✓ axial and reflection symmetries (no  $3^+$  and  $3^-$  states)

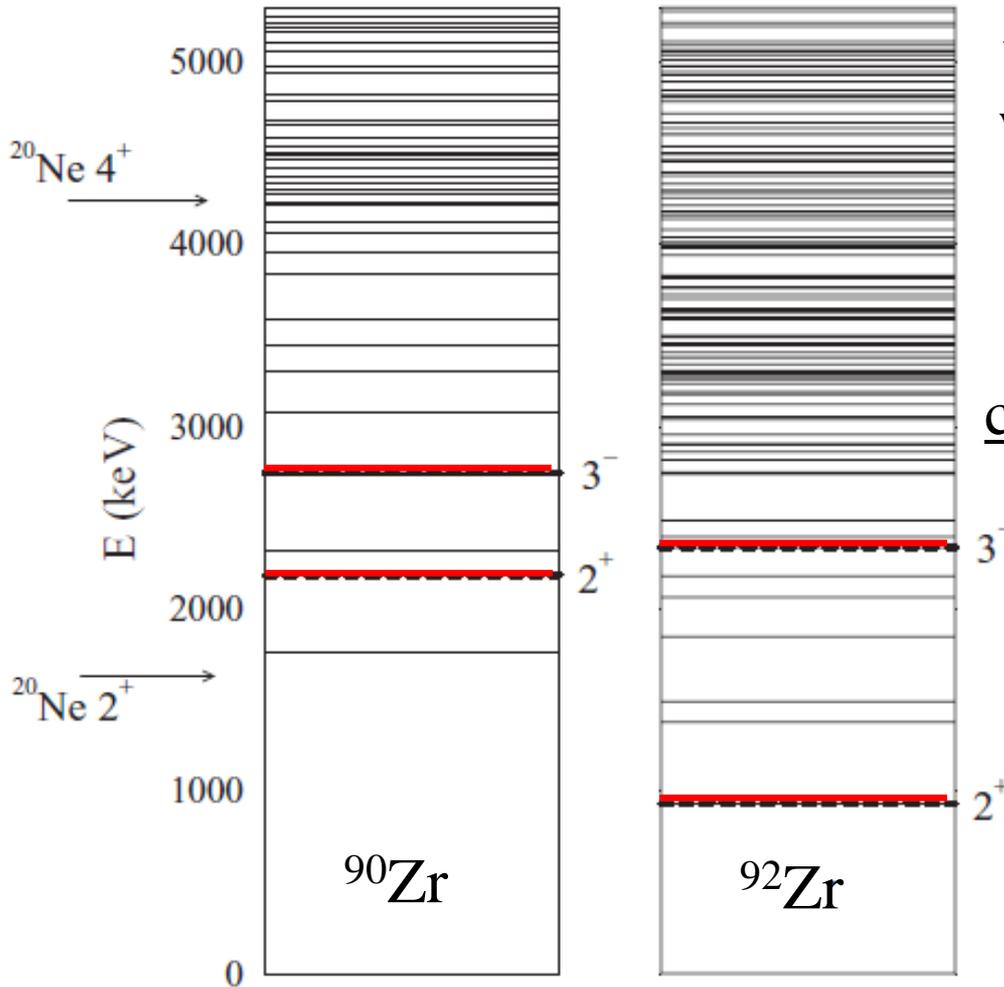




Experimental data:  
D. Bourgin, S. Courtin et al.,  
PRC90('14)044601.

c.f. S. Courtin's talk,  
Tue. afternoon

# Role of non-collective excitations in fusion



✓ many non-collective states:  
weakly coupled, but many levels

35 levels ( $< 5$  MeV) for  $^{90}\text{Zr}$   
87 levels ( $< 5$  MeV) for  $^{92}\text{Zr}$

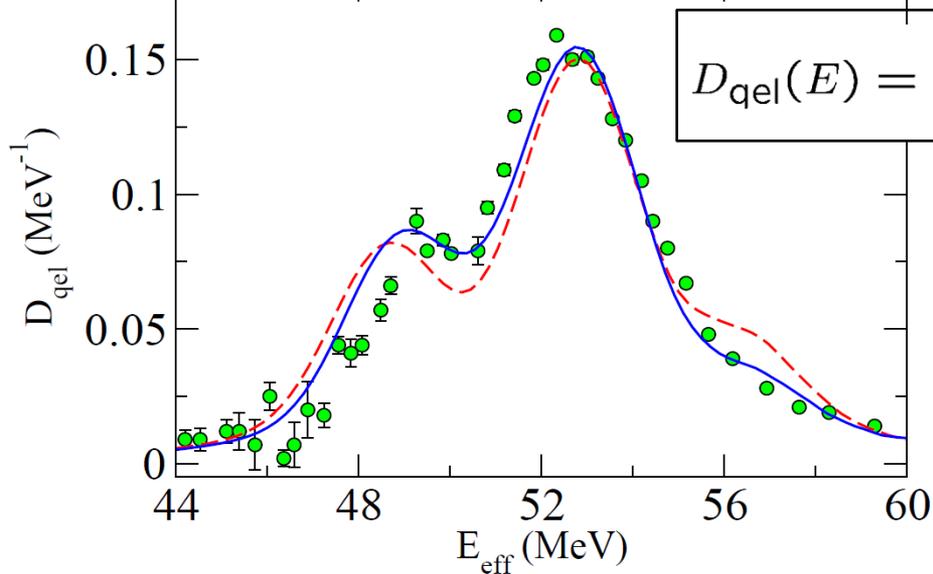
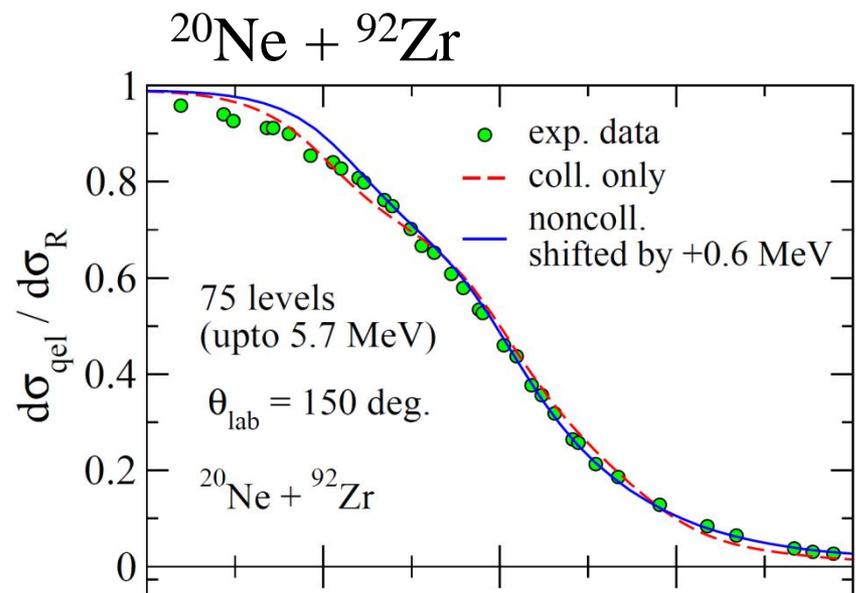
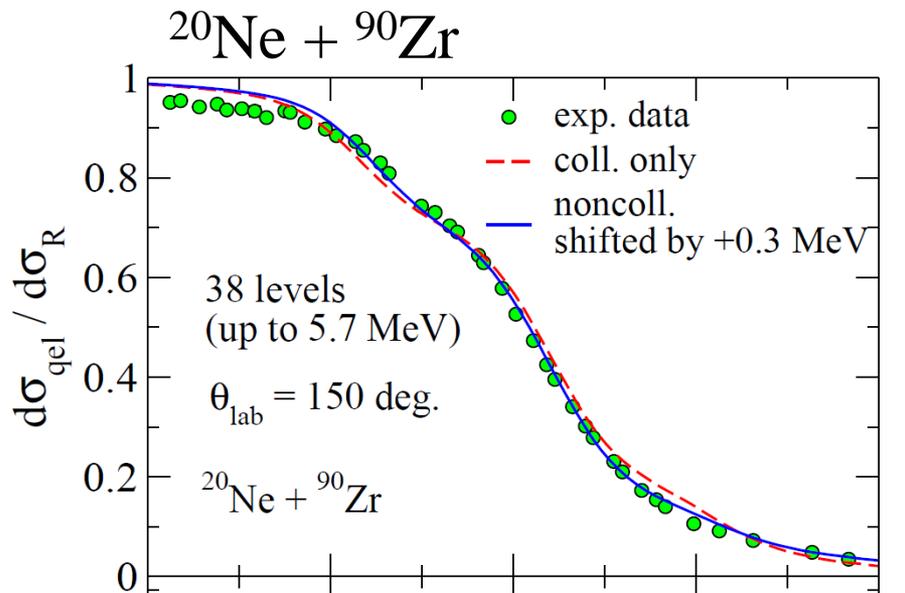
comparison between  $^{90}\text{Zr}$  and  $^{92}\text{Zr}$

$^{90}\text{Zr}$  (Z=40 sub-shell closure,  
N=50 shell closure)

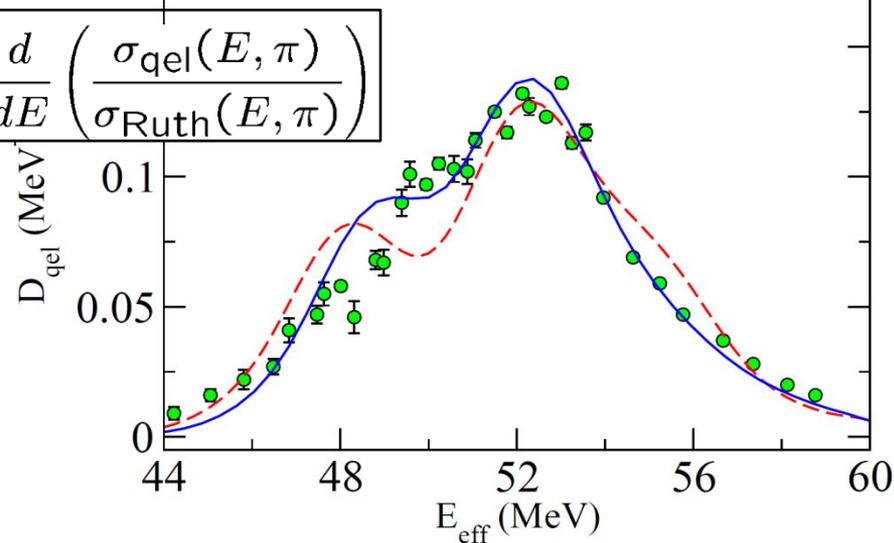
$^{92}\text{Zr} = ^{90}\text{Zr} + 2n$

role of these many  
weak channels?

the coupling strengths for non-collective excitations: poorly known  
→ random numbers (cf. random matrix model)



$$D_{\text{qel}}(E) = -\frac{d}{dE} \left( \frac{\sigma_{\text{qel}}(E, \pi)}{\sigma_{\text{Ruth}}(E, \pi)} \right)$$



Expt.:  
 E. Piasecki et al.,  
 PRC80('09)054613

$$E_{\text{eff}} = 2E \frac{\sin(\theta_{\text{c.m.}}/2)}{1 + \sin(\theta_{\text{c.m.}}/2)}$$

S. Yusa, K.H., and N. Rowley, PRC88('13)054621

### ➤ Role of non-collective excitations

- ✓ difference between  $^{20}\text{Ne} + ^{90}\text{Zr}$  and  $^{20}\text{Ne} + ^{92}\text{Zr}$
- ✓ heavy systems relevant to SHE

### ➤ Damping of collective excitations

- ✓ overlapping region
- ✓ deep sub-barrier fusion hindrance

### ➤ C.C. calculations with MR-DFT method

- ✓ anharmonicity
- ✓ truncation of phonon states
- ✓ octupole vibrations and tri-axiality: in progress

### more flexibility:

- application to transitional nuclei
- a good guidance to the Q-moment of excited states