Chiral Nucleon-Nucleon Forces in Nuclear Structure Calculations

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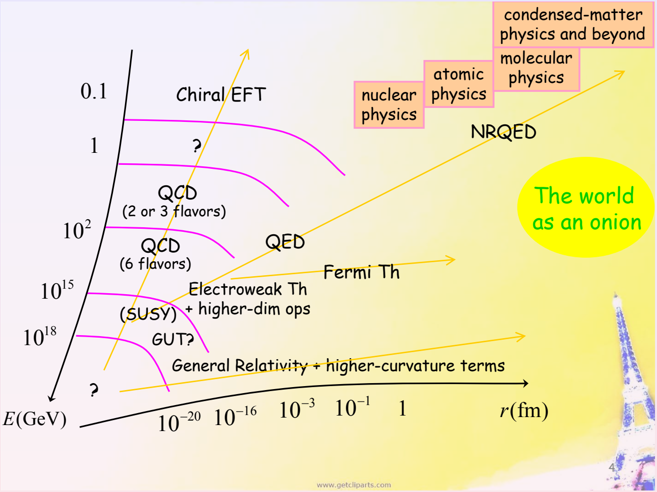


Courtesy of U. van Kolck

lnQ

perturbative QCD

~ 1 GeV $M_{QCD} \sim m_N,\ m_\rho,\ 4\pi f_\pi,\ \ldots$

hadron theory with chiral simmetry

~ 100 MeV $M_{nuc} \sim f_\pi,\ 1/r_{NN},\ m_\pi,\ \ldots$

~ 30 MeV $\xi \sim 1/a_{NN}$

Nuclear physics exhibits a separation of scales

$\Downarrow$

To resort to EFT could be a valuable way to describe the physics of nuclei, since the underlying theory QCD is not solvable

Weinberg's "theorem"

"If one writes down the most general possible Lagrangian, including all terms consistent with assumed symmetry principles, and then calculates S-matrix elements with this Lagrangian to any order in perturbation theory, the result will simply be the most general possible S-matrix consistent with analyticity, perturbative unitarity, cluster decomposition and the assumed symmetry principles" [1]

[1] *S. Weinberg, Physica A* **96** *327 (1979)*

Chiral EFT for nuclear theory

- Identify the relevant degrees of freedom (nucleons,pions, deltas) and symmetries of the problem (chiral symmetry) .
- Build up the most general Lagrangian consistent within these constraints.

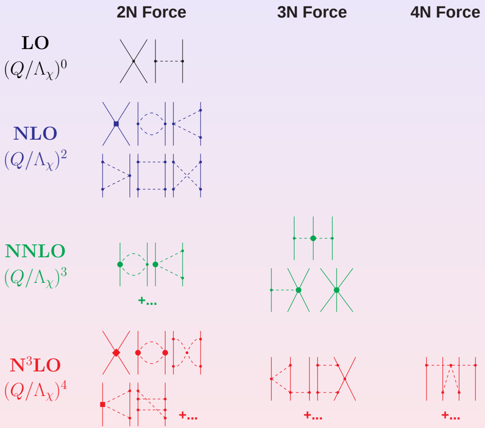
$$\mathcal{L}_{\pi\pi} = \frac{f_\pi^2}{4} \operatorname{tr} \left[\partial_\mu U \partial^\mu U^\dagger + m_\pi^2 (U + U^\dagger) \right] + \dots$$

$$\mathcal{L}_{\pi N} = \bar{\Psi} \left(i\gamma^\mu D_\mu - M_N + \frac{g_A}{2} \gamma^\mu \gamma_5 u_\mu + \dots \right) \Psi$$

$$\mathcal{L}_{NN} = -\frac{1}{2} C_S \bar{N} N \bar{N} N - \frac{1}{2} C_T (\bar{N} \vec{\sigma} N) \cdot (\bar{N} \vec{\sigma} N) + \dots$$

- Perform a perturbative expansion of this Lagrangian for momenta $q < \Lambda$, and adjust the coefficients to the physical observables (renormalization).

The chiral perturbative expansion



An important observation: ChPT allows the contruction of nuclear two- and many-body forces on an equal footing.

More precisely: most interaction vertices in the 3NF, as well as in the 4NF, occur in the 2NF too.

Consequently, the corresponding parameters LECs are consistently the same in the 2NF and 3NF.

A main issue: ChPT introduces a cutoff Λ and the calculated observables depends on its choice

Necessarily, the chiral hamiltonian has to be renormalized for each chosen cutoff via fixing the chosen LECs to fit the available experimental data (*NN* scattering data, deuteron and triton binding energies, ...).

Cutoff invariance can be then guaranteed, at least for the two- and three-body systems.

What about the many-body systems?

Infinite nuclear matter: this is an interesting environment to study the dependence on Λ of the results of a many-mody calculation

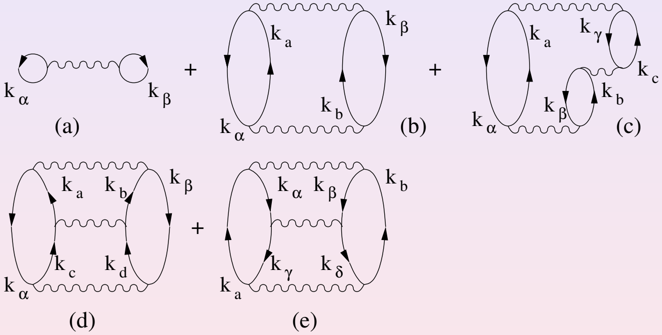
We calculate infinite nuclear matter EOS starting from chiral 2NF and 3NF defined within different cutoffs

Perturbative approach: we perform a Goldstone expansion of the binding energy per nucleon E/A up to third order in the energy.

L. C., J. W. Holt, N. Itaco, R. Machleidt, F. Sammarruca, Phys. Rev. C **87***, 014322 (2013)*
L. C., J. W. Holt, N. Itaco, R. Machleidt, L. Marcucci, F. Sammarruca, Phys. Rev. C **89***, 044321 (2014)*

- M. Hoferichter *et al*, Phys. Lett. B **746**, 410 (2015).
- A. Carbone *et al*, Phys. Rev. C **90**, 054322 (2014).
- G. Hagen *et al*, Phys. Rev. C **89**, 014319 (2014).
- T. Krüger *et al*, Phys. Rev. C **88**, 025802 (2013).
- ... and many, many others ...

The perturbative expansion



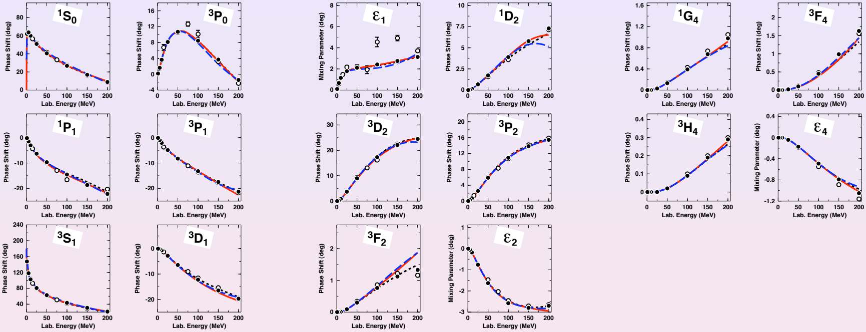
The N^3LO two-body potential

We consider for our study three N^3LO two-body potentials with different cutoffs and different regulator functions $f(p', p) = \exp[-(p'/\Lambda)^{2n} - (p/\Lambda)^{2n}]$:

- $\Lambda = 500$ MeV using $n = 2$ in the regulator function[1]
- $\Lambda = 450$ MeV using $n = 3$ in the regulator function
- $\Lambda = 414$ MeV using $n = 10$ in the regulator function (sharp cutoff)[2]

[1] *R. Machleidt and D.R. Entem, Phys. Rep.* ***503*** *1 (2011)*
[2] *L. C., A. Covello, and A. Gargano, N. Itaco, D. R. Entem, T. T. S. Kuo, and R. Machleidt Phys. Rev. C* ***75****, 024311 (2007)*

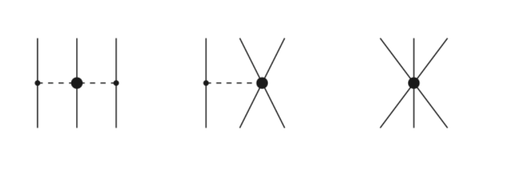
Phase shifts



- Dotted black curve: $\Lambda = 500$ MeV
- Dashed blue curve: $\Lambda = 450$ MeV
- Solid red curve: $\Lambda = 414$ MeV

The N^{2}LO three-body potential

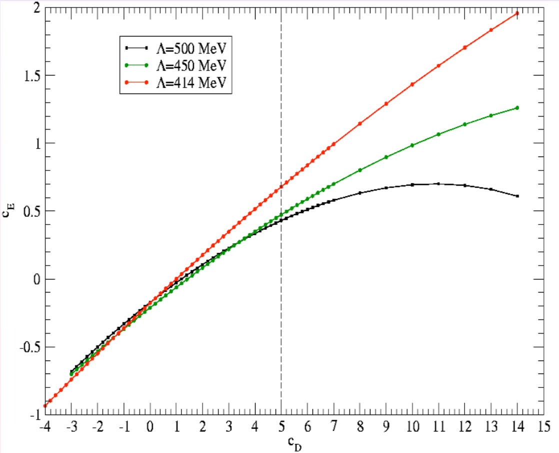
Aside the N^{3}LO two-body potentials we consider also the contribution from their corresponding three-body potentials, calculated at N^{2}LO:



They bring two more coefficients (c_D, c_E) corresponding to the contact and the 1π-exchange terms, to be adjusted to the physics of the three-nucleon system.

The fit of c_D, c_E parameters

The binding energy of the triton:



The fit of c_D, c_E parameters

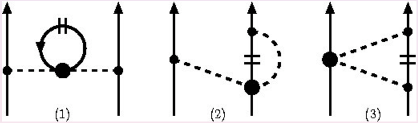
The triton Gamow-Teller matrix elements of the triton via the μ capture in $^2\mathrm{H}(\mu^-,\nu_\mu)nn$ and $^3\mathrm{He}(\mu^-,\nu_\mu)^3\mathrm{H}$ [1]:

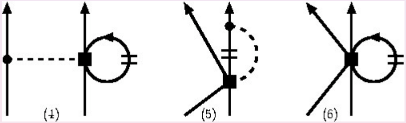
Λ cutoff (in MeV)	c_D	c_E
500	0.0	-0.17
450	-0.24	-0.11
414	-0.4	-0.07

[1] *L. E. Marcucci, A. Kievsky, S. Rosati, R. Schiavilla, and M. Viviani, Phys. Rev. Lett.* **108**, *052502 (2012)*

The density-dependent effective *NN* potential

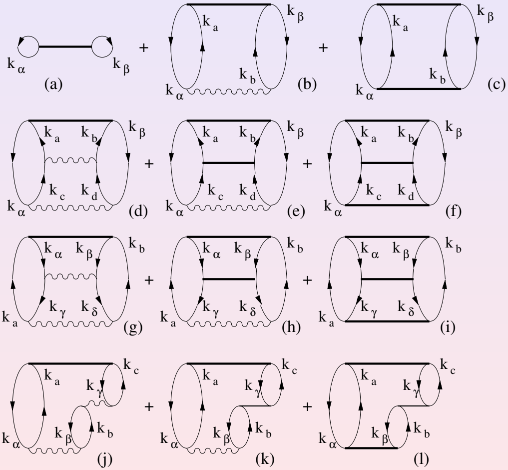
The effect of the chiral N^{2}LO 3NF has been taken into account adding a density-dependent two-body potential $\overline{V}_{NNN}$[1] to the N^{3}LO two-body one:

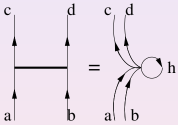




[1] *J. W. Holt, N. Kaiser, and W. Weise, Phys. Rev. C* **81**, *024002 (2010)*

The perturbative expansion





The equation of state for infinite neutron matter

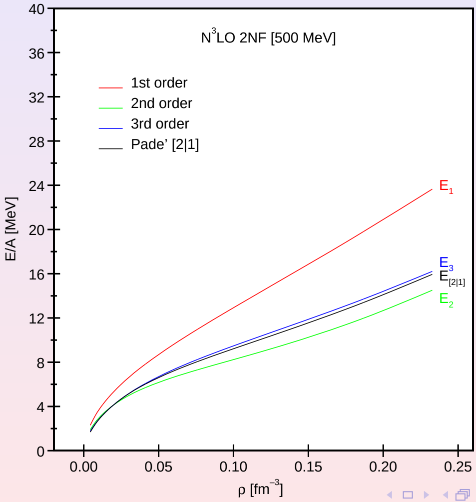
The infinite neutron matter EOS

A keypoint: in pure neutron matter the contact interaction, V_E, and the 1π-exchange term, V_D, that appear in the N^{2}LO three-body force, vanish.

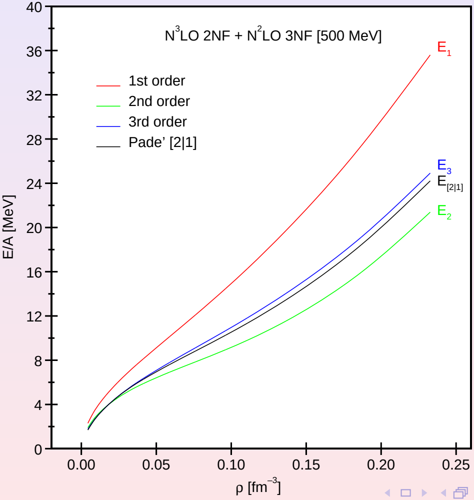
Therefore, the low-energy constants of V_E and V_D c_E, c_D do not play any role in the determination of the EOS of infinite neutron matter.

The EOS depends, when using with chiral 3NFs up to N^{2}LO, only on the parameters that have been fixed in the two-nucleon system.

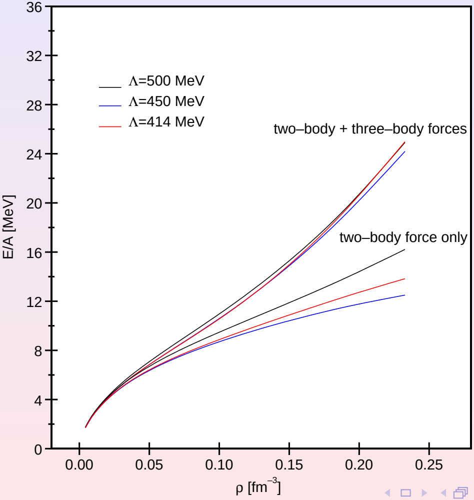
The infinite neutron matter EOS



The infinite neutron matter EOS

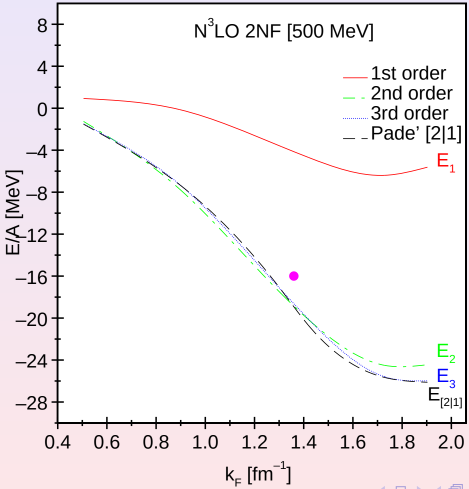


The infinite neutron matter EOS

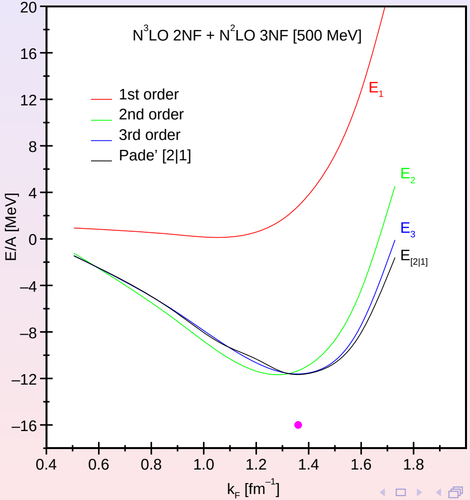


The equation of state for infinite symmetric nuclear matter

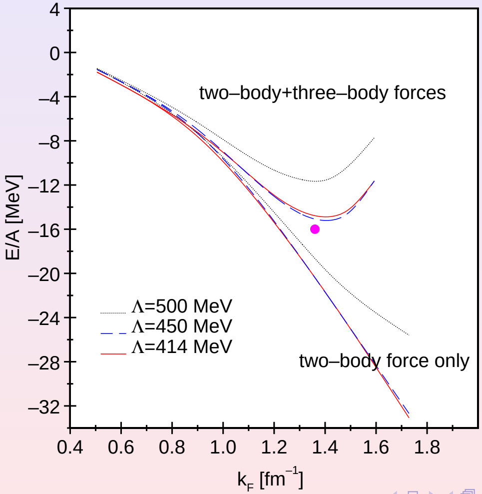
The infinite nuclear matter EOS



The infinite nuclear matter EOS

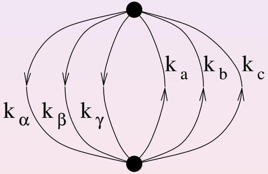


The infinite nuclear matter EOS



What has been left out ...

$3p - 3h$ 3BF diagrams:



Results for $k_F = 1.3 \text{ fm}^{-1}$

Perturbative contributions only 2NF

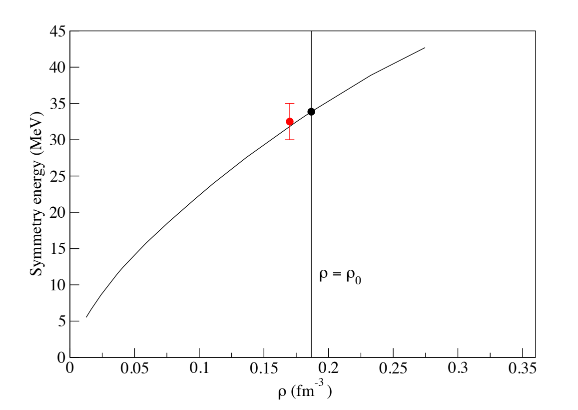
	Cutoff parameter Λ (MeV)		
	414	450	500
HF contribution	-35.507	-32.786	-25.066
2nd order *pp* diagram	-5.736	-8.551	-14.060
3rd order *pp* diagram	0.017	-0.022	0.653
3rd order *hh* diagram	-0.022	-0.021	-0.027
3rd order *ph* diagram	1.040	1.200	-0.279

Results for $k_F = 1.3\ \mathrm{fm}^{-1}$

Perturbative contributions 2NF + 3NF

	Cutoff parameter Λ (MeV)		
	414	450	500
HF contribution	-28.792	-25.688	-19.503
2nd order *pp* diagram	-7.388	-11.273	-13.511
3rd order *pp* diagram	0.563	0.745	1.642
3rd order *hh* diagram	-0.010	-0.008	-0.008
3rd order *ph* diagram	0.581	0.152	-1.516

The symmetry energy with N^{3}LO (414)



$$K_0 = 279 \text{ (empirical value } 230 \pm 30)$$
$$L = 68 \text{ (empirical value 70)}$$

Part 1: concluding remarks

- Chiral potentials with a cutoff $\Lambda \leq 500$ MeV exhibit a perturbative behavior both for infinite neutron and nuclear matter calculations
- The EOS for infinite neutron matter shows substantial regulator independence when including 3NF contributions
- The EOS for infinite nuclear matter shows lesser regulator independence

Part 1: concluding remarks

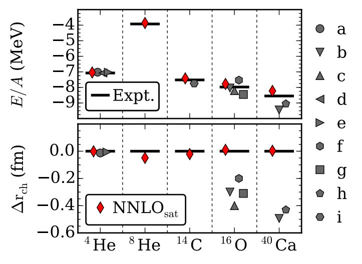
- Chiral potentials with a cutoff $\Lambda \leq 500$ MeV exhibit a perturbative behavior both for infinite neutron and nuclear matter calculations
- The EOS for infinite neutron matter shows substantial regulator independence when including 3NF contributions
- The EOS for infinite nuclear matter shows lesser regulator independence

Perspectives

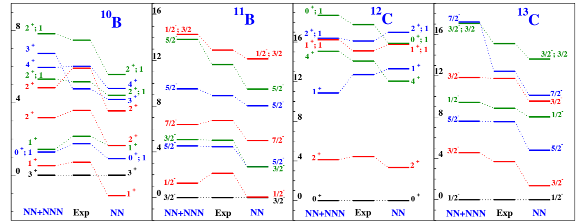
- Improve the calculation of the perturbative expansion
- Need of a N^{3}LO three-body force?
- Need to include four-body force effects?

Calculations for finite nuclei: the realistic shell model

- A. Ekstrom *et al*, Phys. Rev. C **91**, 051301 (2015).
- S. K. Bogner *et al*, Phys. Rev. Lett. **113**, 142501 (2014).
- V. Somá *et al*, Phys. Rev. C **89**, 061301 (2014).
- P. Navratil *et al*, Phys. Rev. Lett. **99**, 042501 (2007).
- ... and many, many others ...



E/A (MeV)
Expt.
Δr_ch (fm)
NNLO_sat
4He
8He
14C
16O
40Ca
a
b
c
d
e
f
g
h
i



10B
11B
12C
13C
NN+NNN
Exp
NN

The realistic shell model

- The starting point is a realistic potential V_{NN}
- An effective shell-model hamiltonian H_{eff} is then derived by way of the many-body theory of the effective hamiltonian
- The shell model calculation is performed using only quantities obtained from the effective shell-model hamiltonian, both single-particle energies and residual two-body interaction are derived from the theory

The shell-model effective hamiltonian

A very useful way to derive $H_{\rm eff}$ is the time-dependent perturbative approach as developed by Kuo and his co-workers in the 1970s *(see T. T. S. Kuo and E. Osnes, Lecture Notes in Physics vol. 364 (1990))*
In this approach the effective hamiltonian $H_{\rm eff}$ is expressed as

$$H_{\rm eff} = \hat{Q} - \hat{Q}' \int \hat{Q} + \hat{Q}' \int \hat{Q} \int \hat{Q} - \hat{Q}' \int \hat{Q} \int \hat{Q} \int \hat{Q} \cdots ,$$

- The integral sign represents a generalized folding operation (folded diagrams are summed up at all orders using Lee-Suzuki iterative technique)
- The $\hat{Q}$-box is a collection of irreducible valence-linked Goldstone diagrams that takes into account core-polarization effects for the valence nucleons in the model space

The shell-model effective hamiltonian

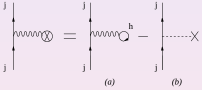
$\hat{Q}$-box diagrams and all effective operators (electric quadrupole transitions, magnetic dipole transitions, ...) are expanded up to third order in perturbation theory.

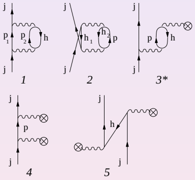
We calculate the Padè approximant $[2|1]$ of the $\hat{Q}$-box, in order to obtain a better estimate of the value to which the perturbation series should converge

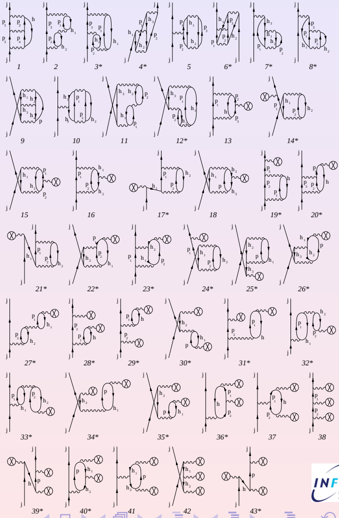
$$[2|1] = V^0_{Qbox} + V^1_{Qbox} + V^2_{Qbox}(1 - (V^2_{Qbox})^{-1} V^3_{Qbox})^{-1} \quad ,$$

We include enough intermediate states so that the H_{eff} has a flat dependence on them

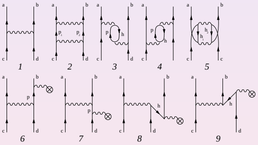
$\hat{Q}$-box perturbative expansion: 1-body diagrams

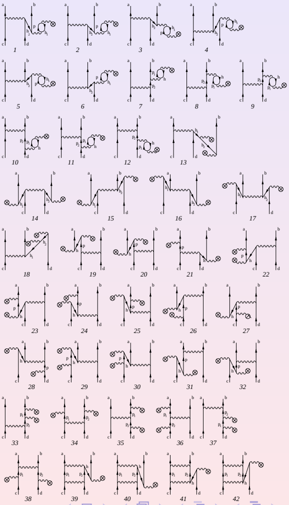




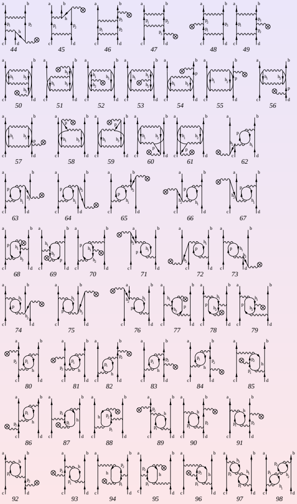


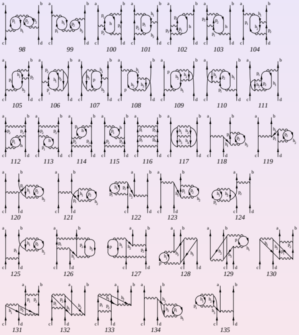
$\hat{Q}$-box perturbative expansion: 2-body diagrams





$\hat{Q}$-box perturbative expansion: 2-body diagrams





L.C., A. Covello, A. Gargano, N. Itaco, and T. T. S. Kuo, Ann. Phys. **327** *, 2125-2151 (2012)*

The shell-model effective hamiltonian

A benchmark calculation: shell-model deals with open-shell nuclei, a basic test is a nucleus that can be described as 2 nucleons outside a close-shell core

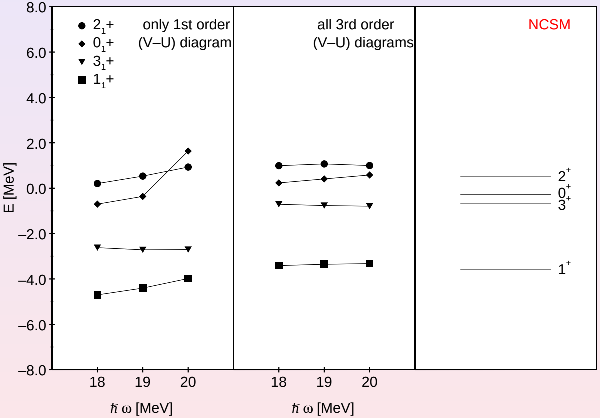
We have chosen to test our calculations with those of NCSM for ^{6}Li with N^{3}LO potential: its structure should be made up by the ^{4}He core plus one valence proton and one valence neutron

The comparison with NCSM needs another upgrade for our calculations, we have to start from a purely intrinsic many-body hamiltonian, so to avoid center-of-mass spurious motion:

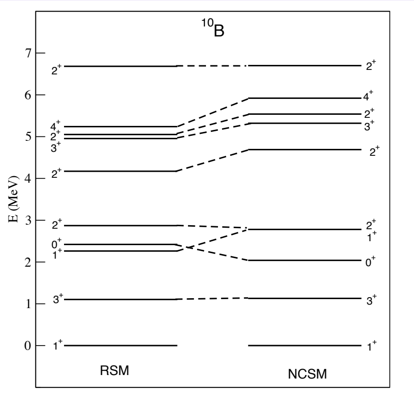
$$H = (1 - \frac{1}{A}) \sum_i \frac{p_i^2}{2M} + \sum_{i<j} (V_{ij} - \frac{\mathbf{p_i} \cdot \mathbf{p_j}}{MA}) =$$

$$\sum_i (\frac{p_i^2}{2M} + \frac{1}{2} M\omega^2 r_i^2) + \sum_{i<j} (V_{ij} - \frac{1}{2} M\omega^2 r_i^2 - \frac{p_i^2}{2MA} - \frac{\mathbf{p_i} \cdot \mathbf{p_j}}{MA}) = H_0 + H_I$$

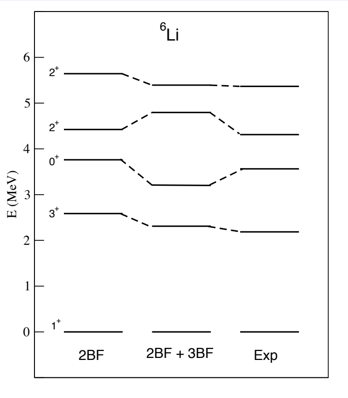
Test: ^{6}Li first excited states with N^{3}LO potential

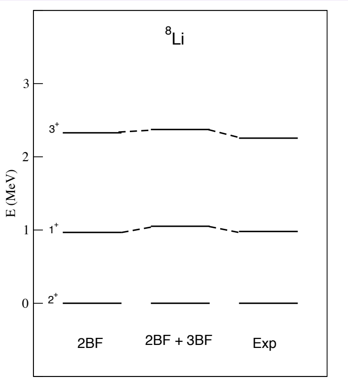


Test: ^{10}B first excited states with N^{3}LO potential

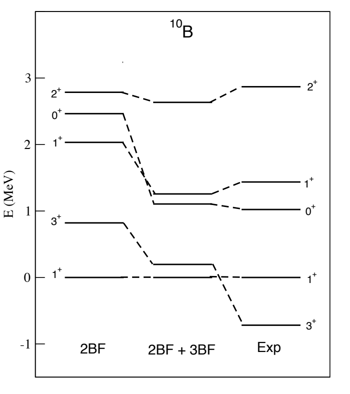


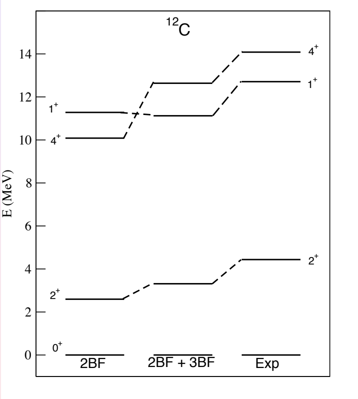
The effect of three-nucleon forces: N^3LO (500)



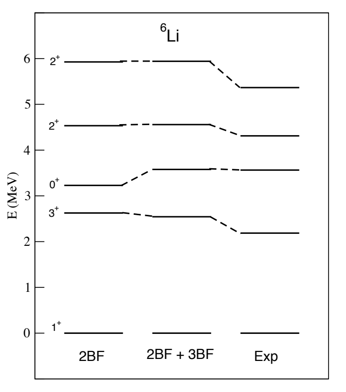


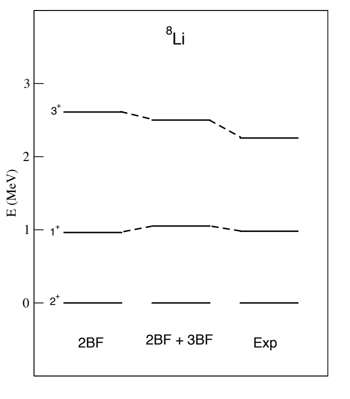
The effect of three-nucleon forces: N^{3}LO (500)



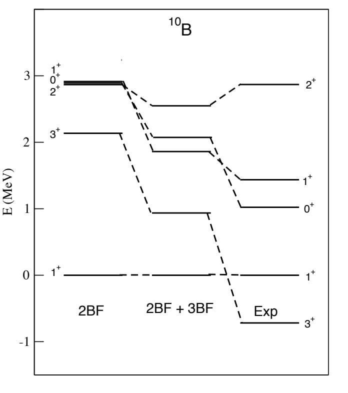


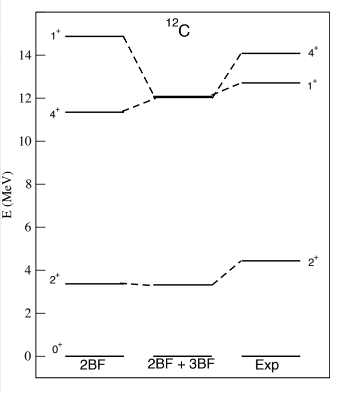
The effect of three-nucleon forces: N^{3}LO (414)



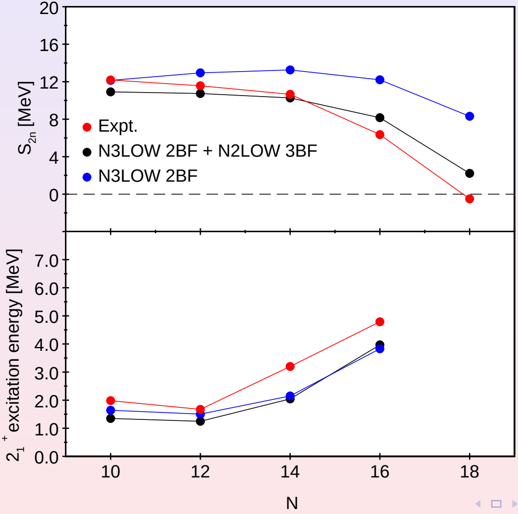


The effect of three-nucleon forces: N^{3}LO (414)





The effect of 3NF: oxygen isotopes with N^3LO (414)



Concluding remarks

- The agreement of our results with the experimental data testifies the reliability of a microscopic shell-model calculation with chiral potentials.
- Pure three-body forces contribute positively to the improvement of theoretical results.
- Role of three-body correlations should be investigated.
- Perspectives: benchmark calculations with other many-body approaches.