



## Clusters in Heavy Ion Collisions and the Symmetry Energy

Hermann Wolter, Univ. of Munich

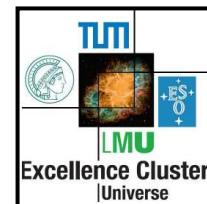
**Aim of this short report:**

- observation of yield and isotopic content of clusters in heavy ion collisions is a way to study the nuclear symmetry energy
- light clusters (n,p,d,t,<sup>3</sup>He,α) and heavier clusters (intermediate mass fragments, IMF, (3≤Z≤≈20) have to be treated differently and carry different information
- characteristic examples for both cases

**connected to talks by: Y.X. Zhang (Monday), P. Napolitani (earlier today)  
S. Yenello and A. Ono (plenary session tomorrow)**

**with**

**Malgorzata Zielinska-Pfabe, Piotr Decowski<sup>†</sup> (Smith Coll., USA),  
Maria Colonna (LNS Catania),  
and support from Remi Bougault (LPC Caen), Abdou Chbihi  
(GANIL, Caen)**



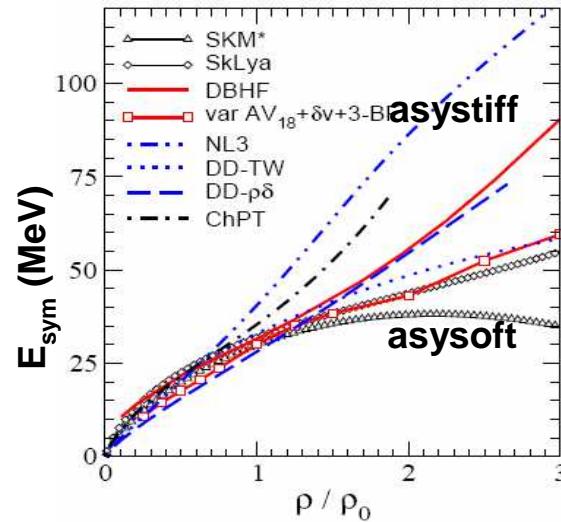
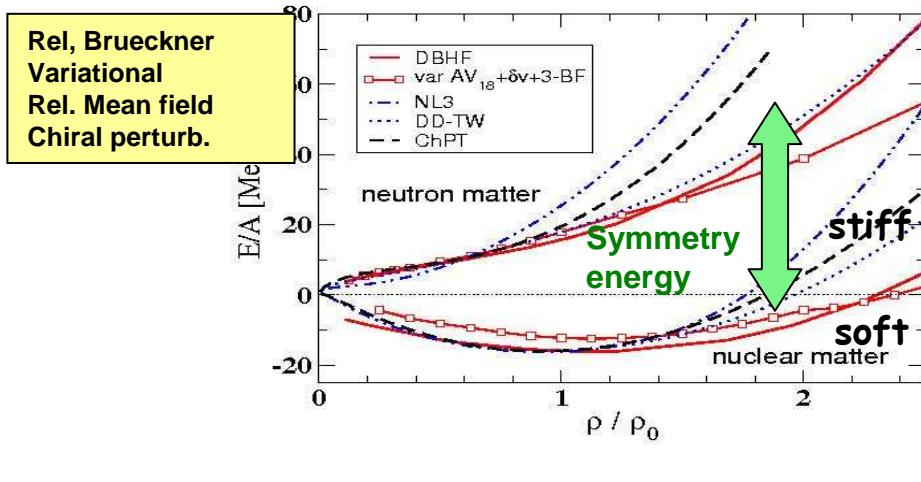
# Equation-of-State and Symmetry Energy

density-asymmetry dep.  
of nucl.matt.

$$E(\rho_B, \delta)/A = E_{nm}(\rho_B) + E_{sym}(\rho_B)\delta^2 + O(\delta^4) + \dots$$

$$\delta = \frac{\rho_n - \rho_p}{\rho_n + \rho_p}$$

$$a_s \approx E_{sym}(\rho_0)$$



C. Fuchs, H.H. Wolter, EPJA 30(2006)5

Why is symmetry energy so uncertain??  
 → In-medium p mass, and short range isovector tensor correlations (e.g. B.A. Li, PRC81 (2010));  
 → use heavy ion collisions to investigate in the laboratory

## Symmetry potentials and effective masses

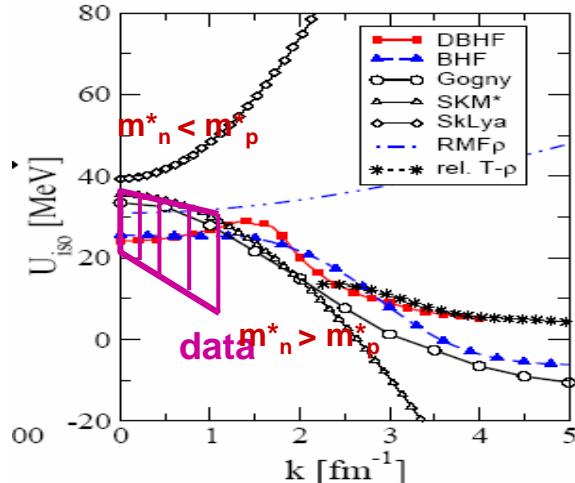
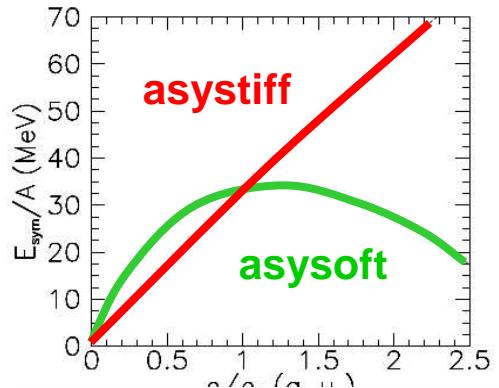
Momentum dependence of symmetry potential (isoscalar and isovector)

$$\varepsilon(\rho[f(\rho, k)]; \delta) \rightarrow U(\rho, k; \delta) = \frac{\partial \varepsilon(\rho, \delta)}{\partial f(\rho, k)} = \underbrace{U_0(\rho, k) + U_{\text{sym}}(\rho, k)(\tau\delta) + \dots}_{U_\tau(\rho, k)}$$

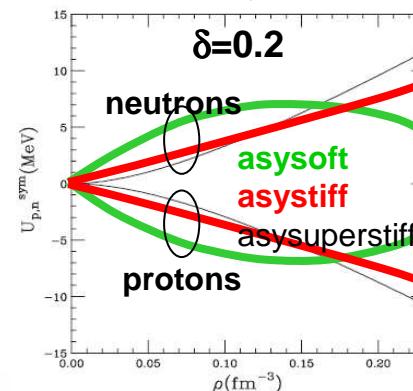
Proton-Neutron effective mass splitting

$$\frac{m^*_\tau}{m} = \left( 1 + \frac{m}{\hbar^2 k} \frac{\partial U_\tau}{\partial k} \right)^{-1}$$

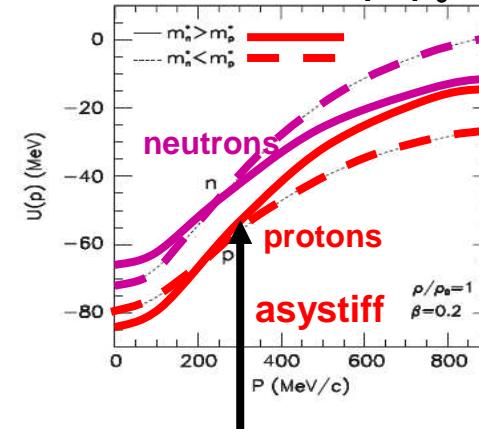
Symmetry energy (BGBD)



Neutron/proton potentials as a fct of density for  $p=p_F$



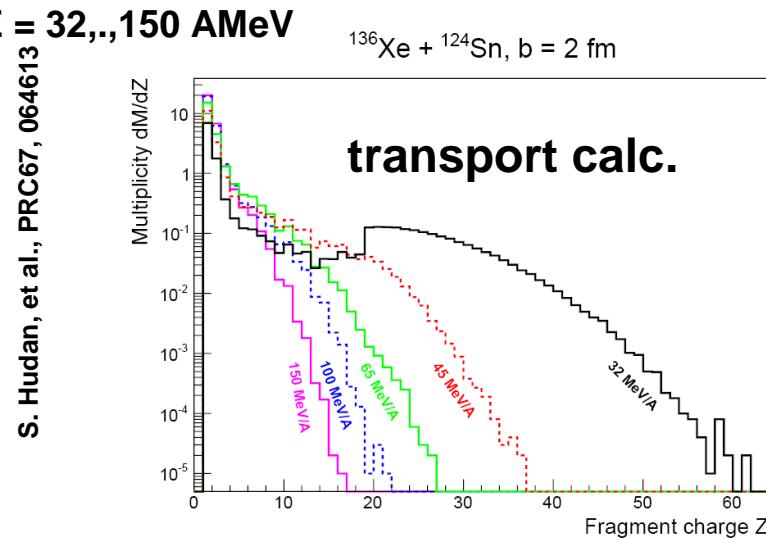
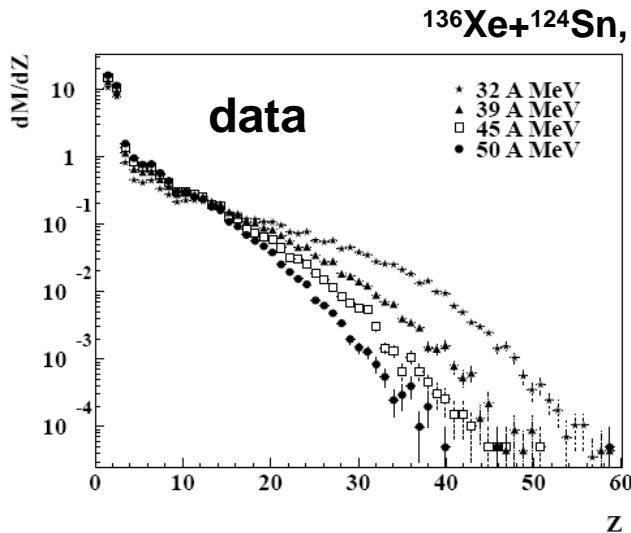
momentum for  $\rho=\rho_0$



crossing of potentials  
→ leads to crossing of emission spectra

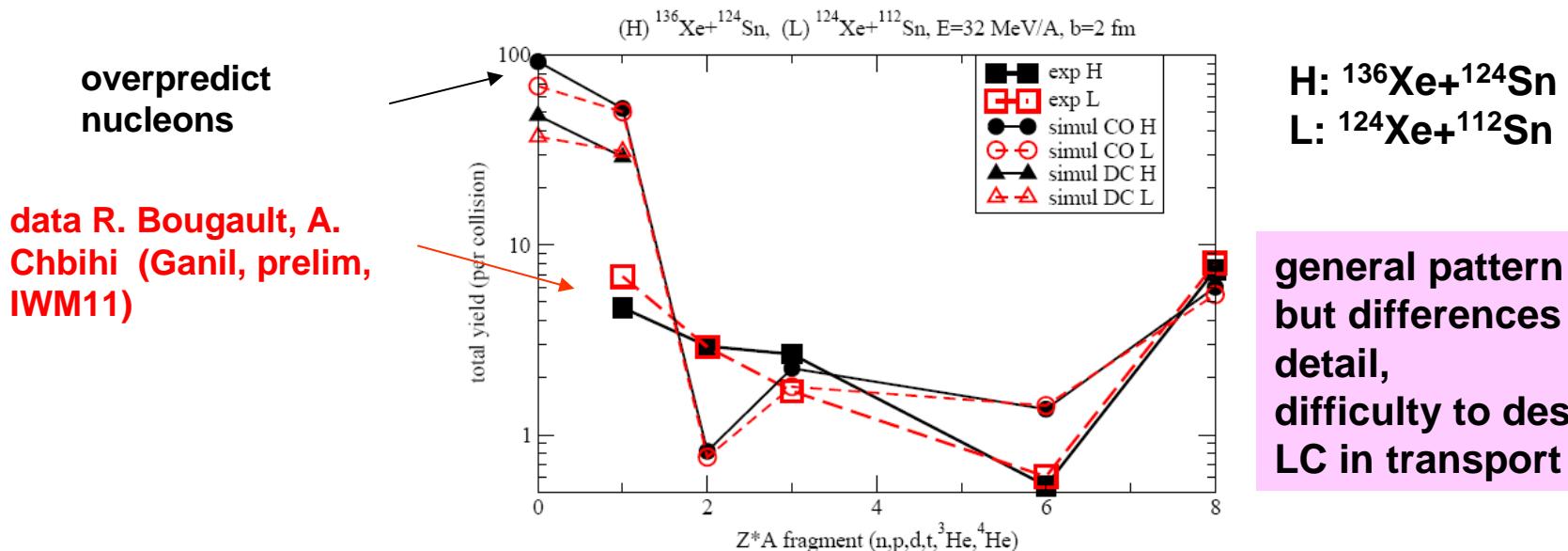
No agreement on behavior and even ordering of n/p effective masses between Skyrme, RMF, BHF, DBHF

# Clusters in heavy ion collisions



H.H. Wolter, et al.,  
EPJ Web of Conf. &&, 03097 (2014)

## a closer look at light clusters (LC)

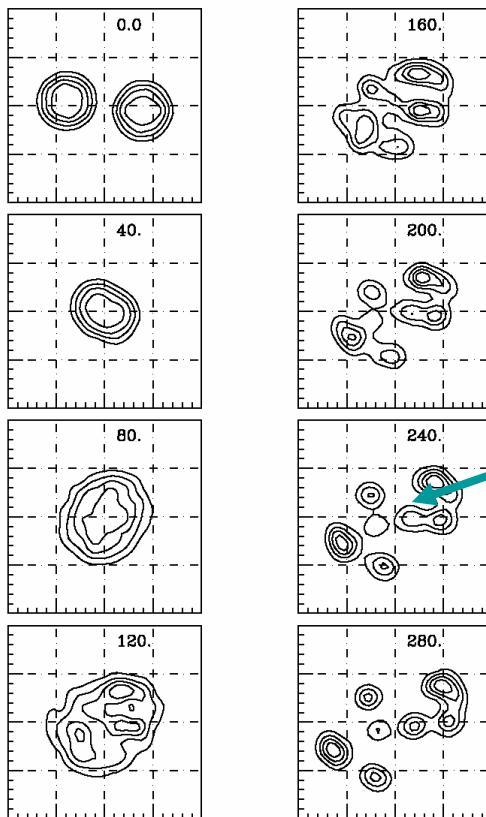


# Symmetry energy drives isospin content of clusters (Isospin dynamics)

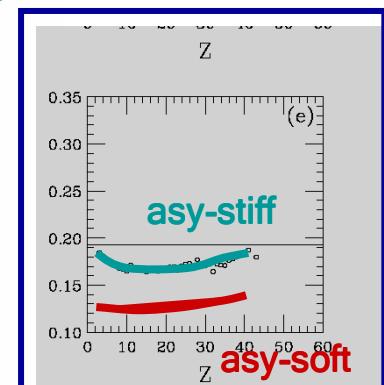
V.Baran et al.,  
NPA703(2002)603  
NPA730(2004)329

$^{124}\text{Sn} + ^{124}\text{Sn}$ , 50 AMeV

Central collision,  $b=2$  fm

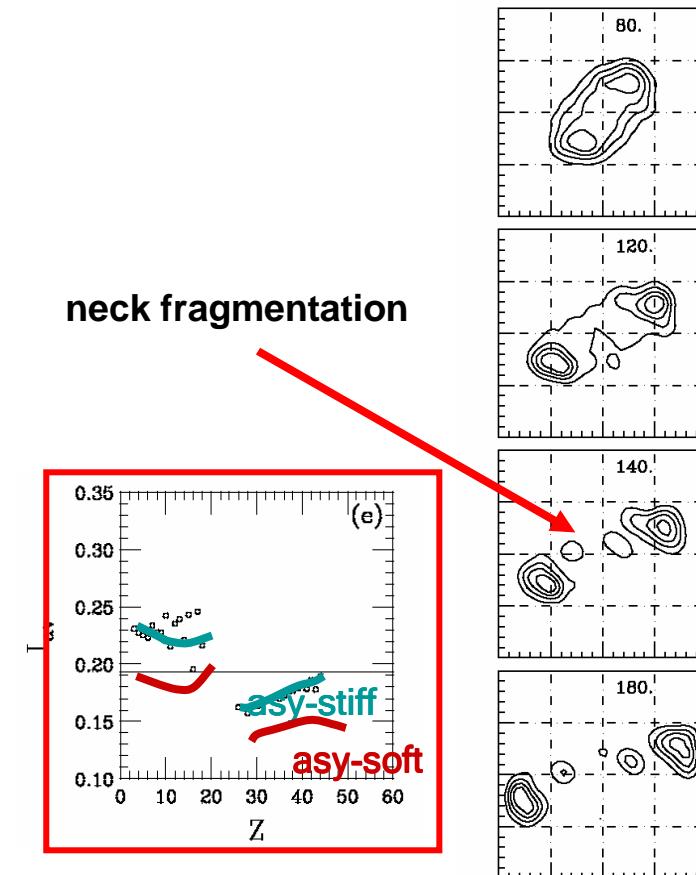


bulk fragmentation



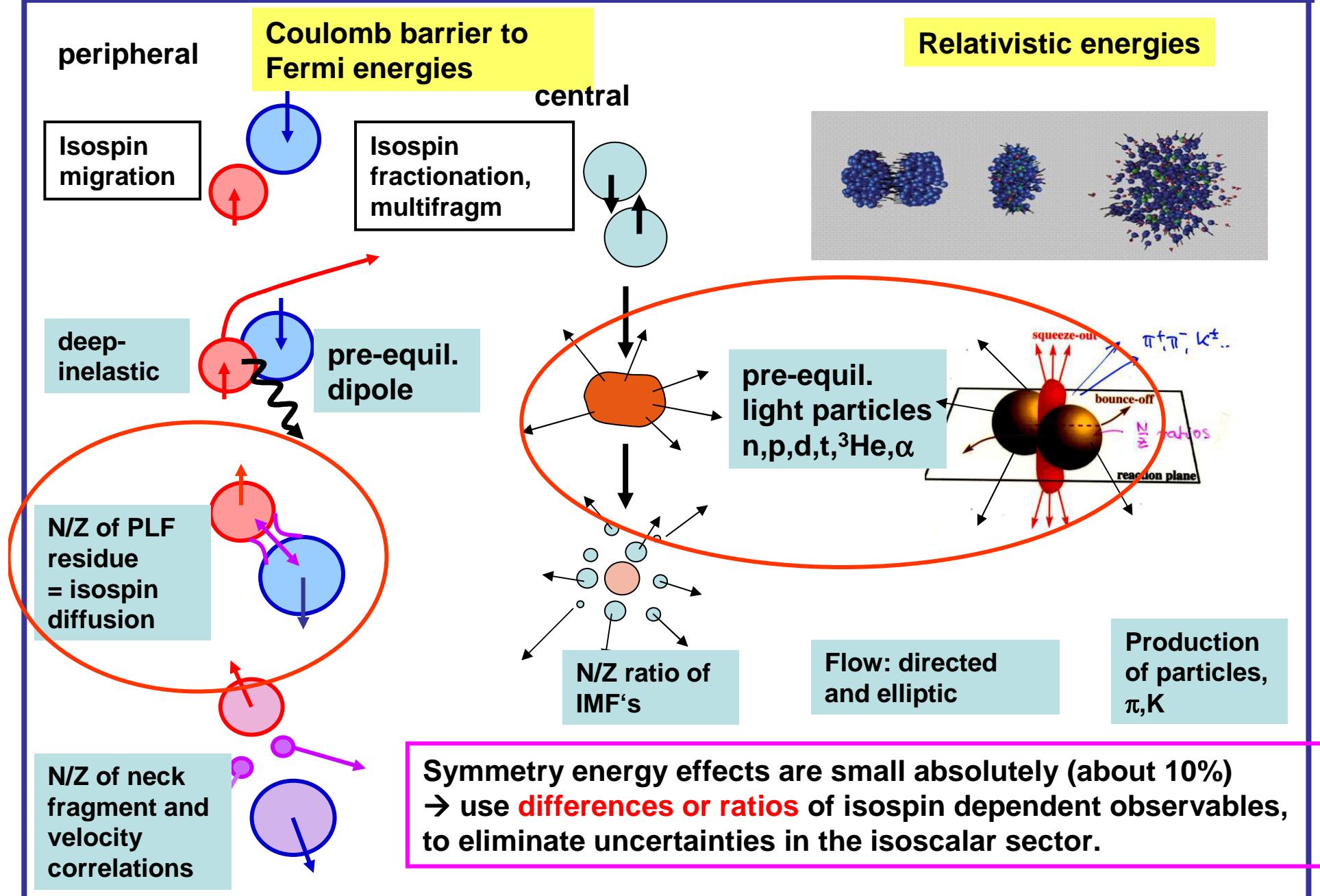
Multifragmentation: Isospin fractionation at low densities

Peripheral collision,  $b=6$  fm

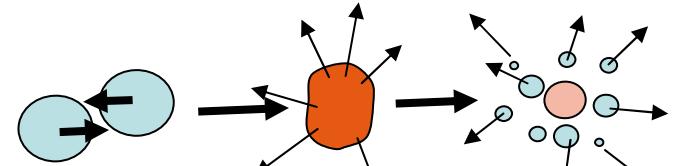


Neck-fragmentation: Isospin migration at interface with normal density

# Isospin sensitive Observables in Heavy Ion Collisions



## Approaches to cluster formation in HIC



1. freeze-out approx.: statistical decay of excited, equilibrated freeze-out config.  
„statistical clusters“

2. „dynamical clusters“: transport approach with fluctuations/correlations:  
seeds of fragment/light cluster formation

**BUU/BLE/BLOB**

$$\left( \frac{\partial}{\partial t} + \frac{\vec{p}}{m} \vec{\nabla}^{(r)} - \vec{\nabla} U(r) \vec{\nabla}^{(p)} \right) f(\vec{r}, \vec{p}; t) = I_{coll} [\sigma^{in-med}] + \delta I_{fluct}$$

deterministic, dissipative 1-body  
equation + fluctuation

**QMD/AMD**

$$|\Phi\rangle = A \prod_{i=1}^A \phi(r; r_i, p_i) |0\rangle$$

$$\dot{r}_i = \{r_i, H\}; \quad \dot{p}_i = \{p_i, H\}; \quad H = \sum_i t_i + \sum_{i,j} V(r_i - r_j)$$

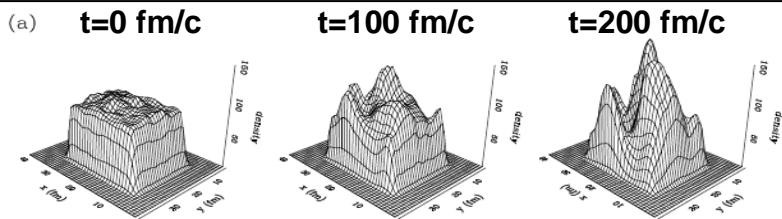
TDHF + stochastic NN collisions

difference in spectrum of fluctuations

→ can be adjusted for IMF (intermediate mass fragments) formation, since IMF formation stabilized by mean field

BUU calculation in a box (i.e. periodic boundary conditions) with initial conditions inside the instability region:  $\rho = \rho_0/3$ ,  $T = 5$  MeV,  $\delta = 0$

→ Formation of „clusters (fragments)“, from small (numerical) fluctuations in the density.  
Time scale = growth time of the unstable modes



But LC (light cluster) formation dominated by correlations: not well described in mean field

## Fragment formation II

### Light clusters (LC:d,t, $^3\text{He}$ , $\alpha$ )

Correlation dominated (esp.Pauli-correlation). not good in BUU and MD models, since simple interactions and classical phase space distribution give bad eigenstates for LC

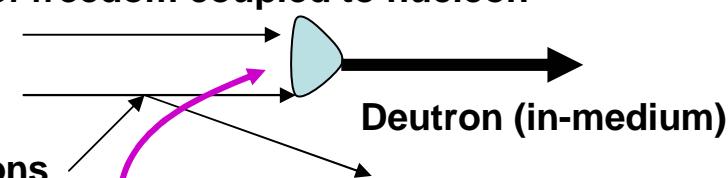
except(!) for AMD: can define realistic wave functions for LC with reasonable BE;

### Solution for BV models:

LC distribution functions as explicit degrees of freedom coupled to nucleon distribution functions by 3-body collisions of type NNN $\rightarrow$ ND

$\rightarrow$  P. Danielewicz and Q. Pan, PRC 46 (1992)

(d,t, $^3\text{He}$ , **but no  $\alpha$ !**) coupled transport equations



### Caveat: Medium properties of LC:

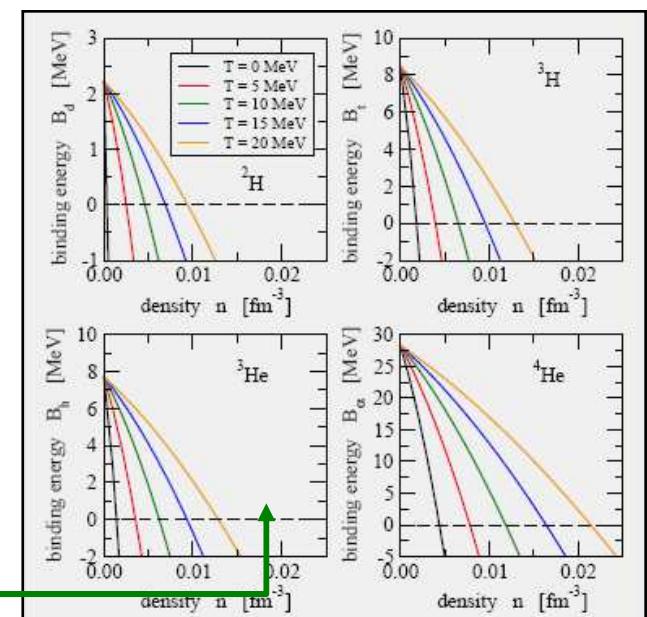
Medium corrections in the formation of light charged particles in heavy ion reactions

[C. Kuhrt](#), Beyer, Danielewicz,..PRC63 (2001) 034605

Calculated in nuclear matter and static nuclei in Generalized RMF approach by Typel, Röpke,...,HW, PRC81 (2010)

similar: transitions amplitudes in medium

Mott density:  
Cluster dissolves



# Isospin diffusion

isospin transport  
through „neck“ in  
peripheral collisions

$^{124}\text{Sn}(\text{H}) + ^{112}\text{Sn}(\text{L})$

Isospin transport ratio R :

$$R = \frac{2X_{AB} - X_{AA} - X_{BB}}{X_{AA} - X_{BB}}$$

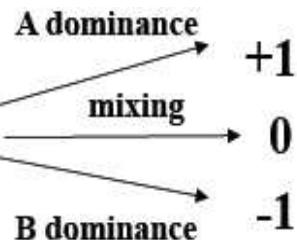
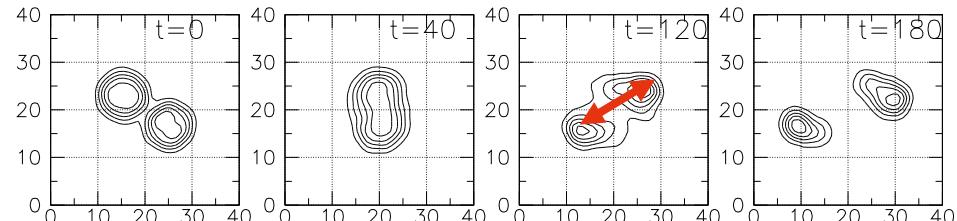
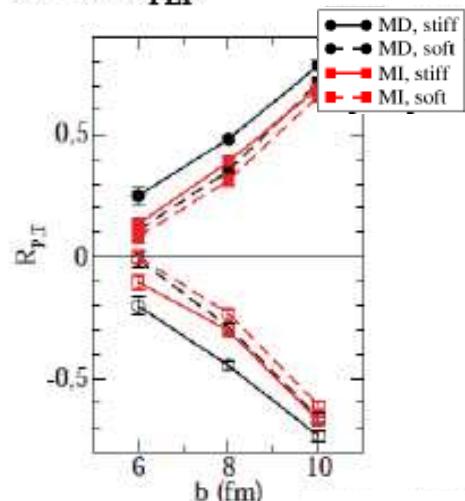
B. Tsang et al., PRL 102 (2009)

$^{124}\text{Sn} + ^{112}\text{Sn}$ , 50 AMeV

SMF calculations

J. Rizzo et al., NPA(2008)

$\mathbf{X} = \mathbf{N}/\mathbf{Z}_{\text{PLF}}$



Simple equil. model

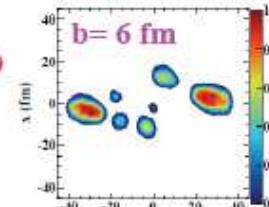
$$\beta_{P,T}^M = \beta^{eq} + (\beta^{H,L} - \beta^{eq}) e^{-t/\tau},$$

$$\Rightarrow R_{P,T}^{\rho} = \pm e^{-t/\tau}$$

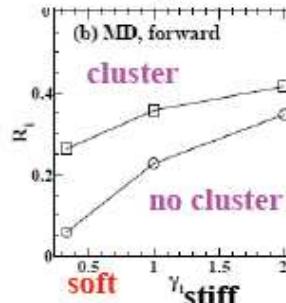
Ratio det. by interact.  
and relax. times

pBUU calculations (Danielewicz)

Coupland et al.,  
PRC 84, 054603 (2011)

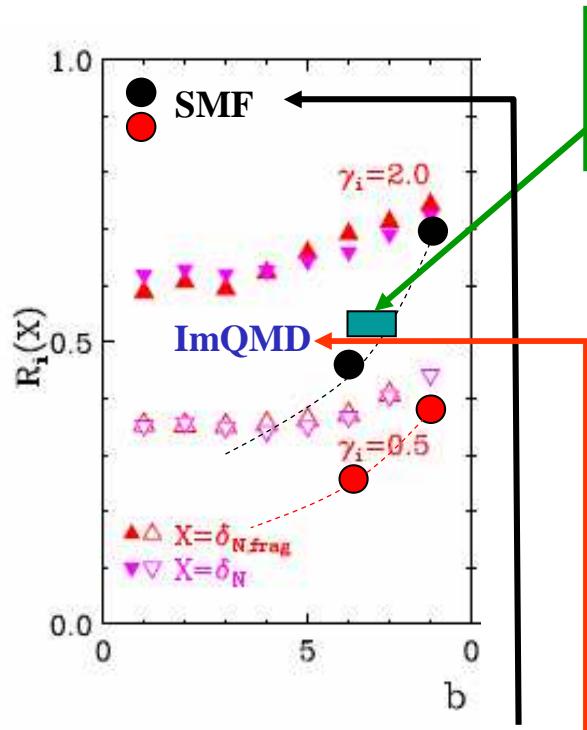


Cluster effects  
(d,t,He<sup>3</sup>)



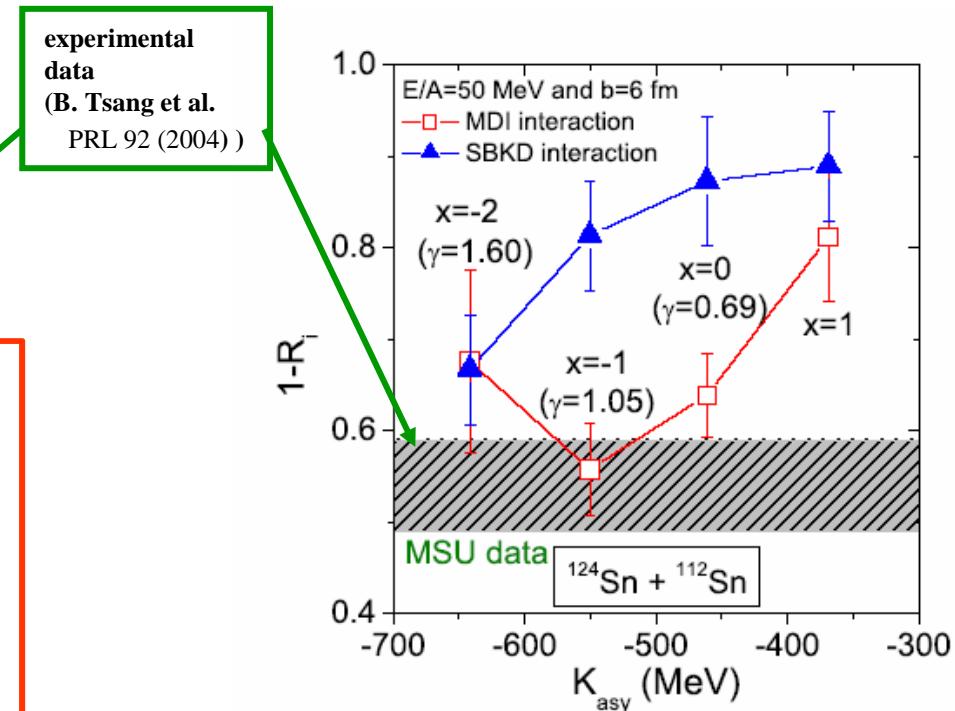
# Transport Ratios for Projectile/Target Residues: $^{112,124}\text{Sn} + ^{112,124}\text{Sn}$ , 50 MeV

## Comparison of models:



J.Rizzo, et al., NPA806 (08) 79

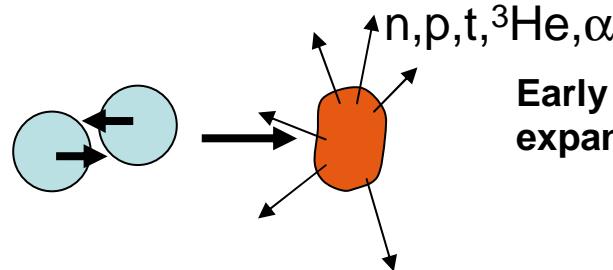
M.B. Tsang, et al., PRL 102 (08)



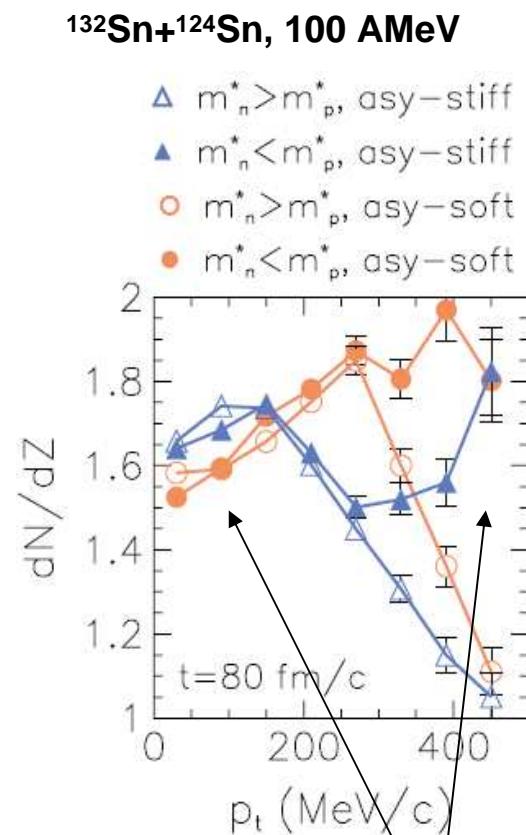
L.W.Chen, C.M.Ko, B.A.Li, PRL 94, 032701 (2005)

1. Qualitative agreement, but not quantitative
2. different impact parameter dependence; to be understood

## Ratios of emitted pre-equilibrium particles in central collisions



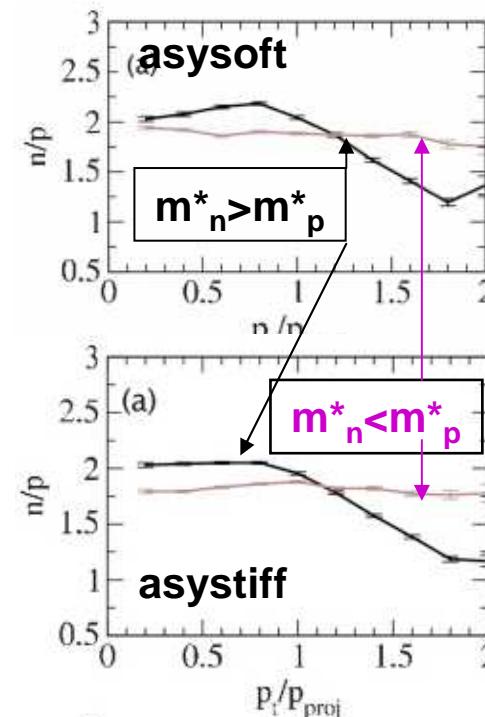
Early emitted Light Clusters reflect difference in potentials in expanded source, e.g. ratio  $Y(n)/Y(p)$ .



splitting dep. on stiffness of symm energy resp. effect, mass

(J. Rizzo et al., PRC72,064609 (2005))

$^{197}\text{Au} + ^{197}\text{Au}$ , 600 AMeV

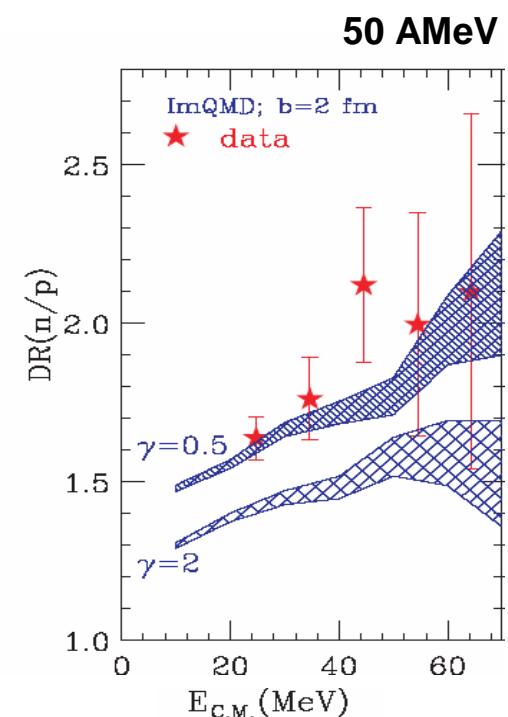


essentially only dep. on eff. mass

V.Giordano, et al., PRC 81(2010))

Double Ratios

$$\frac{^{124}\text{Sn} + ^{124}\text{Sn}}{^{112}\text{Sn} + ^{112}\text{Sn}}$$



no effective mass depend.

M. Famiano, et al., PRL 97,052701 (06)  
M.B.Tsang, et al., PRL 102, 122701 (09)

# Light Cluster Emission: $^{136,124}\text{Xe} + ^{124,112}\text{Sn}$ , $E = 32,..,150$ AMeV ,

## Single ratios

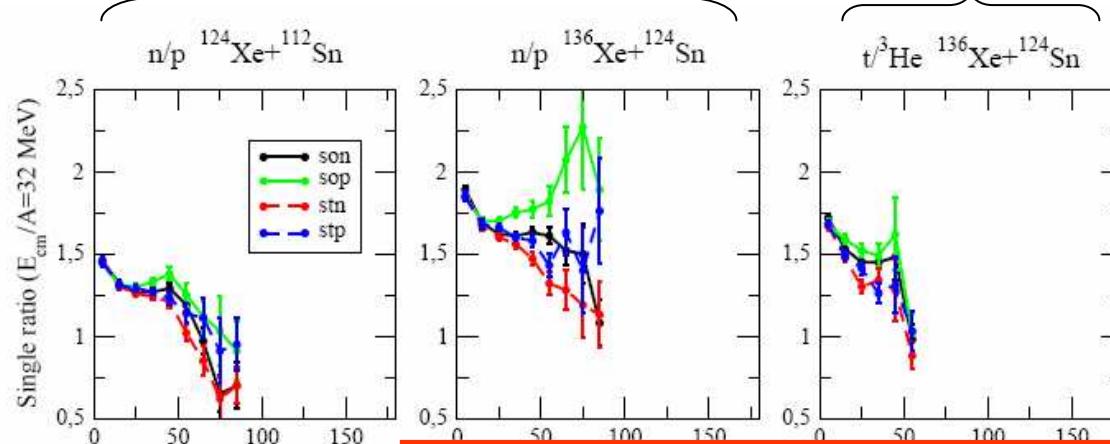
effects smaller for heavier clusters,  
but have to improve statistics  
 $n/p$

$t/3\text{He}$

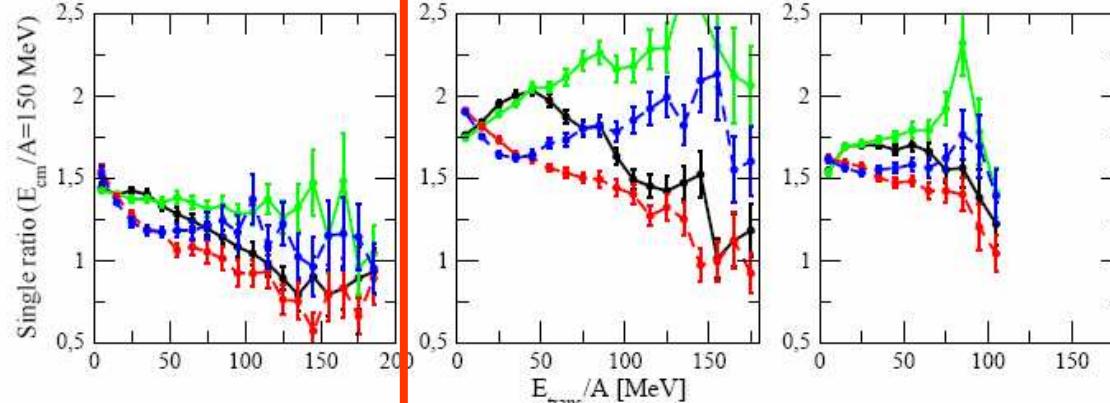
Variation  
of density dep. and  
effective mass splitting  
with isoscalar potential  
held fixed  
(Bombaci-Gale-  
Bertsch-DasGupta  
type)

son: asysoft,  $m_n^* > m_p^*$   
stn: asystiff,  $m_n^* > m_p^*$   
sop: asysoft,  $m_n^* < m_p^*$   
stp: asystiff,  $m_n^* < m_p^*$

$E=32$  AMeV



$E=150$  AMeV



neutron poor  
 $^{124}\text{Xe} + ^{112}\text{Sn}$

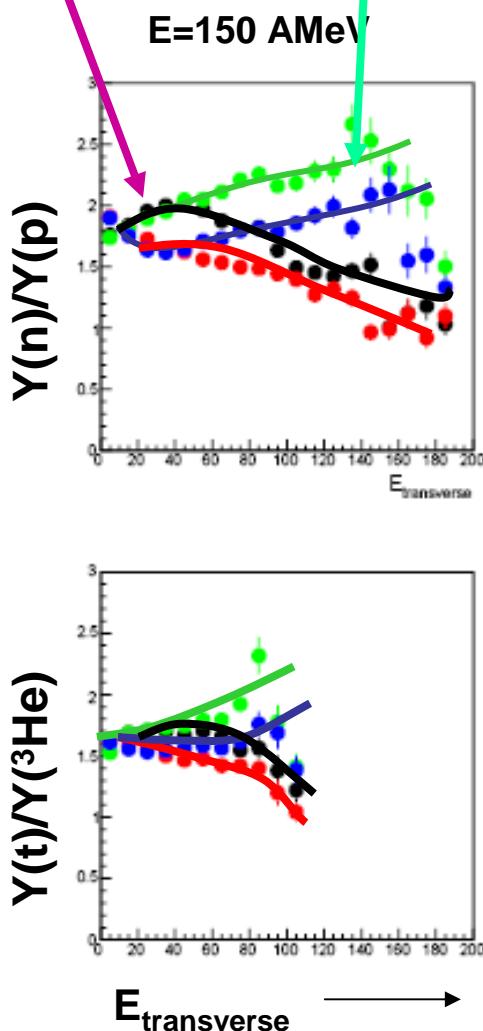
neutron rich  
 $^{136}\text{Xe} + ^{124}\text{Sn}$

effects larger for higher energy and neutron rich system

look in more detail

Asy-EOS dominates  
for slow particles;  
asysoft has larger repulsion  
at lower densities

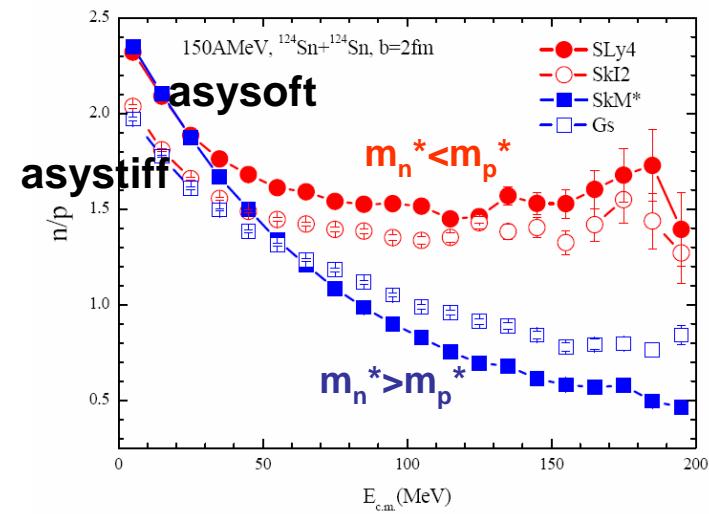
Effective mass dominates  
for fast particles;  
smaller eff. mass favors emission



Effect also exists for  
light clusters (easier  
to measure) but  
somewhat reduced

son: asysoft,  $m_n^* > m_p^*$   
stn: asystiff,  $m_n^* > m_p^*$   
sop: asysoft,  $m_n^* < m_p^*$   
stp: asystiff,  $m_n^* < m_p^*$

similar conclusions for  
Sn+Sn collisions (MSU)

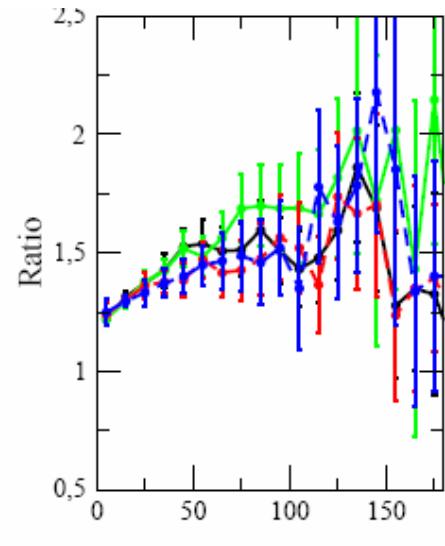


Y. Zhang, M.B.Tsang,et al., PLB 732, 186 (2014)

Possibility to separate  
density and momentum  
dependence of symmetry  
energy

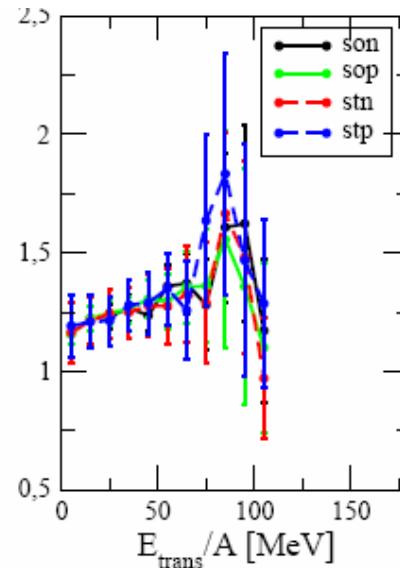
### Double Ratios

$$\frac{n/p_{^{136}\text{Xe} + ^{124}\text{Sn}}}{n/p_{^{124}\text{Xe} + ^{112}\text{Sn}}}$$



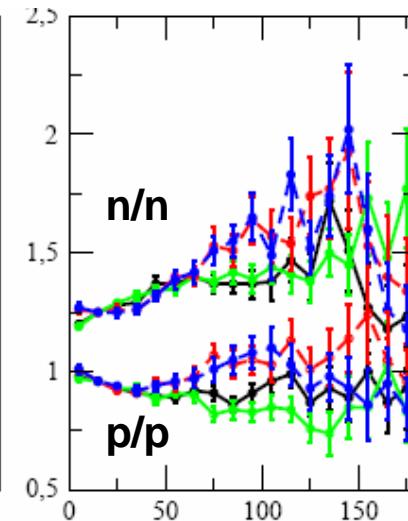
sensitivity much smaller

$$\frac{t/^3\text{He}_{^{136}\text{Xe} + ^{124}\text{Sn}}}{t/^3\text{He}_{^{124}\text{Xe} + ^{112}\text{Sn}}}$$



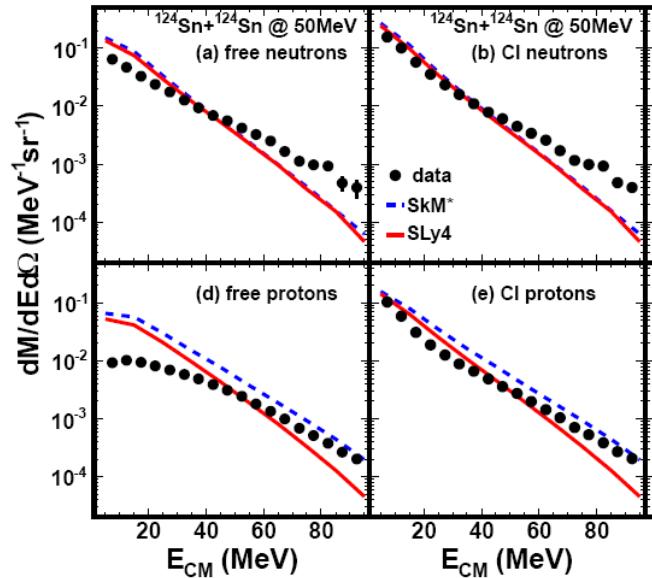
### Singe Ratios

$$\frac{n_{^{136}\text{Xe} + ^{124}\text{Sn}}}{n_{^{124}\text{Xe} + ^{112}\text{Sn}}}, \text{ resp } ., \frac{p_{^{136}\text{Xe} + ^{124}\text{Sn}}}{p_{^{124}\text{Xe} + ^{112}\text{Sn}}}$$



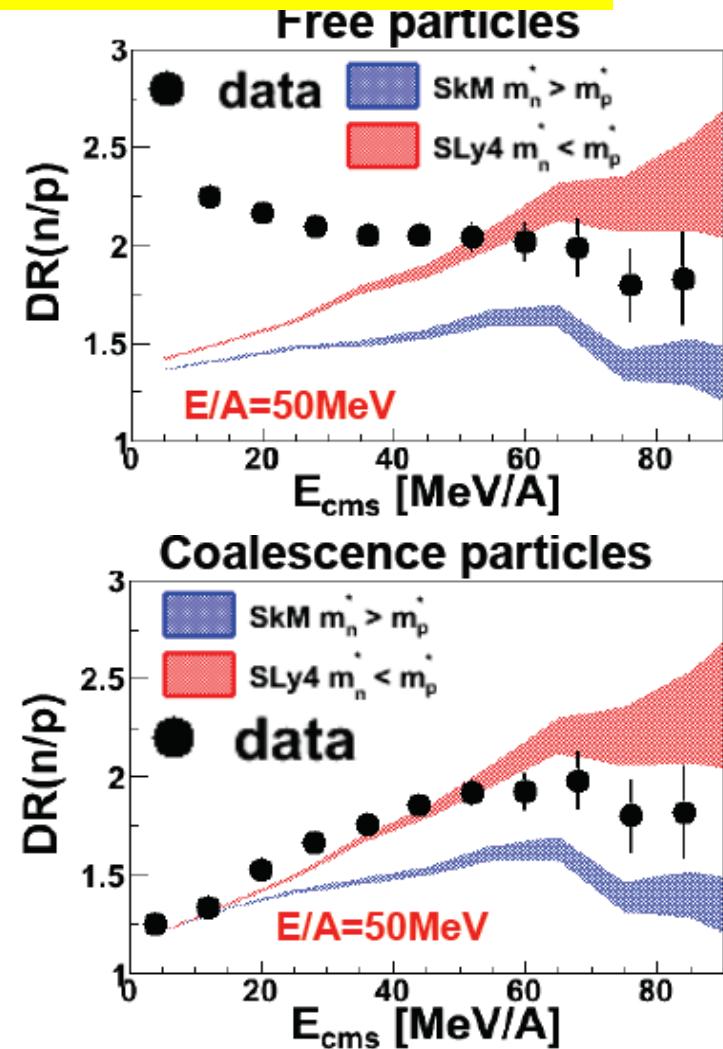
sensitivity only to asy-stiffness

# Comparison with data: problem of light cluster description in transport approaches → „coalescence invariant“ (CI) spectra



coalescence invariant spectra  
agree better with experiment: A poor man's  
substitute for not treating light clusters properly  
in the simulation

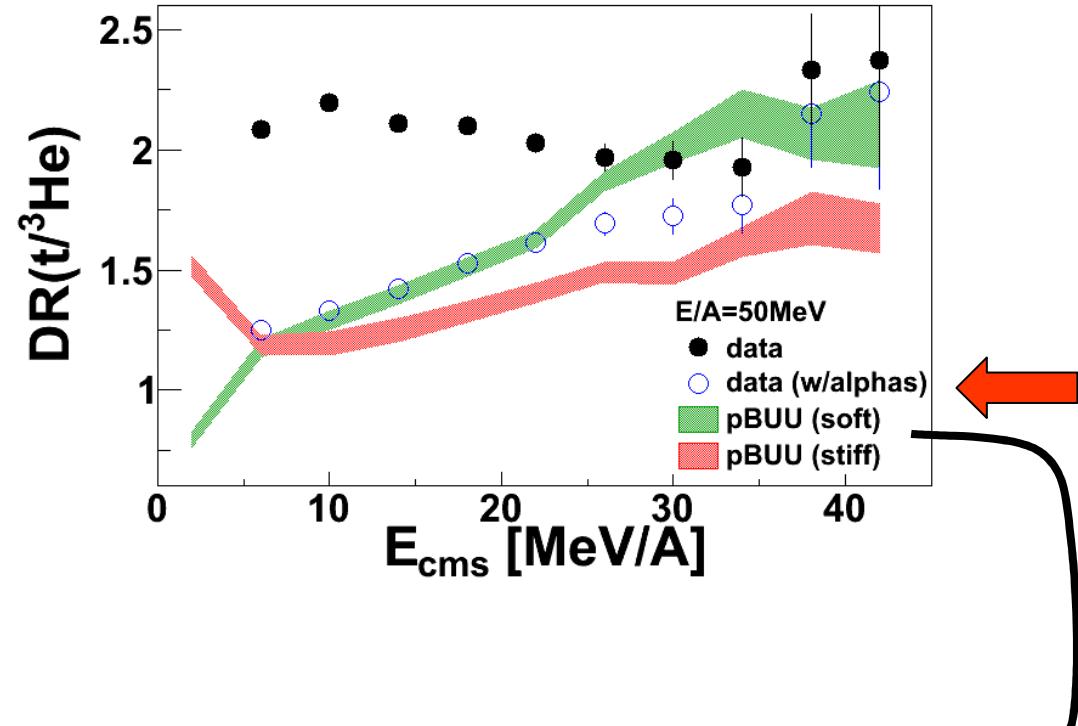
favors  $m_n^* < m_p^*$  (in contrast to  
optical model analyses)  
→ more work required!



Y.X. Zhang, M.B.Tsang,et al., PLB 732, 186 (2014)  
D.D.S.Coupland, arXiv 1406.4546

## Interdependence of light cluster (LC) yields

W.Lynch, INPC, Florence, 2013:  
 $^{124,112}\text{Sn} + ^{124,112}\text{Sn}$ , 50 AMeV;  
Z. Chajecki, NuSYM13



Cluster yields depend on dynamic treatment of LC production in HIC

## Conclusions:

- cluster production in heavy ion collisions is an important source of information on the symmetry energy
- the information is different in bigger fragments (IMF, mean field dominated) and light clusters (LC, mf and effective masses)
- the single ratio spectra of LC may allow to separate the density and momentum dependence.
- more asymmetric systems and the n/p ratio are most sensitive. The t/3He ratio carries a similar information
- more precise data recently available, but not yet precise enough to determine effective masses (at stress with other determinations)
- strongly connected with the question to describe dynamical cluster formation in HIC

Grazie!

