

# Dynamics of Quasifission in TDHF Theory

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## Collaborators:

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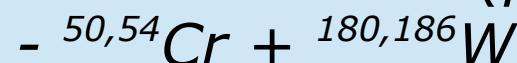
**Frankfurt:** J.A. Maruhn

## Topics Covered:

*Time-dependent DFT theory (TDHF)*

*Capture (DC-TDHF)*

*Quasifission using TDHF*



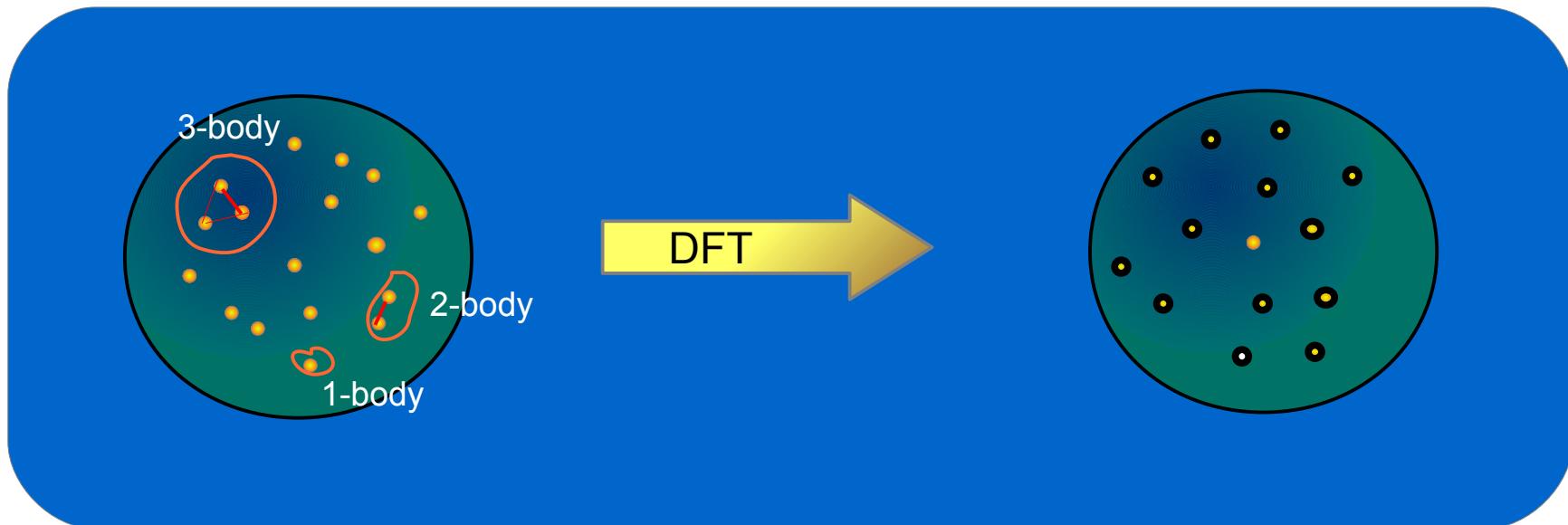
*Dynamical moment of inertia,  $E^*$ ,  $T$*

*$P_{CN}$  from TDHF*

*Dynamics of Fission (time permitting)*



# Nuclear Mean Field or Energy Density Functional (EDF)



ab-initio  
 $\langle \Psi | H | \Psi \rangle = E$

Mean-field - EDF  
 $\Psi \rightarrow \Phi_{Slater}$   
 $H \rightarrow H_{eff}$

$$E = \langle \Phi | H_{eff} | \Phi \rangle = \int d^3r \left\{ H(\rho, \tau, j, s, T, J_{\mu\nu}; \mathbf{r}) + H_{Coulomb}(\rho_p) \right\}$$

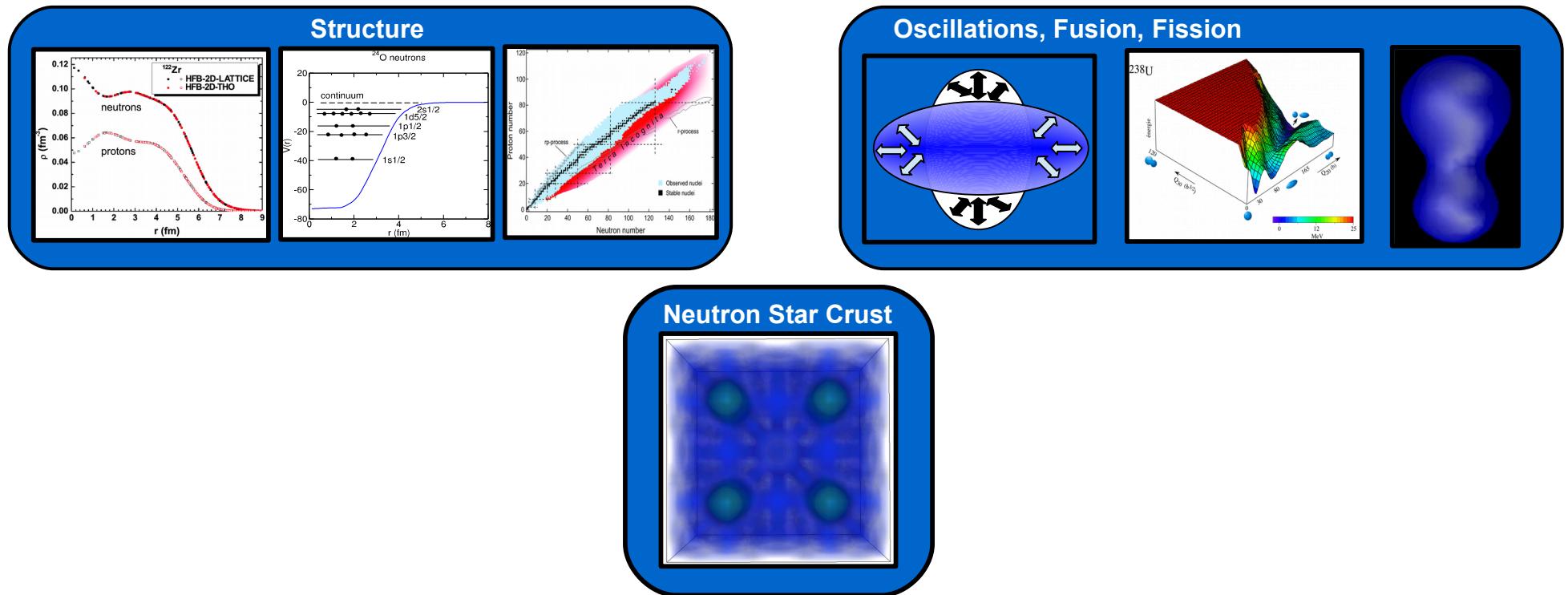
Single-(one-) particle density etc. in terms of s.p. states

$$\rho_q(\mathbf{r}) = \sum_{i=1}^A \sum_{\sigma} \varphi_i^*(\mathbf{r}, \sigma, q) \varphi_i(\mathbf{r}, \sigma, q)$$

EDF in NP more complicated  
 $v = v_{NN-eff} \rightarrow DFT \text{ (Hartee-Fock)}$   
 $v \neq v_{NN-eff} \rightarrow DFT \text{ (Kohn-Sham)}$



# Study Structure, Reactions, and Star Matter in Same Framework



- Time-dependent generalization TDHF or TDDFT (variational or Runge-Gross)

$$\delta S = \delta \int_{t_1}^{t_2} dt \langle \Phi(t) | H_{\text{eff}} - i \hbar \partial_t | \Phi(t) \rangle = 0$$

$$i \frac{\partial}{\partial t} \varphi_\alpha = h(\rho, \tau, j, s, T, J_{\mu\nu}; \mathbf{r}) \varphi_\alpha$$

self-consistent

- TDHF gives the *most probable outcome* – best if x-section dominated by one process



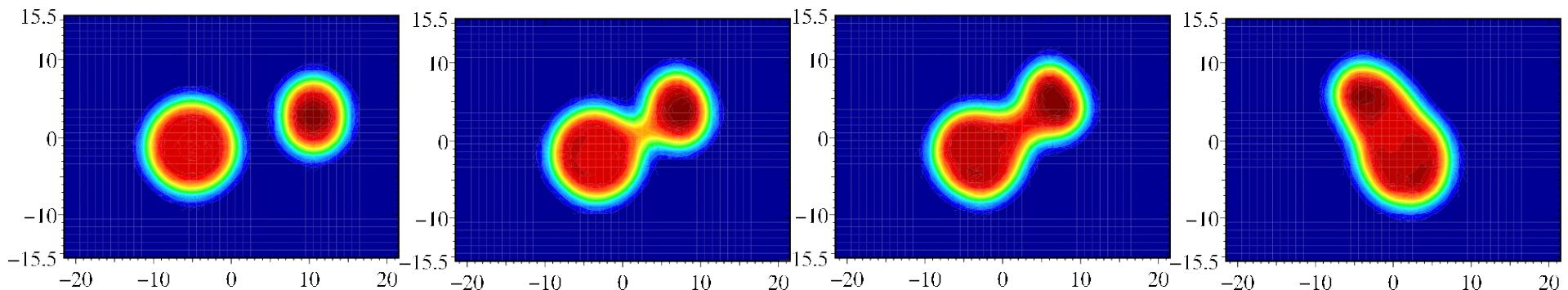
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# Modern TDHF Codes

## VU-TDHF Code

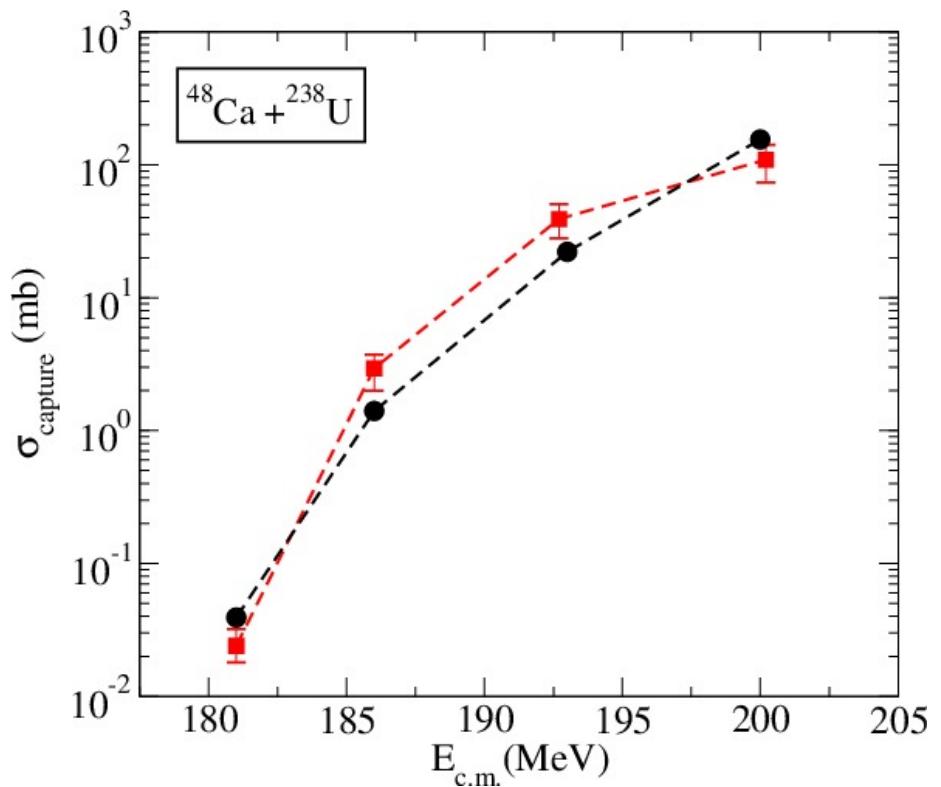
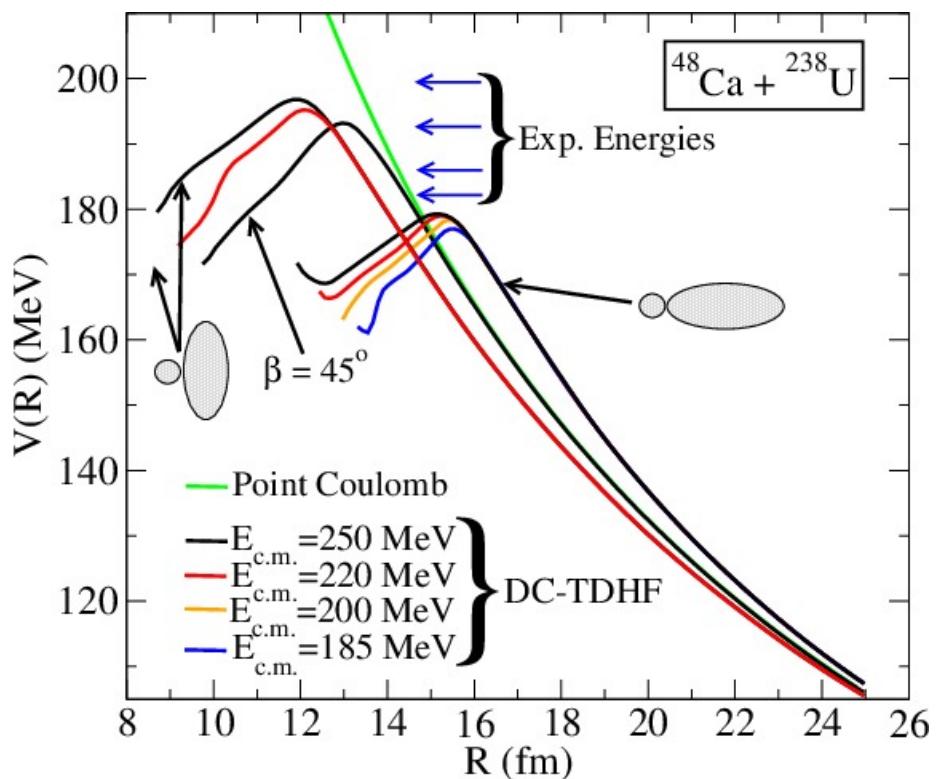
- Basis-Spline discretization for high accuracy
- 3-D Cartesian lattice – no geometrical simplification
- Complete EDF including all terms (time-even, full time-odd)
- Coded in **Fortran-95** and **OpenMP**

1. Umar, Oberacker, VU-TDHF, Phys. Rev. C 73, 054607 (2006)
2. Maruhn, Reinhard, Stevenson, Umar, Sky3D, Comp. Phys. Comm. 85, 2195 (2014)



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# DC-TDHF Barriers – Capture



## Angle average $^{238}\text{U}$ alignment

- x-section falls rapidly for  $\beta > 10^\circ$
- $\sin(\beta)$  multiply small angles
- $P(\beta)$  is in the range 0.4-0.6

### Experimental data:

1. M. G. Itkis et al., J. Nucl. Radiochem. Sci. 3, 57 (2002)
2. M. G. Itkis et al., Nucl. Phys. A 734, 136 (2004)

$$\sigma_f(E_{c.m.}) = \int_0^1 d\beta \sin(\beta) P(\beta) \sigma(E_{c.m.}, \beta)$$

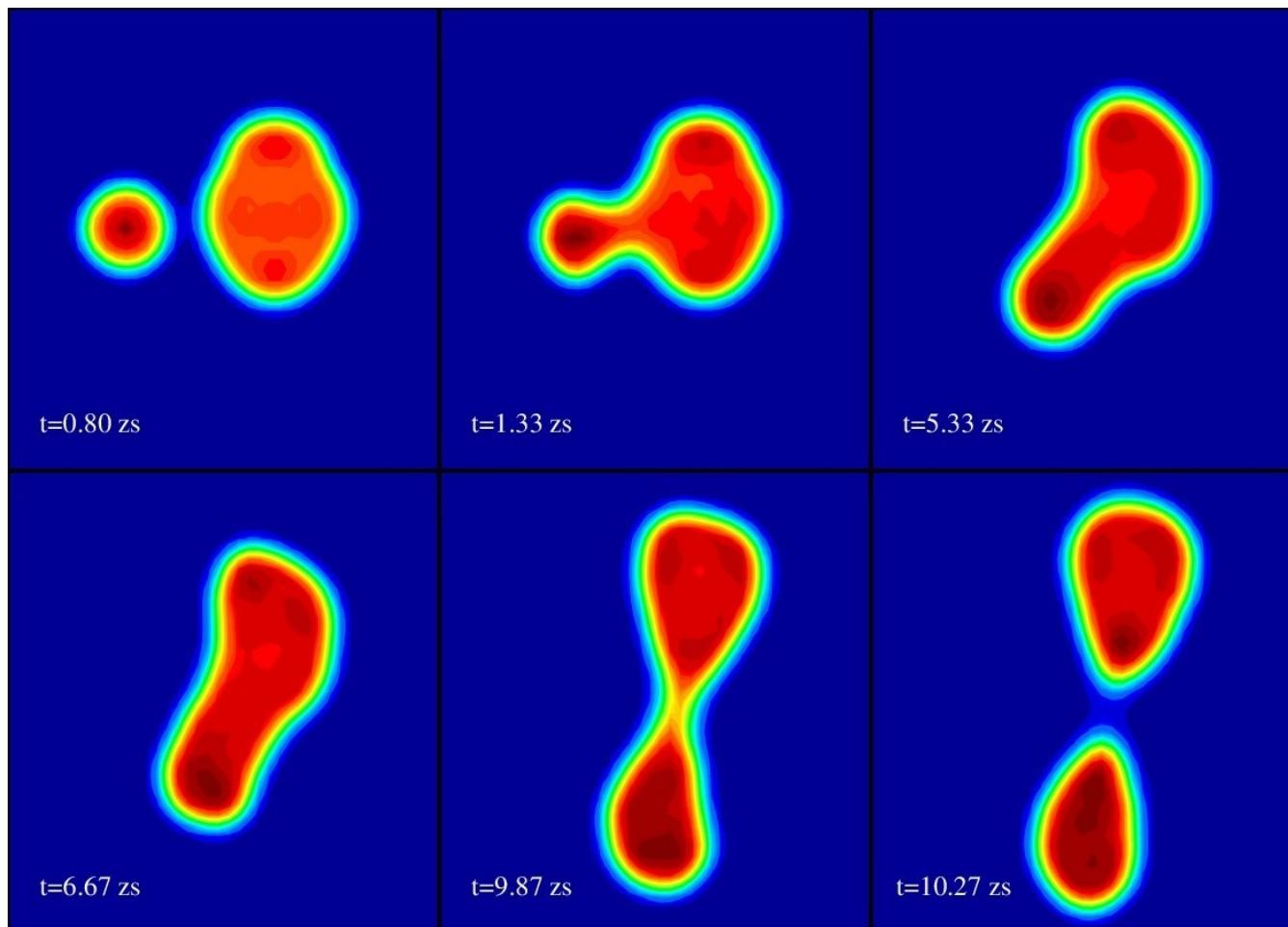


# Quasifission in TDHF – $^{40,48}\text{Ca} + ^{238}\text{U}$

## Heavy Systems

$$\sigma_{capture} = \sigma_{QF} + \sigma_{fusion-fission} + \sigma_{ER}$$

- QF dominant part
- Important for studying SHE dynamics



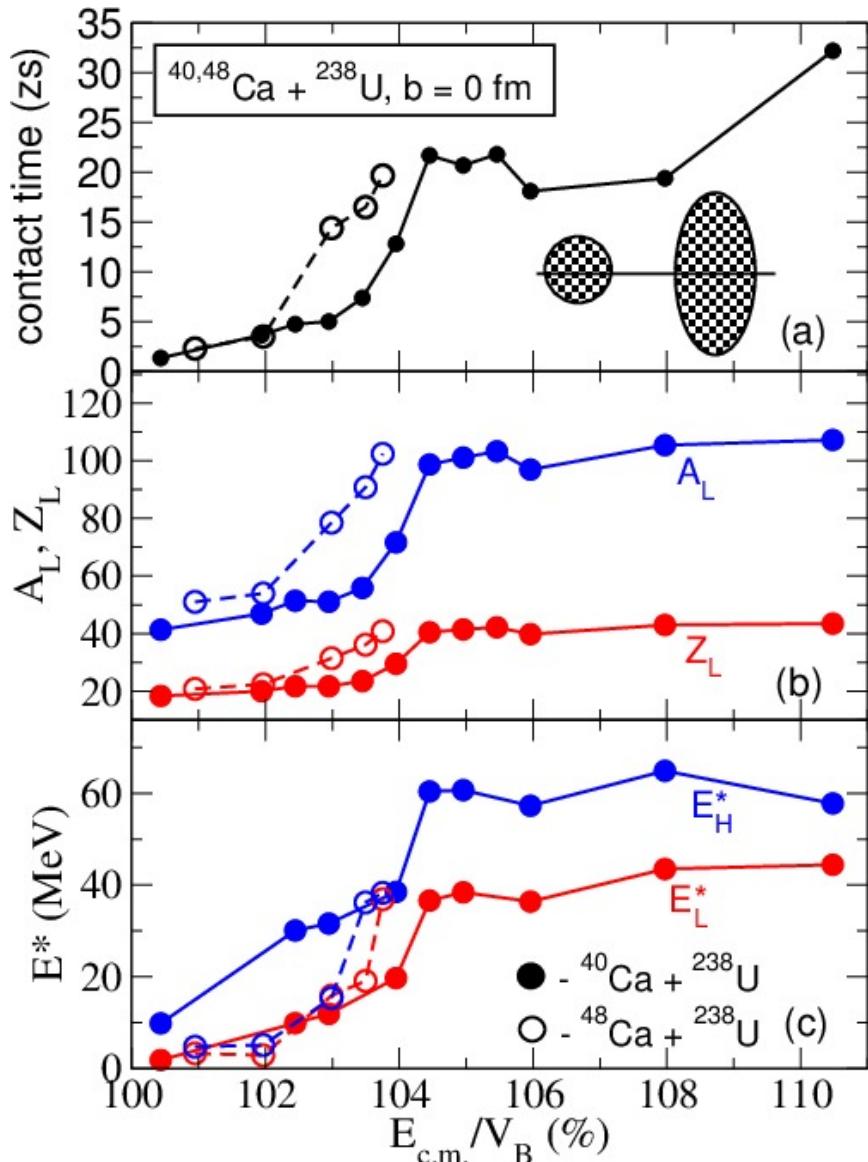
*Final masses:*  
 $A_L = 101, A_R = 177$   
 $Z_L = 41, Z_R = 71$

$E_{cm} = 209 \text{ MeV}$   
 $b = 1.103 \text{ fm (L=20)}$

←  $^{40}\text{Ca} + ^{238}\text{U}$

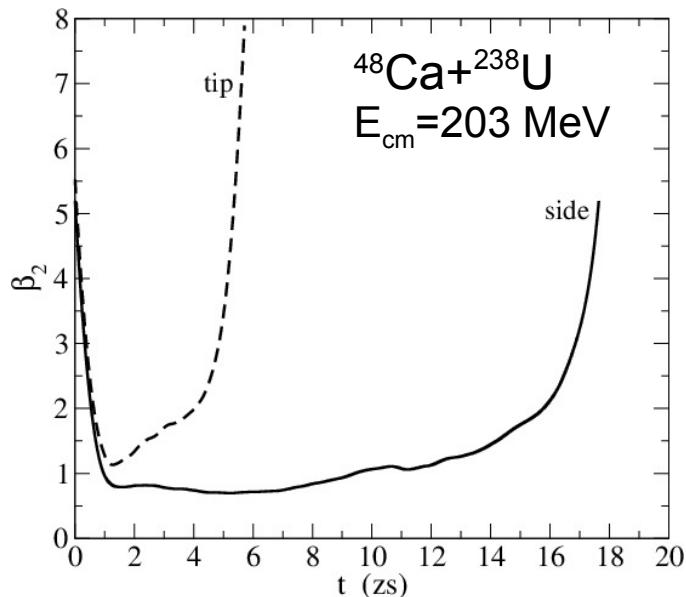


# Quasifission – $^{40,48}\text{Ca} + ^{238}\text{U}$



- Compare  $^{40,48}\text{Ca} + ^{238}\text{U}$  ( $b=0$ )
- **fusion** implies contact-time > 35 zs  
(plus density shows no indication of QF)
- $^{40}\text{Ca} + ^{238}\text{U}$  wider energy range for QF
- $E^*$  sharing seems different  
(calculated dynamically using DC-TDHF)

Each point takes about a week  
on a 16 processor workstation

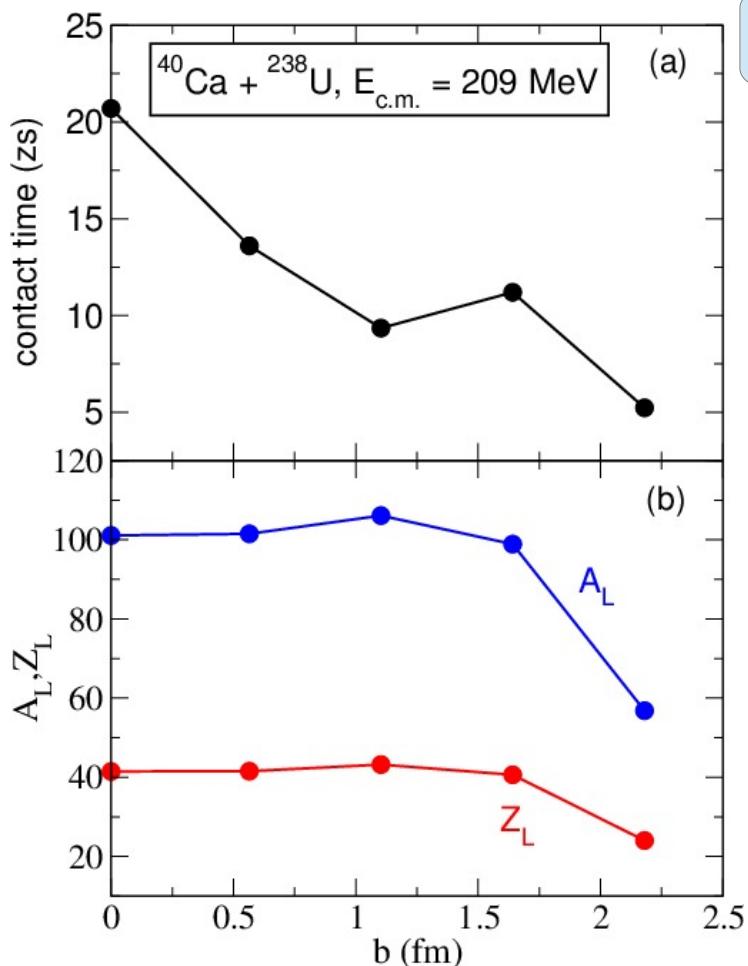


$$\beta_2 = \frac{4\pi}{3} \frac{Q_{20}}{AR_0^2}$$

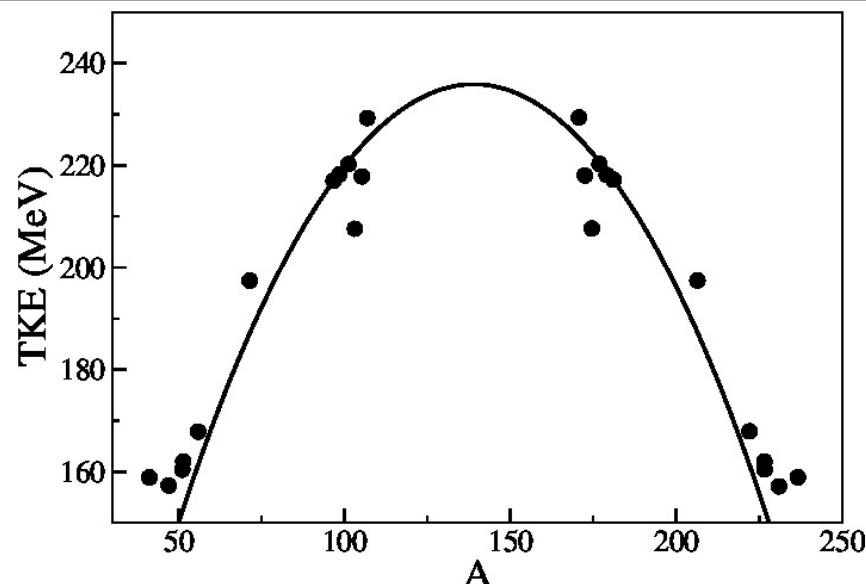
The  $\beta=0^\circ$  orientation of  $^{238}\text{U}$  results in much smaller contact-times and mass transfer



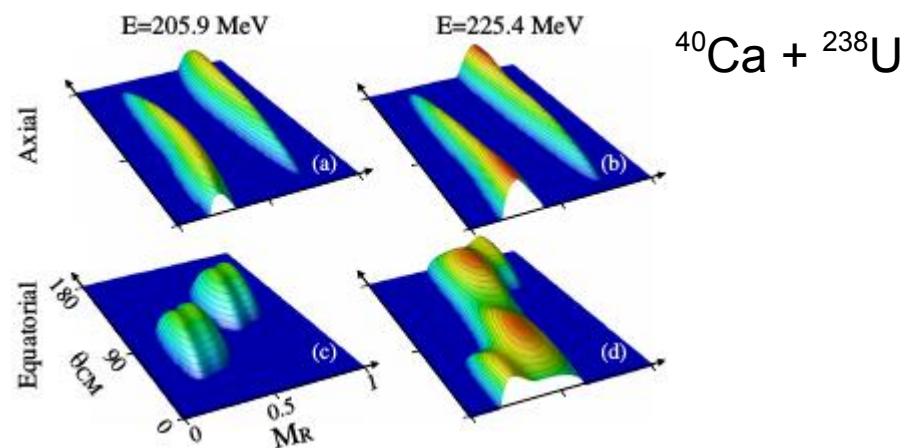
# Impact Parameter Dependence – Viola Systematics



*Final fragment TKE's well described by Viola systematics*



*Narrow range of impact parameters at low energies*



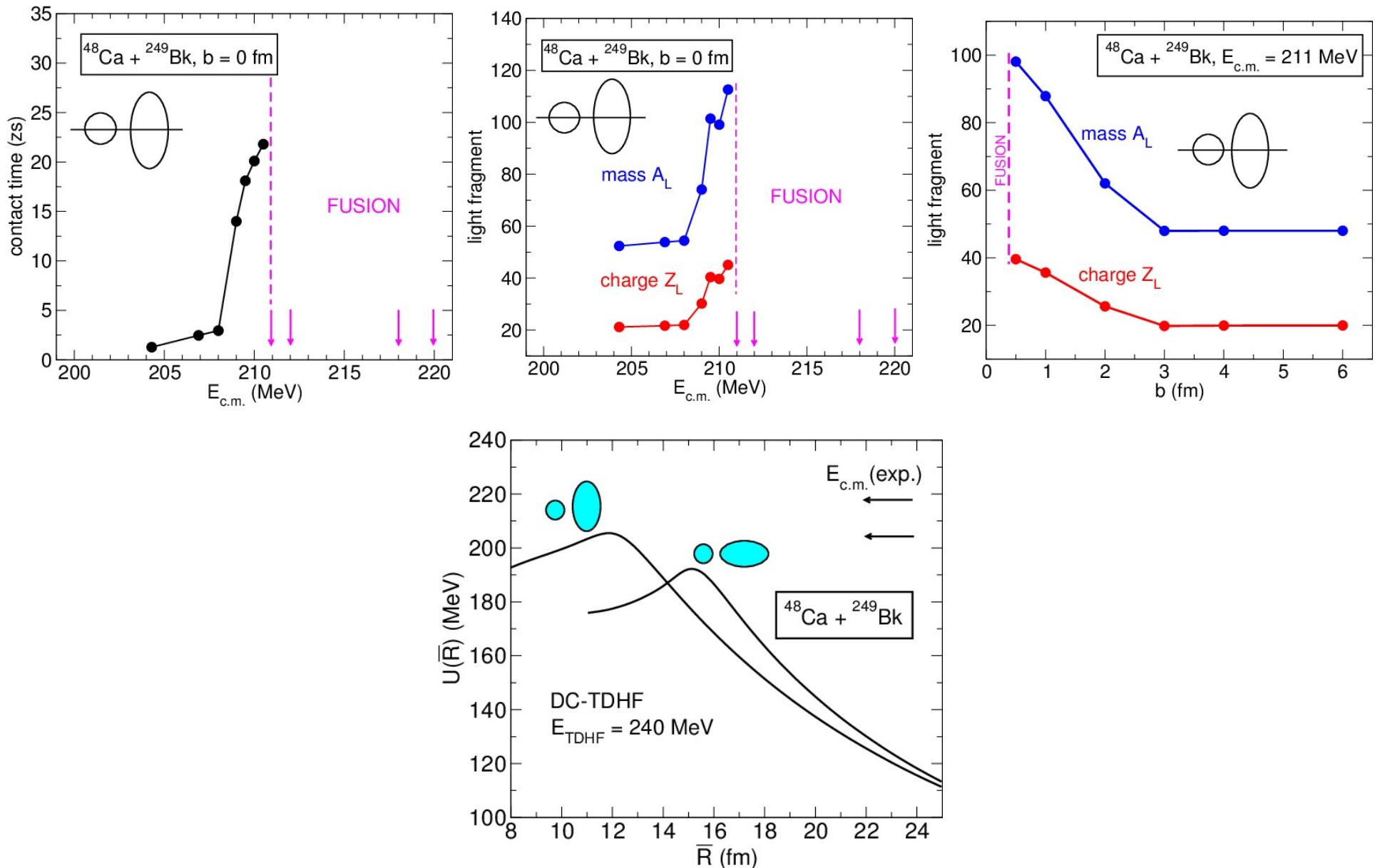
Wakhle et al., PRL 113, 182502 (2014).



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V.E. Oberacker, A.S. Umar, C. Simenel, PRC 90, 054605 (2014).

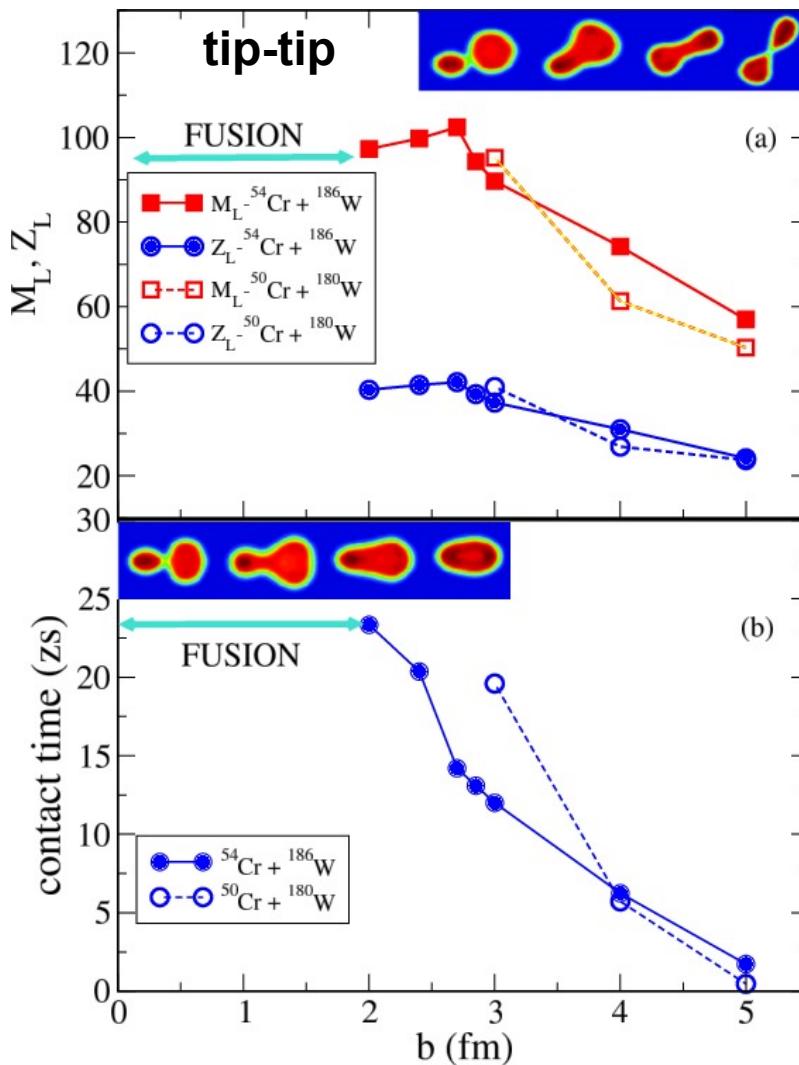
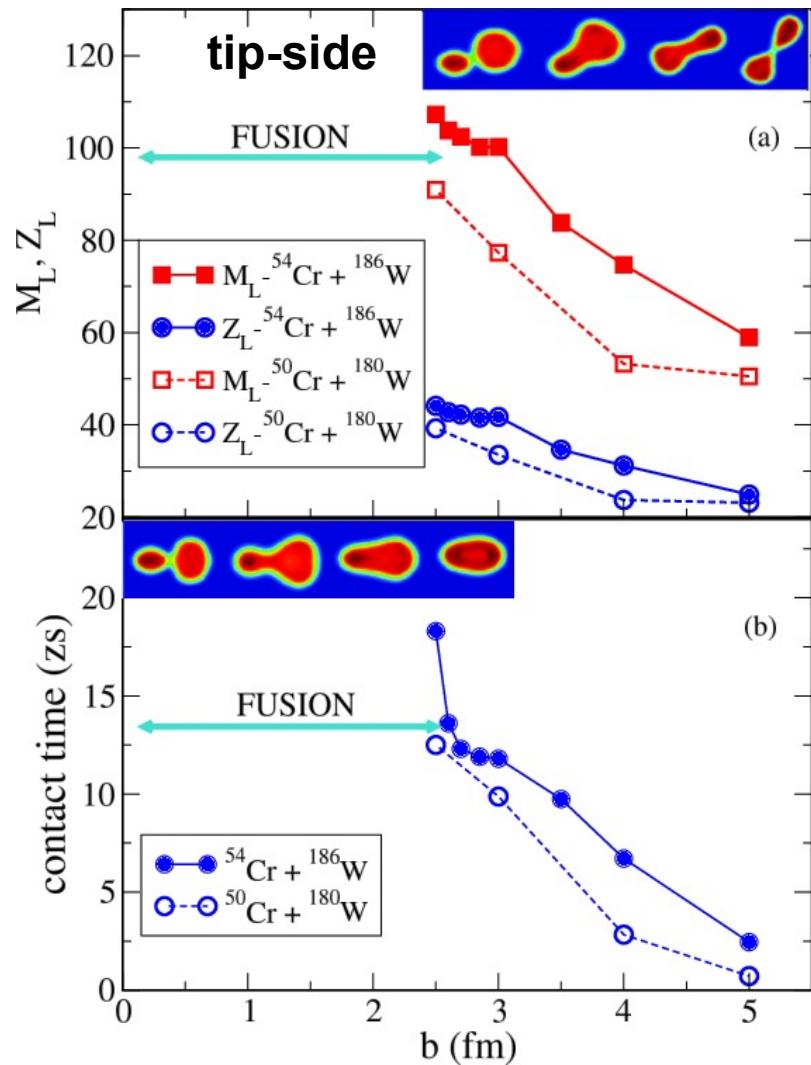
# Quasifission in $^{48}\text{Ca} + ^{249}\text{Bk}$ (ongoing-preliminary)



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# Quasifission in $^{50,54}\text{Cr} + ^{180,186}\text{W}$ ( $E_{\text{c.m.}}/V_B = 1.13$ )

*Two deformed nuclei with smaller mass/charge asymmetry than Ca+U*

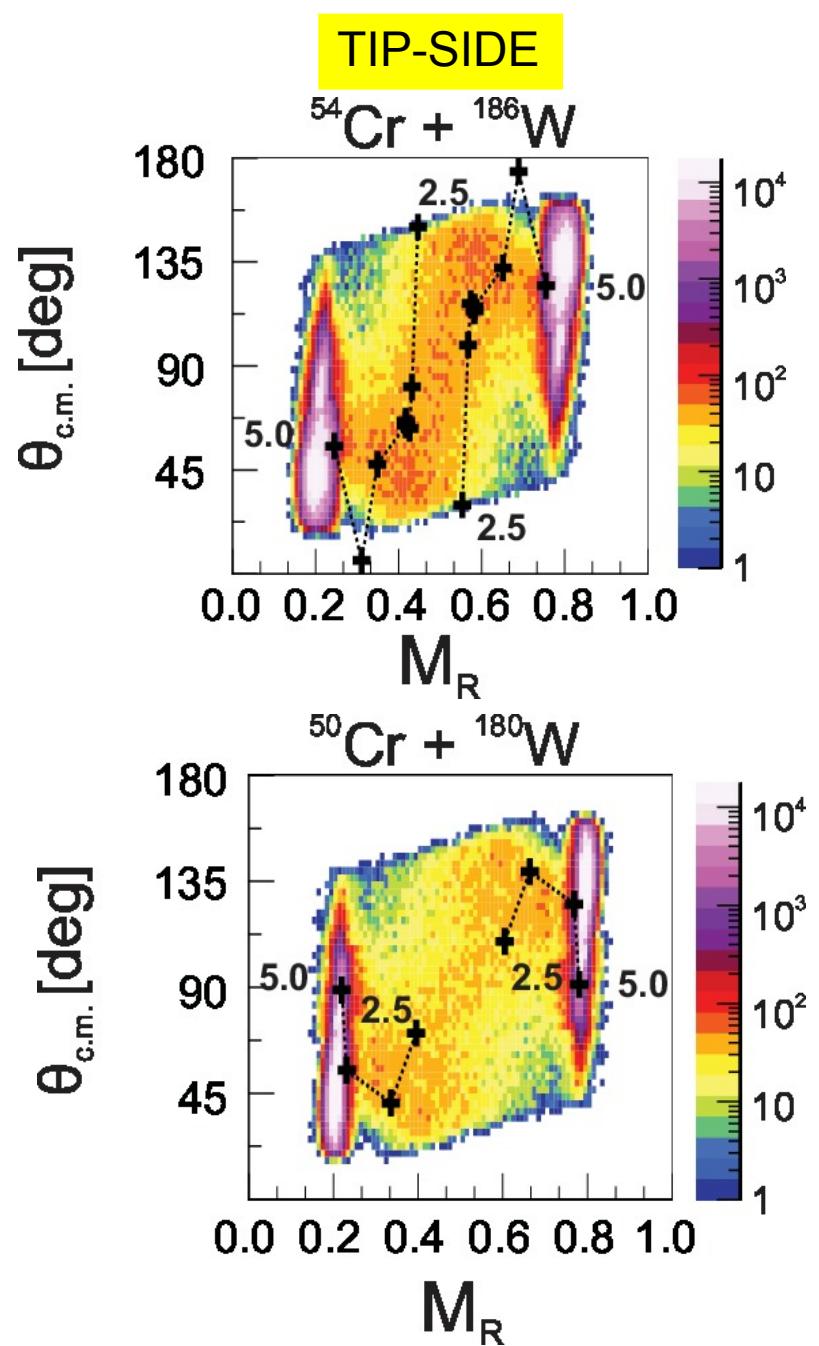
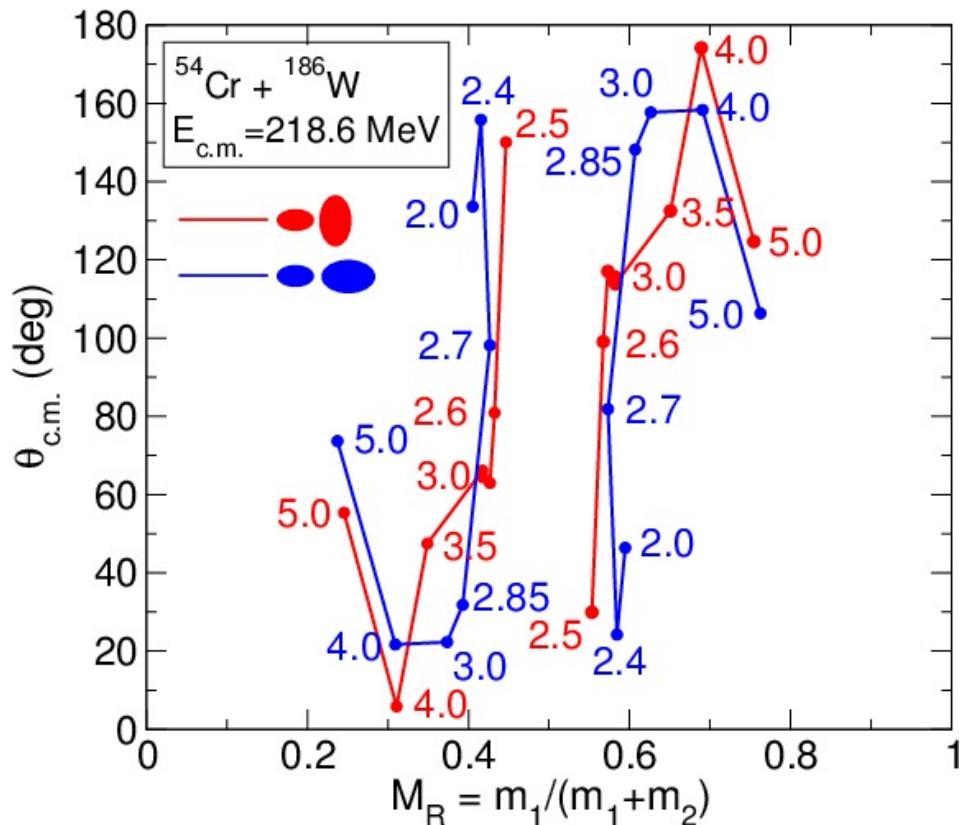


K. Hammerton, Z. Kohley, D. J. Hinde, M. Dasgupta, A. Wakhle, E. Williams, V. E. Oberacker, A. S. Umar, I. P. Carter, K. J. Cook, J. Greene, D. Y. Jeung, D. H. Luong, S. D. McNeil, C. S. Palshetkar, D. C. Rafferty, C. Simenel, and K. Stiefel, PRC 91, 041601(R) (2015)



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# Mass Angle Distributions (MAD's)



# Fission Fragment Angular Distributions - P<sub>CN</sub>

- Angular distribution based on TSM (see Yanez et al. PRC 88, 014606 (2013))

$$W(\theta) = \sum_{J=0}^{J_{CN}} \frac{(2J+1)^2 \exp[-(J+1/2)^2 \sin^2 \theta / 4K_0^2(FF)] J_0[i(J+1/2)^2 \sin^2 \theta / 4K_0^2(FF)]}{\text{erf}[(J+1/2)/(2K_0^2(FF))^{1/2}]} + \sum_{J=J_{CN}}^{J_{\max}} \frac{(2J+1)^2 \exp[-(J+1/2)^2 \sin^2 \theta / 4K_0^2(QF)] J_0[i(J+1/2)^2 \sin^2 \theta / 4K_0^2(QF)]}{\text{erf}[(J+1/2)/(2K_0^2(QF))^{1/2}]}$$

- Parameter  $K_0$  involves shape and temperature

$$K_0^2 = T \mathfrak{S}_{eff} / \hbar^2$$

*QF angular distribution*

$$P_{CN} \approx \frac{\sigma_{FF}}{\sigma_{QF} + \sigma_{FF}}$$

- Need parallel/perpendicular moment of inertia

$$\frac{1}{\mathfrak{S}_{eff}} = \frac{1}{\mathfrak{S}_{\parallel}} - \frac{1}{\mathfrak{S}_{\perp}}$$

Temperature at the saddle point ?

$$\mathcal{T} = \left[ \frac{E^* - B_f - E_{rot} - E_{\nu}}{A/8.5} \right]^{1/2}$$

- Assumption!

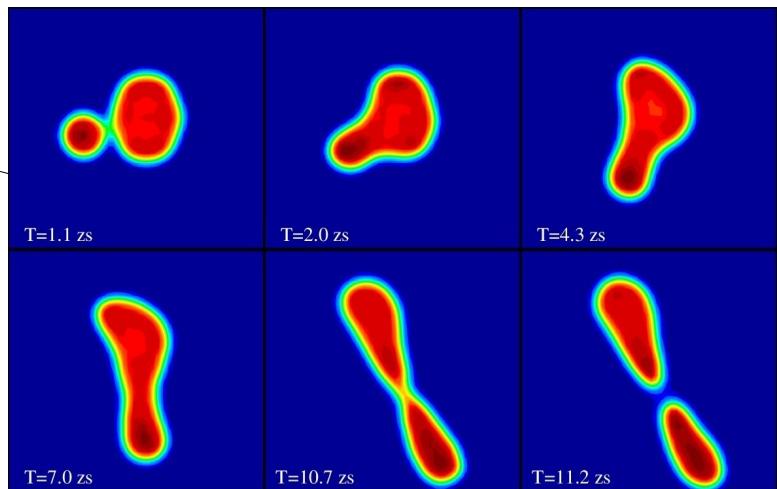
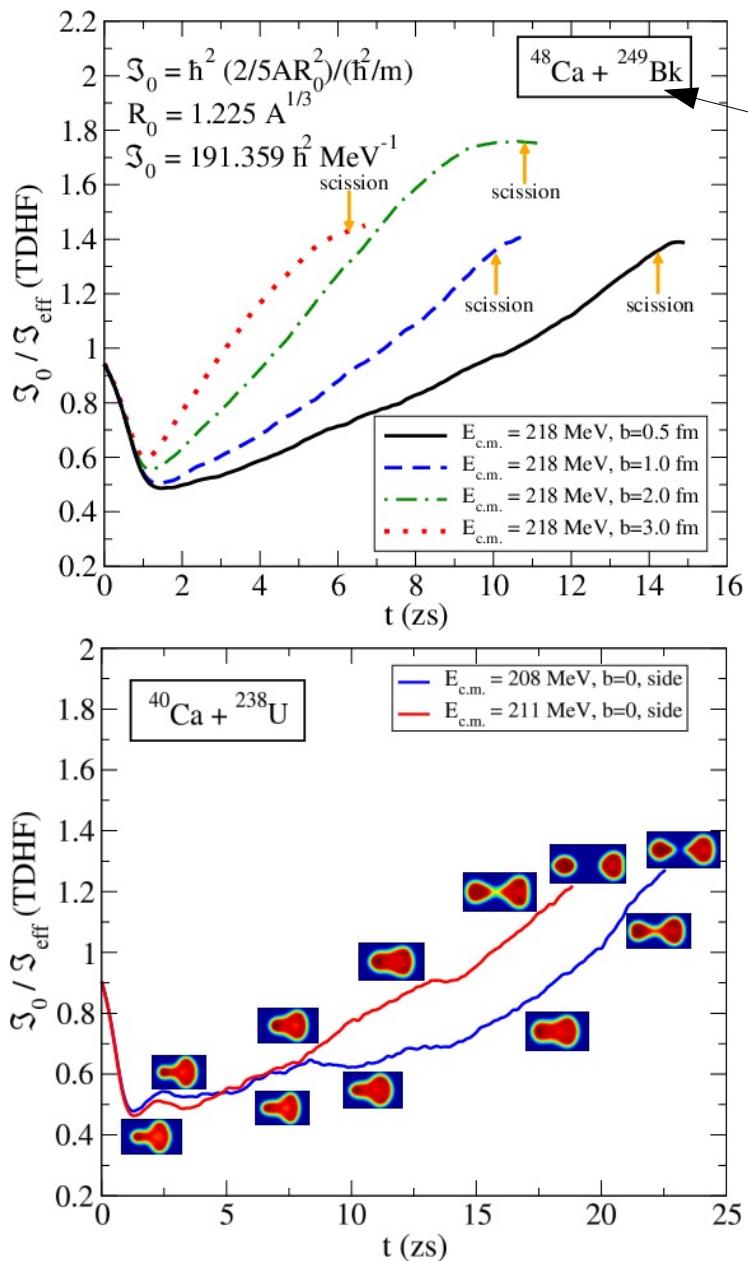
$$\mathfrak{S}_0 / \mathfrak{S}_{eff} = 1.5$$

Can obtain from dynamical  $E^*$  using DC-TDHF

$$\mathcal{T} = \left[ \frac{E_{TDHF}^*}{A/8.5} \right]^{1/2}$$



# Moment of Inertia from TDHF



- Diagonalize the moment of inertia tensor

$$\mathfrak{I}_{ij}/m = \int d^3r \rho_{TDHF}(\mathbf{r}, t)(r^2\delta_{ij} - x_i x_j)$$

- Eigenvalues give the parallel/perpendicular moment of inertia

$$\frac{1}{\mathfrak{I}_{eff}} = \frac{1}{\mathfrak{I}_{\parallel}} - \frac{1}{\mathfrak{I}_{\perp}}$$

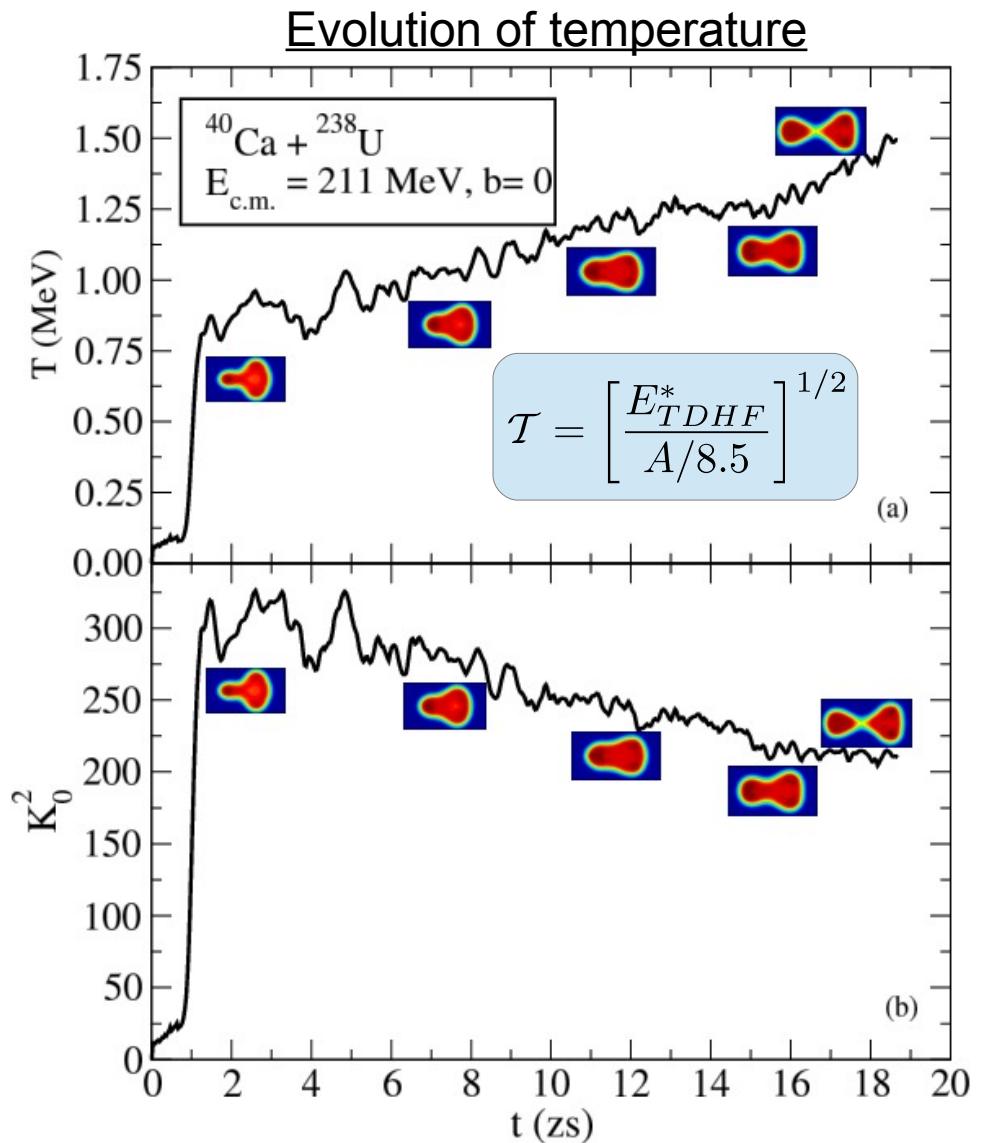
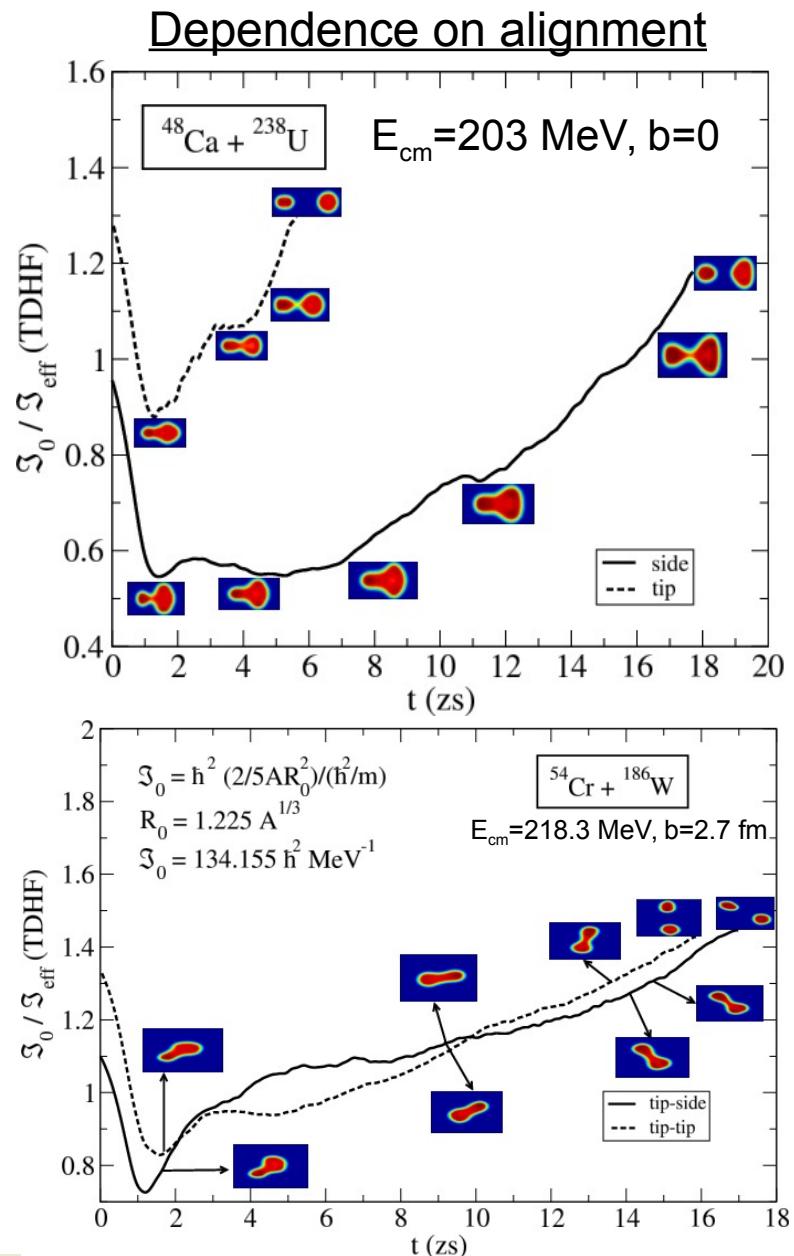
- Ratio  $\mathfrak{I}_0/\mathfrak{I}_{eff}$

$$\mathfrak{I}_0 = \hbar^2(2/5AR_0^2)/(\hbar^2/m)$$

Equivalent sphere



# Moment of Inertia Cont'd

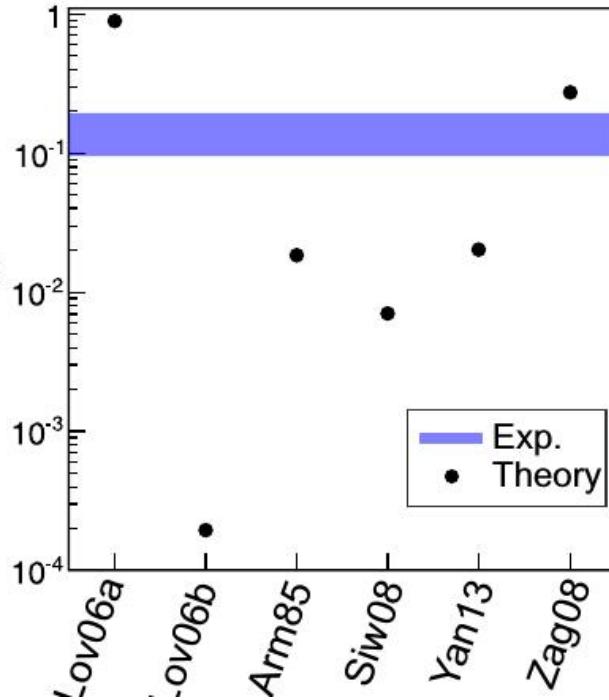
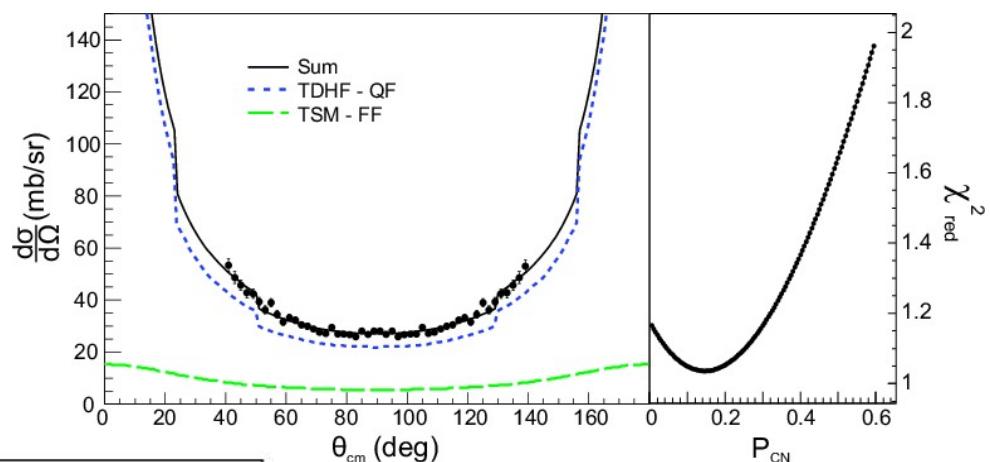
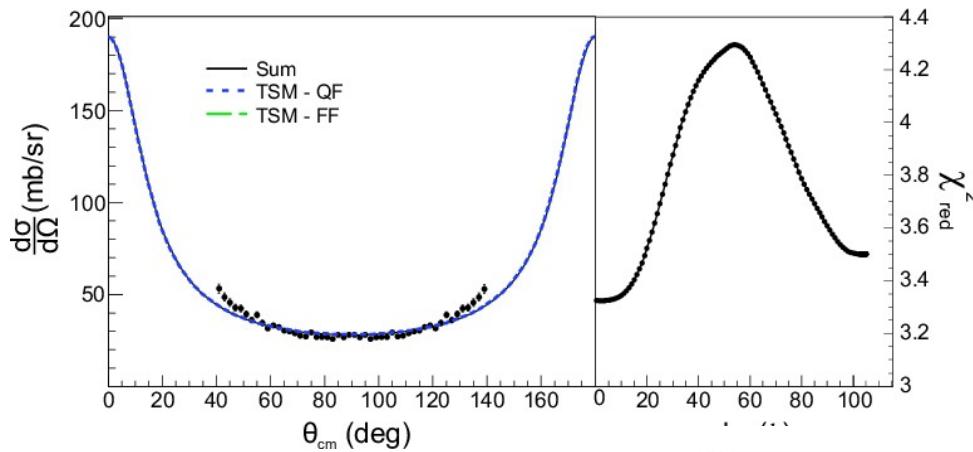


2 months of computing time!  
 Don't need full time-dependence in practice



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# $P_{CN}$ from TDHF Angular Distributions $^{54}\text{Cr} + ^{186}\text{W}$



Comparison with different models



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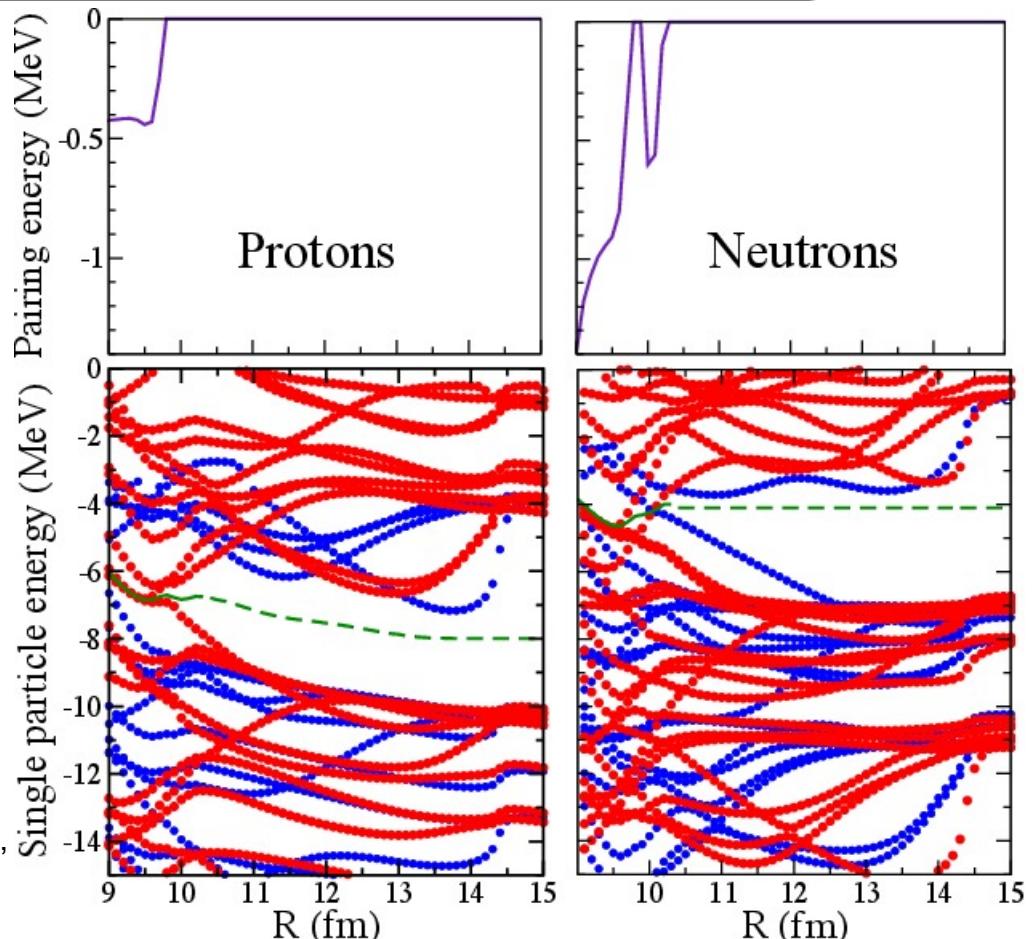
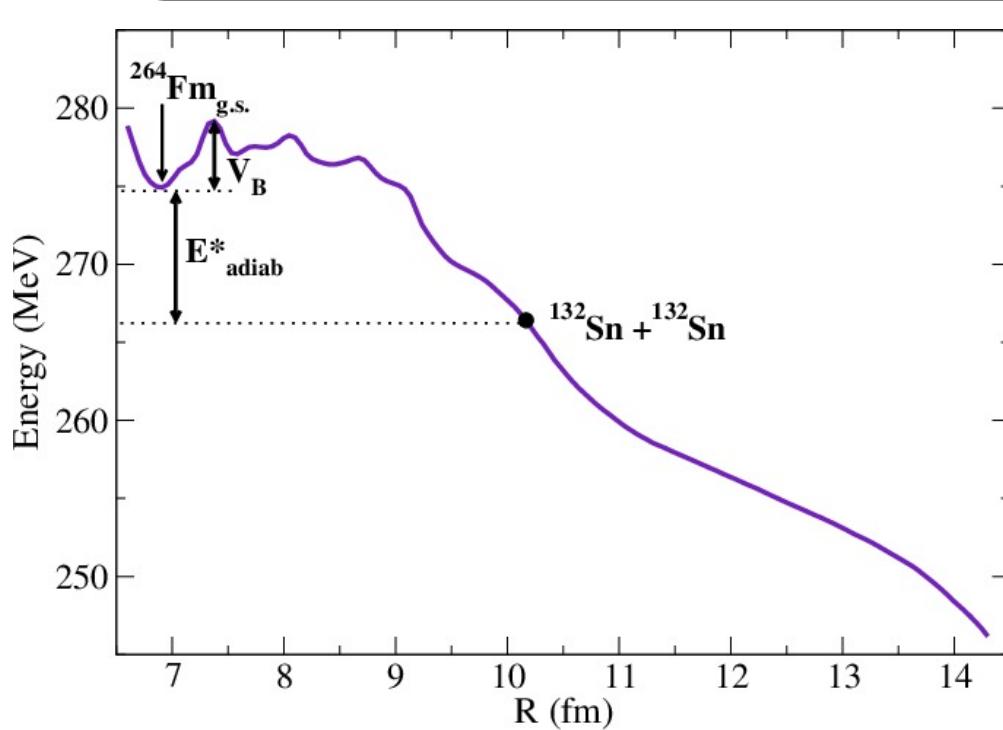
Z. Kohley, private communication

Nucleus-Nucleus 2015 (NN2015)

Catania, Italy

# Scission Dynamics Using TDHF – Fragment Pre-Formation

- Transition from adiabatic motion to non-adiabatic motion of scission
- Follow the single-particle states as a function of deformation, look for the last level crossing.



C. Simenel and A.S. Umar, **PRC** 89 (R), 031601 (2014)

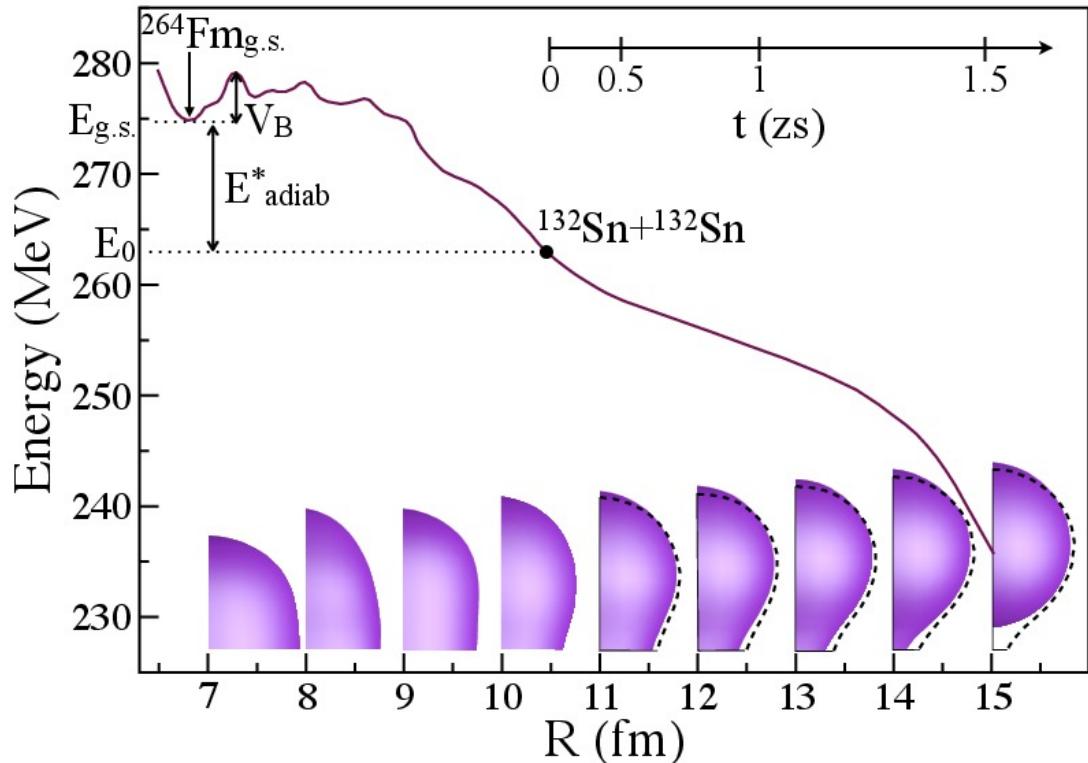
J. Sadhukhan, K. Mazurek, A. Baran, J. Dobaczewski, W. Nazarewicz, and J. A. Sheikh, **PRC** 88, 064314 (2013).



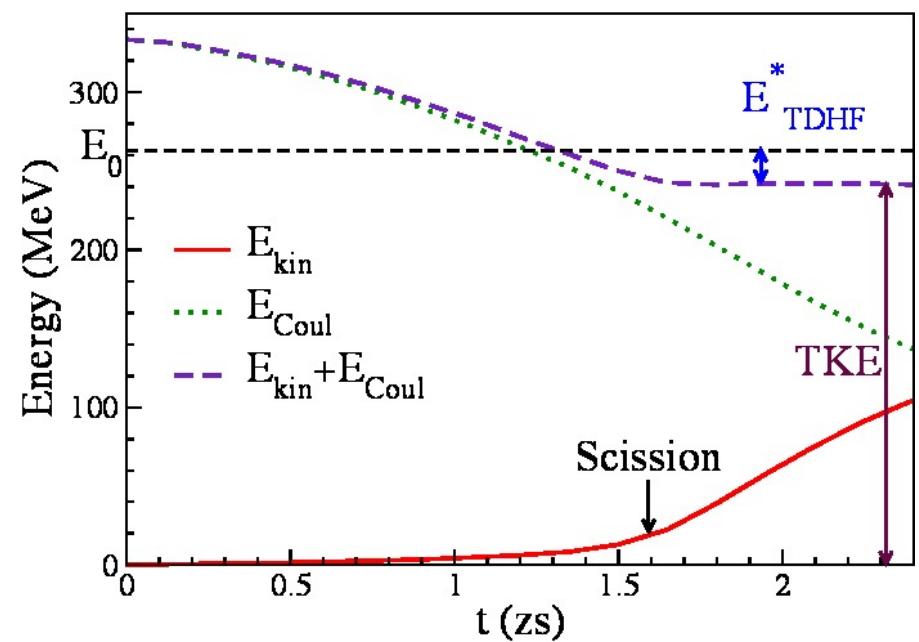
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# Scission Dynamics Using TDHF - Dynamics

- Start TDHF evolution at the point of fragment formation
- Compare results to adiabatic scission, can calculate TKE,  $E^*$  etc.



- Asymptotic TKE is around 241 MeV
- Adiabatic  $E^*$  is about 12 MeV
- Dynamic TDHF  $E^*$  is about 22 MeV leading to a total of 34 MeV excitation
- Also done 258Fm, TKE is 238-241 MeV (exp. Value 235 MeV).



(exp: E.K. Hulet et al. PRL 56, 313 (1986))



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## Summary

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TDDFT have a strong place among the theories needed for future challenges of low-energy nuclear physics

Numerical issues are resolved – limitations only due to theoretical approximations (effective interactions, mean-field theory, etc.)

Quasifission and deep-inelastic reactions are well suited for TDDFT

We may be able to calculate  $P_{CN}$  directly from TDHF without MAD's!

$$P_{CN} \approx \frac{\sigma_{Capture} - \sigma_{QF}}{\sigma_{Capture}}$$

We now have a reasonable handle on above- and sub-barrier fusion employing the DC-TDHF approach

One major and difficult area that needs attention is the dynamics of fission

