Description of the fusion-fission reactions in the framework of dinuclear system conception

Sh. A. Kalandarov

Bogolyubov Lab. Of Theoretical Physics
Joint Institute for Nuclear Research
Dubna

Collaborators:

BLTP JINR: R.V. Jolos, A.K. Nasirov, G.G. Adamian,
N.V. Antonenko, A. Andreev

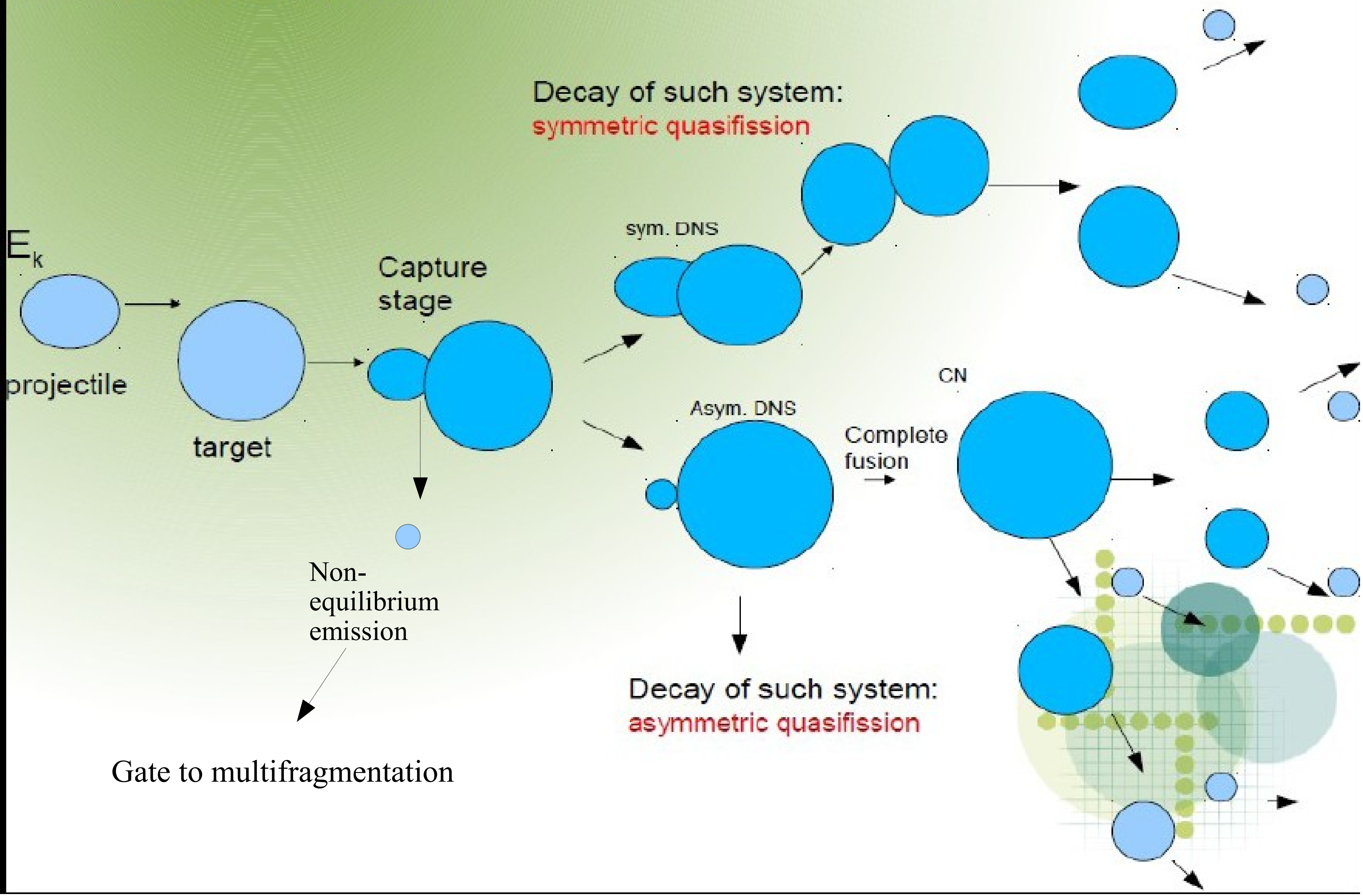
FLNR JINR: V.V. Volkov, E. Cherepanov

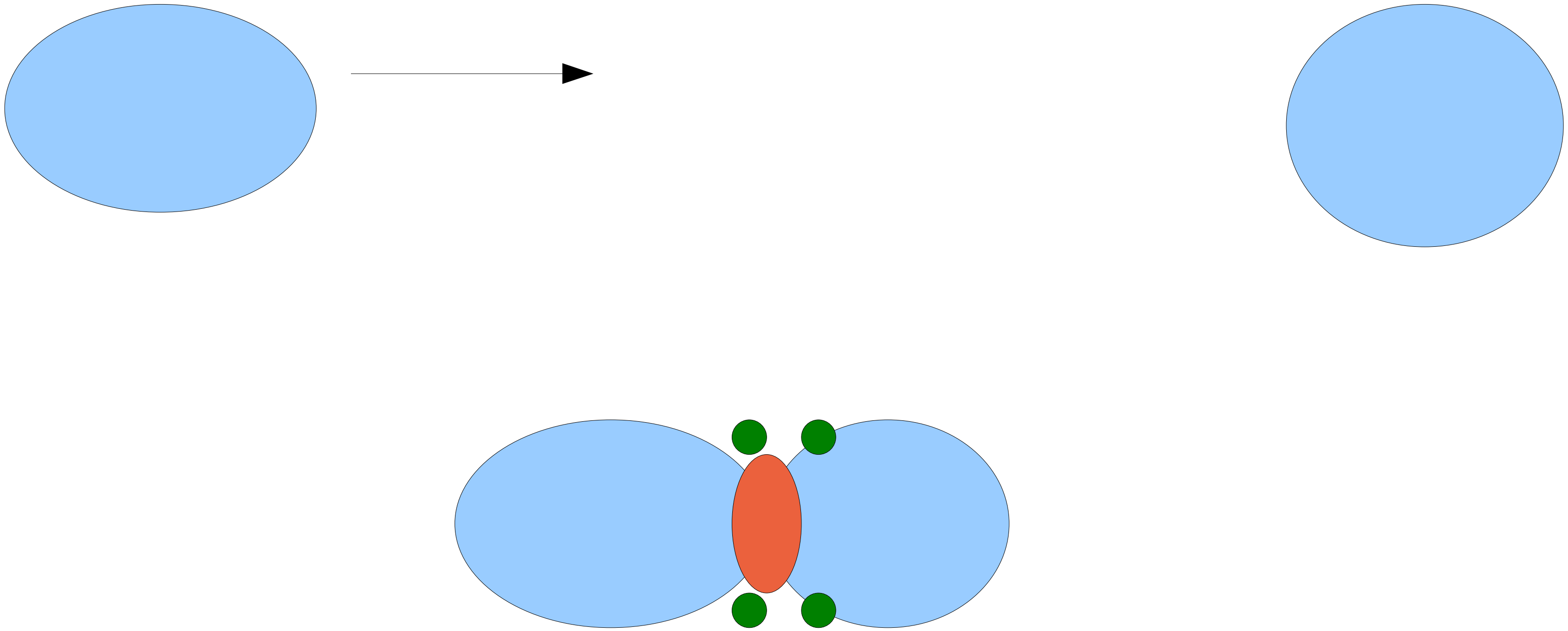
GSI: S. Heinz
GANIL: J.-P. Wieleczko
IPN Orsay: D. Lacroix
LNS INFN, Catania: S. Pirrone, G. Politi, G. Brunilde
INFN , Firenze: G. Casini

Content

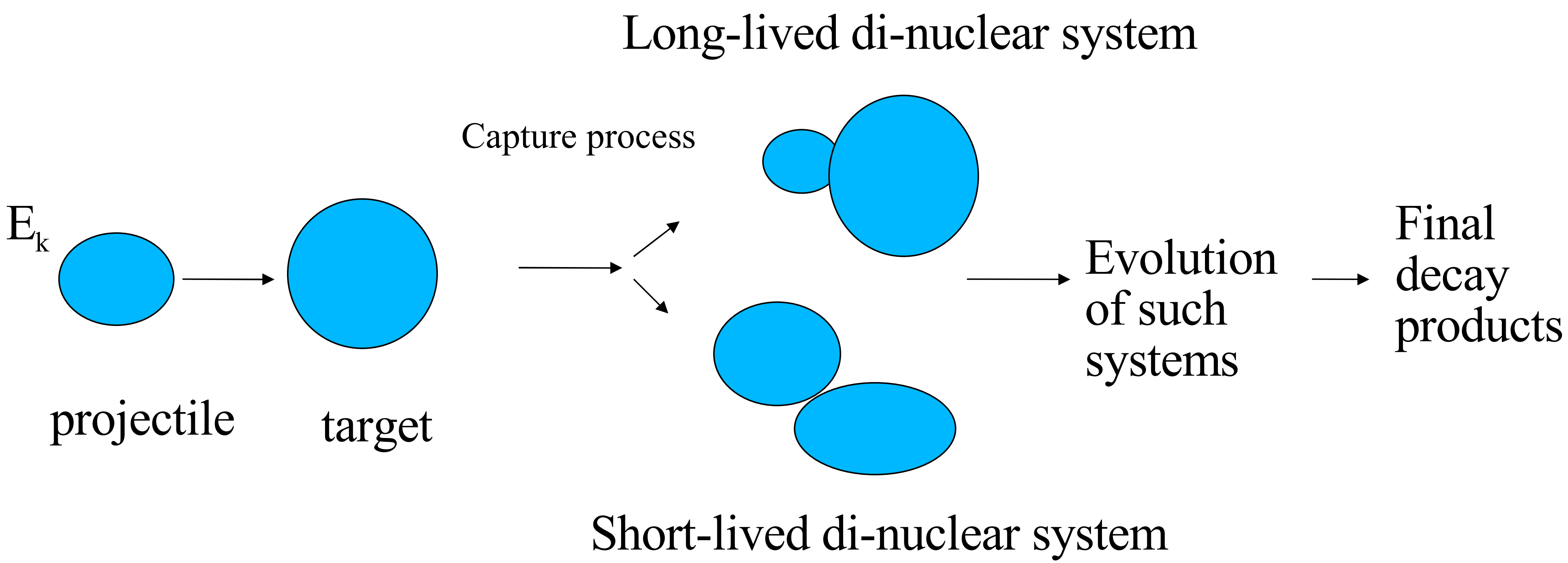
- ***DiNuclear System conception***
- ***Methods and models used for the description of reaction stages***
- ***Applications***
- ***Summary***

Dinuclear system conception





Reactions with medium mass compound nuclei far above the Coulomb barrier



Capture stage $\xrightarrow[\text{overlapping}]{}$ Nucleon exchange stage $\xrightarrow[\text{overlapping}]{}$ Decay stage

- ***Methods and models used for the description of reaction stages***

Capture stage

Methods for calculating the capture cross section:

Simple «transition through barrier» approach

Quasiclassical dynamical approach

Quantum dynamical approach
V.V. Sargsyan, Phys. Rev. C 80,034606(2009)

Lack of the knowledge:

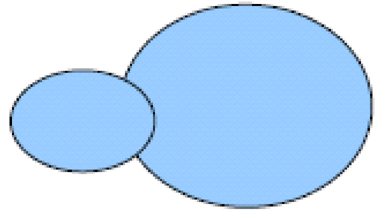
Behaviour of nucleons in overlapping region is not explored yet!
Nuclear molecules
Effects of non-equilibrium processes not considered yet!
A way to multifragmentation
Coupling with nucleon transfer process is not considered !
Overlapping with fusion

Simple «transition through barrier» approach

Capture=formation of di-nuclear system

Partial capture cross section for given **L** is:

$$\sigma_{cap} = \pi \lambda\!\!\!^{-2} (2L+1) P_{cap}(E_{c.m.}, L)$$



The distance between nuclei in DNS system is
$R_{min}=R_1+R_2+0.5$ fm

where $\lambda\!\!\!^{-2} = \hbar^2 / (2 \mu E_{c.m.})$

$$E_{c.m.} = \frac{A_1 A_2}{A_1 + A_2} E_{lab}$$

and

$$P_{cap}(E_{c.m.}, L) = (1+EXP(2\pi(V_b(L)-E_{c.m.})/\hbar\omega_L)^{-1}$$

is a capture probability given by Hill-Wheeler.

$$L_{kin} \hbar = pb$$

b – impact parameter

p - momentum

Quasiclassical dynamical approach

$$\frac{d(\mu(R,\alpha_1,\alpha_2)\dot{R})}{dt} + \gamma_R(R,\alpha_1,\alpha_2)\dot{R}(t) = F(R),$$

$$F(R,\alpha_1,\alpha_2) = -\frac{\partial V(R,\alpha_1,\alpha_2)}{\partial R} - \dot{\mathbf{R}}^2 \frac{\partial \mu(R)}{\partial R},$$

$$\frac{dL}{dt} = \gamma_\theta(R,\alpha_1,\alpha_2)R(t)\big(\dot{\theta}R(t) - \dot{\theta}_1 R_{1\mathrm{eff}} - \dot{\theta}_2 R_{2\mathrm{eff}}\big),$$

$$L_0 = J_R(R,\alpha_1,\alpha_2)\dot{\theta} + J_1\dot{\theta}_1 + J_2\dot{\theta}_2,$$

$$E_{\mathrm{rot}} = \frac{J_R(R,\alpha_1,\alpha_2)\dot{\theta}^2}{2} + \frac{J_1\dot{\theta}_1^2}{2} + \frac{J_2\dot{\theta}_2^2}{2},$$

Nasirov et. al., Nuclear Physics A 759 (2005) 342–369

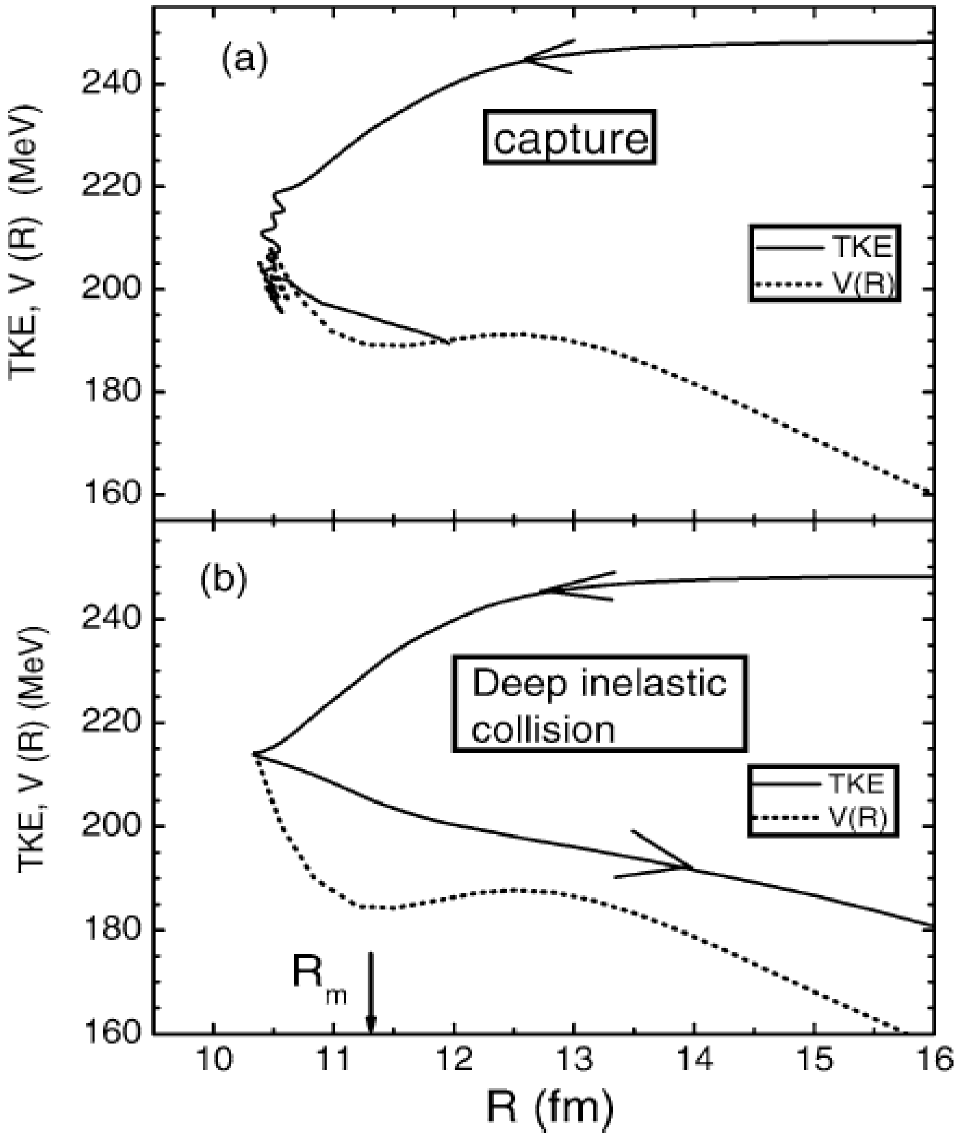


Fig. 2. Illustration of capture (a) and deep inelastic collision (b) at heavy ion collisions. The kinetic energy of the relative motion and the part of nucleus–nucleus potential are shown by solid and dotted curves, respectively.

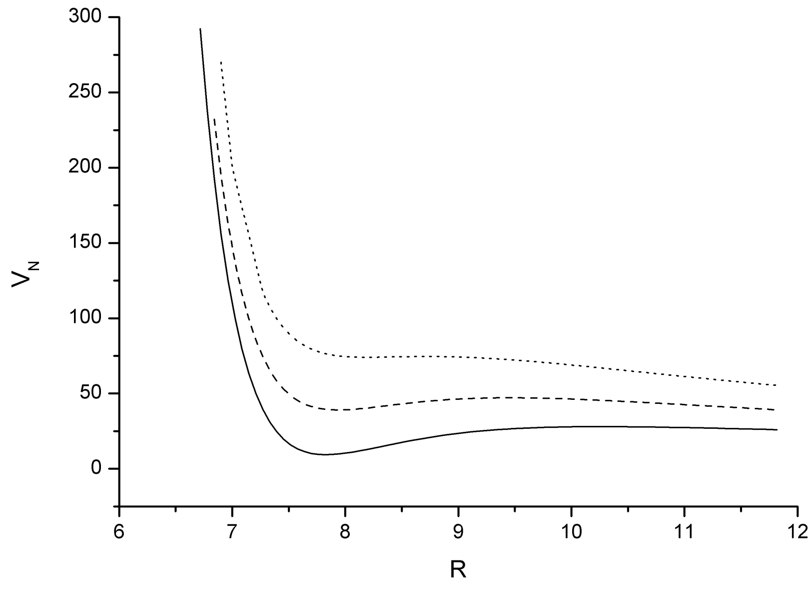
Nasirov et. al., Nuclear Physics A 759 (2005) 342–369

Transition through barrier approach

capture is determined by the presence of potential pocket and kinetic energy for overcoming the coulomb barrier

Dynamical approach

In addition, capture is determined by kinetic energy dissipation;
L window for capture;
orientation effects taken into account;



MultiNucleon transfer/fusion and decay stages

a) Statistical approach;

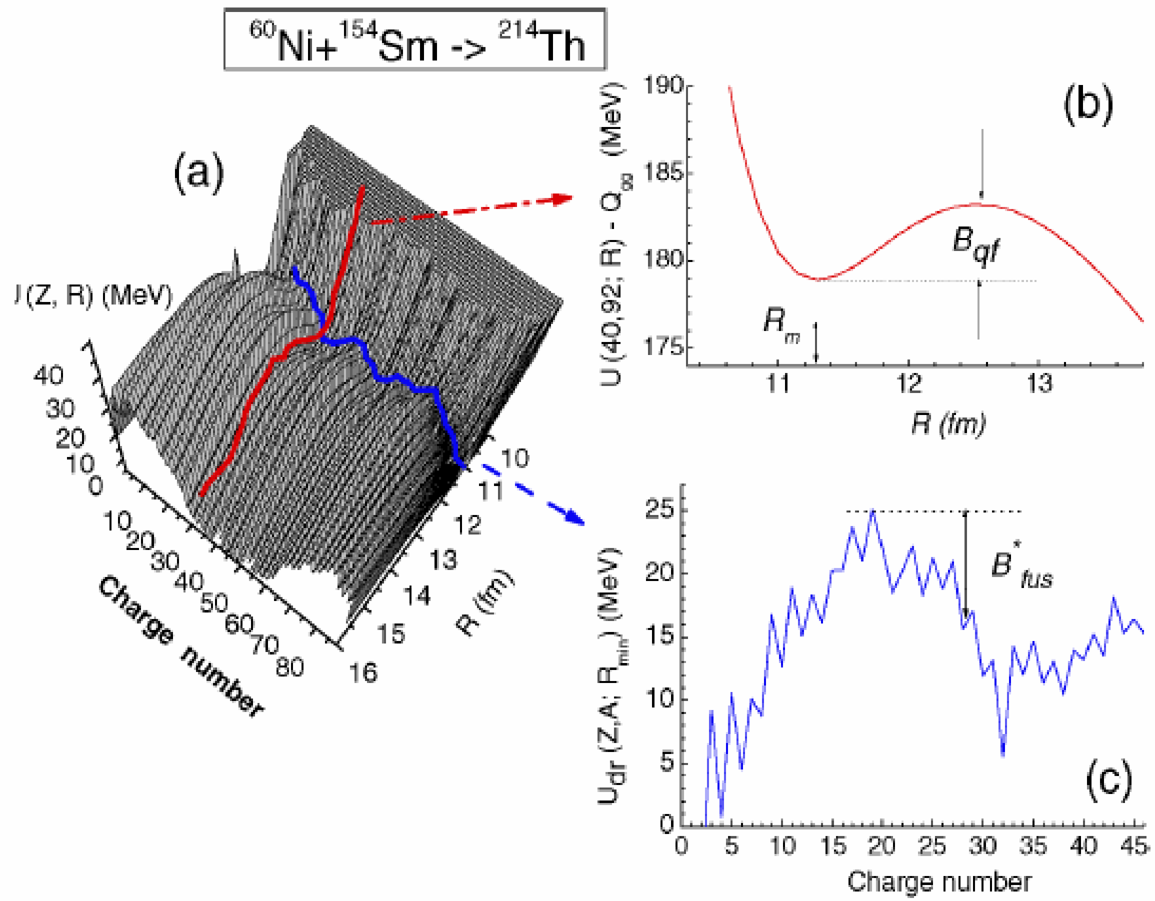
b) Dynamical approach with microscopical and/or phenomenological transport coefficients;

c) coupled «formation-decay» approach($B_{qf} >> B_{local}$ or $t_{qf} >> t_{nucl.\ exchange}$ leads to statistical description)

Statistical approach for fusion and decay

$$P_{CN} = \frac{\rho_{fus}}{\rho_{fus} + \rho_{qfiss} + \rho_{symm}}$$

And then any statistical code for deexcitation of CN and quasifission products



This approach was often applied for the determination of production cross sections for superheavy elements.

Dynamical approach with quantum and/or phenomenological transport coefficients

In di-nuclear system, due to the action of the mean field, the transition of nucleons from one nuclei to another take place. Such a process can be described in the framework of transport model. In this approach the time dependence of the probability $P_{Z,A}(t)$ of finding a system at moment t in a state with charge Z and mass A asymmetries is given by set of master equations:

$$\frac{dP_{Z_1,A_1,Z_2,A_2}}{dt} = \Delta^{(-,0)}_{Z_1+1,A_1+1,Z_2-1,A_2-1} P_{Z_1+1,A_1+1,Z_2-1,A_2-1}(t) + \Delta^{(+,0)}_{Z_1-1,A_1-1,Z_2+1,A_2+1} P_{Z_1-1,A_1-1,Z_2+1,A_2+1}(t)$$

$$+\Delta^{(0,-)}_{Z_1,A_1+1,Z_2,A_2-1} P_{Z_1,A_1+1,Z_2,A_2-1}(t) + \Delta^{(0,+)}_{Z_1,A_1-1,Z_2,A_2+1} P_{Z_1,A_1-1,Z_2,A_2+1}(t)$$

$$-(\Delta^{(-,0)}_{Z_1,A_1,Z_2,A_2} + \Delta^{(+,0)}_{Z_1,A_1,Z_2,A_2} + \Delta^{(0,-)}_{Z_1,A_1,Z_2,A_2} + \Delta^{(0,+)}_{Z_1,A_1,Z_2,A_2} + \Lambda^{d}_{Z_1,A_1,Z_2,A_2}) P_{Z_1,A_1,Z_2,A_2}(t)$$

Number of coupled equations is very large!

G.G. Adamian et.al., Phys. Rev. C **68**, 034601 (2003)

With the microscopical transport coefficients:

$$\Delta_{Z,N}^{(\pm,0)}(\Theta) = \frac{1}{\Delta t} \sum_{P,T}^{Z} |g_{PT}|^2 n_P^T(\Theta)[1 - n_T^P(\Theta)] \times \frac{\sin^2[\Delta t(\epsilon_P - \epsilon_T)/2\hbar]}{(\epsilon_P - \epsilon_T)^2/4},$$

$$\Delta_{Z,N}^{(0,\pm)}(\Theta) = \frac{1}{\Delta t} \sum_{P,T}^{N} |g_{PT}|^2 n_P^T(\Theta)[1 - n_T^P(\Theta)] \times \frac{\sin^2[\Delta t(\epsilon_P - \epsilon_T)/2\hbar]}{(\epsilon_P - \epsilon_T)^2/4},$$

$$g_{PT}(R) = \frac{1}{2} \int d\mathbf{r} \psi_T^*(\mathbf{r})[U_T(\mathbf{r}) + U_P(\mathbf{r} - \mathbf{R})]\psi_P(\mathbf{r} - \mathbf{R})$$

$$\Lambda_{Z,N}^{qf}(\Theta) = \sum_n \Lambda_{Z,N}^{qf}(n) \Phi_{Z,N}(n,\Theta),$$

$$\Lambda_{Z,N}^{fis}(\Theta) = \sum_n \Lambda_{Z,N}^{fis}(n) \Phi_{Z,N}(n,\Theta).$$

$$\Lambda_{Z,N}^{qf}(\Theta) = \frac{\omega}{2\pi\omega^{B_{qf}}} \left(\sqrt{\left(\frac{\Gamma}{2\hbar}\right)^2 + (\omega^{B_{qf}})^2} - \frac{\Gamma}{2\hbar} \right) \times \exp\left(-\frac{B_{qf}(Z,N)}{\Theta(Z,N)} \right),$$

G.G. Adamian et.al., Phys. Rev. C **68**, 034601 (2003)

Or phenomenological transport coefficients:

$$\Delta = 2\pi k \frac{R_1 R_2}{R_1 + R_2} \sqrt{\frac{\rho'_{DNS}}{\rho_{DNS}}}$$

k=0.5 and has dimension 10^{21} 1/(fm s)

$$\rho_{DNS} = \int_0^{Ex_{DNS}-\epsilon} \int_0^{Ex_{DNS}} \rho_1(Ex_1)\,\rho_2(Ex_{DNS}-\epsilon-Ex_1)\,d\epsilon\,dEx_1$$

L.G. Moretto and J.S. Sventek, Physics Letters B 58, 26 (1975)

$$\Lambda^{d}_{Z_{1,}A_{1,}Z_{2,}A_2} = \frac{\int_0^{Ex_{DNS}-B_d-\epsilon'} \int_0^{Ex_{DNS}-B_d} \rho_1(Ex_1)\,\rho_2(Ex_{DNS}-B_d-\epsilon'-Ex_1)\,d\epsilon'\,dEx_1}{h\int_0^{Ex_{DNS}-\epsilon} \int_0^{Ex_{DNS}} \rho_1(Ex_1)\,\rho_2(Ex_{DNS}-\epsilon-Ex_1)\,d\epsilon\,dEx_1}$$

h=4.1356 10^{-21} MeV s

And then the yields for quasifission products:

$$Y_{Z_{1,}A_{1,}Z_{2,}A_2} = \Lambda^{d}_{Z_{1,}A_{1,}Z_{2,}A_2} \int_0^t P_{Z_{1,}A_{1,}Z_{2,}A_2}(t)\,dt$$

For CN formation probability:

$$P_{CN} = \int_0^t P_{Z_{0,}A_{0,}Z_{CN},A_{CN}}(t)\,dt$$

$$\sigma_{Z,A} = \sum \sigma_{cap} * P^{norm}_{Z,A} * W^{sur}_{Z,A}$$

Deexcitation of excited quasifission products and CN is made by use of statistical decay model.

EMPIRE code (Nasirov), GROGI code (Adamian,Antonenko,Zubov) and PACE, GEMINI,HIVAP codes by many others.

Deexcitation and fusion processes are decoupled!

Coupled «formation-decay» approach for fusion and decay processes

In nuclear reactions which lead to medium-mass CN, the decay barriers in R coordinate are very large than corresponding local barriers in mass asymmetry coordinate. Then, for such systems, set of master equations have a stationary solution.
In stationary solution of master equation, the probability of finding system in a given charge and mass asymmetry is proportional to the relevant level density ρ.

The corresponding weights for CN and DNS configurations are:

$$P_{Z,A}(E^*_{CN}, J) = \frac{\exp[-U(R_m, Z, A, J)/T_{CN}(J)]}{1 + \sum_{Z'=2,A'} \exp[-U(R_m, Z', A', J)/T_{CN}(J)]}.$$

Potential energy of DNS

Driving potential:

$$U(R, Z, L) = B_1 + B_2 + V_N(R, Z, L) - B_{12} - V_{rot}^{CN}$$

$$V_N(R, Z, L) = V_{COUL}(R, Z) + V_{NN}(R, Z) + V_{rot}(R, L)$$

Nuclear potential we take as double folding potential:

$$V_{NN}(R, Z) = \int \rho_1(r_1)\rho_2(R - r_2) F(r_1 - r_2)\, dr_1 dr_2$$

Where nucleon-nucleon forces depend on nuclear densities:

$$F(r_1 - r_2) = C_0 \left(F_{in.} \frac{\rho_0(r_1)}{\rho_0} + F_{ex}\left(1 - \frac{\rho_0(r_1)}{\rho_0}\right)\right) \delta(r_1 - r_2)$$

$$V_{NN}(R,Z) = C_0\left[\frac{F_{in}-F_{ex}}{\rho_0}\left(\int \rho_1^2(\mathbf{r})\rho_2(\mathbf{R}-\mathbf{r})d\mathbf{r} + \int \rho_1(\mathbf{r})\rho_2^2(\mathbf{R}-\mathbf{r})d\mathbf{r}\right) + F_{ex}\int \rho_1(\mathbf{r})\rho_2(\mathbf{R}-\mathbf{r})d\mathbf{r}\right]$$

$$F_{in,ex} = f_{in,ex} + f'_{in,ex}\frac{N_1 - Z_1}{A_1}\frac{N_2 - Z_2}{A_2}$$

N_i, Z_i — **neutron and proton numbers**

$C_0 = 300$ **MeV fm**^{3}, $f_{in} = 0.09$, $f_{ex} = -2.59$,

$f'_{in} = 0.42$, $f'_{ex} = 0.54$

$\rho_0 = 0.17\ fm^{-3}$

Transition state method

Particle emission width from excited CN:

$$\Gamma_i = \frac{m_i R^2}{\pi \hbar^2 \rho_{CN}(E^*_{CN})} \int_0^{E^*_{CN}-B_i} \rho_{res}(E^*_{CN} - B_i - \epsilon)\epsilon d\epsilon,$$

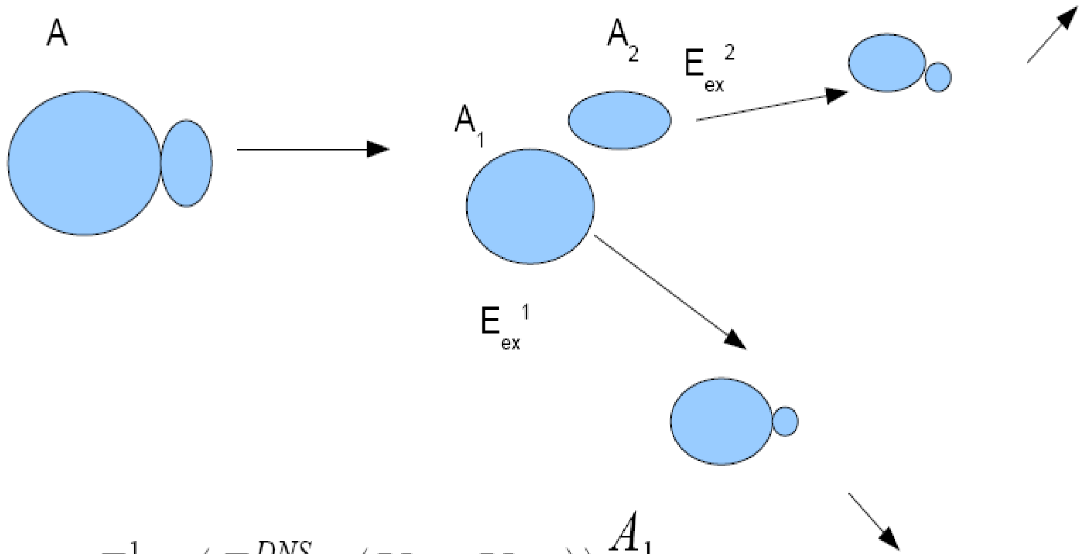
Decay width of DNS:

$$\Gamma_Z = \frac{1}{2\pi\rho(E_{ex})} \int_0^{E_{ex}-B_Z} \rho'(E_{ex} - B_Z - \varepsilon) d\varepsilon$$

Then, normalized probabilities for any given decay channels are:

$$W_{Z,A}(E_{CN},J)=\frac{P_{Z,A}*P^d_{Z,A}}{\sum P_{Z,A}*P^d_{Z,A}}$$

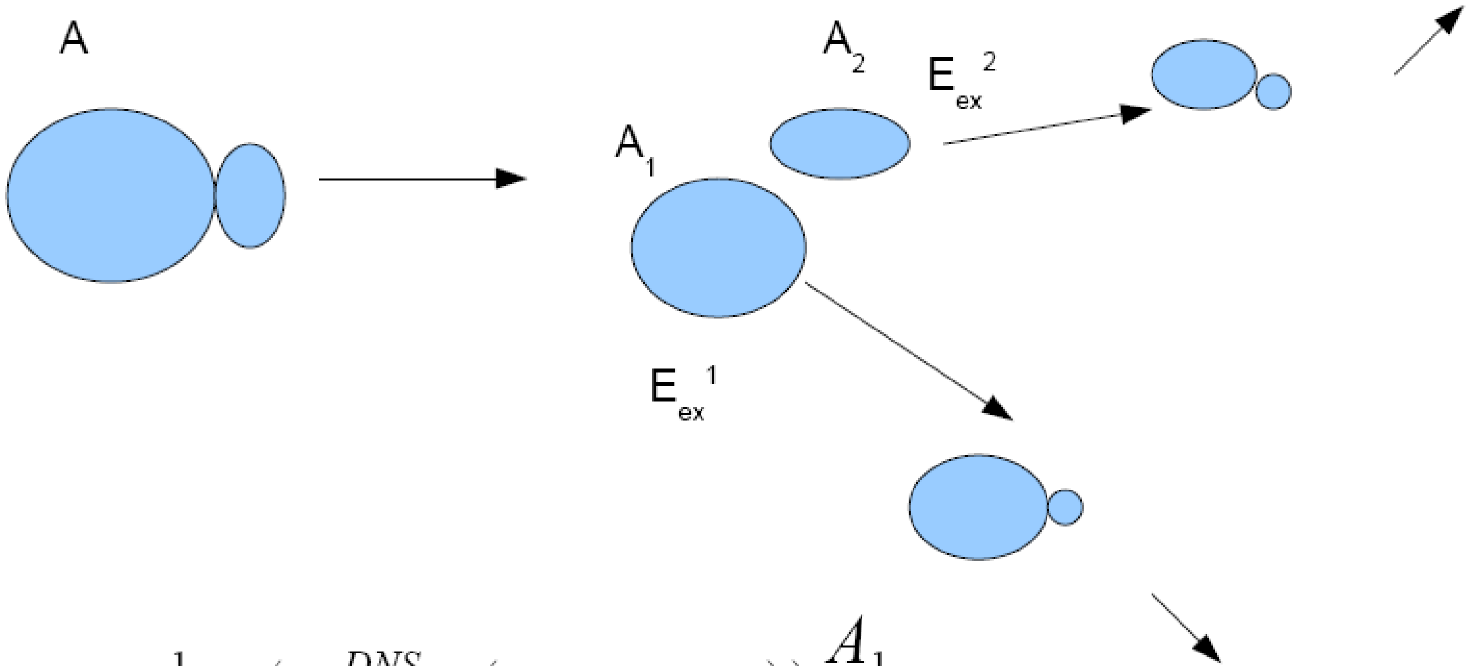
Cascade decay process of excited intermediate system is generated by Monte-Carlo method:



$$E^1_{ex}=(E^{DNS}_{ex}-(U_b-U_{min}))\frac{A_1}{A}$$

$$E^2_{ex}=(E^{DNS}_{ex}-(U_b-U_{min}))\frac{A_2}{A}$$

Cascade decay process of excited intermediate system is generated by Monte-Carlo method:

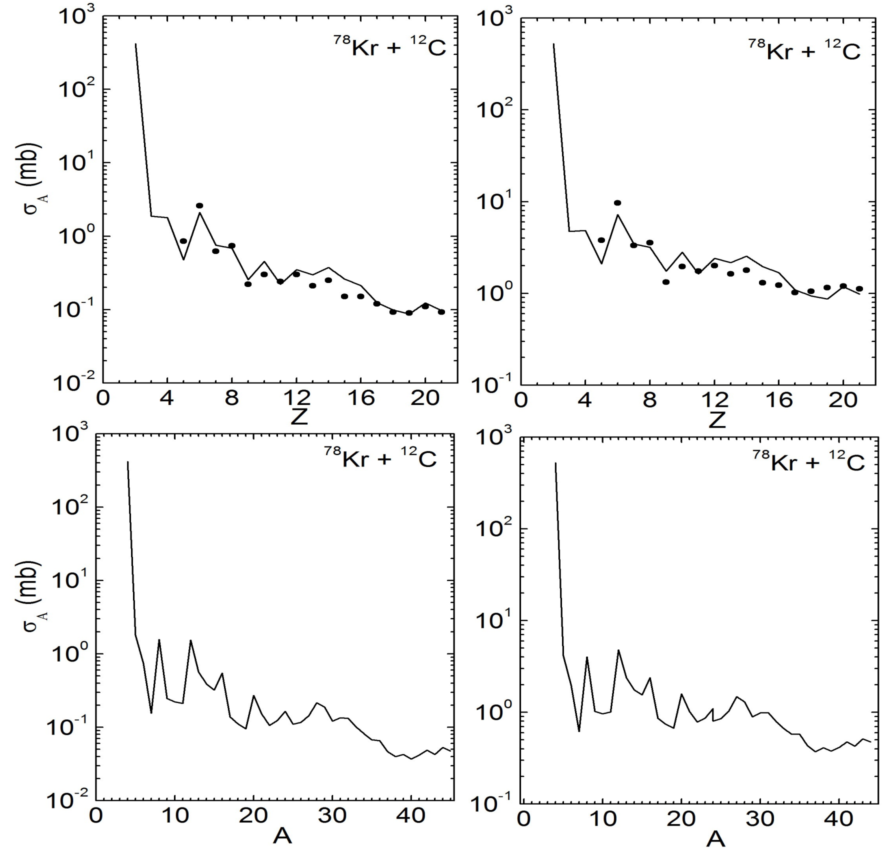


$$E_{ex}^{1} = (E_{ex}^{DNS} - (U_b - U_{min}))\frac{A_1}{A}$$

$$E_{ex}^{2} = (E_{ex}^{DNS} - (U_b - U_{min}))\frac{A_2}{A}$$

Examples of application of the Hill-Wheeler+formation-decay approach

Charge and mass distributions of final decay fragments in 78Kr+12C at 8.52(left) and 11.37MeV/A(right), experimental data are from K. X. Jing *et al.*, Nucl. Phys. A **645**, 203 (1999).



Kalandarov et. al., Phys. Rev. C 82,044603(2010)

Comparison of mass-asymmetric fission barriers

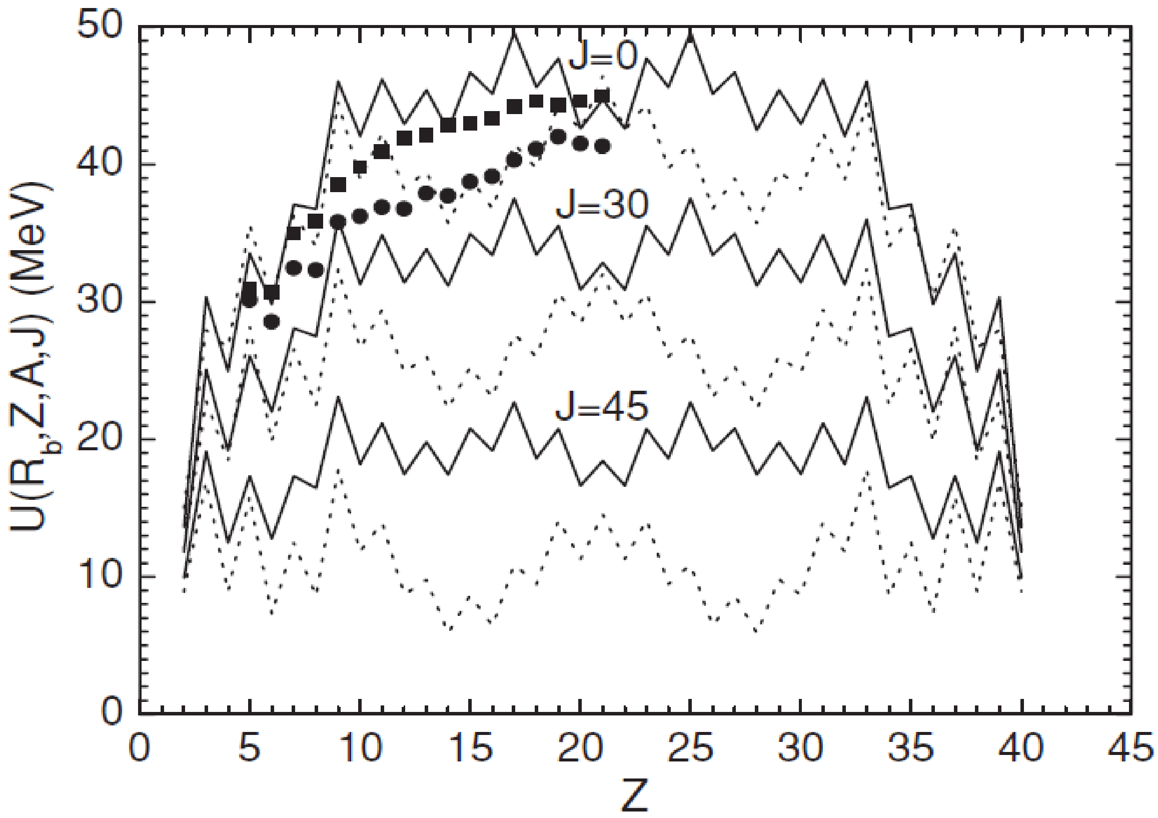
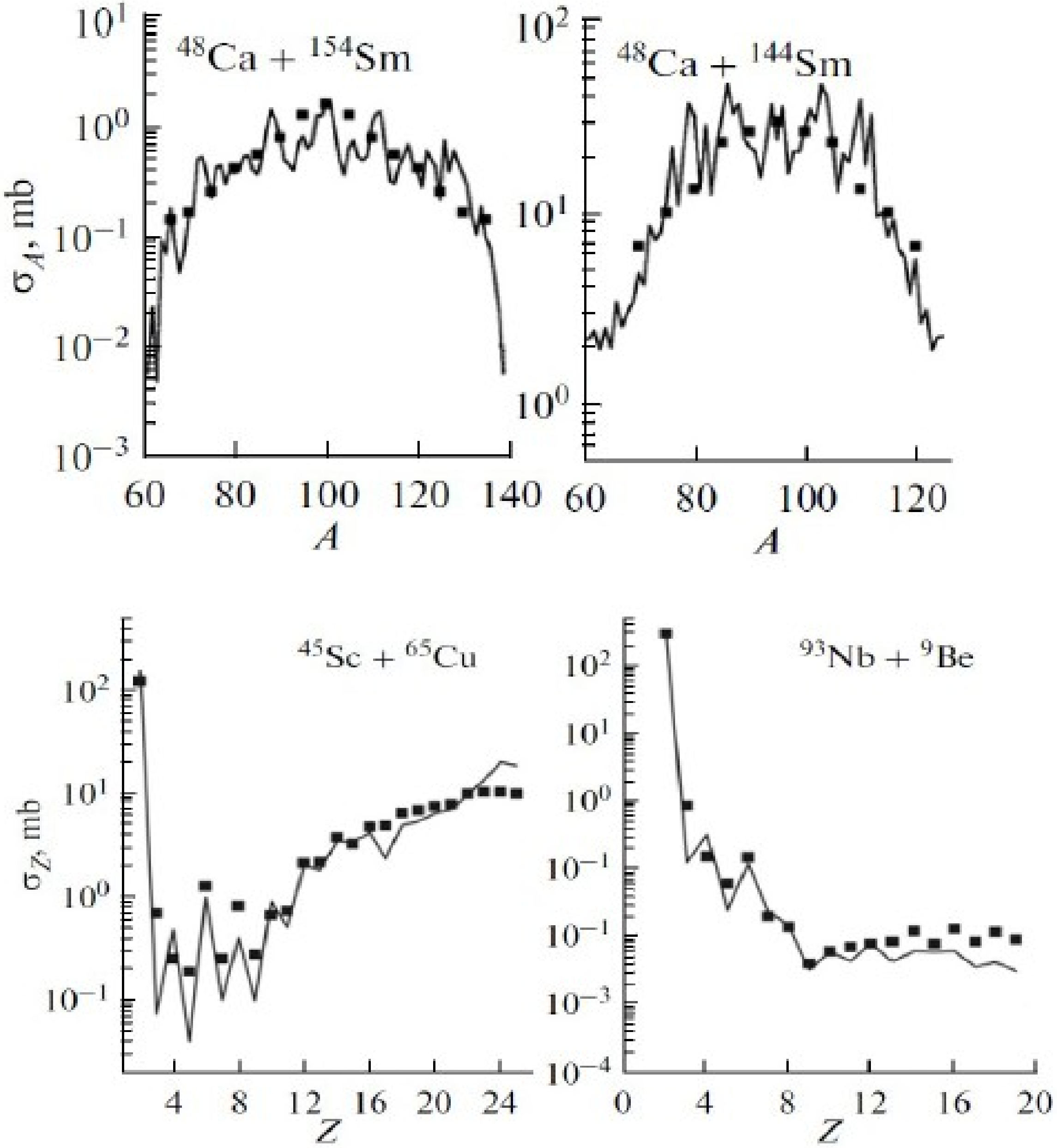


FIG. 2. Potential energies $U(R_b, Z, A, J)$ of the DNS at position $R = R_b$ of the Coulomb barrier of V versus charge asymmetry (expressed by the Z value of one of the fragments) are presented at different values of angular momentum J for the reactions ^{78}Kr + ^{12}C (dashed line) and ^{86}Kr + ^{12}C (solid line). The value of A relates to Z to supply the minimum of U. The value of U is normalized to the energy of the rotating compound nucleus. Mass-asymmetric macroscopic fission barriers extracted from the experimental cluster decay cross sections in Ref. [17] are shown for the reactions ^{78}Kr + ^{12}C (filled circles) and ^{86}Kr + ^{12}C (filled squares).

Kalandarov et. al., Phys. Rev. C 82,044603(2010)



^{48}Ca + ^{154}Sm
^{48}Ca + ^{144}Sm
σ_A, mb
A
^{45}Sc + ^{65}Cu
^{93}Nb + ^{9}Be
σ_Z, mb
Z

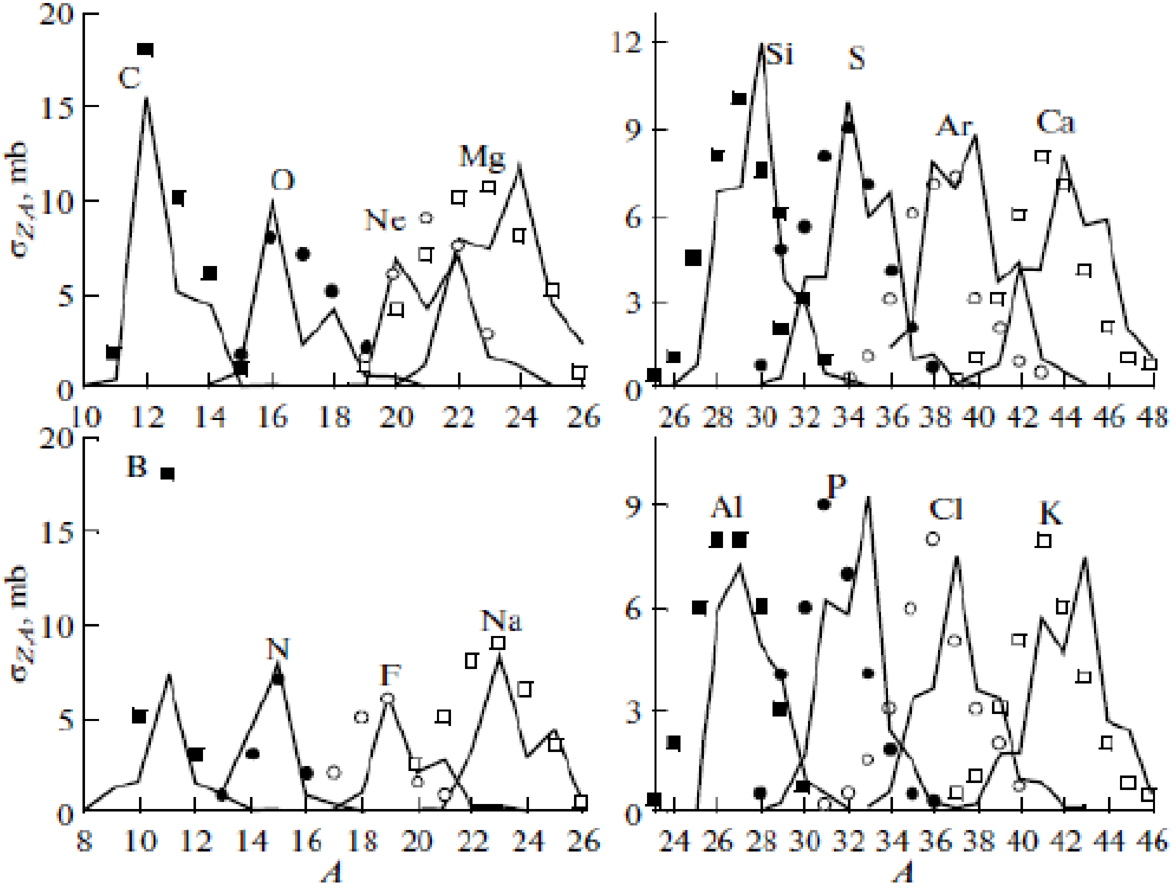


Fig. 28. Calculated (solid lines) and measured [41] (symbols) isotopic distributions of products originating from the ^{84}Kr + ^{27}Al reaction at $E_{lab} = 10.6$ MeV/nucleon that are indicated in the figure.

Sh. A. Kalandarov et al.,

PHYSICAL REVIEW C **82**, 044603 (2010)
PHYSICAL REVIEW C **83**, 054619 (2011)
PHYSICAL REVIEW C **84**, 054607 (2011)
PHYSICAL REVIEW C **84**, 064601 (2011)

78,82Kr+40Ca at 5.5MeV/nucleon

(G. Ademard et al., Phys Rev C 83 054619,(2011))

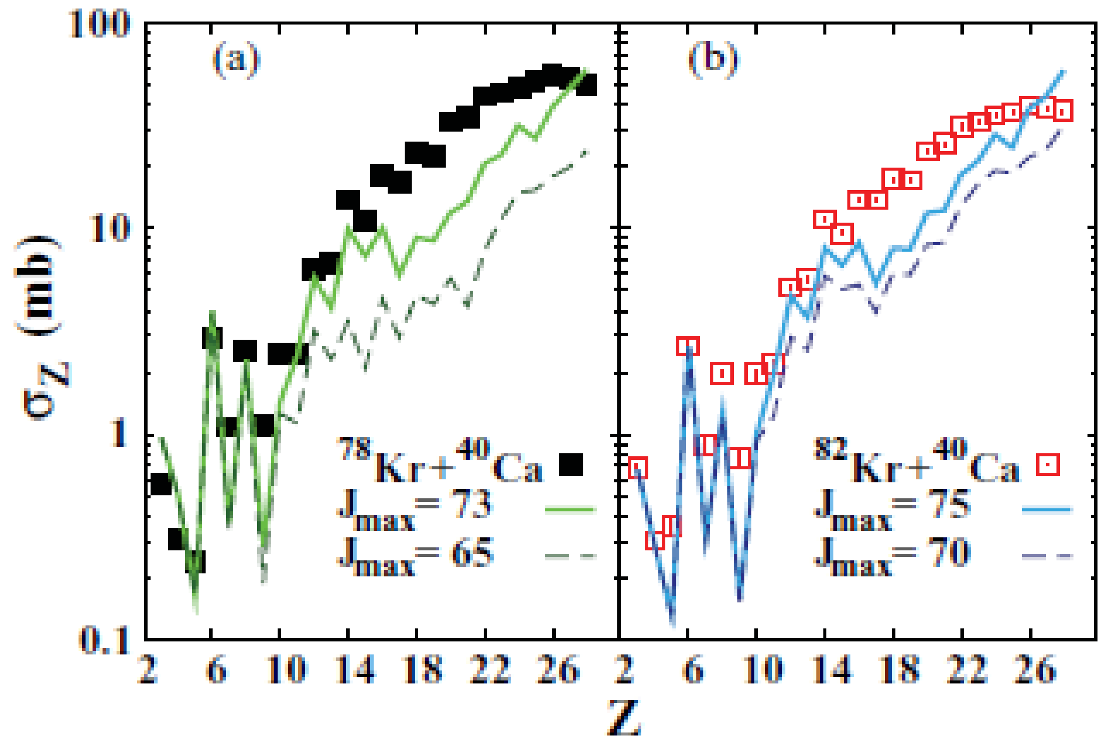
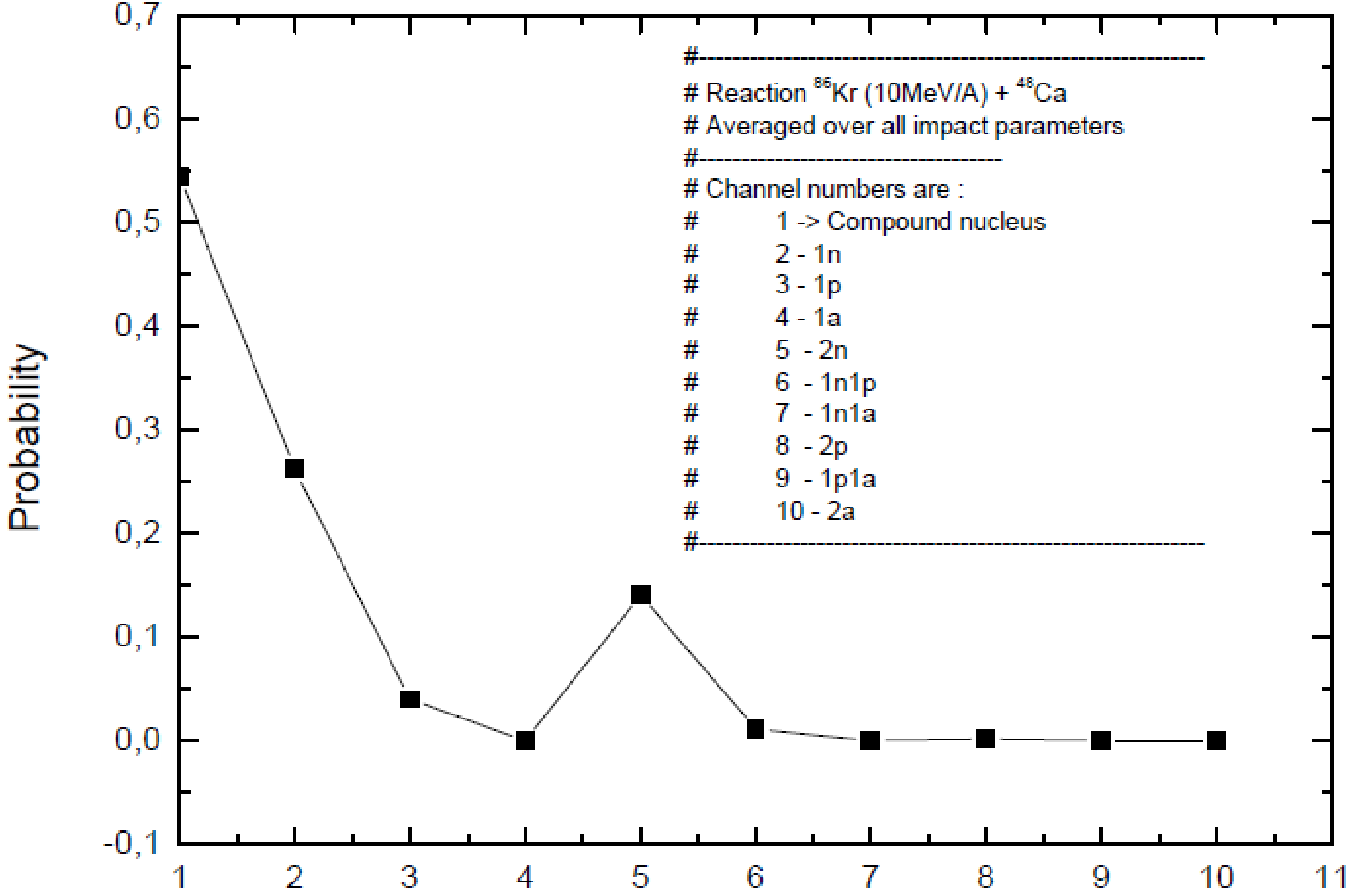


FIG. 13. (Color online) Comparison between measured and calculated cross sections. The calculated results with $J_{\max} = 65$ ($J_{\max} = 73$) for the ^{78}Kr+^{40}Ca reaction and $J_{\max} = 70$ ($J_{\max} = 75$) for the ^{82}Kr+^{40}Ca reaction are shown by dashed (solid) lines in panel (a) [(b)], respectively. Full (open) squares are data from the ^{78}Kr+^{40}Ca (^{82}Kr+^{40}Ca) reaction, respectively.

Recent improvements and applications:

Reactions 78Kr+40Ca and 86Kr+48Ca at 10 MeV/nucleon

In collaboration with D. Lacroix(IPN,Orsay), we employ the event generator code designed for nuclear reactions at intermediate energies, HIPSE(D. Lacroix et al., PHYSICAL REVIEW C **69**, 054604 (2004)) to take into account the non-equilibrium emission of light particles.



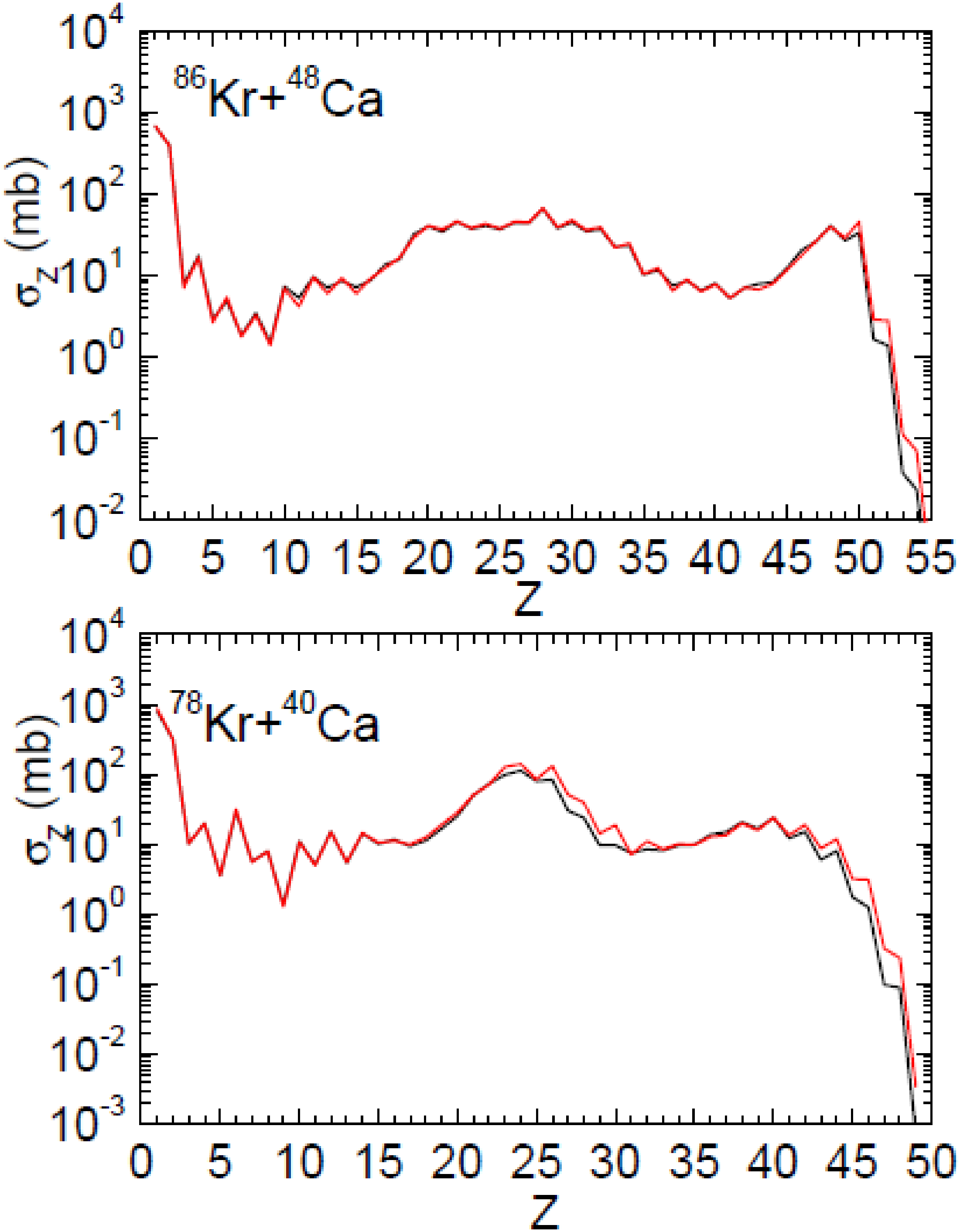
Probability
0,7
0,6
0,5
0,4
0,3
0,2
0,1
0,0
-0,1
1
2
3
4
5
6
7
8
9
10
11
#--
Reaction ^{86}Kr (10MeV/A) + ^{48}Ca
Averaged over all impact parameters
#------------------------------------
Channel numbers are :
1 -> Compound nucleus
2 - 1n
3 - 1p
4 - 1a
5 - 2n
6 - 1n1p
7 - 1n1a
8 - 2p
9 - 1p1a
10 - 2a
#--

HIPSE

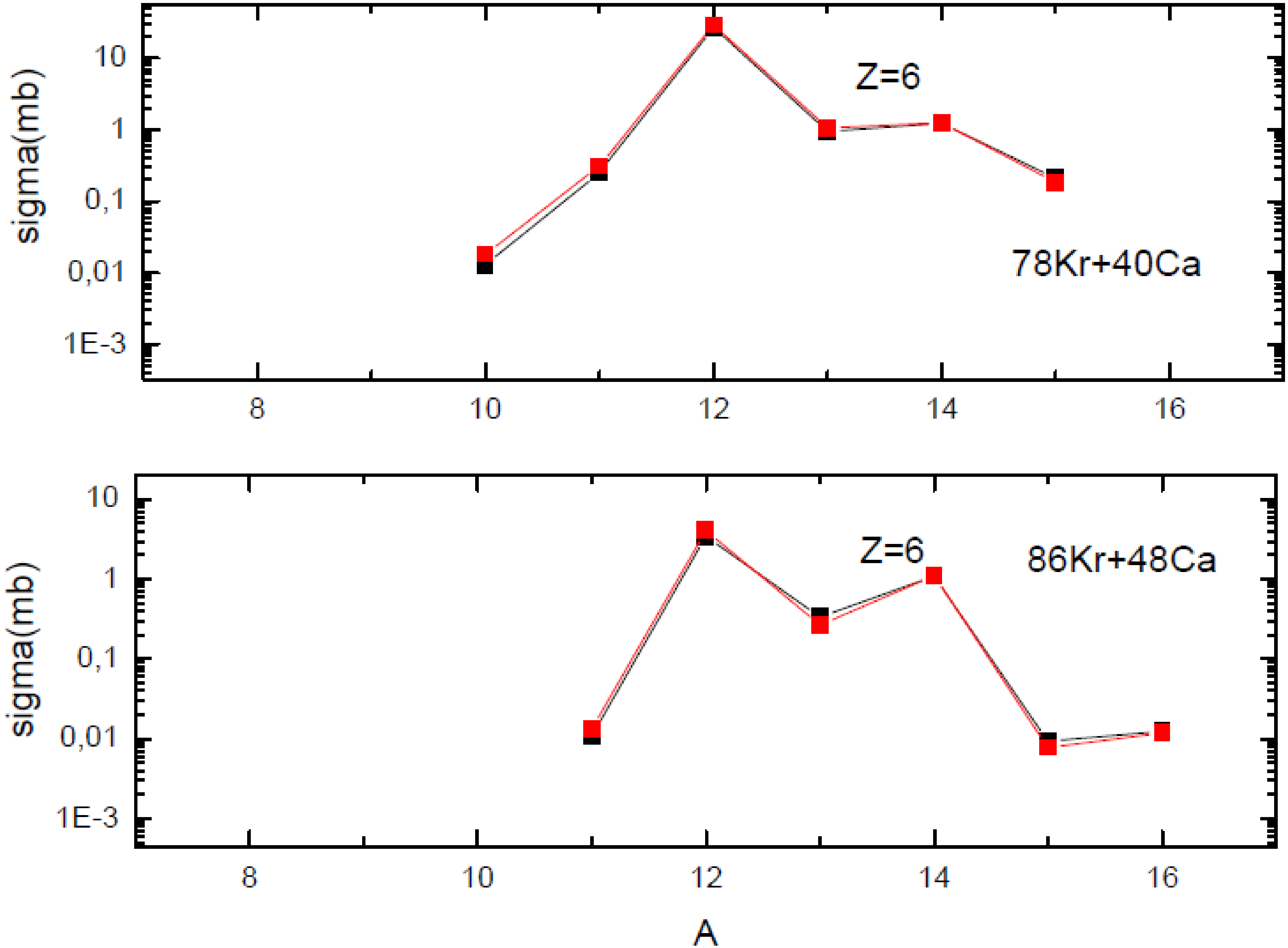
For each non-equilibrium channel, loss of L, E*, Z, N → DNS code

For each initial DNS systems after non-equilibrium emission, calculate the equilibrium emission and decay

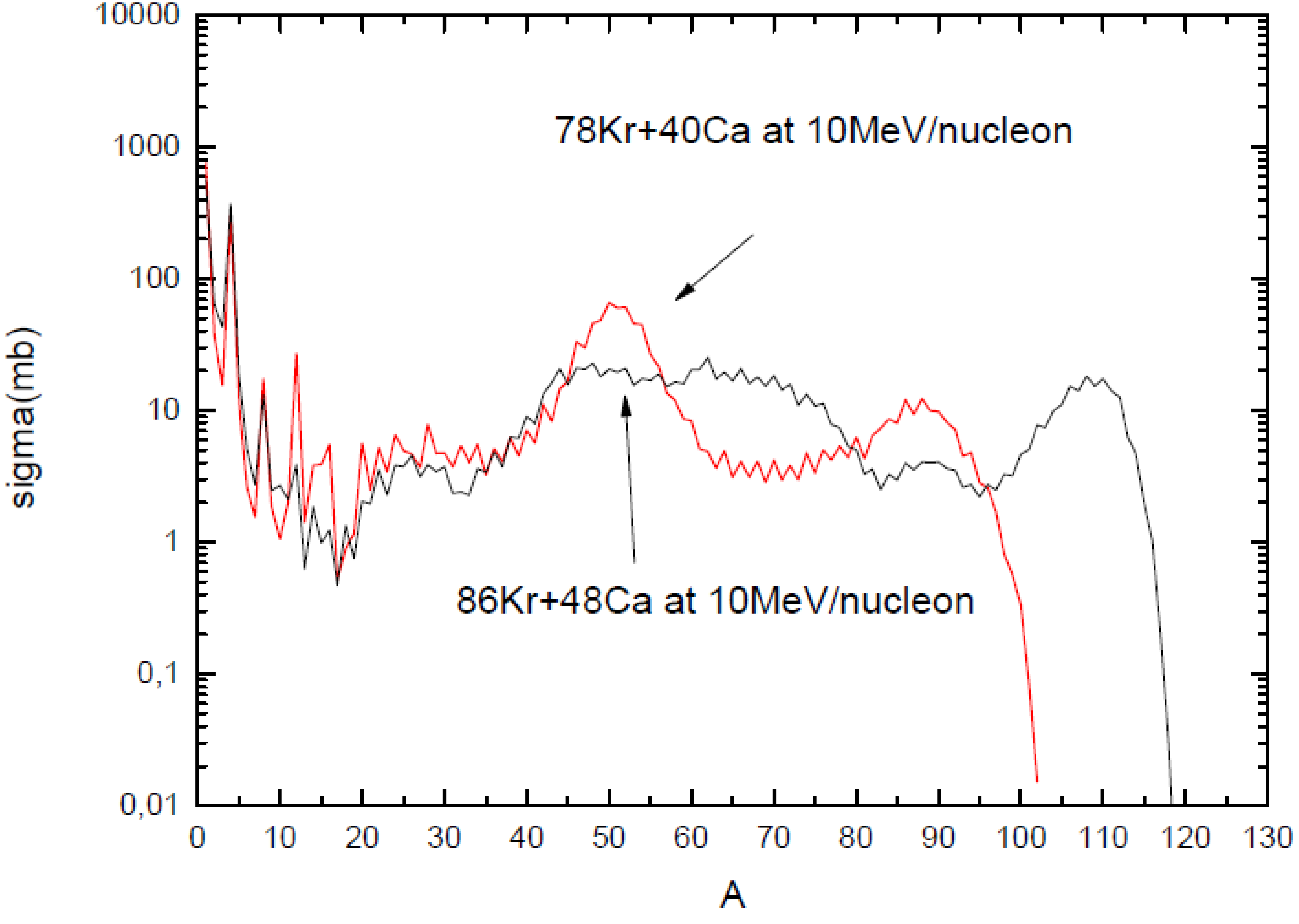
Effect: fusion occurs at broader impact parameters



^{86}Kr+^{48}Ca
σ_Z (mb)
Z
^{78}Kr+^{40}Ca
σ_Z (mb)
Z



Z=6
78Kr+40Ca
86Kr+48Ca
sigma(mb)
A



78Kr+40Ca at 10MeV/nucleon
86Kr+48Ca at 10MeV/nucleon
sigma(mb)
A
10000
1000
100
10
1
0,1
0,01
0
10
20
30
40
50
60
70
80
90
100
110
120
130

Summary

Dinuclear system conception and its application to fusion-fission reactions leading to medium mass compound CN is presented.

HIPSE code is incorporated to DNS model in decoupled mode with a purpose of taking into account non-equilibrium emission of light particles.

Calculated charge,mass and isotopic distributions are obtained for the reactions 78,86Kr+40,48Ca reactions at 10MeV/nucleon, comparison with experimental results obtained in LNS INFN,Catania is in progress.

Thank you for your attention!

Expressions for the friction coefficients

$$\gamma_R\big(R(t),\alpha_1,\alpha_2\big)=\sum_{i,i'}\left|\frac{\partial V_{ii'}(R(t),\alpha_1,\alpha_2)}{\partial R}\right|^2 B^{(1)}_{ii'}(t), \tag{A.5}$$

$$\gamma_\theta\big(R(t),\alpha_1,\alpha_2\big)=\frac{1}{R^2}\sum_{i,i'}\left|\frac{\partial V_{ii'}(R(t),\alpha_1,\alpha_2)}{\partial \theta}\right|^2 B^{(1)}_{ii'}(t), \tag{A.6}$$

and the dynamic contribution to the nucleus–nucleus potential

$$\delta V\big(R(t),\alpha_1,\alpha_2\big)=\sum_{i,i'}\left|\frac{\partial V_{ii'}(R(t),\alpha_1,\alpha_2)}{\partial R}\right|^2 B^{(0)}_{ii'}(t), \tag{A.7}$$

were obtained in [45] by estimating the evolution of the coupling term between relative motion of nuclei and nucleon motion inside nuclei; $B^{(0)}_{ii'}(t)$ is given by Eq. (A.10). The dynamic contribution to the nucleus–nucleus potential (A.7) is taken into account at calculation of the capture stage of reaction

$$V(R,\alpha_1,\alpha_2)=V_0(R,\alpha_1,\alpha_2)+\delta V(R,\alpha_1,\alpha_2). \tag{A.8}$$

The mass of inertia for the relative motion ($\mu=\mu_0+\delta\mu$) is determined as a sum of the DNS reduced mass and the dynamic correction $\delta\mu(R(t))$. The last is calculated using the expression

$$\begin{aligned}\delta\mu\big(R(t),\alpha_1,\alpha_2\big)=&\sum_{i,i'}\left|\frac{\partial V_{ii'}(R(t),\alpha_1,\alpha_2)}{\partial R}\right|^2 B^{(2)}_{ii'}(t)\\ &-\mu_0\frac{2}{A_{\rm tot}}\int\frac{\rho^{(0)}_1(r-r_1)\rho^{(0)}_2(r-r_2)}{\rho^{(0)}_1(r-r_1)+\rho^{(0)}_2(r-r_2)}\,d^3r,\end{aligned} \tag{A.9}$$

where $B^{(2)}_{ii'}(t)$ is determined by

$$B_{ik}^{(n)}(t) = \frac{2}{\hbar}\int_0^t dt'\,(t-t')^n \exp\left(\frac{t'-t}{\tau_{ik}}\right)\sin\bigl[\omega_{ik}\bigl(R(t')\bigr)(t-t')\bigr]\bigl[\tilde{n}_k(t')-\tilde{n}_i(t')\bigr], \tag{A.10}$$

$$\hbar\omega_{ik} = \varepsilon_i + \Lambda_{ii} - \varepsilon_k - \Lambda_{kk}. \tag{A.11}$$

Here $\tilde{n}_i$ is the diagonal matrix element of the density matrix which is calculated according to the model presented elsewhere [45,46]; $\tau_{ik} = \tau_i\tau_k/(\tau_i + \tau_k)$; τ_i is the life time of the quasiparticle excitations in the single-particle state i of the nucleus. It determines the damping of single-particle motion. τ_i is calculated using the results of the quantum liquid theory [47] and the effective nucleon–nucleon forces from [44]

$$\frac{1}{\tau_i^{(\alpha)}} = \frac{\sqrt{2}\pi}{32\hbar\varepsilon_{F_K}^{(\alpha)}}\left[(f_K-g)^2 + \frac{1}{2}(f_K+g)^2\right]\left[(\pi T_K)^2 + \left(\tilde{\varepsilon}_i - \lambda_K^{(\alpha)}\right)^2\right]$$
$$\times\left[1+\exp\left(\frac{\lambda_K^{(\alpha)}-\tilde{\varepsilon}_i}{T_K}\right)\right]^{-1}, \tag{A.12}$$