Neutron Star Structure from the Quark-Model Baryon-Baryon Interaction

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References

``Nuclear Matter from Effective Quark-Quark Interaction'', M. Baldo and K. Fukukawa, Phys. Rev. Lett. 113, 242501 (2014)

1. Introduction

One of the most fundamental issues in Nuclear Physics

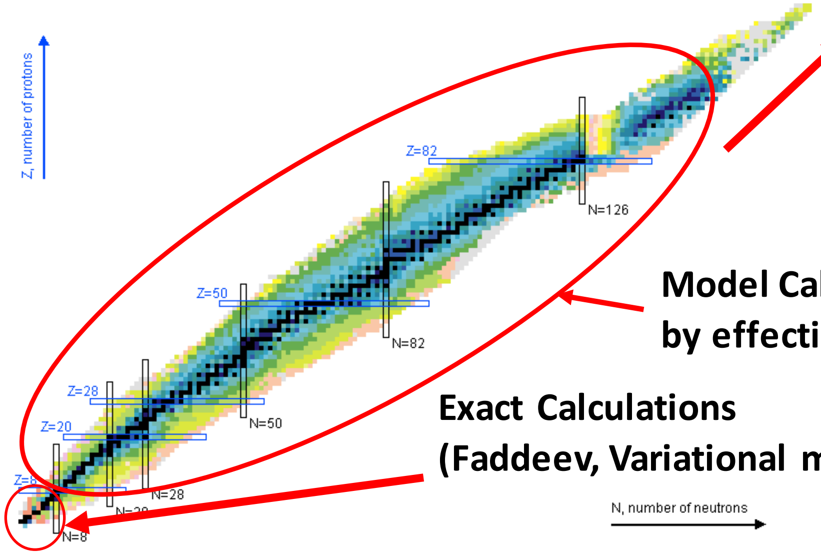
Better understanding of nuclear many-body systems from realistic forces

Realistic forces reproduce NN phase shifts (χ^2~1) very accurately

(Meson Exchange, Chiral EFT theory, Quark-Model and (Lattice QCD))

Mesons and baryons **constituent quark-model** **first principle calculations**

+ three-body force (Urbana, Illinoi, microscopic…. (2π, πρ, N bar…))



Nuclear matter

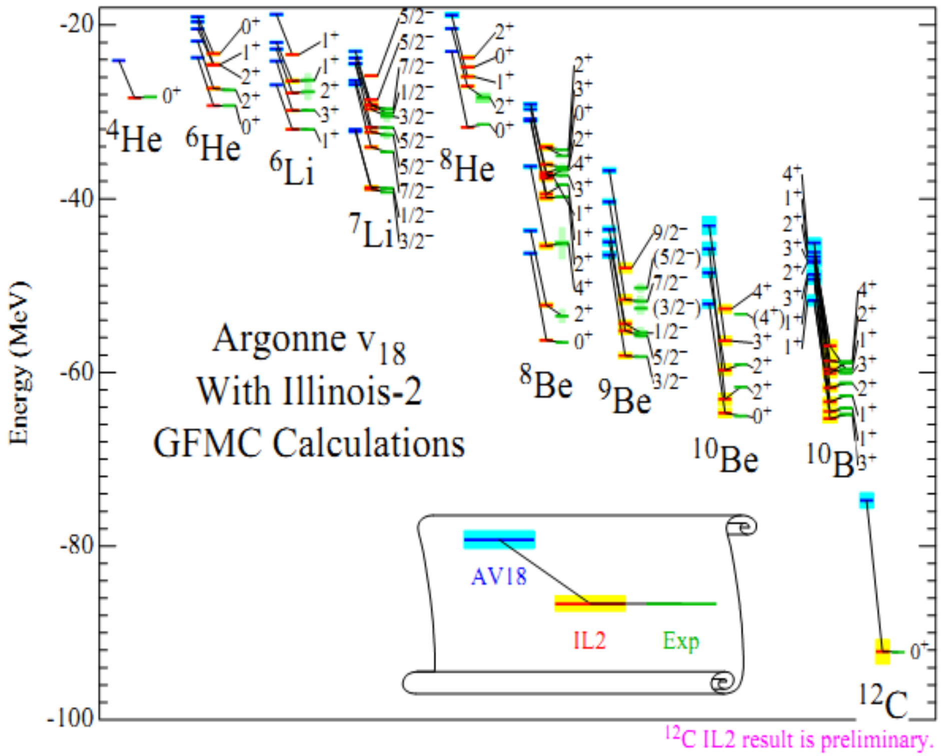
Model Calculations (Cluster model, Shell model, AMD …)
by effective NN interactions (Gogny, Skyrme, G-matrix)

Exact Calculations
(Faddeev, Variational method, No-core shell model, GFMC etc.) using bare NN interaction

Cited from the BNL Website (nudat 2)

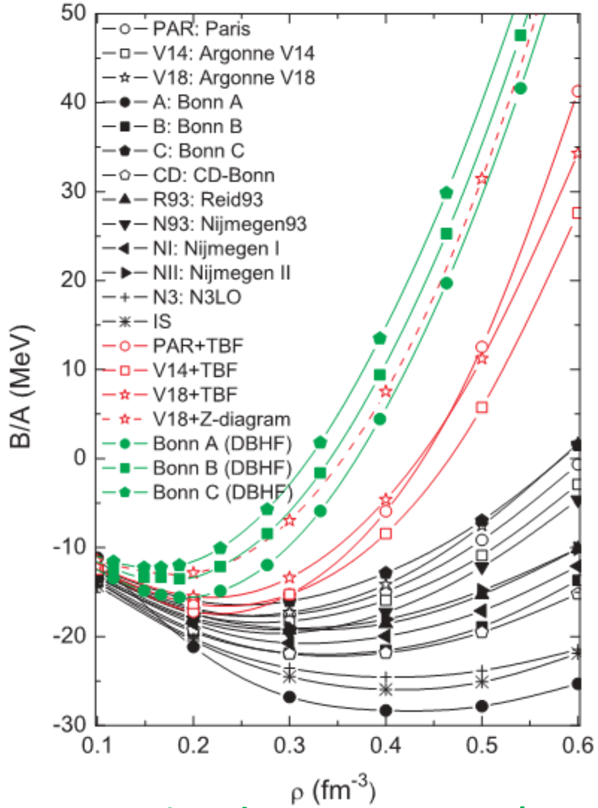
The Importance of three-nucleon forces

in light nuclei



S.C. Pieper, NPA 751. 516c - 532c (2005)

in the symmetric nuclear matter



Z. H. Li et al., PRC 74, 047304 (2006).

How about Quark Models NN interactions?

2. Quark-Model Baryon-Baryon Interaction and Few-Nucleon Systems

Review article for FSS and fss2
Y. Fujiwara, Y. Suzuki, and C. Nakamoto, Prog. Part. Nucl. Phys. 58, 439 (2007).

(3q)-(3q) resonating group method (RGM) firstly solved by M. Oka and K. Yazaki, PLB 90, 41 (1980)

RGM equation for the relative wave function χ(R)

Anti-symmetrization is the origin of non-locality.

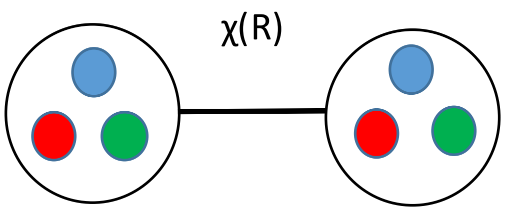
$$\langle \phi(3q)\phi(3q)|E-H|A\{\phi(3q)\phi(3q)\chi(R)\}\rangle = 0$$
$$\leftrightarrow [H_0 + V_D + V_{EX}]\chi(R) = \varepsilon N \chi(R)$$
$$\leftrightarrow [\varepsilon - H_0 - V_{RGM}(\varepsilon)]\chi(R) = 0$$

with $V_{RGM}(\varepsilon) = V_D + V_{EX} + \varepsilon N_{EX}$

nonlocal and energy dependent potential

ε : the energy measured from the NN threshold.

$$N \equiv \langle \phi(3q)\phi(3q)|A|\phi(3q)\phi(3q)\rangle$$
$$N_{EX} = 1 - N$$

ψ={ 

} φ(3q): $(0s)^3$ wave function with common width parameter

(3q)-(3q) Hamiltonian H

$$H = \sum_{i=1}^{6}\left(m_i c^2 + \frac{p_i^2}{2m_i}\right) - T_G$$
$$+\left(U_{ij}^{Cf} + U_{ij}^{FB} + \sum_\beta U_{ij}^{S\beta} + \sum_\beta U_{ij}^{\mathrm{PS}\beta} + \sum_\beta U_{ij}^{V\beta}\right)$$

Quark Part short-range part
Confinement + Fermi-Breit (OGEP)

Effective Meson Exchange Potential (EMEP)
medium- and long-range part
Pseudo-scalar, Scalar, and vector nonets

Parameters

The number of free parameter for fss2 is reasonable. (20 parameters)

QM parameters

i) b (width parameter for the $(0s)^3$ (3q) cluster)
ii) m_{ud} iii) $\lambda = m_s/m_{ud}$
iv) α (quark-gluon coupling constant)

EMEP parameters

i) $f_1^{PS,S,Ve,Vm}$ ii) $f_8^{PS,S,Ve,Vm}$

iii) The angle of the singlet-octet meson mixing θ

iv) Some meson masses $(\epsilon, S^*, \delta, \kappa)$

v) Other parameters to improve

the fit of NN phase shifts $c_\delta, c_{qss}, c_{qT}$

Y. Fujiwara et al. Phys. Rev. C 65, 014002 (2001).

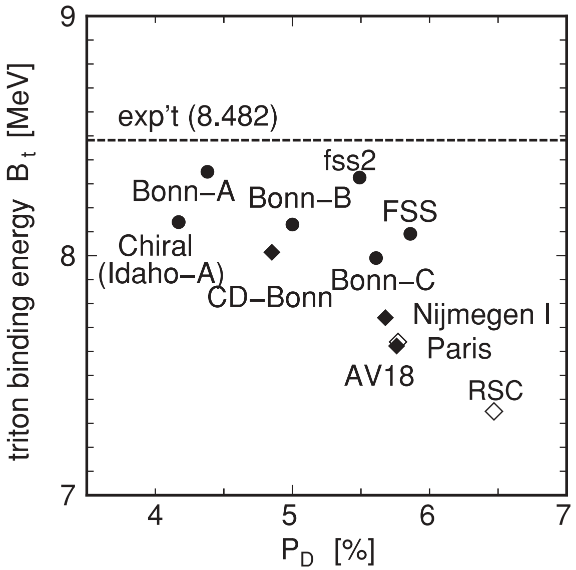
TABLE I. Quark-model parameters, SU_3 parameters of the EMEPs, S-meson masses, and some reduction factors c_δ, etc., for the models fss2 and FSS. The ρ meson in fss2 is treated in the two-pole approximation, for which m_1 (β_1) and m_2 (β_2) are shown below the table.

	b (fm)	m_{ud} (MeV/c^2)	α_S	$\lambda = m_s/m_{ud}$
fss2	0.5562	400	1.9759	1.5512
FSS	0.616	360	2.1742	1.526
	f_1^S	f_8^S	θ^S	θ_4^S [a]
fss2	3.48002	0.94459	33.3295°	55.826°
FSS	2.89138	1.07509	27.78°	65°
	f_1^{PS}	f_8^{PS}	θ^{PS}	
fss2	–	0.26748	–	(no η, η')
FSS	0.21426	0.26994	−23°	
	f_1^{Ve}	f_8^{Ve}	f_1^{Vm}	f_8^{Vm} [b]
fss2	1.050	0	1.000	2.577
(MeV/c^2)	m_ϵ	m_{S^*}	m_δ	m_κ
fss2	800	1250	846 [c]	936
FSS	800	1250	970	1145
	c_δ	c_{qss}	c_{qT} [e]	
fss2	0.4756 [d]	0.6352	3.139	
FSS	0.381	–	–	

[a] θ_4^S is used only for $\Sigma N(I=3/2)$.
[b] $\theta^V = 35.264°$ (ideal mixing) and two-pole ρ meson with m_1 (β_1) = 664.56 MeV/c^2 (0.34687) and m_2 (β_2) = 912.772 MeV/c^2 (0.48747) [30] are used.
[c] For the NN system, $m_\delta = 720$ MeV/c^2 is used.
[d] Only for π, otherwise 1.
[e] The enhancement factor for the Fermi-Breit tensor term.

The quark-model NN interaction have an attractive feature in 3-nucleon systems

Triton binding energy



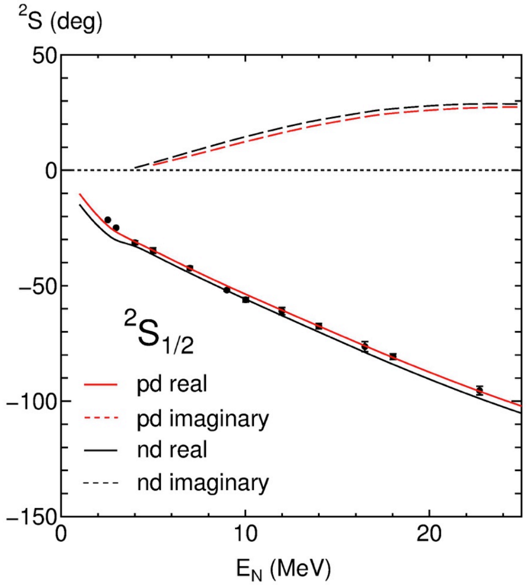
Y. Fujiwara et al., Phys. Rev. C 77 027001 (2008)

◆ take into account the charge dependence

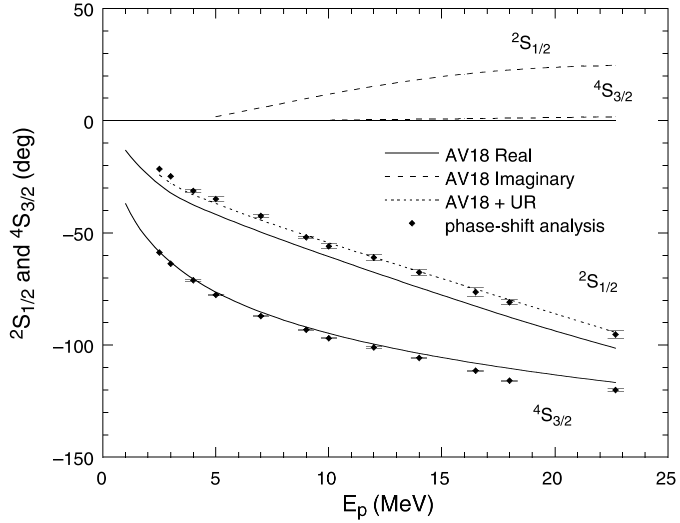
● do not take into account the charge dependence

The energy deficiency by fss2 **~350 keV**

$^2S_{1/2}$ phase shifts in proton-deuteron elastic scattering



K. Fukukawa, Doctor thesis



Z.M. Chen, W. Tornow and A. Kievsky, Few-Body Systems 35,15 (2004).

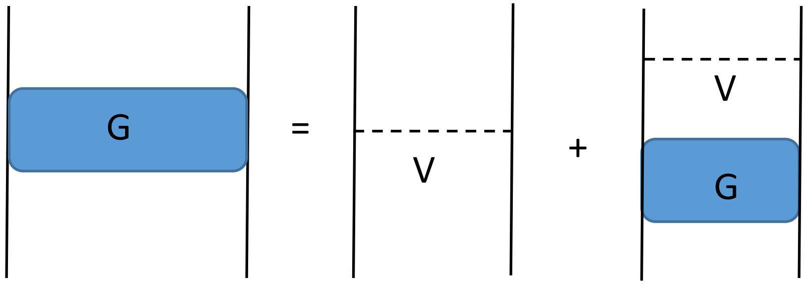
The model fss2 reproduces the $^2S_{1/2}$ pd phase shift

3. Bethe-Bruckner-Goldstone (BBG) Expansion and Equation of State

The lowest order (2 hole-line) expansion: Bethe-Goldstone equation

K. A. Brueckner and J. L. Gammel Phys. Rev. 109, 1023 (1958)

$$\langle k_1 k_2 | G(\omega) | k_3 k_4 \rangle_A = \langle k_1 k_2 | v | k_3 k_4 \rangle_A + \sum_{k_3' k_4'} \langle k_1 k_2 | v | k_3' k_4' \rangle \frac{\left(1-\Theta_F(k_3')\right)\left(1-\Theta_F(k_4')\right)}{\omega - e_{k_3'} - e_{k_4'}} \langle k_3' k_4' | G(\omega) | k_3 k_4 \rangle_A$$



Self-consistently solved

v : bare NN interaction

k_i: momentum and spin-isospin variable

$|k_1 k_2\rangle_A = |k_1 k_2\rangle - |k_2 k_1\rangle$

ω: starting energy

$e_k = \frac{\hbar^2 k^2}{2M_N} + U(k)$: Single-particle energy

$U(k) = \sum_{k' < K_F} \langle k k' | G(e_k + e_{k'}) | k k' \rangle$: Single-particle potential

Two somewhat opposite choices

Gap (or standard) choice assume U(k)=0 for $k > k_F$

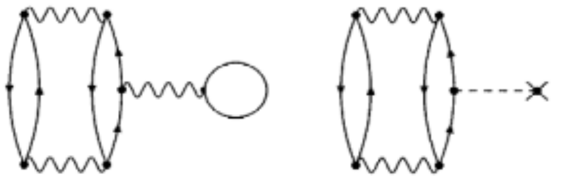
Continuous choice adopt the above expression for all k

Three hole-line approximation

(a) (b)

BHF approximation

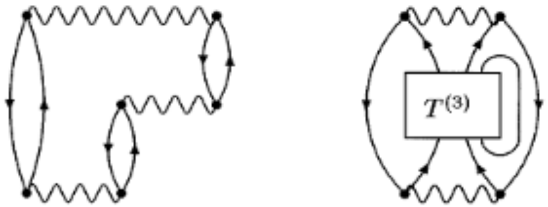
(c) (d)



Bubble diagram **U-insertion**

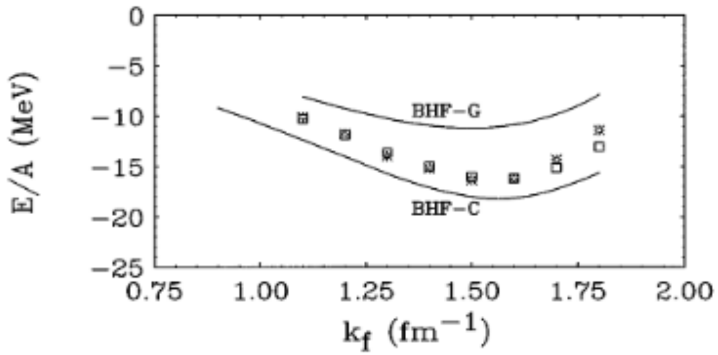
$H=(H_0+U)+(H_1-U)$

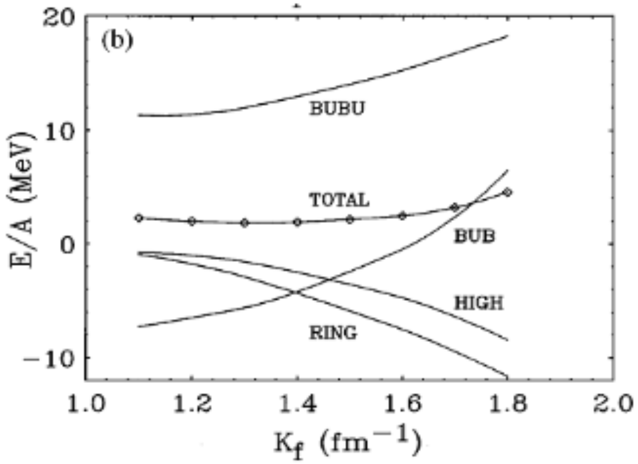
(e) (f)



Ring diagram **higher order diagram (Bethe-Faddeev Equation)**

R. Rajarman, RMP 39, 745 (1967)
B. D. Day, PRC 24, 1203 (1981) (Reid potential)



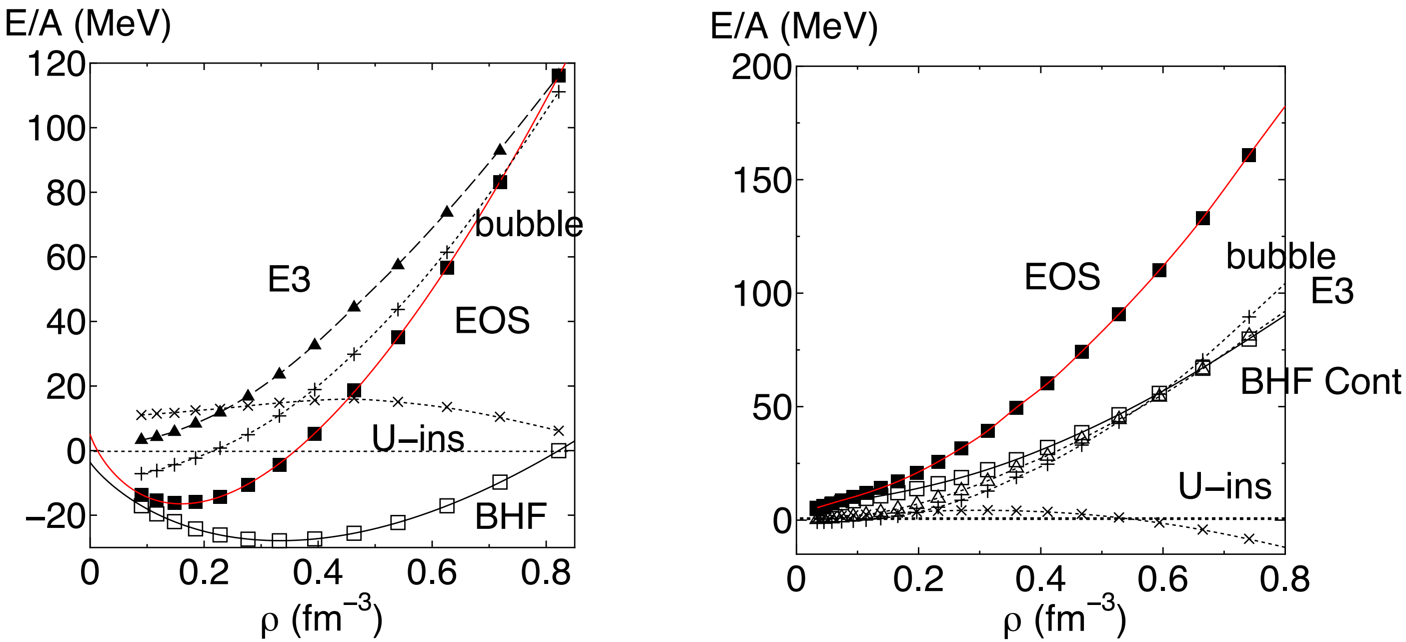


AV14 case: H. Q. Song et al. PRL 81, 1584 (1998)

Three hole contribution is not large because of the cancelation.

Contribution of Each Diagram (Continuous choice case)

Symmetric Nuclear Matter (SNM) Pure Neutron Matter (PNM)



Fitted EOS

$$\frac{E}{A}(\rho) = a\rho + b\rho^c + d$$

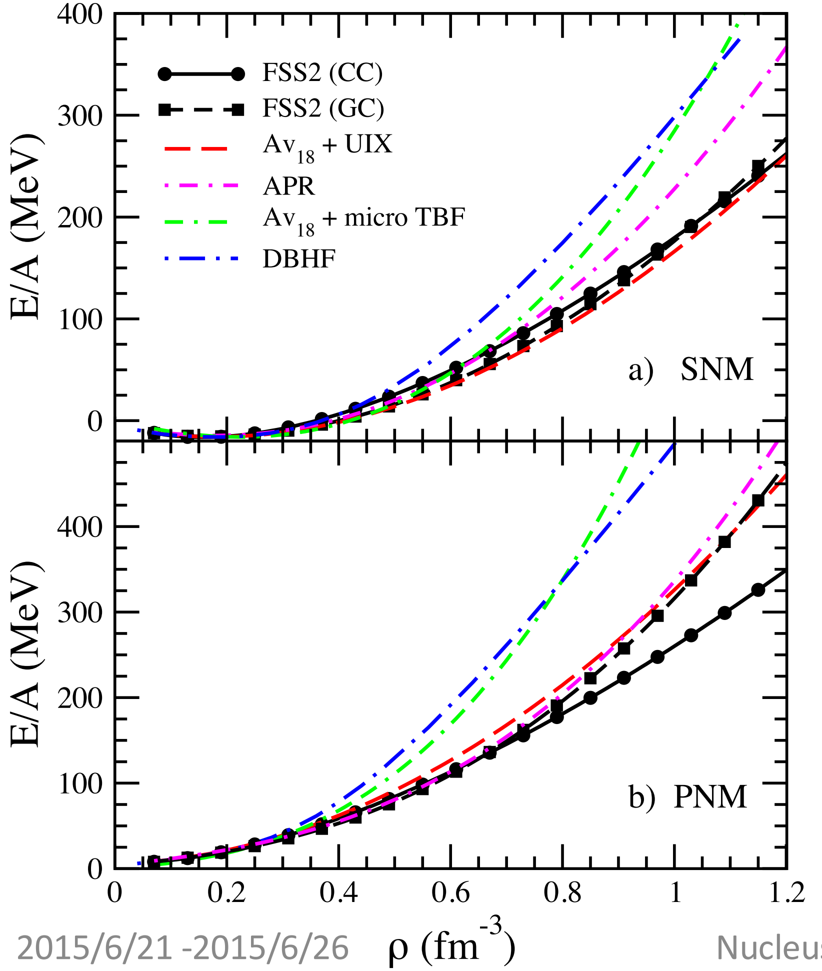
1. BHF calculation makes the SNM and PNM EOS rather soft
2. **Three hole-line contributions have a substantial effect for saturation**

   (E_0, ρ_0, K, E_{sym}, L : slope of the symmetry energy at saturation)

   Saturation properties E_0 = -16.3 MeV, ρ_0 = 0.157 fm^{-3} , K = 219 MeV, E_{sym} = 31.4 MeV
3. In high-density region, the bubble diagrams contributions are rapidly repulsive.

   ➪ transition to quark phase?

Comparison with other calculations

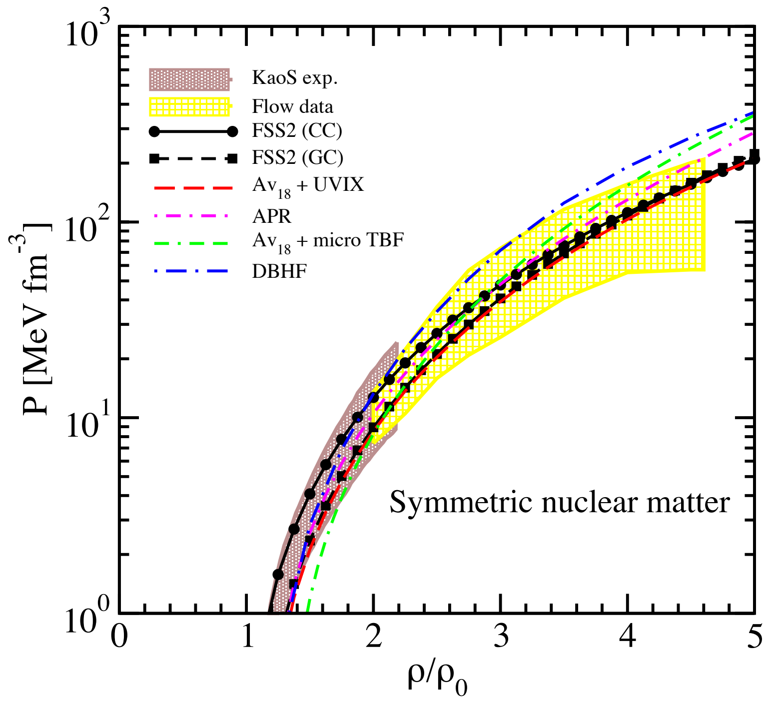


1. The gap choice EOS and continuous choice EOS agree well up to 0.7 fm^{-3}. This fact supports the convergence of the expansion.

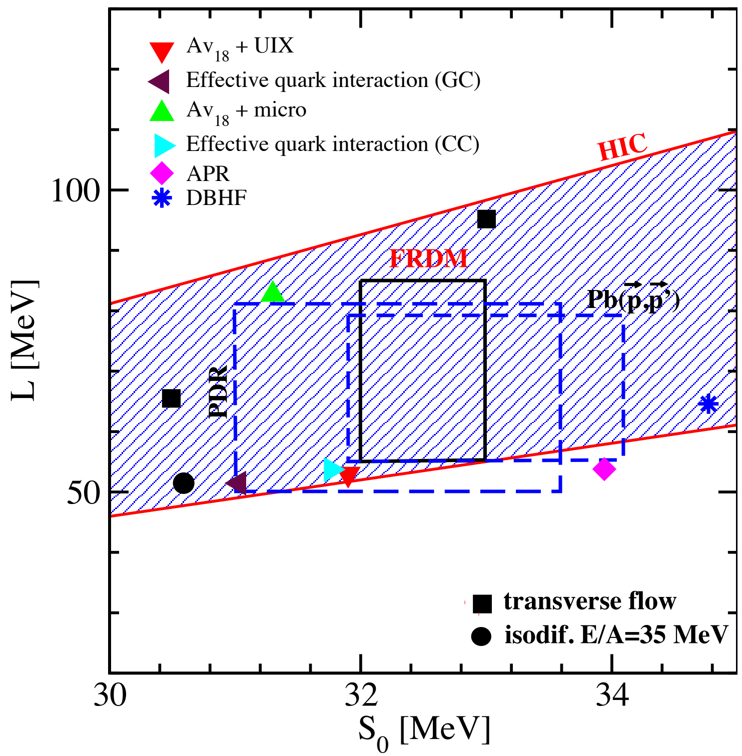
2. EOS agree well up to 0.5 fm^{-3}. QM EOS are isosoft EOS and similar to AV18 + Urbana 3NF.

Comparison with phenomenology

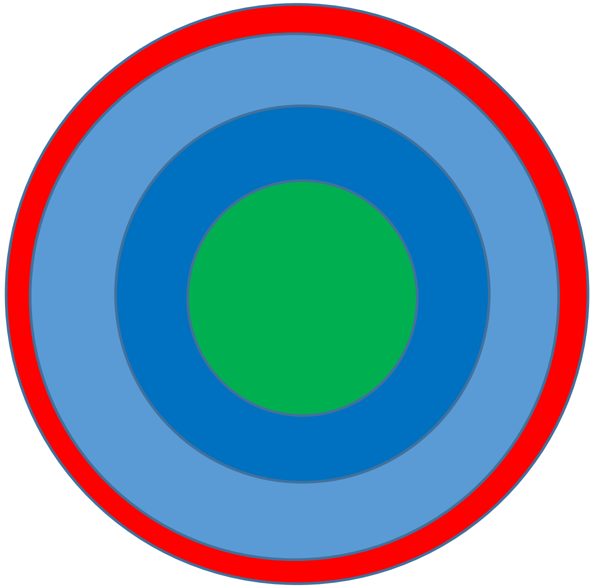
The pressure of SNM



L vs S_0



The Structure of Neutron Stars



- outer crust : nuclei and electron gas ($\rho < \rho_{drip}$)
- inner crust: asymmetric nuclei, neutron gas and electron gas ($\rho_{drip} < \rho < \rho_{NM}$)
- outer core: asymmetric nuclear matter, electron and muons
- inner core: Nucleon, Hyperon meson, quarks (from $2\rho_0 \sim 3\rho_0$)

Typical Values Radius 10 ~ 12 km

Recent Observational Development ($2M_{sun}$ neutron star)

2010 P. B. Demorest et al., Nature 467, 1081 (2010). $1.97 \pm 0.04\, M_{sun}$

The mass measurement using the Shapiro delay (the delay of the pulsar by the relativistic effect)

2013 J. Antoniaids et al., Science 340, 1233232 (2013). $2.01 \pm 0.04\, M_{sun}$

This was not explained using realistic NN+3NF and YN interaction (hyperon puzzle)

⇨ YNN, YNN, YYY force? Transition to quark phase ?

Structure Calculation of Neutron Stars

Step I: Composition of each particle

- Charge neutrality $\sum_i \rho_i q_i = 0$
- Beta equilibrium $\mu_i = b_i \mu_n - q_i \mu_e$

Chemical potential $\mu_i = \dfrac{\partial \epsilon(\rho_p, \rho_n, \rho_e, \rho_\mu)}{\partial \rho_i}$

Parabolic Approximation

$$\frac{E}{A}(\rho, x_p) = \frac{E}{A}(\rho, x_p = 0.5) + (1 - 2x_p)^2 S(\rho)$$

I. Bombaci, U. Lombardo, Phys. Rev. C 44, 1892 (1991).

Electron: ultra-relativistic Approximation

Muon: non-relativistic approximation

Step II: Tolman-Oppenheimer-Volkoff equations

S. L. Shapiro and S. A. Teukolsky, Black Holes, White Dwarfs, and Neutron Stars

R. C. Tolman, PR 55, 364 (1939), J. R. Oppenheimer and G. M. Volkoff, PR 55, 374 (1939)

$$\frac{dp}{dr} = -G\frac{\epsilon m}{r^2}\left(1 + \frac{P}{\varepsilon}\right)\left(1 + \frac{4\pi P r^3}{m}\right)\left(1 - \frac{2Gm}{r}\right)^{-1}$$

$$\frac{dm}{dr} = 4\pi r^2 \epsilon$$

To close the equation we need the relationship Between pressure and radius (**Equation of state**).

P=P(ε)

Equation of state

$$P(\rho) = \rho^2 \frac{d}{d\rho}\frac{\epsilon(\rho_i(\rho))}{\rho}$$

(Upper panel) Proton fraction x_p

In the continuous choice, the proton fraction is law even in the low-density region, reflecting the small symmetry energy.

⇨ the direct Urca process starts does not happen (or happen in the very high-density region)

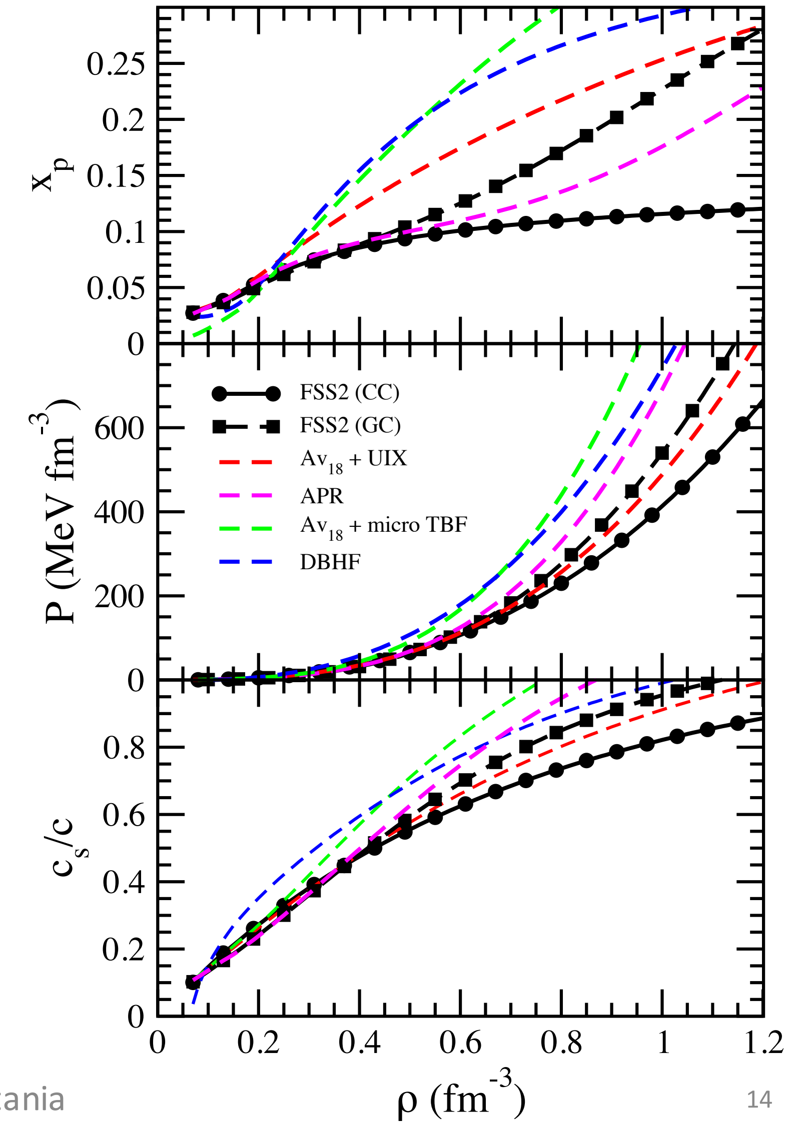
$n \to p + e^- + \nu_e \qquad p + e^- \to n + \nu_e$

(Middle Panel) Pressure

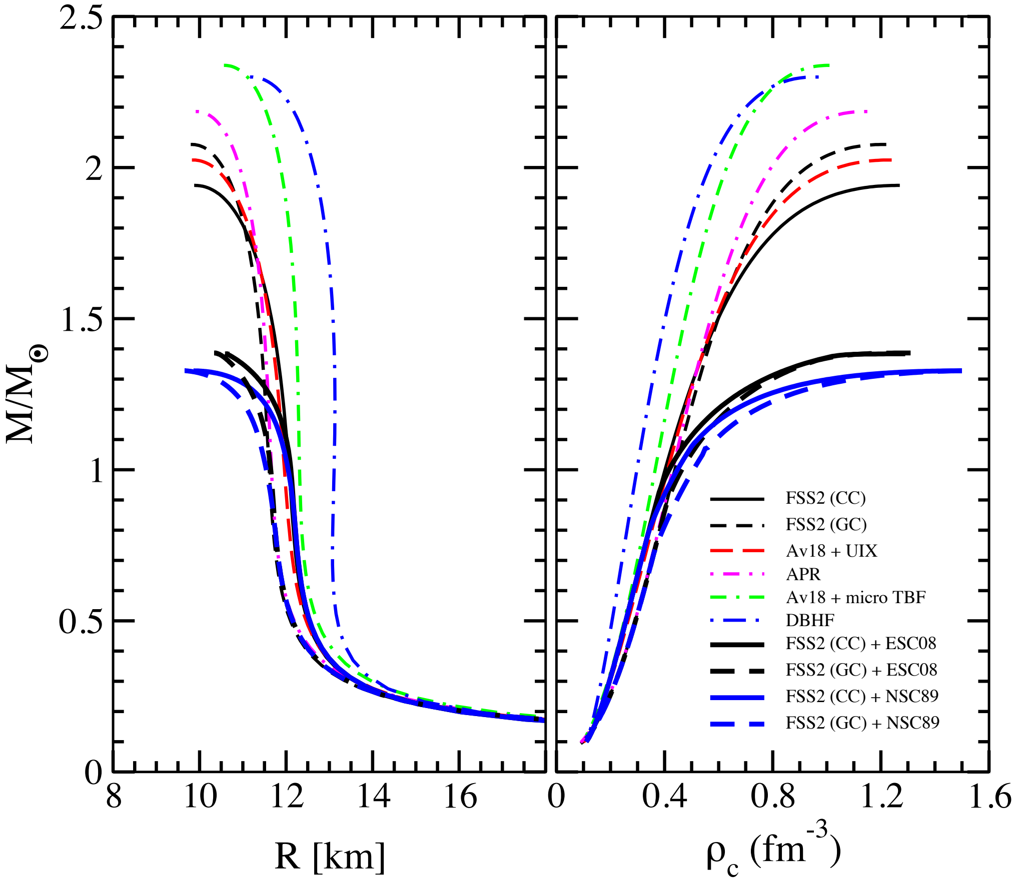
Beta-Equilibrium EOSs are comparable to other EOS because the neutron fraction is larger than other equation.

(Lower Panel) Sound velocity (in unit of light speed)

We can find that QM EOS is not superluminal until very high-density region.



Mass-Radius relation



The observed mass is about 2 solar mass, despite the EOS is relatively soft.

The maximum mass is slightly different between the gap case and continuous case, which reflects the stiffness in the high density region.

4. Summary

We have applied the QM NN interaction fss2 to the SNM and PNM EOs and solved the TOV equations.

0. The model fss2 gives a fairly better description in the low-energy region, three- and four-nucleon systems.
1. **At variance with other potentials, fss2 gives rather repulsive 3 hole-line contributions, especially at high density in the SNM and PNM. The saturation properties (Saturation point, incompressibility, and the symmetry energy and its slope) without three-body forces.**
2. The maximum mass is close to 2 solar mass in both gap and continuous choice, which is compatible with experimental data.
3. At the high-density region, it is questionable of the validity of the BBG expansion.
   ⇨ Transition to quark matter (How far are EOSs valid?)

Outlook

1. Low-density EOS ($^1S_0$ superfluidity, clustering phenomena, pasta structure etc.)
2. Finite nuclei calculation taking into 3-nucleon correlations
3. Including hyperon (but very difficult to solve Bethe-Faddeev Equation)