Cluster production within transport theory

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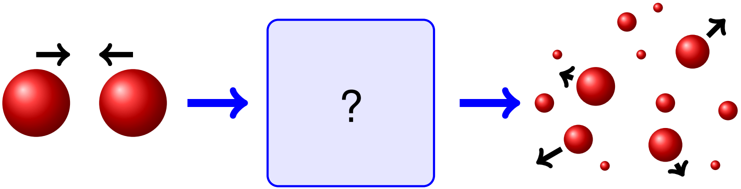
- Importance of clusters in low-density matter and HIC
- Extended AMD with cluster correlations
- Impacts of clusters on HIC

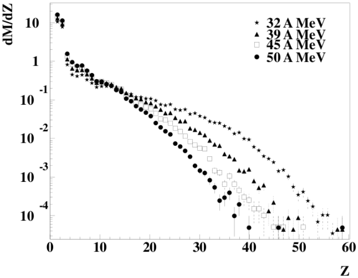
How to understand the dynamics of heavy-ion collisions

Collisions of two nuclei (e.g., Xe + Sn at 50 MeV/nucleon)



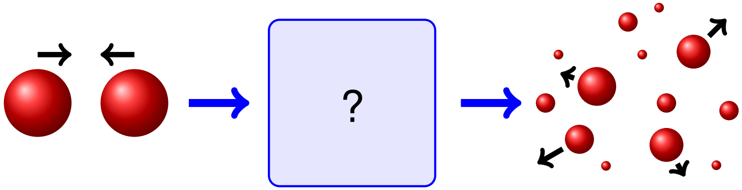
- Clusters are important in the final states.
- What about at earlier times?

  ⇒ Transport theory with clusters



INDRA data, Hudan et al., PRC67 (2003) 064613.

Collisions of two nuclei (e.g., Xe + Sn at 50 MeV/nucleon)



- Clusters are important in the final states.
- What about at earlier times?

  ⇒ Transport theory with clusters

Partitioning of protons

	Xe + Sn 50 MeV/u	Au + Au 250 MeV/u
p	≈10%	21%
α	≈20%	20%
d, t, ^{3}He	≈10%	40%
$A > 4$	≈60%	18%

INDRA data, Hudan et al., PRC67 (2003) 064613.

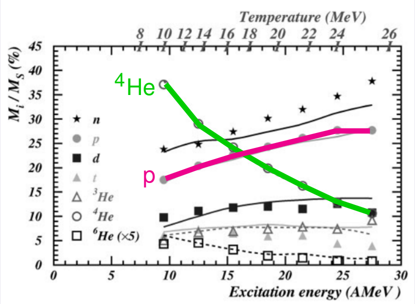
FOPI data, Reisdorf et al., NPA 848 (2010) 366.

Clusters at low densities

Heavy-Ion Collisions

Experimental data of cluster abundance in $^{36}Ar + {}^{58}Ni$ for the events where the quasi-projectile is vaporized.

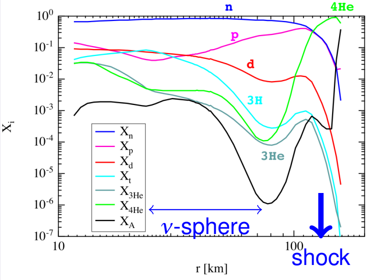
Borderie et al., EPJA6 (1999) 197, PLB388 (1996) 224.



Supernova

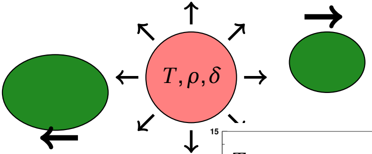
Mass fraction of light clusters in the post-bounce supernova core, based on nuclear statistical equilibrium.

Sumiyoshi and Röpke, PRC77 (2008) 055804.



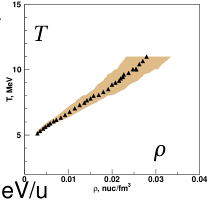
Clusters at low densities (comparison of HIC and EOS)

(Talk by K. Hagel)



Qin et al.,

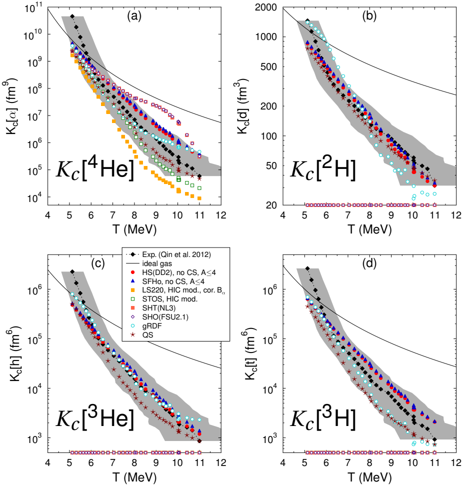
PRL108(2012)172701



Ar, Zn + Sn @ 47 MeV/u

Comparison of HIC and various EOS's.

Hempel et al., PRC 91 (2015) 045805.



Equilibrium Constants

$$K_c(N,Z) = \frac{\rho(N,Z)}{\rho_p^Z \rho_n^N} \quad \text{for cluster } (N,Z)$$

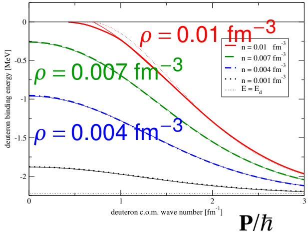
Consistent description of collision dynamics and EOS is desirable.

Equation for a deuteron in uncorrelated medium

$$\left[e(\tfrac{1}{2}\mathbf{P}+\mathbf{p})+e(\tfrac{1}{2}\mathbf{P}-\mathbf{p})\right]\tilde{\psi}(\mathbf{p})$$
$$+\left[1-f(\tfrac{1}{2}\mathbf{P}+\mathbf{p})-f(\tfrac{1}{2}\mathbf{P}-\mathbf{p})\right]\int\frac{d\mathbf{p}'}{(2\pi)^3}\langle\mathbf{p}|v|\mathbf{p}'\rangle\tilde{\psi}(\mathbf{p}')$$
$$=E\tilde{\psi}(\mathbf{p})$$

$$G_2(12,1'2',\omega)=\int\frac{S(12,1'2',E)}{\omega-E+i\eta}dE$$

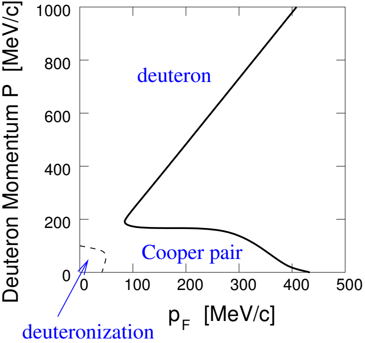
$E_d = -\text{B.E.}$



Momentum ($\mathbf{P}$) dependence of B.E.

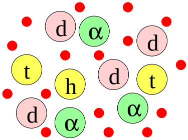
Röpke, NPA867 (2011) 66.

Deuteron in medium (at $T=0$)



Danielewicz and Bertsch, NPA 533 (1991) 712.

Chemical reactions in cluster gas (and in HIC)

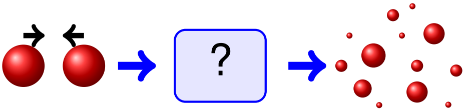


Cluster productions/reactions are important

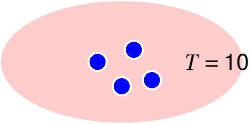
- in low density matter
- in the dynamics of heavy-ion collisions

Equilibrium $\Longleftrightarrow$ Dynamics

Transport models with clusters



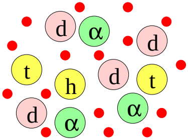
Example: Four nucleons at $T = 10$ MeV



- Without correlations:
  $\langle E \rangle = \frac{3}{2} T \times 4 = 60$ MeV
- If they always form an α:
  $\langle E \rangle = \frac{3}{2} T \times 1 - 28.3\,\text{MeV} = -13.3$ MeV

Balance of energy changes very much by clusters.

Chemical reactions in cluster gas (and in HIC)



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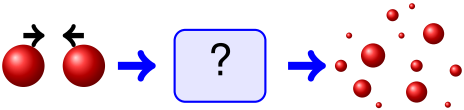


- $n+p+X \leftrightarrow d+X'$
- $d+n+X \leftrightarrow t+X'$
- $d+p+X \leftrightarrow h+X'$
- $t+p+X \leftrightarrow \alpha+X'$
- $h+n+X \leftrightarrow \alpha+X'$
- $d+d+X \leftrightarrow \alpha+X'$
- $2n+p+X \leftrightarrow t+X'$
- $n+2p+X \leftrightarrow h+X'$
- $d+n+p+X \leftrightarrow \alpha+X'$
- $2n+2p+X \leftrightarrow \alpha+X'$

- $d+d \leftrightarrow p+t$
- $d+d \leftrightarrow n+h$
- $p+t \leftrightarrow n+h$
- $d+t \leftrightarrow n+\alpha$
- $d+h \leftrightarrow p+\alpha$
- $d+t \leftrightarrow 2n+h$
- $d+h \leftrightarrow 2p+t$
- $d+\alpha \leftrightarrow t+h$

Transport models for heavy-ion collisions

Mean field models

Phase space distribution $f(\mathbf{r}, \mathbf{p}, t)$

$$\frac{\partial f}{\partial t} + \mathbf{v} \cdot \frac{\partial f}{\partial \mathbf{r}} - \frac{\partial U}{\partial \mathbf{r}} \cdot \frac{\partial f}{\partial \mathbf{p}} = I_{\text{coll}}$$

BNV, VUU, BUU, . . .

- Extension to include fluctuations (Boltzmann-Langevin, SMF, . . .)
- Extension to include clusters (Danielewicz et al., NPA 533 (1991) 712.)

Molecular dynamics models

Nucleon (wave packet) $\mathbf{R}_k(t)$, $\mathbf{P}_k(t)$

$$\frac{d\mathbf{R}_k}{dt} = \frac{\partial H}{\partial \mathbf{P}_k}, \quad \frac{d\mathbf{P}_k}{dt} = -\frac{\partial H}{\partial \mathbf{R}_k} + \text{(NN coll)}$$

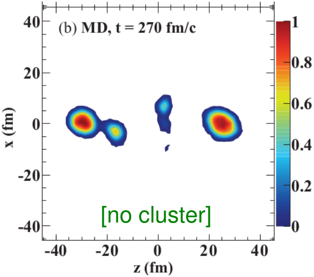
.*QMD, AMD, CoMD, . . .

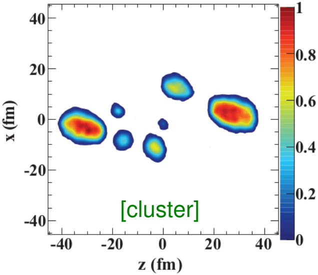
- Improvements for single-particle motion (a version of AMD) or for Pauli principle (CoMD)
- Elastic scattering of existing clusters (Ono et al., PRC 47 (1993) 2652.) (H.Zheng et al., NPA 892 (2012) 43.)
- AMD with cluster production

- All the models are based on the single-particle motions of nucleons.
- Only some models handle cluster correlations explicitly.
- “Coalescence” prescription in same cases. — not consistent with dynamics.

Transport with clusters (pBUU)

Coupland, Lynch, Tsang, Danielewicz, Zhang, PRC 84 (2011) 054603.





BUU with clusters

Danielewicz et al., NPA 533 (1991) 712.

Coupled equations for $f_n(\mathbf{r},\mathbf{p},t)$, $f_p(\mathbf{r},\mathbf{p},t)$, $f_d(\mathbf{r},\mathbf{p},t)$, $f_t(\mathbf{r},\mathbf{p},t)$, $f_h(\mathbf{r},\mathbf{p},t)$ are solved by the test particle method.

$$\frac{\partial f_n}{\partial t} + \mathbf{v}\cdot\frac{\partial f_n}{\partial \mathbf{r}} - \frac{\partial U_n}{\partial \mathbf{r}}\cdot\frac{\partial f_n}{\partial \mathbf{p}} = I_n^{\text{coll}}[f_n, f_p, f_d, f_t, f_h]$$

$$\frac{\partial f_p}{\partial t} + \mathbf{v}\cdot\frac{\partial f_p}{\partial \mathbf{r}} - \frac{\partial U_p}{\partial \mathbf{r}}\cdot\frac{\partial f_p}{\partial \mathbf{p}} = I_p^{\text{coll}}[f_n, f_p, f_d, f_t, f_h]$$

$$\frac{\partial f_d}{\partial t} + \mathbf{v}\cdot\frac{\partial f_d}{\partial \mathbf{r}} - \frac{\partial U_d}{\partial \mathbf{r}}\cdot\frac{\partial f_d}{\partial \mathbf{p}} = I_d^{\text{coll}}[f_n, f_p, f_d, f_t, f_h]$$

$$\frac{\partial f_t}{\partial t} + \mathbf{v}\cdot\frac{\partial f_t}{\partial \mathbf{r}} - \frac{\partial U_t}{\partial \mathbf{r}}\cdot\frac{\partial f_t}{\partial \mathbf{p}} = I_t^{\text{coll}}[f_n, f_p, f_d, f_t, f_h]$$

$$\frac{\partial f_h}{\partial t} + \mathbf{v}\cdot\frac{\partial f_h}{\partial \mathbf{r}} - \frac{\partial U_h}{\partial \mathbf{r}}\cdot\frac{\partial f_h}{\partial \mathbf{p}} = I_h^{\text{coll}}[f_n, f_p, f_d, f_t, f_h]$$

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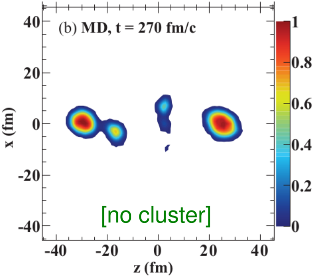
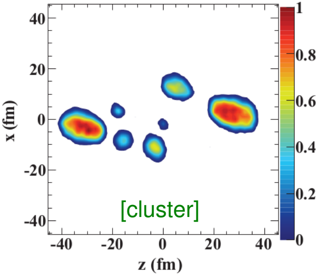
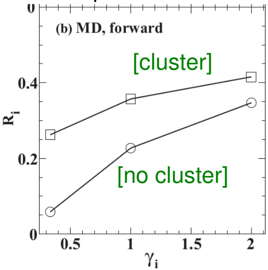
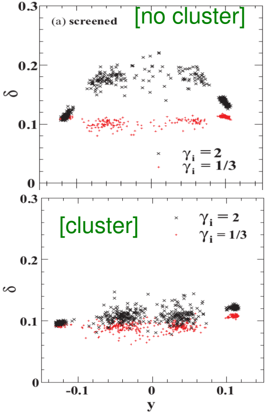




Isospin Diffusion

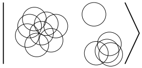


γ_i: density-dep. of E_{sym}



Distribution of fragments ($A > 2$) in isospin asymmetry and rapidity

Antisymmetrized Molecular Dynamics (very basic version)



AMD wave function

$$|\Phi(Z)\rangle = \det_{ij}\left[\exp\left\{-\nu\left(\mathbf{r}_j - \frac{\mathbf{Z}_i}{\sqrt{\nu}}\right)^2\right\}\chi_{\alpha_i}(j)\right]$$

$$\mathbf{Z}_i = \sqrt{\nu}\mathbf{D}_i + \frac{i}{2\hbar\sqrt{\nu}}\mathbf{K}_i$$

ν : Width parameter = (2.5 fm)$^{-2}$

χ_{α_i} : Spin-isospin states = $p\uparrow, p\downarrow, n\uparrow, n\downarrow$

Equation of motion for the wave packet centroids Z

$$\frac{d}{dt}\mathbf{Z}_i = \{\mathbf{Z}_i, \mathcal{H}\}_{\text{PB}} \quad + \quad \text{(NN collisions)}$$

$\{\mathbf{Z}_i, \mathcal{H}\}_{\text{PB}}$: Motion in the mean field

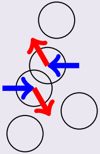
$$\mathcal{H} = \frac{\langle\Phi(Z)|H|\Phi(Z)\rangle}{\langle\Phi(Z)|\Phi(Z)\rangle} + \text{(c.m. correction)}$$

H: Effective interaction (e.g. Skyrme force)

NN collisions

$$W_{i\to f} = \frac{2\pi}{\hbar}|\langle\Psi_f|V|\Psi_i\rangle|^2\delta(E_f - E_i)$$

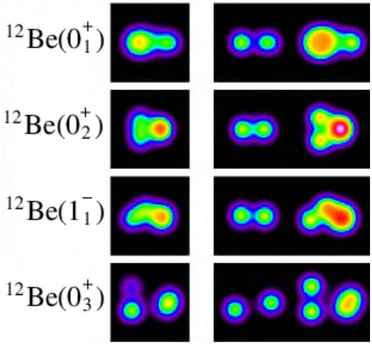
- $|V|^2$ or σ_{NN} (in medium)
- Pauli blocking

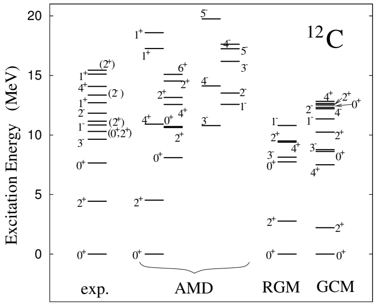


Ono, Horiuchi et al., Prog. Theor. Phys. 87 (1992) 1185.

Nuclear structure studies by AMD

Kanada-En'yo et al., Prog. Theor. Exp. Phys. 2012 01A202 (2012)

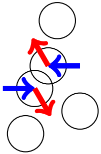




- AMD can describe the states in which nucleons are correlated to form clusters.
  However, it is another problem whether such states appear in reactions with the correct probabilities.
- In structure calculations, multiple AMD wave functions are superposed (including the parity and angular-momentum projection, orthogonalization with other states).

Multifragmentation(?) without cluster correlations

$$W_{i\to f} = \frac{2\pi}{\hbar}|\langle\Psi_f|V|\Psi_i\rangle|^2\delta(E_f-E_i)$$



In the usual way of NN collision, only the two wave packets are changed.

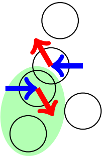
$$\left\{|\Psi_f\rangle\right\} = \left\{|\varphi_{k_1}(1)\varphi_{k_2}(2)\Psi(3,4,\ldots)\rangle\right\}$$

(ignoring antisymmetrization for simplicity of presentation.)

Xe + Sn central collisions at 50 MeV/u

	AMD	INDRA
$M(p)$	40.2	8.4
$M(\alpha)$	2.5	10.1

$$W_{i \to f} = \frac{2\pi}{\hbar} |\langle \Psi_f | V | \Psi_i \rangle|^2 \delta(E_f - E_i)$$



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$$\left\{ |\Psi_f\rangle \right\} = \left\{ |\varphi_{k_1}(1)\varphi_{k_2}(2)\Psi(3,4,\ldots)\rangle \right\}$$

(ignoring antisymmetrization for simplicity of presentation.)

Extension for cluster correlations

Include correlated states in the set of the final states of each NN collision.

$$\left\{ |\Psi_f\rangle \right\} \ni |\varphi_{k_1}(1)\psi_d(2,3)\Psi(4,\ldots)\rangle, \ \ldots$$

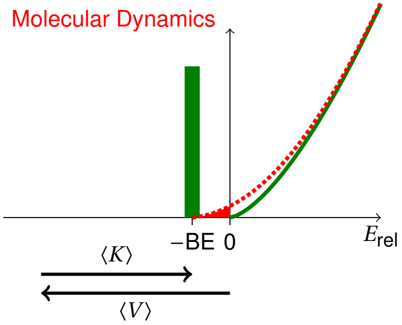
Xe + Sn central collisions at 50 MeV/u

	AMD	INDRA
$M(p)$	40.2	8.4
$M(\alpha)$	2.5	10.1

Density of states for two nucleon system

Exact Quantum Mechanics

Molecular Dynamics

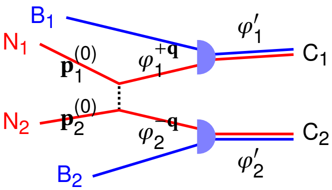


NN collisions with cluster correlations

Similar to Danielewicz et al., NPA533 (1991) 712.

$$\mathrm{N}_1 + \mathrm{B}_1 \ + \ \mathrm{N}_2 + \mathrm{B}_2 \ \to \mathrm{C}_1 + \mathrm{C}_2$$

- $\mathrm{N}_1, \mathrm{N}_2$: Colliding nucleons
- $\mathrm{B}_1, \mathrm{B}_2$: Spectator nucleons/clusters
- $\mathrm{C}_1, \mathrm{C}_2$: N, $(2N)$, $(3N)$, $(4N)$ (up to α cluster)



Transition probability

$$W(\mathrm{NBNB} \to \mathrm{CC}) = \frac{2\pi}{\hbar} |\langle \mathrm{CC}|V|\mathrm{NBNB}\rangle|^2 \delta(E_f - E_i)$$

$$v\,d\sigma \propto |\langle \varphi_1' | \varphi_1^{+\mathbf{q}} \rangle|^2 |\langle \varphi_2' | \varphi_2^{-\mathbf{q}} \rangle|^2 |M|^2 \delta(E_f - E_i) p_{\mathrm{rel}}^2 \, dp_{\mathrm{rel}} \, d\Omega$$

$|M|^2 = |\langle \mathrm{NN}|V|\mathrm{NN}\rangle|^2$: Matrix elements of NN scattering

$\Leftarrow (d\sigma/d\Omega)_{\mathrm{NN}}$ in medium (or in free space)

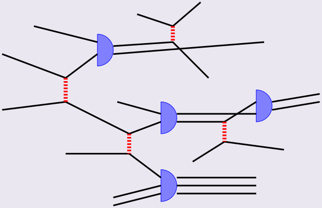
$$\mathbf{p}_{\mathrm{rel}} = \tfrac{1}{2}(\mathbf{p}_1 - \mathbf{p}_2) = p_{\mathrm{rel}} \hat{\mathbf{\Omega}}$$

$$\mathbf{q} = \mathbf{p}_1 - \mathbf{p}_1^{(0)} = \mathbf{p}_2^{(0)} - \mathbf{p}_2$$

$$\varphi_1^{+\mathbf{q}} = \exp(+i\mathbf{q} \cdot \mathbf{r}_{\mathrm{N}_1}) \varphi_1^{(0)}$$

$$\varphi_2^{-\mathbf{q}} = \exp(-i\mathbf{q} \cdot \mathbf{r}_{\mathrm{N}_2}) \varphi_2^{(0)}$$

NN collisions with cluster correlations (more explanations)



For each NN collision, cluster formation is considered.

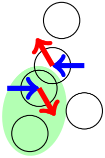
$$\mathrm{N}_1 + \mathrm{B}_1 + \mathrm{N}_2 + \mathrm{B}_2 \to \mathrm{C}_1 + \mathrm{C}_2$$

$$W_{i\to f} = \frac{2\pi}{\hbar} |\langle \mathrm{CC}|V_{NN}|\mathrm{NBNB}\rangle|^2 \delta(E_f - E_i)$$

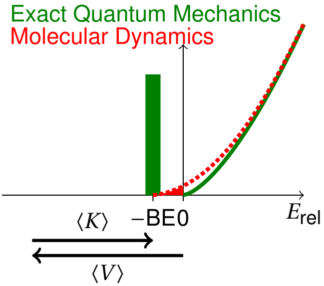
AO, J. Phys. Conf. Ser. 420 (2013) 012103

- We always have a Slater determinant of nucleon wave packets. Clusters in the final states are represented by placing wave packets at the same phase space point.
- Consequently the processes such as $d + X \to n + p + X'$ and $d + X \to d + X'$ are automatically taken into account.
- No parameters have been introduced to adjust individual reactions.
- Cluster formation is suppressed when the momentum transfer is small, by the probability $1 - e^{-\mathbf{q}^2/(64\ \mathrm{MeV}/c)^2}$.
- There are many possibilities to from clusters in the final states. Non-orthogonality of the final states should be carefully handled.

Does AMD need correction for clusters? — phase space



Density of states for two nucleon system



For the relative motion of the two nucleons,

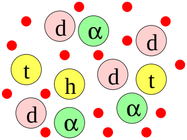
For the wave-packet centroid $(\mathbf{R}, \mathbf{P})$

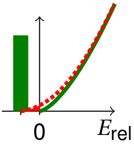
$$\frac{d}{dt} Z = \{Z, \mathcal{H}\}_{\mathrm{PB}} \quad \Leftrightarrow \quad \mathcal{H}(\mathbf{R}, \mathbf{P}) = \langle \hat{H} \rangle$$

$$\iint \theta\Big(\mathcal{H}(\mathbf{R}, \mathbf{P}) < 0\Big) d\mathbf{R} d\mathbf{P} \ll (2\pi\hbar)^3$$

The bound phase space for the wave packet centroids is too small ($\ll h^3$).

$\Rightarrow$ Too small prorability of cluster formation in final states of NN collisions.

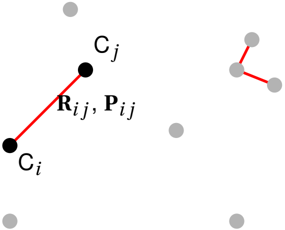




Clusters may form a loosely bound state.

e.g., $^7\text{Li} = \alpha + t - 2.5$ MeV

Need more probability of $|\alpha + t\rangle \to |^7\text{Li}\rangle$



Step 1 Clusters (and nucleons) C_i and C_j are *linked*,

- if C_i is one of the 4 clusters closest to C_j, and $(i \leftrightarrow j)$,
- and if the distance is $1 < |\mathbf{R}_{ij}| < 5$ fm,
- and if they are slowly moving away, $\mathbf{P}_{ij}^2/2\mu_{ij} < E_{\rm rel}^{\rm max}$ and $\mathbf{R}_{ij}\cdot\mathbf{P}_{ij} < 0$.

Step 2 *Transition* of the internal state of CC by eliminating the internal momentum

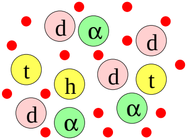
$$\mathbf{P}_i = \mathbf{P}_{i\parallel} + \mathbf{P}_{i\perp} \to \begin{cases} 0 & A_{\rm CC} \le 10 \\ \mathbf{P}_{i\perp} & A_{\rm CC} > 10 \end{cases}$$

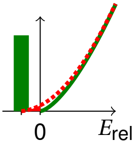
$$\mathbf{P}_{i\parallel} = (\mathbf{P}_i \cdot \hat{\mathbf{R}}_i)\hat{\mathbf{R}}_i$$

for $i \in$ CC in the c.m. of CC, with some care of the momentum conservation.

Next *Energy conservation.*

Correlations to bind several clusters

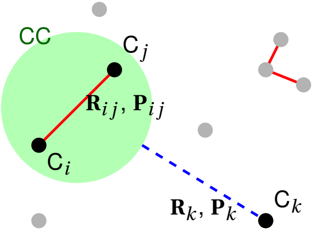




Clusters may form a loosely bound state.

e.g., $^{7}\text{Li} = \alpha + t - 2.5\ \text{MeV}$

Need more probability of $|\alpha + t\rangle \to |^{7}\text{Li}\rangle$



Step 3: Search a third particle for E-conservation

- Cluster C_k that has the smallest value of f_k is selected.

$$f_k = \frac{(|\mathbf{R}_k| + 7.5\ \text{fm}) \times (1.2 - \hat{\mathbf{P}}_k \cdot \hat{\mathbf{R}}_k)}{\min(\mathbf{P}_{k\parallel}^2/2\mu_k,\ 5\ \text{MeV})}$$

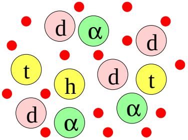
- If the selected C_k already belongs to a CC′, this whole CC′ is treated as the third particle for E-conservation.

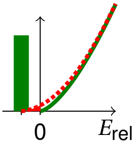
Step 4: Scale the radial component of the relative momentum between CC and C_k for the total energy conservation.

$$\mathbf{P}_k = \mathbf{P}_{k\parallel} + \mathbf{P}_{k\perp} \to \beta\mathbf{P}_{k\parallel} + \mathbf{P}_{k\perp}$$

$$\mathbf{P}_{k\parallel} = (\mathbf{P}_k \cdot \hat{\mathbf{R}}_k)\hat{\mathbf{R}}_k$$

Correlations to bind several clusters





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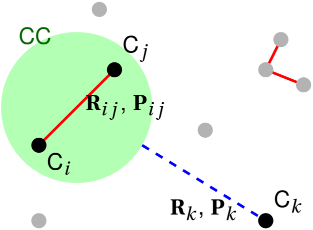
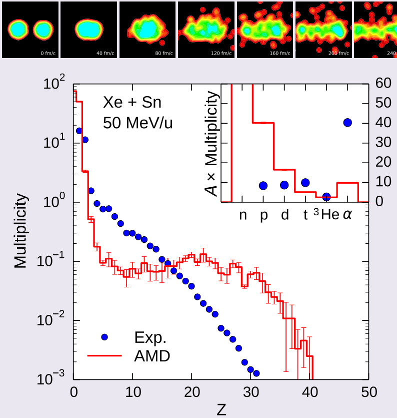


Notes

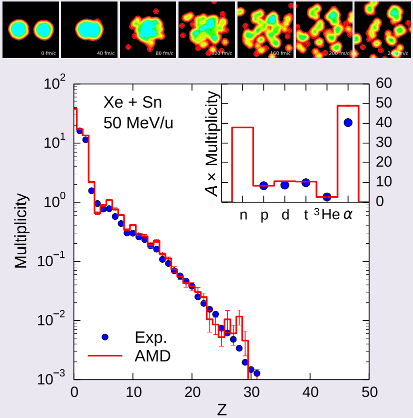
- Two nucleons are not linked. Clusters are not linked if they can form an α or a lighter cluster.
- Clusters are linked if they already form a CC and if the distance conditions are met. Clusters in different CC's are not linked.
- $E_{\text{rel}}^{\text{max}} = 8$ MeV is chosen, but it is reduced to suppress the formation of too large CC. $E_{\text{rel}}^{\text{max}} := E_{\text{rel}}^{\text{max}} \exp[-(A_{\text{CC}}/35)^2]$
- Canceled for CC to form an overbound nucleus ($10 \le A_{\text{CC}} \le 18$). Also canceled for $A_{\text{CC}} < 6$ and $A_{\text{CC}} > 100$.
- As always, clusters (and therefore CC's) are broken by NN collisions and mean-field propagation.

Xe + Sn central collisions at 50 MeV/nucleon

Without clusters

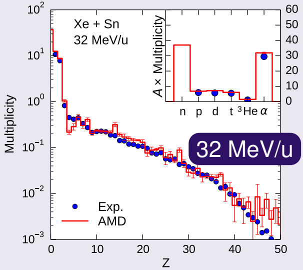


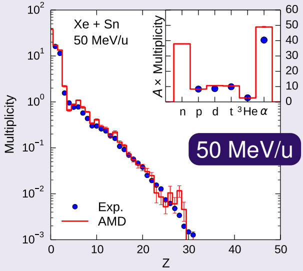
With clusters



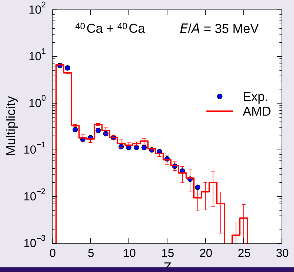
Results for multifragmentation in central collisions

Xe + Sn

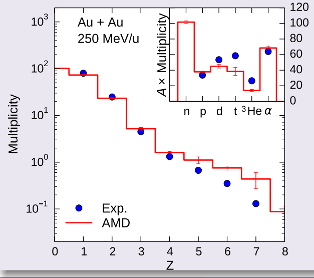




Ca + Ca at 35 MeV/u

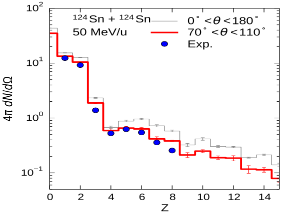


Au + Au at 250 MeV/u

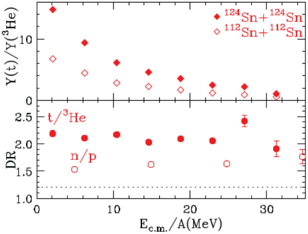


Data:
Hudan et al., PRC 67 (2003) 064613.
Hagel et al., PRC 50 (1994) 2017.
Reisdorf et al., NPA 848 (2010) 366.

Fragments and clusters in ^{124}Sn + ^{124}Sn at 50 MeV/nucleon



Large $t/^3$He ratio in experiment



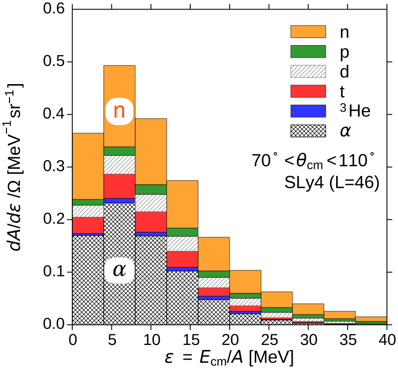
MSU Data:

Liu et al., PRC 69 (2004) 014603.

Liu et al., PRC 86 (2012) 024605.

Coupland et al., arXiv:1406.4546.

Cluster mass compositions as functions of $\varepsilon = E_{\rm cm}/A$

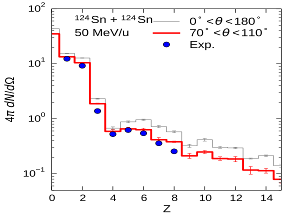


$$\left(\frac{N}{Z}\right)_{\rm system} < \left(\frac{N}{Z}\right)_{\rm gas} < \frac{N_{\rm gas}-2n_\alpha}{Z_{\rm gas}-2n_\alpha} \approx \frac{\rm n}{\rm p}, \frac{\rm t}{^3{\rm He}}$$

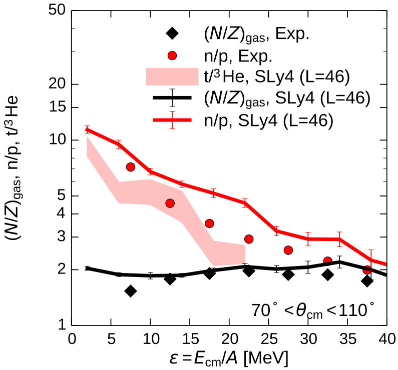
1.48 ≈ 2 ∼ 10

fractionation

Fragments and clusters in ^{124}Sn + ^{124}Sn at 50 MeV/nucleon



Ratios: n/p, t/^{3}He, $(N/Z)_{\text{gas}}$



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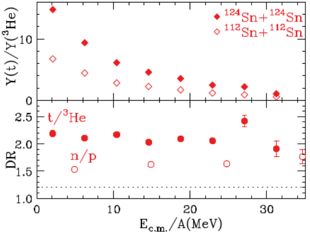

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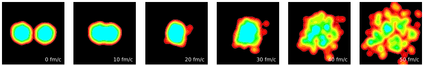
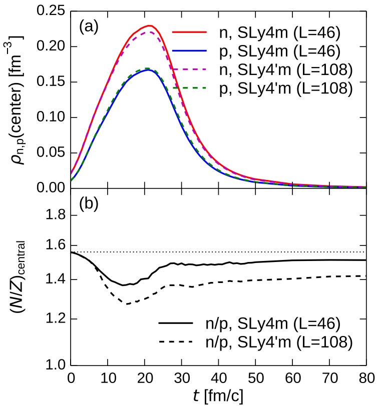
fractionation

Compression and expansion in collisions at 300 MeV/nucleon



^{132}Sn + ^{124}Sn, $E/A = 300$ MeV, $b \sim 0$



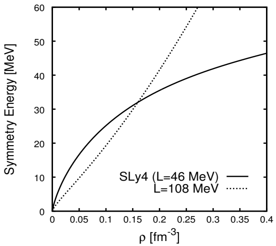
Momentum dependence of Skyrme (SLy4) interaction has been corrected for high energy collisions, in a similar way to Gale, Bertsch, Das Gupta, PRC 35 (1987) 1666.

Nuclear EOS (at $T = 0$)

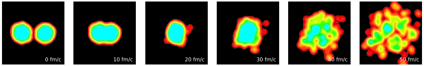
$$(E/A)(\rho_p, \rho_n) = (E/A)_0(\rho) + S(\rho)\delta^2 + \cdots$$

$$\rho = \rho_p + \rho_n, \quad \delta = \frac{\rho_n - \rho_p}{\rho_n + \rho_p}$$

- $S_0 = S(\rho_0)$
- $L = 3\rho_0 (dS/d\rho)_{\rho=\rho_0}$



Talks by P. Russotto and S. Yenello



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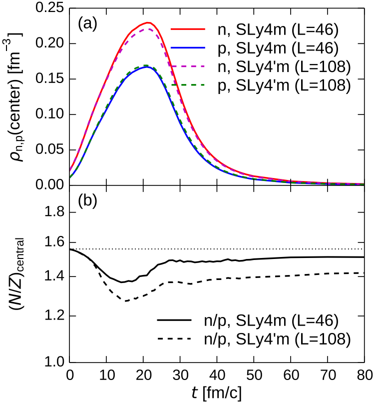

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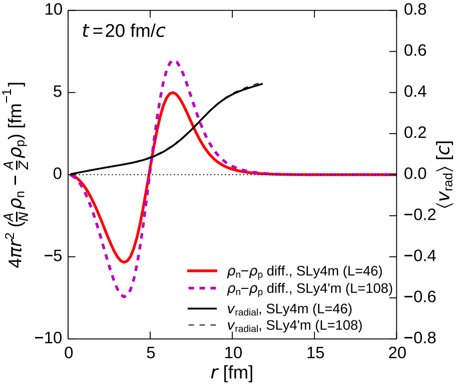
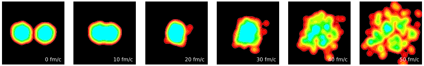
- Neutron-proton density diff. (fn of r)

$$4\pi r^2 \left[\frac{A}{N} \rho_n(r) - \frac{A}{Z} \rho_p(r) \right]$$

- Radial expansion velocity $v_{\rm rad}(r)$



Compression and expansion in collisions at 300 MeV/nucleon



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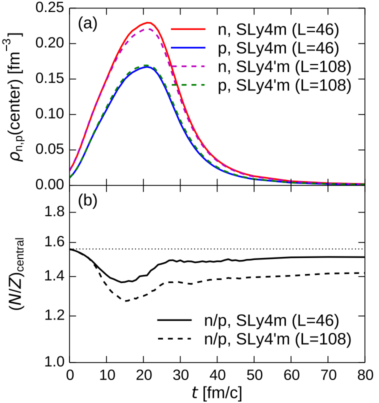

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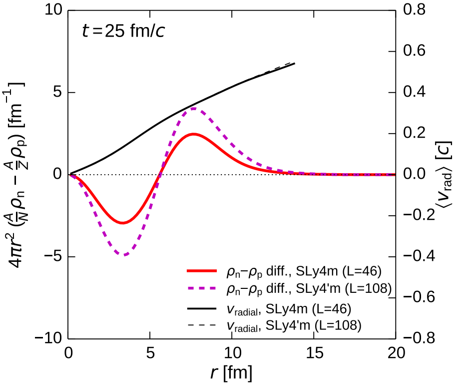

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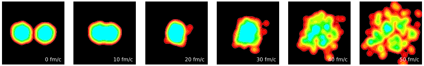

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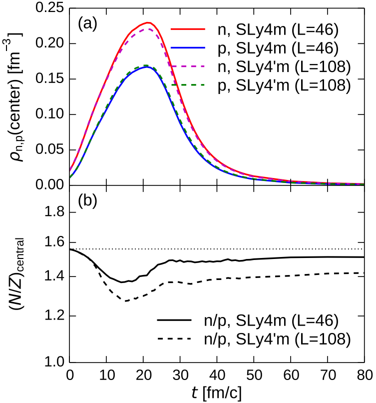

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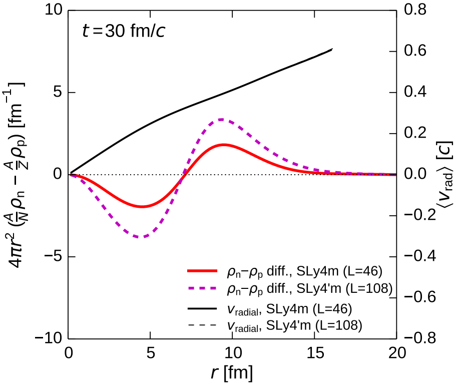

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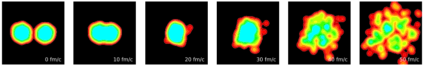

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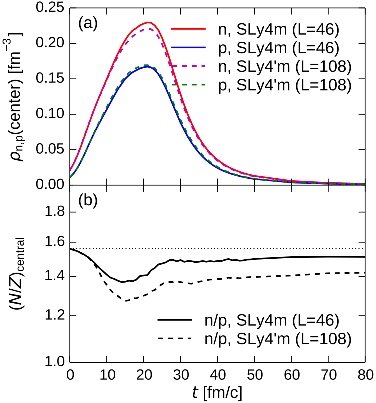

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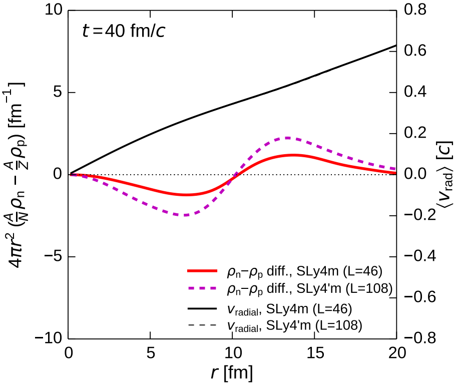

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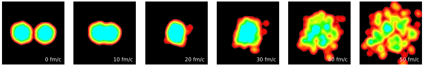

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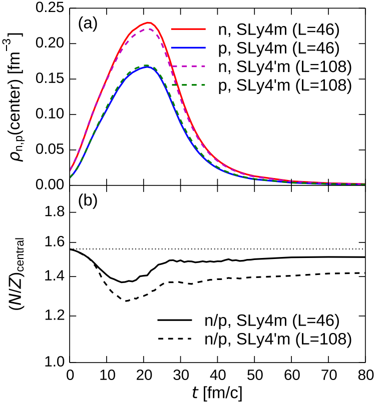

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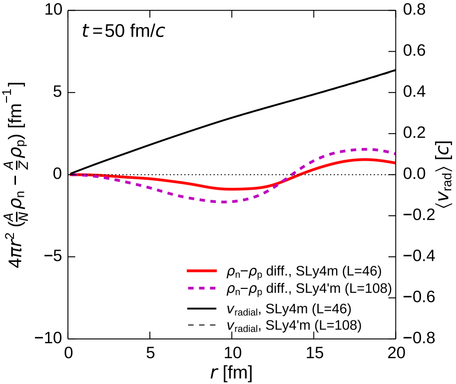

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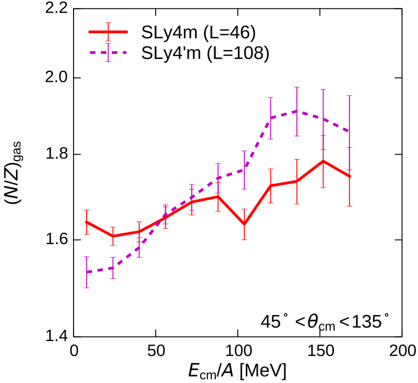
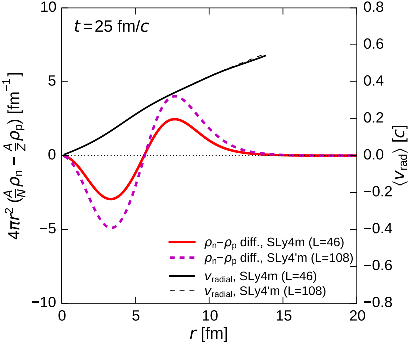
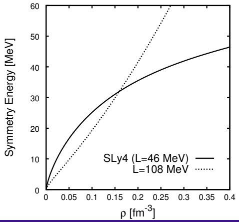
N/Z Spectrum Ratio — an observable





$$\left(\frac{N}{Z}\right)_{\text{gas}} = \frac{Y_n(v) + Y_d(v) + 2Y_t(v) + Y_h(v) + 2Y_\alpha(v)}{Y_p(v) + Y_d(v) + Y_t(v) + 2Y_h(v) + 2Y_\alpha(v)}$$



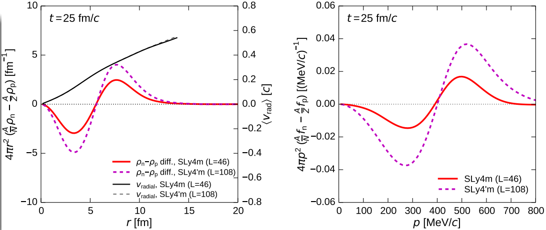
N/Z of spectrum of emitted particles is similar to the neutron-proton density difference at the compression stage.

Nucleon distributions in $\mathbf{r}$- and $\mathbf{p}$-spaces, With/without clusters

$n - p$ in $\mathbf{r}$-space $n - p$ in $\mathbf{p}$-space

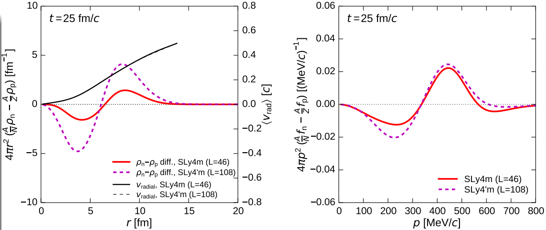
With Clusters

- Stiff symmetry energy
  - $\Leftrightarrow$ High density part: $N/Z \Downarrow$
  - $\Leftrightarrow$ High momentum part: $N/Z \Uparrow$
- Expansion is simple:
  $\mathbf{r}$-space $\Leftrightarrow$ $\mathbf{p}$-space $\Rightarrow$ Obs.



Without Clusters

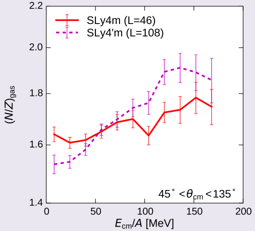
- Stiff symmetry energy
  - $\Leftrightarrow$ High density part: $N/Z \Downarrow$
  - $\Leftrightarrow$ High momentum part: ???
- Expansion is not so simple.
  $\mathbf{r}$-space $\not\Leftrightarrow$ n/p spectrum



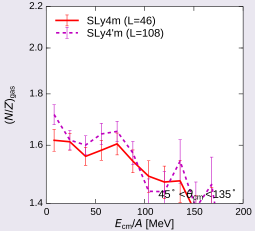
c.f. Comparison of AMD and SMF at 50 MeV/u: Colonna, Ono, Rizzo, PRC82 (2010) 054613.

N/Z Spectrum Ratio — effect of clusters

With clusters

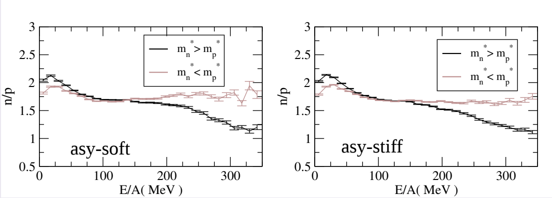


Without clusters



- Effect of $E_{\text{sym}}(\rho)$ is weak in calculations without clusters.
- Dependence on the neutron-proton effective mass splitting, i.e., $m_n^* > m_p^*$ or $m_n^* < m_p^*$. (Talks by Y.X. Zhang and H. Wolter)
- Other observables such as π^-/π^+

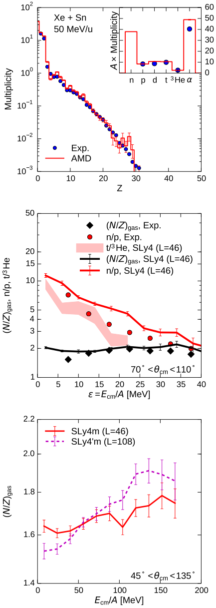
Stochastic Mean Field calculation



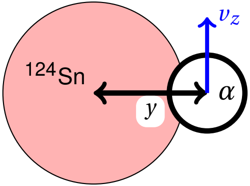
Au + Au at 400 MeV/u, Giordano et al., PRC 81 (2010) 044611.

Summary

- Clusters are important, not only because they are emitted, but also because formation and existence of light clusters influence very much the global reaction dynamics and the bulk nuclear matter properties.
- AMD has been extended to include cluster correlations in the final states of two-nucleon collisions. The binding of several clusters to form nuclei should also be considered.
- Some observables, such as n/p and $t/^3$He ratios, are sensitive to the α-particle formation.
- If cluster correlation is strong, the expansion is simple in collsions at 300 MeV/nucleon so that the high-density effect of the symmetry energy is reflected almost directly in the $(N/Z)_{\text{gas}}$ spectrum ratio.



Cluster put into a nucleus in AMD



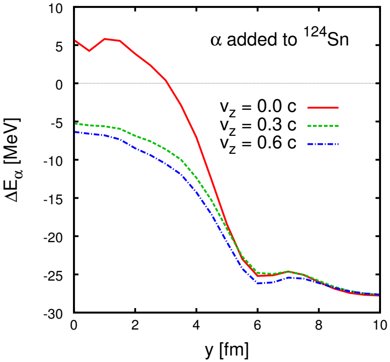
α cluster $|\alpha,\mathbf{Z}\rangle$: Four wave packets with different spins and isospins at the same phase space point $\mathbf{Z}$.

$$E_\alpha : \quad \mathscr{A}\, |\alpha,\mathbf{Z}\rangle|^{124}\mathrm{Sn}\rangle$$

$$E_\mathrm{N} : \quad \mathscr{A}\, |\mathbf{Z}\rangle|^{124}\mathrm{Sn}\rangle \quad (\mathrm{N} = p\uparrow, p\downarrow, n\uparrow, n\downarrow)$$

$$-B_\alpha = \Delta E_\alpha = E_\alpha - (E_{p\uparrow} + E_{p\downarrow} + E_{n\uparrow} + E_{n\downarrow})$$

(Energies are defined relative to $|^{124}\mathrm{Sn}\rangle$.)



$$\frac{\mathrm{Re}\,\mathbf{Z}}{\sqrt{\nu}} = (0, y, 0),$$

$$\frac{2\hbar\sqrt{\nu}\,\mathrm{Im}\,\mathbf{Z}}{M} = (0, 0, v_z)$$

- Distance from the center y
  $\approx$ Dependence on density
- Dendence on $P_\alpha = M_\alpha v_z$
- Due to the density dependence of the Skyrme force, the interaction between nucleons in the α cluster is weakened in the nucleus.

Energy is OK, but the probability is . . .