Reaction theory: status and perspectives.

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Outline

1. Intro
2. The CDCC method and its extensions
3. CDCC and Faddeev
4. Core excitations
5. Four-body CDCC
6. Inclusive breakup
Why reaction theory is important?

- Reaction theory provides the necessary framework to extract meaningful **structure** information from measured **cross sections** and also permits the understanding of the **dynamics** of nuclear collisions.

- The many-body scattering problem is not solvable in general, so specific models tailored to specific types of reactions are used (elastic, breakup, transfer, knockout...) each of them emphasizing some particular degrees of freedom.

- In particular, exotic nuclei close to driplines are usually weakly-bound and **breakup** (coupling to the continuum) is important and must be taken into account in the reaction model.

- **Few-body** models provide an appealing simplification of this complicated problem.
Nuclear reaction theory over the world
From the many-body problem to the few-body picture

Microscopic models

- Fragments described microscopically
- Realistic NN interactions (Pauli properly accounted for)
- Numerically demanding / not simple interpretation.
From the many-body problem to the few-body picture

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**Inert cluster models**
- Ignores cluster excitations (only few-body d.o.f).
- Phenomenological inter-cluster interactions (aprox. Pauli).
- Exactly solvable (in some cases).
- Achieved for 3-body and 4-body (coupled-channels, semiclassical).
From the many-body problem to the few-body picture

Microscopic models
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- Numerically demanding / not simple interpretation.

Non-inert-core few-body models
- Few-body + some relevant collective d.o.f.
- Pauli approximately accounted for.
- Achieved for 3-body problems (coupled-channels).

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Inert-core models: the CDCC method example
The **Continuum-Discretized Coupled-Channels method (CDCC)**

- **Three-body reaction** model: two-body projectile with inert cores on inert target.

- Breakup treated as inelastic excitations to two-body continuum.

- Continuum states are represented by a finite set of square-integrable functions (discretization).

Three-body wavefunction (after discretization): \([H - E]\psi_{\text{CDCC}} = 0\).

\[
\psi_{\text{CDCC}}(r, R) = \phi_0(k_0, r)\chi_0(K_0, R) + \sum_{n', j, \pi} \phi_{n', \pi}^j(k_{n'}, r')\chi_{n', j, \pi}(K_{n'}, R')
\]

- Only provides **elastics** and **breakup** (not transfer)
The origins of CDCC


Full numerical implementation by Kyushu group (Sakuragi, Yahirou, Kamimura, and co.): Prog. Theor. Phys. (Kyoto) 68, 322 (1982)
Application of the CDCC formalism: d$^+\ {^{58}\text{Ni}}$

Coupling to continuum states produce:

- Polarization of the projectile (modification of real part)
- Flux removal (absorption) from the elastic channel (imaginary part)

$$(\frac{d\sigma}{d\Omega})/(\frac{d\sigma_R}{d\Omega})$$

Exp. (80.0 MeV)
Exp. (79.0 MeV)
CDCC: No continuum

$\epsilon_{\text{max}}$
$\epsilon_{\text{min}}$

$\phi_{l,n}$

l=0, 2
n=1, 2, 3

ground state

$\epsilon = -2.22$ MeV

Limited to two-body projectiles, with inert fragments
Application of the CDCC formalism: d+ $^{58}$Ni

Coupling to continuum states produce:
- Polarization of the projectile (modification of real part)
- Flux removal (absorption) from the elastic channel (imaginary part)

Limited to two-body projectiles, with inert fragments
Some extensions of the standard CDCC method

1. Inclusion of target excitations, eg. $d + A \rightarrow p + n + A^*$

2. Inclusion of core excitations in the projectile (eg. $^{11}\text{Be}=^{10}\text{Be} + n$).

3. Extension to 4-body reactions (3+1) ($^6\text{He}$, $^{11}\text{Li}$, $^9\text{Be}$), eg. $^{11}\text{Li}=^9\text{Li}+n+n$.
   - [ Mahir Hussein’s talk on Thursday, poster by Jesús Casal ]

4. Use of microscopic projectile WFs (microscopic CDCC):
   - Brussels: Descouvemont and Hussein, PRL 111, 082701 (2013)
Testing CDCC against the Faddeev formalism
In the 60s L. Faddeev provided the rigorous (exact) solution of a pure three-body scattering problem fulfilling correct boundary conditions.

\[ \psi_{\text{Fad}} = \psi_1 + \psi_2 + \psi_3 \]

- Treats elastic, inelastic, breakup and transfer simultaneously and on equal footing.
- But...numerically involved (not easily applicable to heavy ions), but very useful for benchmark of approximate models (eg. CDCC, DWBA...).
The CDCC method can be derived as an approximation of the Faddeev equations in a truncated model space for a selected Jacobi set [Austern, Yaho, Kawai, PRL63, 2649 (1989)].

\[ \Psi_{\text{Fad}} = \Psi_1 + \Psi_2 + \Psi_3 \approx \Psi_1 \]

Does the CDCC solution converge to Faddeev, when the model space is increased?

- Austern, Yaho, Kawai, PRL63, 2649 (1989): YES
- Sawada, Thushima, PTP76, 440 (1986): NO
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CDCC as an approximation to Faddeev

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**Goal:** test CDCC and Faddeev solutions using the same three-body Hamiltonian.

### Elastic:

\[ ^{58}\text{Ni}(d,d)^{58}\text{Ni} \]

### Exclusive breakup:

\[ d + ^{12}\text{C} \rightarrow p + n + ^{12}\text{C} \]

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Breakdown of the CDCC ansatz: $d^+{}^{10}\text{Be} \rightarrow p+n^+{}^{10}\text{Be}$

But...differences have been evidenced for breakup at small incident energies (N.J. Upadhyay, A. Deltuva, F.M. Nunes, PRC85, 054621 (2012))

CDCC can be (and must be!) improved
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CDCC can be (and must be!) improved
Beyond the strict few-body picture: the effect of core excitation

- Existing few-body models (CDCC, Faddeev, Glauber, etc) usually rely on inert core models.

- However, core excitations are expected to affect:
  1. the **structure** of the projectile (c.f. talk by Silvia Leoni)
  2. the **dynamics** (interplay of few-body and collective excitations)
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New questions for reaction theory:

- Are these core excitations well simulated within the effective fragment-fragment interactions.
- How to extend existing few-body models to incorporate these effects?
Core excitation in breakup: *frozen-halo* picture

\[ \Psi_{JM}(\vec{r}, \xi) = [\varphi_{\ell,j}(\vec{r}) \otimes \Phi_I(\xi)]_{JM} \]

- \( \varphi_{\ell,j}(\vec{r}) \) = valence particle wavefunction
- \( \Phi_I(\xi) \) = core wavefunction (frozen)
Core excitation in breakup: *frozen-halo* picture

\[
\Psi_{JM}(\mathbf{r}, \xi) = \left[ \varphi_{\ell,j}^{J}(\mathbf{r}) \otimes \Phi_{I}(\xi) \right]_{JM}
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- \( \varphi_{\ell,j}^{J}(\mathbf{r}) \) = valence particle wavefunction
- \( \Phi_{I}(\xi) \) = core wavefunction (*frozen*)

10\(^{+}\)Be(I) \(\rightarrow\) 11\(^{+}\)Be(J)

\[
\begin{array}{c}
3/2^{+} \\
5/2^{+} \\
1/2^{-} \\
1/2^{+}
\end{array}
\begin{array}{c}
3.41\, \text{MeV} \\
1.78\, \text{MeV} \\
0.32\, \text{MeV} \\
\text{Be}
\end{array}
\begin{array}{c}
10\text{Be}^{(0^{+})} \times 1d_{3/2} \\
10\text{Be}^{(0^{+})} \times 1d_{5/2} \\
10\text{Be}^{(0^{+})} \times 1p_{1/2} \\
11\text{Be}^{(0^{+})} \times 2s_{1/2}
\end{array}
\]

10\(^{+}\)Be(I) \(\rightarrow\) 11\(^{+}\)Be(J) \(\rightarrow\) Pb \(\rightarrow\) 10\(^{+}\)Be \(\rightarrow\) n
How do core excitations affect the breakup of weakly-bound nuclei?

\[ \Psi_{JM}(\vec{r}, \xi) = \sum_{\ell, j, I} \left[ \varphi_{\ell, j, I}(\vec{r}) \otimes \Phi_I(\xi) \right]_{JM} \]
How do core excitations affect the breakup of weakly-bound nuclei?

\[ \Psi_{JM}(\vec{r}, \xi) = \sum_{\ell,j,l} \left[ \varphi^J_{\ell,j,l}(\vec{r}) \otimes \Phi^I_{l}(\xi) \right]_{JM} \]

Core excitations may affect the structure and the dynamics of the reaction.
Extending CDCC to include core excitations

- **Standard CDCC** ⇒ use coupling potentials:

\[
V_{\alpha;\alpha'}(R) = \langle \Psi^\alpha_{J'M'}(\vec{r}) | V_{vt}(r_{vt}) + V_{ct}(r_{ct}) | \Psi^\alpha_{JM}(\vec{r}) \rangle
\]

- **Extended CDCC (XCDCC)** ⇒ use generalized coupling potentials

\[
V_{\alpha;\alpha'}(R) = \langle \Psi^\alpha_{J'M'}(\vec{r}, \xi) | V_{vt}(r_{vt}) + V_{ct}(r_{ct}, \xi) | \Psi^\alpha_{JM}(\vec{r}, \xi) \rangle
\]

- \(\Psi^\alpha_{JM}(\vec{r}, \xi)\): projectile WFs involving core-excited admixtures (structure).

- \(V_{ct}(r_{ct}, \xi)\): non-central potential allowing for core excitations/de-excitations (dynamic core excitation).

- Summers *et al*, PRC74 (2006) 014606 (bins)
Evidence of dynamical core excitations in $p^{(11}\text{Be},p')$ at 64 MeV/u (MSU)

Data: Shrivastava et al, PLB596 (2004) 54 (MSU)

- $E_{\text{rel}} = 0$–2.5 MeV contains $5/2^+$ resonance (expected single-particle mechanism)
- $E_{\text{rel}} = 2.5$–5 MeV contains $3/2^+$ resonance (expected core excitation mechanism)

| State           | Model     | $|0^+ \otimes (\ell s)\rangle|$ | $|2^+ \otimes s_{1/2}\rangle|$ |
|-----------------|-----------|---------------------------------|---------------------------------|
| $1/2^+$ (g.s.)  | PRM       | 0.857                           | -                               |
|                 | SM (WBT)  | 0.76                            | -                               |
| $5/2^+$ (1.78 MeV) | PRM        | 0.702                          | 0.177                           |
|                 | SM (WBT)  | 0.682                          | 0.177                           |
| $3/2^+$        | PRM       | 0.165                          | 0.737                           |
|                 | SM (WBT)  | 0.068                          | 0.534                           |
Evidence of *dynamical* core excitations in $p(^{11}\text{Be},p')$ at 64 MeV/u (MSU)

**Data:** Shrivastava et al, PLB596 (2004) 54 (MSU)

Dynamic core excitations gives additional (and significant!) contributions to breakup

R.de Diego et al, PRC85, 054613 (2014)
Extension to 3-body projectiles (4-body reactions)
Extension to 3-body projectiles (4b-CDCC)

To extend the CDCC formalism, one needs to evaluate the new coupling potentials:

\[ V_{n,n'}(R) = \int dr \, \phi^*_n(x,y) \left\{ V_{nt}(r_1) + V_{nt}(r_2) + V_{ct}(r_3) \right\} \phi_{n'}(x,y) \]

\[ \phi_n(x,y) \text{ 3-body WFs for bound and continuum states (difficult to calculate!)}. \]
Extension to 4-body reactions: $^{11}\text{Li}$ elastics and breakup

Large 3-body breakup ($^{11}\text{Li} \rightarrow ^9\text{Li}+n+n$) and subsequent strong reduction of elastics mostly due to strong E1 response of $^{11}\text{Li}$ (Coulomb polarizability).

- Both effects well accounted for by 4b-CDCC, but the underlying 3b model of $^{11}\text{Li}$ predicts a B(E1) larger than previously reported.
Some conclusions on 4b-CDCC

✔ 4-body reactions with inert cores are nowadays feasible within the CDCC scheme.

✖ What about core excitations? (not included in present reaction calculations).

✖ Upcoming experiments in full kinematics provide more detailed observables and with higher accuracy:

For inclusive measurements (eg. $^9$Li singles after $^{11}$Li) non-elastic breakup and transfer may contribute also (not included by present models)
The problem of non-elastic breakup
Elastic and non-elastic breakup

CDCC/Faddeev provides only the EBU part \( \Rightarrow \) How to compute the NEB part?
Why studying inclusive breakup?

- **Surrogate reactions** at low energies to extract $n + A$ CN cross sections:
  
  E.g.: $d + A \rightarrow p + B^*$

- Understanding of large “singles” yields in breakup of weakly-bound nuclei.
  
  E.g.: $^6,^7\text{Li} + A \rightarrow \alpha + X$

- Evaluation of **incomplete fusion** (part of the NBU cross section)

- Benchmark of semiclassical approaches for **knockout reactions** at intermediate energies (**stripping**)
Why studying inclusive breakup?

- **Surrogate reactions** at low energies to extract cross sections:
  E.g.: $d + A \rightarrow p + B$

- Understanding of large "singles" yields in breakup of weakly-bound nuclei.
  E.g.: $^6,^7\text{Li} + A$

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- Benchmark of semiclassical approaches for knockout reactions at intermediate energies (*stripping*)

*Eg: $\alpha$ production in $^6\text{Li} + ^{209}\text{Bi} \rightarrow \alpha + A$ CN cross section*
Searching in the eighties for inclusive breakup models

- **Baur & co**: DWBA sum-rule with surface approximation.

- **Hussein & McVoy**: extraction of singles cross section combining the spectator model with sum rule over final states.

- **Ichimura, Austern, Vincent (IAV)**: Post-form DWBA.

- **Udagawa, Tamura (UT)**: prior-form DWBA.

Most of these theories have fallen into disuse and should be revisited.
Searching in the eighties for inclusive breakup models

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Formal expression for non-elastic breakup (NEBU)

- Inclusive breakup: \((b + x) + A \rightarrow b + (x + A)^*\)
- Inclusive differential cross section: \(\sigma_{inc}^b = \sigma_{EBU}^b + \sigma_{NEBU}^b\)
- Within an spectator model, \(\sigma_{NEBU}^b\) can be interpreted as the absorption occurring in the \(x + A_{gs}\) channel, following \(a \rightarrow b + x\)

\[
\frac{d\sigma_{NEBU}}{d\Omega_b dE_b} = -\frac{2}{\hbar v_a} \rho_b(E_b) \langle \varphi_x | W_{xA} | \varphi_x \rangle
\]

- \(\varphi_x\) describes the evolution of \(x\) after dissociating from \(a\).

\[
[K_x + U_{xA} - E_x] \varphi_x(r_x) = (\chi_b^{(-)} | V_{bx} | \psi_{xb}^{(+)}
\]

- \(\psi_{xb}^{(+)}\) can be approximated by the DWBA, CDCC or Faddeev wavefunctions.
Application of IAV model to deuteron inclusive breakup

Pioneering applications by Baur et al. in the 1970s using zero-range DWBA.

Pampus et al, NPA311 (1978)141

Ongoing applications to weakly-bound nuclei using CDCC for the EBU part and the IAV model for NEB part (finite-range DWBA.)

J. Lei (talk on Tuesday)
Standard few-body models and their extensions

- Few-body models
  - Core excitations
    - ✔ Deltuva (2013)
  - “Exclusive” models
    - ✔ P. Batham, Thompson, Tostevin (2005)
  - “Inclusive” models
    - ✔ knockout reactions
    - Hussein et al (never applied?)
    - Austern et al (never applied?)

- Faddeev
- CDCC / CRC
- Semiclassical
- DWBA (transfer)
Summary and conclusions

**CDCC and its extensions**

- **Core excitations** implemented (XCDCC), but with (too) simple structure models $\Rightarrow$ more microscopic inputs needed.
- **4b-CDCC** feasible, but numerical convergence is still challenging.
- Required evaluation of 2-3 particles **correlations** after breakup.

**Faddeev**

- Feasible for light targets *(Deltuva et al. )* 😊
- Ongoing initiatives to develop Faddeev codes (eg. TORUS at USA) 😊
- Limited so far to light targets (Coulomb) 😞

**Evaluation of inclusive cross sections**

- Old theories (1980s) are being revisited and implemented 😊.
- So far, they use DWBA, but may need to go beyond for halo nuclei. 😊
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