Reaction theory: status and perspectives.

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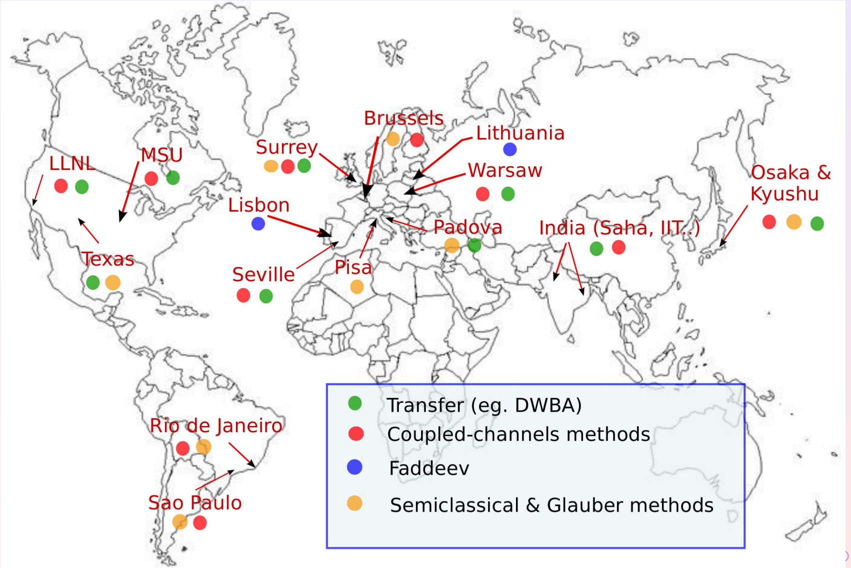
Outline

1. Intro
2. The CDCC method and its extensions
3. CDCC and Faddeev
4. Core excitations
5. Four-body CDCC
6. Inclusive breakup

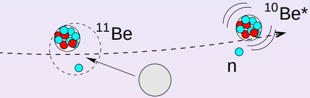
Why reaction theory is important?

- Reaction theory provides the necessary framework to extract meaningful structure information from measured cross sections and also permits the understanding of the dynamics of nuclear collisions.
- The many-body scattering problem is not solvable in general, so specific models tailored to specific types of reactions are used (elastic, breakup, transfer, knockout...) each of them emphasizing some particular degrees of freedom.
- In particular, exotic nuclei close to driplines are usually weakly-bound and breakup (coupling to the continuum) is important and must be taken into account in the reaction model.
- Few-body models provide an appealing simplification of this complicated problem.

Nuclear reaction theory over the world



From the many-body problem to the few-body picture



Microscopic models

- ✔ Fragments described microscopically
- ✔ Realistic NN interactions (Pauli properly accounted for)
- ✖ Numerically demanding / not simple interpretation.

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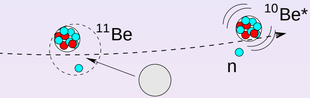

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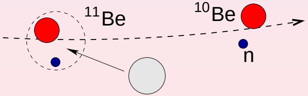


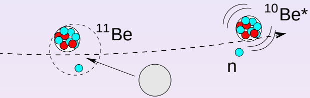
Inert cluster models

- ✖ Ignores cluster excitations (only few-body d.o.f).
- ✖ Phenomenological inter-cluster interactions (aprox. Pauli).
- ✔ Exactly solvable (in some cases).
- ✔ Achieved for 3-body and 4-body (coupled-channels, semiclassical).

From the many-body problem to the few-body picture

Many-body

Microscopic models



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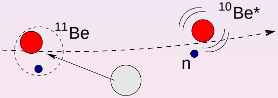
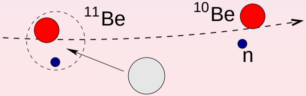
Non-inert-core few-body models



- ✔ Few-body + some relevant collective d.o.f.
- ✔ Pauli approximately accounted for.
- ✔ Achieved for 3-body problems (coupled-channels).

Inert cluster models



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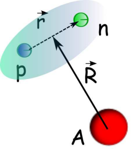
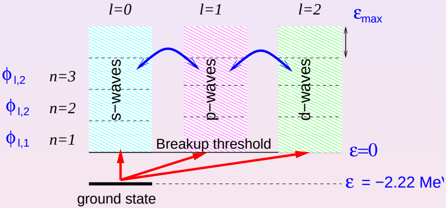
Few-body

Inert-core models: the CDCC method example

The Continuum-Discretized Coupled-Channels method (CDCC)

- Three-body reaction model: two-body projectile with inert cores on inert target.
- Breakup treated as inelastic excitations to two-body continuum.
- Continuum states are represented by a finite set of square-integrable functions (discretization).





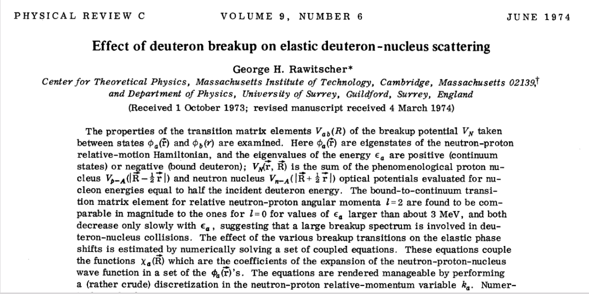
- Three-body wavefunction (after discretization): $[H - E]\Psi^{\mathrm{CDCC}} = 0$.

$$\Psi^{\mathrm{CDCC}}(\mathbf{r},\mathbf{R}) = \underbrace{\phi_0(k_0,\mathbf{r})}_{\text{Bound state(s)}}\chi_0(\mathbf{K}_0,\mathbf{R}) + \sum_{n',j,\pi} \underbrace{\phi_{n'}^{j\pi}(k_{n'},\mathbf{r}')}_{\text{Unbound states}}\chi_{n',j,\pi}(\mathbf{K}_{n'},\mathbf{R}')$$

- Only provides elastics and breakup (not transfer)

The origins of CDCC

- Continuum discretization method proposed by G.H. Rawitscher [PRC9, 2210 (1974)] and Farrell, Vincent and Austern [Ann.Phys.(New York) 96, 333 (1976)] to describe deuteron scattering.



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Effect of deuteron breakup on elastic deuteron-nucleus scattering

George H. Rawitscher*

Center for Theoretical Physics, Massachusetts Institute of Technology, Cambridge, Massachusetts 02139,† *and Department of Physics, University of Surrey, Guildford, Surrey, England*

(Received 1 October 1973; revised manuscript received 4 March 1974)

The properties of the transition matrix elements $V_{ab}(R)$ of the breakup potential V_N taken between states $\phi_a(\vec{r})$ and $\phi_b(r)$ are examined. Here $\phi_a(\vec{r})$ are eigenstates of the neutron-proton relative-motion Hamiltonian, and the eigenvalues of the energy ϵ_a are positive (continuum states) or negative (bound deuteron); $V_N(\vec{r},\vec{R})$ is the sum of the phenomenological proton nucleus $V_{p-A}(|\vec{R}-\frac{1}{2}\vec{r}|)$ and neutron nucleus $V_{n-A}(|\vec{R}+\frac{1}{2}\vec{r}|)$ optical potentials evaluated for nucleon energies equal to half the incident deuteron energy. The bound-to-continuum transition matrix element for relative neutron-proton angular momenta $l=2$ are found to be comparable in magnitude to the ones for $l=0$ for values of ϵ_a larger than about 3 MeV, and both decrease only slowly with ϵ_a, suggesting that a large breakup spectrum is involved in deuteron-nucleus collisions. The effect of the various breakup transitions on the elastic phase shifts is estimated by numerically solving a set of coupled equations. These equations couple the functions $\chi_a(\vec{R})$ which are the coefficients of the expansion of the neutron-proton-nucleus wave function in a set of the $\phi_a(\vec{r})$'s. The equations are rendered manageable by performing a (rather crude) discretization in the neutron-proton relative-momentum variable k_a. Numer-

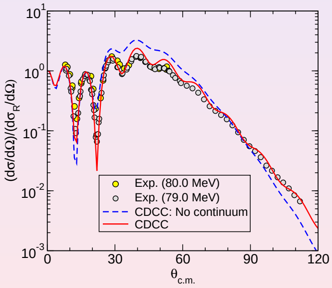
George Rawitscher

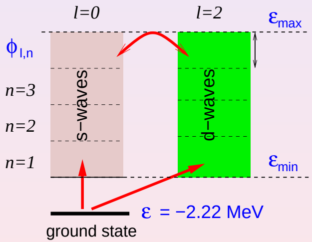
- Full numerical implementation by Kyushu group (Sakuragi, Yahiro, Kamimura, and co.): Prog. Theor. Phys.(Kyoto) 68, 322 (1982)

Application of the CDCC formalism: d+ ^{58}Ni

Coupling to continuum states produce:

- Polarization of the projectile (modification of real part)
- Flux removal (absorption) from the elastic channel (imaginary part)





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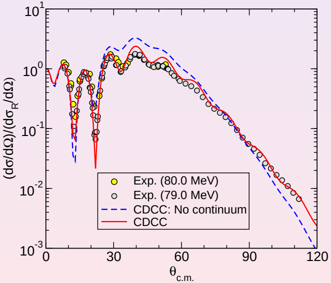

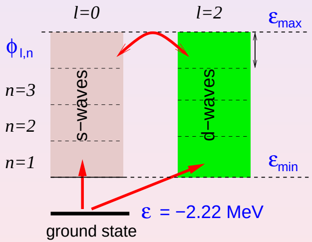

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Some extensions of the standard CDCC method

1. Inclusion of target excitations, eg. $d + A \to p + n + A^*$
   - ☞ Kyushu: Yahiro *et al*, Prog. Theor. Phys. Suppl. 89, 32 (1986)

2. Inclusion of core excitations in the projectile (eg. ^{11}Be=^{10}Be +n).
   - ☞ Michigan/Surrey (bins) : Summers *et al*, PRC74, 014606 (2006)
   - ☞ Seville (pseudo-states): R. de Diego *et al*, PRC 89, 064609 (2014)

3. Extension to 4-body reactions (3+1) (^{6}He, ^{11}Li, ^{9}Be), eg. ^{11}Li= ^{9}Li+n+n
   - ☞ Kyushu: Matsumoto *et al*, NPA738, 471 (2004), PRC70, 061601(R) (2004).
   - ☞ Seville: Rodriguez-Gallardo *et al*, PRC72 (2005) 024007, PRC77, 064609 (2008).
   - ☞ Brussels: Descouvemont *et al*, PRC91, 024606 (2015)

   [Mahir Hussein's talk on Thursday, poster by Jesús Casal]

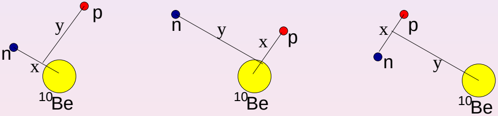
4. Use of microscopic projectile WFs (microscopic CDCC):
   - ☞ Kyushu: Sakuragi et al, Prog. Theor. Phys. Suppl. 89, 136 (1986).
   - ☞ Brussels: Descouvemont and Hussein, PRL 111, 082701 (2013)

Testing CDCC against the Faddeev formalism

Faddeev & nuclear reactions

In the 60s L. Faddeev provided the rigorous (exact) solution of a pure three-body scattering problem fulfilling correct boundary conditions.

$$\boxed{\Psi^{\mathrm{Fad}} = \Psi_1 + \Psi_2 + \Psi_3}$$

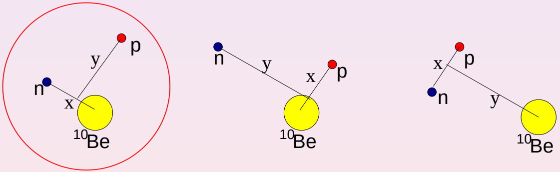


- Treats elastic, inelastic, breakup and transfer simultaneously and on equal footing.
- But...numerically involved (not easily applicable to heavy ions), but very useful for benchmark of approximate models (eg. CDCC, DWBA...).

CDCC as an approximation to Faddeev

- The CDCC method can be derived as an approximation of the Faddeev equations in a truncated model space for a selected Jacobi set [Austern,Yahiro,Kawai, PRL63, 2649 (1989)].

$$\boxed{\Psi^{\mathrm{Fad}} = \Psi_1 + \Psi_2 + \Psi_3 \approx \Psi_1}$$



- Does the CDCC solution converge to Faddeev, when the model space is increased?
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    - Sawada, Thushima, PTP76, 440 (1986): NO

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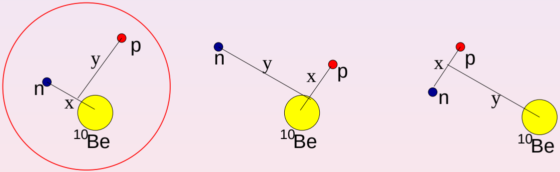

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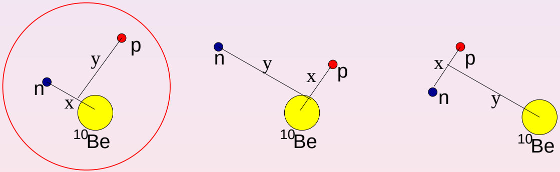

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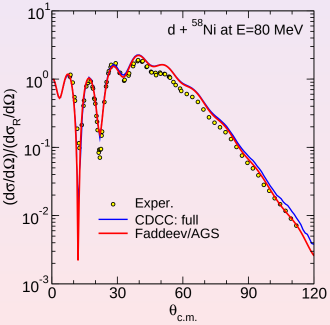
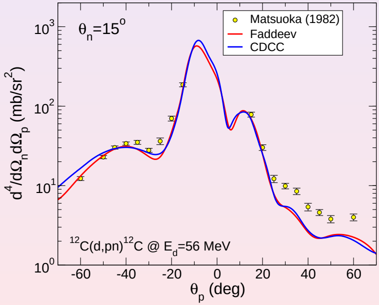
Testing CDCC against Faddeev for elastic scattering

Goal: test CDCC and Faddeev solutions using the same three-body Hamiltonian.

Elastic:
^{58}Ni(d,d)^{58}Ni



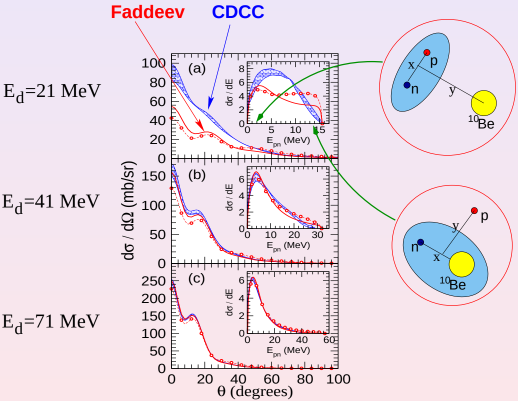
Exclusive breakup:
d+^{12}C $\to$ p+n+^{12}C



A.Deltuva, A.M.M., E.Cravo, F.M.Nunes, A.C.Fonseca, PRC 76, 064602 (2007)

Breakdown of the CDCC ansatz: d+ ^{10}Be → p+n+ ^{10}Be

☞ But...differences have been evidenced for breakup at small incident energies (N.J. Upadhyay, A. Deltuva, F.M. Nunes, PRC85, 054621 (2012))



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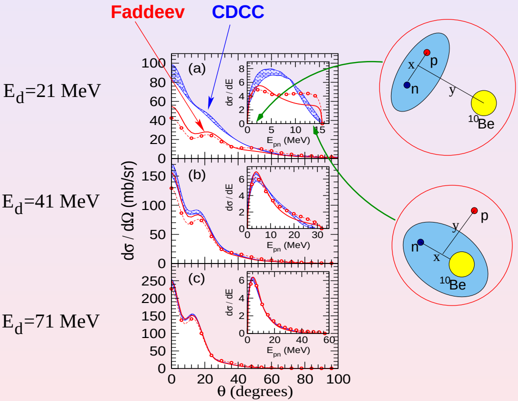

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Beyond the strict few-body picture: the effect of core excitation

- Existing few-body models (CDCC, Faddeev, Glauber, etc) usually rely on inert core models.
- However, core excitations are expected affect:
  1. the structure of the projectile (c.f. talk by Silvia Leoni)
  2. the dynamics (interplay of few-body and collective excitations)

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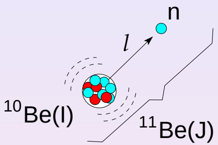
  2. the dynamics (interplay of few-body and collective excitations)

New questions for reaction theory:

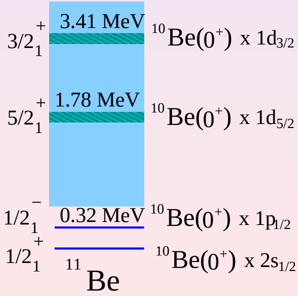
- Are these core excitations well simulated within the effective fragment-fragment interactions.
- How to extend existing few-body models to incorporate these effects?

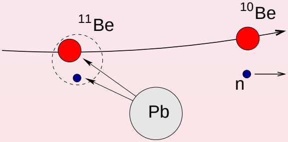
Core excitation in breakup: *frozen-halo* picture

$$\boxed{\Psi_{JM}(\vec{r},\xi) = \left[\varphi^{J}_{\ell,j}(\vec{r}) \otimes \Phi_I(\xi)\right]_{JM}}$$



- $\varphi^{J}_{\ell,j}(\vec{r})$= valence particle wavefunction
- $\Phi_I(\xi)$= core wavefunction (*frozen*)





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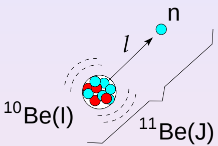

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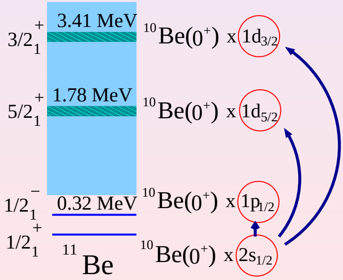

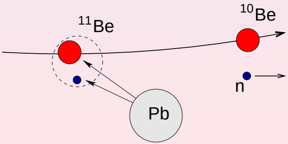

How do core excitations affect the breakup of weakly-bound nuclei?

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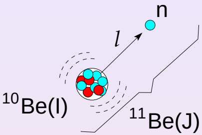

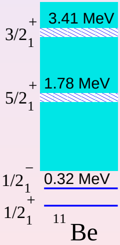

$e[\,^{10}\mathrm{Be}(0^+) \times 1d_{3/2}\,] + f\,[\,^{10}\mathrm{Be}(2^+) \times 2s_{1/2}\,]$

$c[\,^{10}\mathrm{Be}(0^+) \times 1d_{5/2}\,] + d\,[\,^{10}\mathrm{Be}(2^+) \times 1d_{5/2}\,]$

$A[\,^{10}\mathrm{Be}(0^+) \times 1p_{1/2}\,] + B\,[\,^{10}\mathrm{Be}(2^+) \times 1p_{3/2}\,]$

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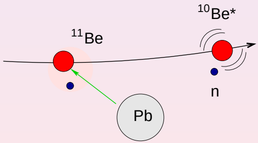

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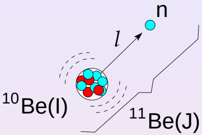
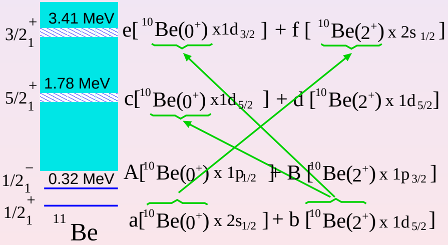
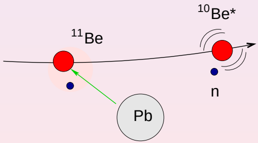






☞ *Core excitations may affect the structure and the dynamics of the reaction*

Extending CDCC to include core excitations

- Standard CDCC $\Rightarrow$ use coupling potentials:

$$\boxed{V_{\alpha;\alpha'}(\mathbf{R}) = \langle \Psi^{\alpha'}_{J'M'}(\vec{r}) | V_{vt}(r_{vt}) + V_{ct}(r_{ct}) | \Psi^{\alpha}_{JM}(\vec{r}) \rangle}$$

- Extended CDCC (XCDCC) $\Rightarrow$ use generalized coupling potentials

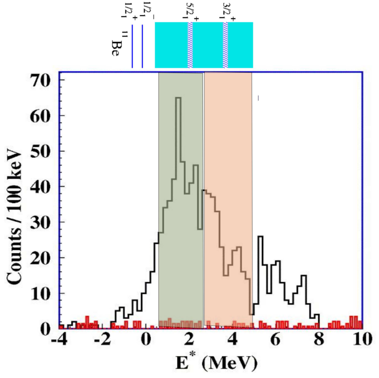
$$\boxed{V_{\alpha;\alpha'}(\mathbf{R}) = \langle \Psi^{\alpha'}_{J'M'}(\vec{r},\xi) | V_{vt}(r_{vt}) + V_{ct}(r_{ct},\xi) | \Psi^{\alpha}_{JM}(\vec{r},\xi) \rangle}$$

☞ $\Psi^{\alpha}_{JM}(\vec{r},\xi)$: projectile WFs involving core-excited admixtures (structure).
☞ $V_{ct}(r_{ct},\xi)$: non-central potential allowing for core excitations/de-excitations (dynamic core excitation).

- Summers *et al*, PRC74 (2006) 014606 (bins)
- R. de Diego *et al*, PRC 89, 064609 (2014) (THO pseudo-states)

Evidence of *dynamical* core excitations in p(^{11}Be,p') at 64 MeV/u (MSU)

Data: Shrivastava et al, PLB596 (2004) 54 (MSU)



State	Model	$\lvert 0^+ \otimes (\ell s)j \rangle$	$\lvert 2^+ \otimes s_{1/2} \rangle$
$1/2^+$ (g.s.)	PRM	0.857	–
	SM (WBT)	0.76	–
$5/2^+$ (1.78 MeV)	PRM	0.702	0.177
	SM(WBT)	0.682	0.177
$3/2^+$ (3.41 MeV)	PRM	0.165	0.737
	SM(WBT)	0.068	0.534

- E_{rel}=0–2.5 MeV contains $5/2^+$ resonance (expected single-particle mechanism)
- E_{rel}=2.5–5 MeV contains $3/2^+$ resonance (expected core excitation mechanism)

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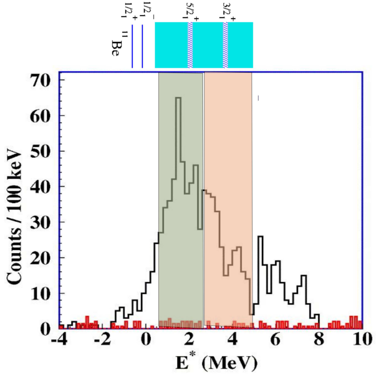
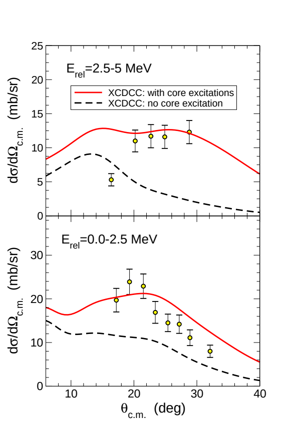
Data: Shrivastava et al, PLB596 (2004) 54 (MSU)



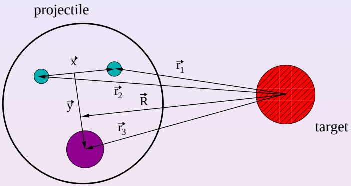


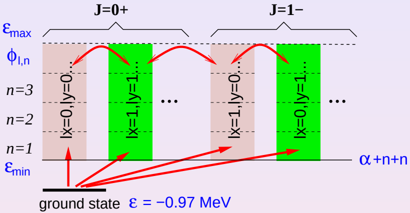
R.de Diego et al, PRC85, 054613 (2014)

☞ *Dynamic core excitations gives additional (and significant!) contributions to breakup*

Extension to 3-body projectiles (4-body reactions)

Extension to 3-body projectiles (4b-CDCC)



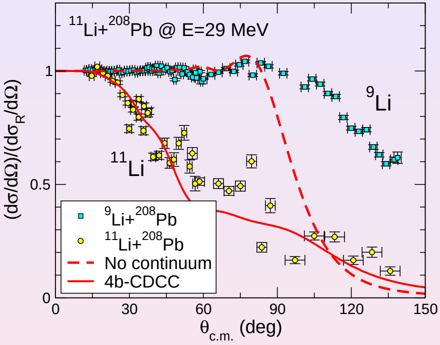


☞ To extend the CDCC formalism, one needs to evaluate the new coupling potentials:

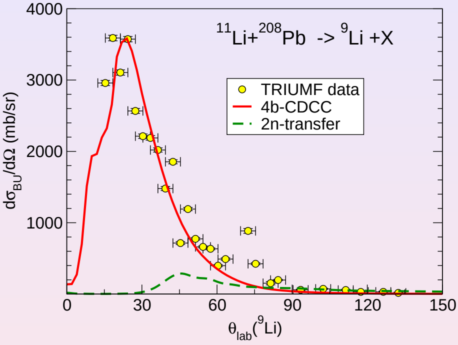
$$\boxed{V_{n;n'}(\mathbf{R}) = \int d\mathbf{r}\, \phi_n^*(\mathbf{x},\mathbf{y}) \left\{ V_{nt}(\mathbf{r}_1) + V_{nt}(\mathbf{r}_2) + V_{ct}(\mathbf{r}_3) \right\} \phi_{n'}(\mathbf{x},\mathbf{y})}$$

☞ $\phi_n(\mathbf{x},\mathbf{y})$ 3-body WFs for bound and continuum states (difficult to calculate!).

Extension to 4-body reactions: ^{11}Li elastics and breakup



M Cubero et al, PRL109, 262701 (2012)

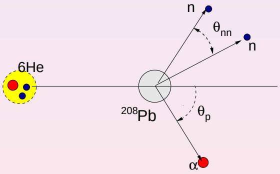


J. Fernández-García et al, PRL110,142701(2013)

- Large 3-body breakup (^{11}Li $\rightarrow$ ^{9}Li+n+n) and subsequent strong reduction of elastics mostly due to strong E1 response of ^{11}Li (Coulomb polarizability).
- Both effects well accounted for by 4b-CDCC, but the underlying 3b model of ^{11}Li predicts a B(E1) larger than previously reported.

Some conclusions on 4b-CDCC

- ✔ 4-body reactions with inert cores are nowadays feasible within the CDCC scheme.
- ✖ What about core excitations? (not included in present reaction calculations).
- ✖ Upcoming experiments in full kinematics provide more detailed observables and with higher accuracy:

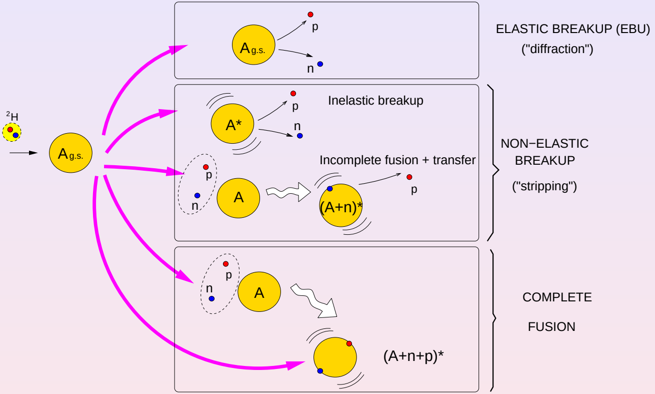


$$\frac{d\sigma}{d\Omega_{n_1}\, d\Omega_{n_2}\, d\Omega_\alpha\, dE_{n_1}\, E_{n_2}}?$$

- ✖ For inclusive measurements (eg. ^{9}Li singles after ^{11}Li) non-elastic breakup and transfer may contribute also (not included by present models)

The problem of non-elastic breakup

Elastic and non-elastic breakup



☞ CDCC/Faddeev provides only the EBU part ⇒ How to compute the NEB part?

Why studying inclusive breakup?

- Surrogate reactions at low energies to extract $n + A$ CN cross sections:
  E.g.: $d + A \rightarrow p + B^*$
- Understanding of large "singles" yields in breakup of weakly-bound nuclei.
  E.g.: ^{6,7}Li + A $\rightarrow \alpha$ + X
- Evaluation of incomplete fusion (part of the NBU cross section)
- Benchmark of semiclassical approaches for knockout reactions at intermediate energies (*stripping*)

Why studying inclusive breakup?

- Surrogate rea $+\ A$ CN cross sections:

  E.g.: $d + A$ –

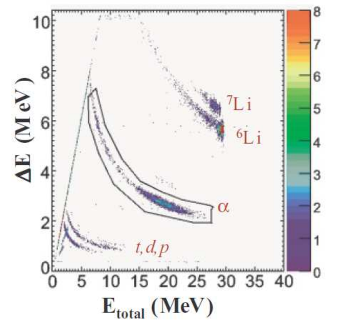
- Understandin up of weakly-bound

  E.g.: ^{6,7}Li + A

- Evaluation of cross section)

- Benchmark of semiclassical approaches for knockout reactions at intermediate energies (*stripping*)

Eg: α production in ^{6}Li+^{209}Bi @ 32 MeV



(Santra *et al*, PRC85,014612(2008))

Searching in the eighties for inclusive breakup models

- Baur & co: DWBA sum-rule with surface approximation.
  - Baur et al, PRC21, 2668 (1980).
- Hussein & McVoy: extraction of singles cross section combining the spectator model with sum rule over final states.
  - Nucl. Phys. A445, 124 (1985).
- Ichimura, Austern, Vincent (IAV): Post-form DWBA.
  - Ichimura, Austern, Vincent, PRC32, 431 (1985).
  - Austern *al*, Phys. Rep.154, 125 (1987).
- Udagawa, Tamura (UT): prior-form DWBA.
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☞ *Most of these theories have fallen into disuse and should be revisited*

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Formal expression for non-elastic breakup (NEBU)

- Inclusive breakup: $\underbrace{(b+x)}_{a}+A \rightarrow b+(x+A)^*$
- Inclusive differential cross section: $\sigma_b^{inc} = \sigma_b^{\mathrm{EBU}} + \sigma_b^{\mathrm{NEBU}}$
- Within an spectator model, σ_b^{NEBU} can be interpreted as the absorption occurring in the $x + A_{\mathrm{gs}}$ channel, following $a \rightarrow b + x$

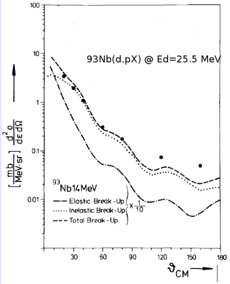
$$\boxed{\frac{d\sigma^{\mathrm{NEBU}}}{d\Omega_b dE_b} = -\frac{2}{\hbar v_a}\rho_b(E_b)\langle\varphi_x|W_{xA}|\varphi_x\rangle}$$

- φ_x describes the evolution of x after dissociating from a.

$$\boxed{[K_x + U_{xA} - E_x]\varphi_x(\mathbf{r}_x) = (\chi_b^{(-)}|V_{bx}|\Psi_{xb}^{(+)}\rangle}$$

- $\Psi_{xb}^{(+)}$ can be approximated by the DWBA, CDCC or Faddeev wavefunctions.

Application of IAV model to deuteron inclusive breakup

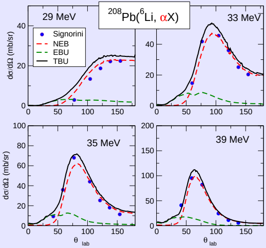


Pioneering applications by Baur *et al.* in the 1970s using zero-range DWBA.

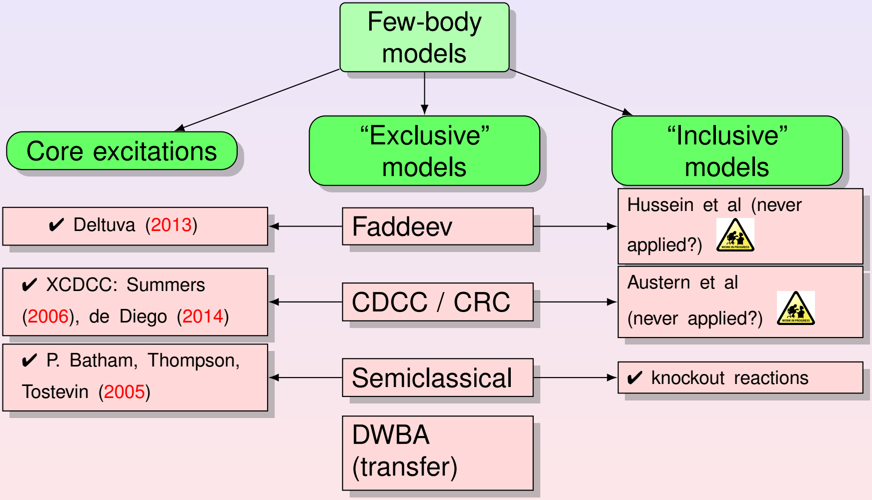
Pampus et al, NPA311 (1978)141

Ongoing applications to weakly-bound nuclei using CDCC for the EBU part and the IAV model for NEB part (finite-range DWBA.)

J. Lei (talk on Tuesday)



Standard few-body models and their extensions



Summary and conclusions

CDCC and its extensions

- Core excitations implemented (XCDCC), but with (too) simple structure models $\Rightarrow$ more microscopic inputs needed.
- 4b-CDCC feasible, but numerical convergence is still challenging.
- Required evaluation of 2-3 particles correlations after breakup.

Faddeev

- Feasible for light targets (Deltuva *et al.*) ☺
- Ongoing initiatives to develop Faddeev codes (eg. TORUS at USA) ☺
- Limited so far to light targets (Coulomb) ☹

Evaluation of inclusive cross sections

- Old theories (1980s) are being revisited and implemented ☺.
- So far, they use DWBA, but may need to go beyond for halo nuclei. ☹


Grazie 1000 !
Cartoline Parlanti