Four-dimensional dynamical simulations of fusion-fission reactions


Omsk State University, Russia

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Evidences of dynamical effects

- Excess of pre-scission particles
- Much wider mass distribution for heavy nuclei
- Anisotropy of fission fragments angular distribution for medium and heavy fissioning nuclei

**Dynamical effect:** path from equilibrium to scission slowed-down by the nuclear viscosity with respect to statistical model
Open questions

• Strength of dissipation and its deformation dependence for shape coordinates and orientation degree of freedom (K-coordinate), which is the projection of nuclear spin onto symmetry axis of fissioning nucleus

• Significance of the K-coordinate for fission dynamics and its influence on the predicted dissipation strength for the shape coordinates

• Correlation between dissipation for the shape collective coordinates and dissipation for the orientation degree of freedom
The stochastic approach

Collective degrees of freedom, which describe actual fission paths, in surrounding heat bath (analogy with a Brownian motion, Kramers, 1940)

Transport equations: Fokker-Planck, Langevin equations

Multi-dimensional Langevin classical equations of motion describe time evolution of the collective variables like the evolution of Brownian particle that interacts stochastically with a “heat bath”.

- Energy dissipation (friction)
- Fluctuations (diffusion)

collective degrees of freedom

intrinsic degrees of freedom ("heat bath")
The stochastic approach

- $E_{\text{coll}}$ - the collective energy
- $E_{\text{int}}$ - the internal energy
- $E_{\text{evap}}$ - the energy carried away by the evaporated particles
The \( (c, h, \alpha) \)-parametrization of nuclear shape

**Funny-Hills parametrization** M. Brack et al., Rev. Mod. Phys. 44 (1972) 320

- \( c \) - elongation of the nucleus
- \( h \) - neck formation
- \( \alpha \) - masses of nascent fragments

\[
\rho_s^2(z) = \begin{cases} 
(c^2 - z^2) \left( A_s + Bz^2/c^2 + \alpha z/c \right), & \text{if } B \geq 0; \\
(c^2 - z^2) \left( A_s + \alpha z/c \right) \exp(Bcz^2), & \text{if } B < 0,
\end{cases}
\]

\[
B = 2h + \frac{c - 1}{2}.
\]

\[
A_s = \begin{cases} 
c^{-3} - \frac{B}{5}, & \text{if } B \geq 0; \\
-\frac{B}{3 \exp(Bc^2) + (1 + \frac{1}{2Bc}) \sqrt{-7Bcz^2c^2}}, & \text{if } B < 0.
\end{cases}
\]

\[
q_1 = c
\]

\[
q_2 = \frac{h + 3/2}{2c^3 + \frac{1-c}{4} + 3/2}
\]

\[
q_3 = \begin{cases} 
\alpha/(A_s + B), & \text{if } B \geq 0 \\
\alpha/A_s, & \text{if } B < 0
\end{cases}
\]
The Langevin equations

\[ \frac{dq_i}{dt} = \mu_{ij} p_j, \]
\[ \frac{dp_i}{dt} = -\frac{1}{2} p_i p_k \frac{\partial \mu_{jk}}{\partial q_i} - \frac{\partial F}{\partial q_i} - \gamma_{ij} \mu_{jk} p_k + \theta_{ij} \xi_j(t) \]

\( q \) - collective coordinate; \( p \) - conjugate momentum

\( m_{ij}(\|\mu_{ij}\| = \|m_{ij}\|^{-1}) \) - inertia tensor Werner-Wheeler approx.

\( F(q) = V(q) - a(q)T^2 \) - Helmholtz free energy

\( V \) - potential energy FRLDM \( a(q) \) - level density Fermi-gas


\( T = \sqrt{E_{\text{int}}/a(q)} \) - temperature \( \gamma_{ij} \) -friction tensor

\( \theta_{ij} \xi_j \) - random force \( \xi_j \) - random variable

The rotational energy

\[ E_{\text{rot}}(q, I, K) = \frac{\hbar^2 I(I + 1)}{2J(q)} + \frac{\hbar^2 K^2}{2J_{\text{eff}}(q)} \]

Influence of K-coordinate on driving force

\[ Q_i^{(4D)} - Q_i^{(3D)} = -\frac{\partial}{\partial q_i} \frac{\hbar^2 K^2}{2J_{\text{eff}}} = \frac{\hbar^2 K^2}{2J_{\text{eff}}^2} \frac{\partial J_{\text{eff}}}{\partial q_i} \]

\( I \) – total angular momentum
\( K \) – spin about the fission (symmetry) axis

At present many models assumes K=0
Transition-state model (at saddle or scission) commonly used for calculations of fission fragment angular distributions
Kubo-Anderson and Metropolis Algorithms for K treatment (Eremenko et al. (2006), Karpov et al. (2007)).

Formalism of dynamical treatment:
T. Døssing and J. Randrup, (1985)

Set of Langevin equations (\( \{q_1, q_2, q_3\} \)-shape coordinates and K-coordinate) integrated together, until scission or ER formation
Dissipation for shape coordinates

\[ \beta = \frac{\gamma}{m} \]  
(dissipation rate)

The one-body dissipations
(large mean free path)

collisions of independent particles with moving potential well. Wall and Wall-and-Window formulas.

J. Blocki et al, Ann Phys (1978)

\[ k_s = 0.27 \]


\[ k_s(q) \] – scaling factor found on the basis of the “chaos-weighted wall” formula

G. Chaudhuri and S. Pal, PRC (2001)

One-body mechanism is expected to dominate. Due to Pauli blocking principle two-body interactions are less probable.
Dissipation coefficient for the K-coordinate

Estimation of dissipation coefficient $\gamma_K$:

$$\gamma_K = \frac{1}{R_N R_{c.m.} \sqrt{2\pi^3 n_0}} \sqrt{\frac{J || J_{eff} || J_R}{J_R^3}}$$

(in case of a dinucleus)
J. P. Lestone et al.

1) Constant value
$$\gamma_K = 0.077 \ (\text{MeV zs})^{-1/2}$$

2) Constant value for compact shapes
$$\gamma_K^{\text{const}} + \gamma_K(q)$$
for dinucleus

3) $\gamma_K(q)$ for dinucleus extrapolated for compact shapes
Orientation degree of freedom

Influence on

fission barrier height

Substantial decrease of potential energy stiffness with respect to mass-asymmetry for heavy nuclei

mass asymmetry coordinate

Shift of Businaro-Gallone point towards larger $Z^2/A$
$^{16}O^{208}$Pb→$^{224}$Th (Elab = 90, 110, 130, 148 and 215 MeV)

4D calculations with $k_s=0.25$ and $\gamma_K = 0.06$ (MeV zs)$^{-1/2}$ allow to describe $\sigma_M^2$ consistently with $\langle n_{\text{pre}} \rangle$ and anisotropy.
Influence of orientation degree of freedom

4D calculations provide more appropriate description of exp. data
Calculated anisotropy for $^{252}\text{Fm}$

$E_{\text{lab}} = 125$ MeV

$E_{\text{lab}} = 142$ MeV

$4\text{D calculations with:}$

1) const. $\gamma_K \approx 0.08$ (MeV zs)$^{-1/2}$ or
2) const. $\gamma_K \approx 0.2$ (MeV zs)$^{-1/2}$ (for compact shapes) + $\gamma_K(q)$ (for dinucleus)

allow to describe exp. data
Calculated results for $^{200}$Pb

$K_s = 1.0$ reproduce the exp. data on fission and ER cross sections independently on $\gamma_K$ used.

Anisotropy is reproduced with const. $\gamma_K \approx 0.08 \ (\text{MeV zs})^{-1/2}$ or const. $\gamma_K \approx 0.4 \ (\text{for compact shapes}) + \gamma_K(q)$ (for dinucleus)
Calculated results for $^{204}$Po

$K_s = 0.5$ or $K_s(q)$ reproduce well the exp. data on fission cross sections independently on $\gamma_K$ used.

Anisotropy is reproduced with const. $\gamma_K \approx 0.08 \ (\text{MeV zs})^{-1/2}$ or const. $\gamma_K \approx 0.2$ (for compact shapes) + $\gamma_K(q)$ (for dinucleus)
Conclusions

- The 4D calculations for heavy nuclei allow consistent description of MED parameters and prescission particles multiplicities, which is impossible in 3D.

- The estimation of constant $\gamma_K=0.077 \, (\text{MeV zs})^{-1/2}$ is good for fissioning nuclei from $^{200}\text{Pb}$ to $^{248}\text{Cf}$.

- It is possible to use the deformation dependent $\gamma_K$ coefficient, calculated according to Lestone et. al, for the shapes featuring a neck, which predicts quite a small values of $\gamma_K=0.0077 \, (\text{MeV zs})^{-1/2}$, however in order to reproduce experimental data on the anisotropy it is necessary to increase the $\gamma_K$ coefficient up to 0.2-0.4 $(\text{MeV zs})^{-1/2}$ for compact shapes featuring no neck.