Four-dimensional dynamical simulations of fusion-fission reactions

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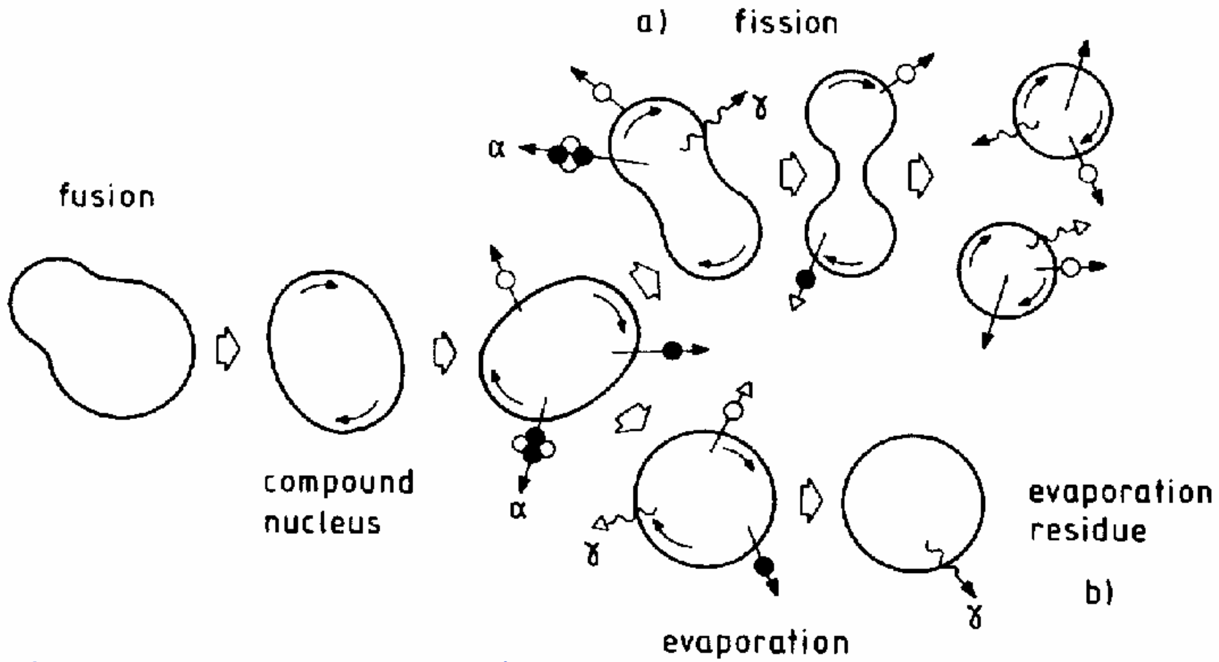
Evidences of dynamical effects



- Excess of prescission particles
- Much wider mass distribution for heavy nuclei
- Anisotropy of fission fragments angular distribution for medium and heavy fissioning nuclei

with respect to statistical model

Dynamical effect: path from equilibrium to scission slowed-down by the nuclear viscosity

Open questions

- Strength of dissipation and it’s deformation dependence for shape coordinates and orientation degree of freedom (K-coordinate), which is the projection of nuclear spin onto symmetry axis of fissioning nucleus

- Significance of the K-coordinate for fission dynamics and its influence on the predicted dissipation strength for the shape coordinates

- Correlation between dissipation for the shape collective coordinates and dissipation for the orientation degree of freedom

The stochastic approach

Collective degrees of freedom, which describe actuall fission paths, in surrounding heat bath (analogy with a Brownian motion, Kramers, 1940)

Transport equations: Fokker-Planck, Langevin equations

collective degrees of freedom

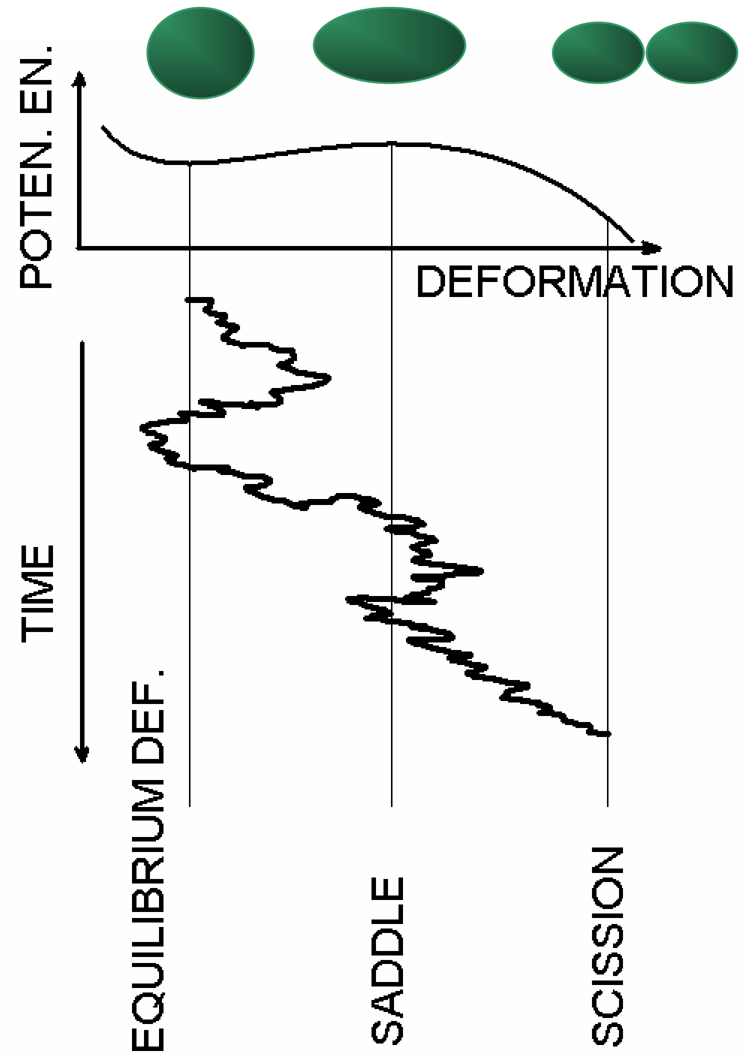
Energy dissipation (friction)

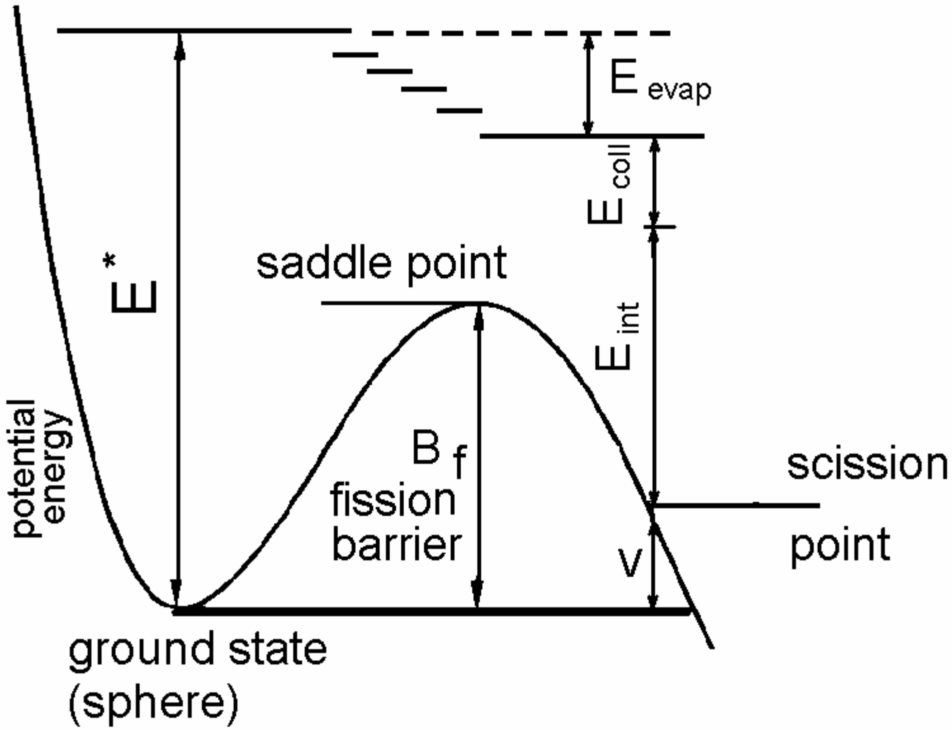
Fluctuations (diffusion)

intrinsic degrees of freedom (“heat bath”)

Multi-dimensional Langevin classical equations of motion describe time evolution of the collective variables like the evolution of Brownian particle that interacts stochastically with a “heat bath”.

The stochastic approach





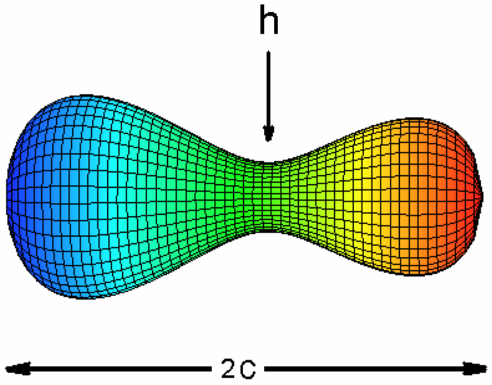
E_{coll} - the collective energy

E_{int} - the internal energy

E_{evap}- the energy carried away by the evaporated particles

The (c, h, α)-parametrization of nuclear shape

Funny-Hills parametrization M. Brack et al., Rev. Mod. Phys. 44 (1972) 320



c - elongation of the nucleus

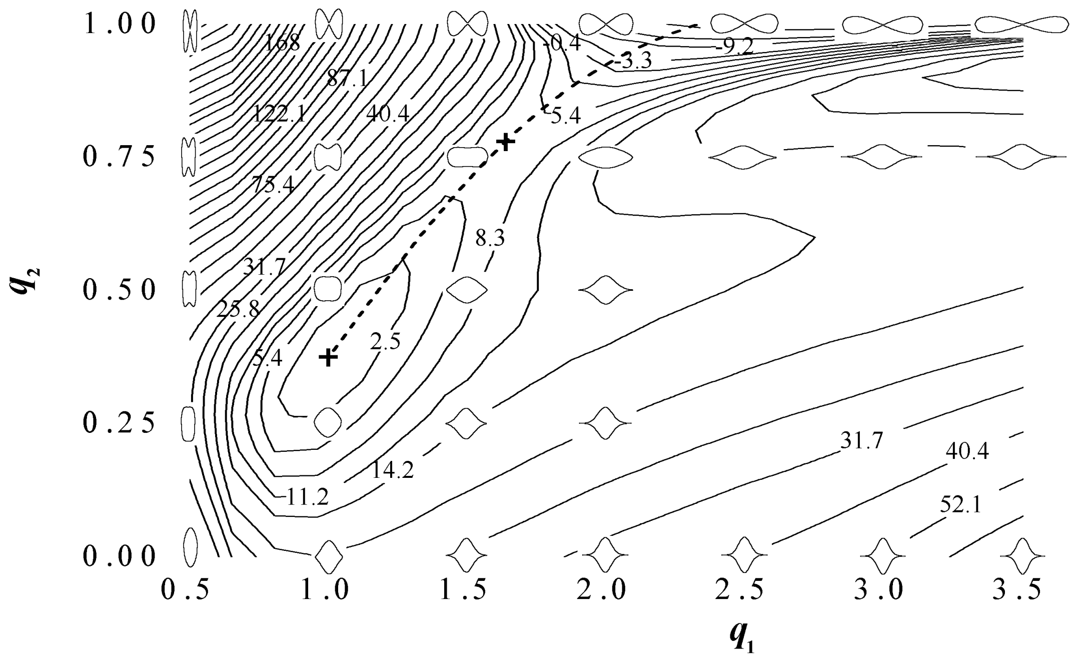
h - neck formation

α - masses of nascent fragments

$$\rho_s^2(z) = \begin{cases} (c^2 - z^2)\left(A_s + Bz^2/c^2 + \alpha z/c\right), & \text{if} \quad B \geq 0; \\ (c^2 - z^2)\left(A_s + \alpha z/c\right)\exp(Bcz^2), & \text{if} \quad B < 0, \end{cases}$$

$$B = 2h + \frac{c-1}{2}.$$

$$A_s = \begin{cases} c^{-3} - \frac{B}{5}, & \text{if} \quad B \geq 0; \\ -\frac{4}{3}\frac{B}{\exp(Bc^3) + \left(1 + \frac{1}{2Bc^3}\right)\sqrt{-\pi Bc^3}\,\mathrm{erf}\left(\sqrt{-Bc^3}\right)}, & \text{if} \quad B < 0. \end{cases}$$



$$q_1 = c$$

$$q_2 = \frac{h + 3/2}{\frac{5}{2c^3} + \frac{1-c}{4} + 3/2}$$

$$q_3 = \begin{cases} \alpha/(A_s + B), & B \geq 0 \\ \alpha/A_s, & B < 0 \end{cases}$$

The Langevin equations

$$\frac{dq_i}{dt} = \mu_{ij} p_j,$$
$$\frac{dp_i}{dt} = -\frac{1}{2} p_i p_k \frac{\partial \mu_{jk}}{\partial q_i} - \frac{\partial F}{\partial q_i} - \gamma_{ij} \mu_{jk} p_k + \theta_{ij} \xi_j (t)$$

q - collective coordinate; **p** - conjugate momentum

$m_{ij}(\|\mu_{ij}\| = \|m_{ij}\|^{-1})$ - inertia tensor Werner-Wheeler approx.

$F(\mathbf{q}) = V(\mathbf{q}) - a(\mathbf{q})T^2$ - Helmholtz free energy

V - potential energy FRLDM $a(\mathbf{q})$ - level density Fermi-gas

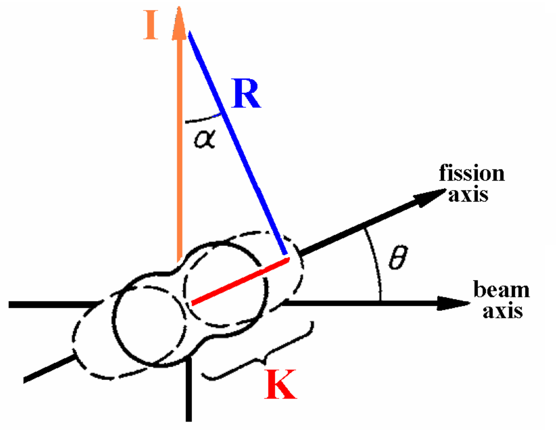
Sierk, Phys. Rev. C 33 (1986) A. V. Ignatyuk et al, Yad. Fiz. 21 (1975)

$T = \sqrt{E_{\text{int}}/a(\mathbf{q})}$ - temperature γ_{ij} -friction tensor

$\theta_{ij}\xi_j$ - random force ξ_j - random variable

Link with discret scheme of particle evaporation at each step of numerical integration according to N.D. Mavlitov et al., Z. Phys. A342, 195 (1995).

Orientation degree of freedom



$$E_{\text{rot}}(\mathbf{q}, I, K) = \frac{\hbar^2 I(I+1)}{2J_\perp(\mathbf{q})} + \frac{\hbar^2 K^2}{2J_{\text{eff}}(\mathbf{q})}$$

The rotational energy

Influence of K-coordinate on driving force

$$Q_i^{(4D)} - Q_i^{(3D)} = -\frac{\partial}{\partial q_i}\frac{\hbar^2 K^2}{2J_{\text{eff}}} = \frac{\hbar^2 K^2}{2J_{\text{eff}}^2}\frac{\partial J_{\text{eff}}}{\partial q_i}.$$

I – total angular momentum

K – spin about the fission (symmetry) axis

At present many models assumes K=0
Transition-state model (at saddle or scission) commonly used for calculations of fission fragment angular distributions

Kubo-Anderson and Metropolis Algorithms for K treatment
(Eremenko et al. (2006), Karpov et al. (2007)).

Formalism of dynamical treatment:

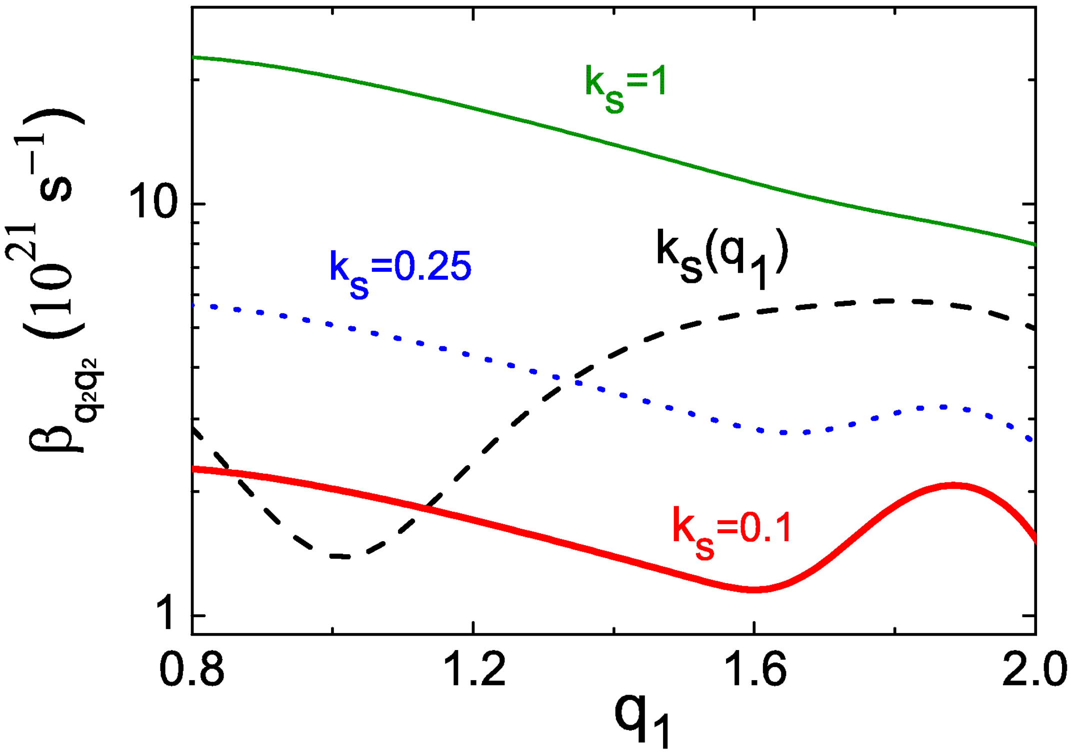
T. Døssing and J. Randrup, (1985)
J. P. Lestone and S.G. McCalla, (2009)

$$K^{(n+1)} = K^{(n)} - \frac{\gamma_K^2 I^2}{2}\left(\frac{\partial V}{\partial K}\right)^{(n)} \tau + \gamma_K I\sqrt{T\tau}\,\xi^{(n)}$$

Set of Langevin equations ($\{q_1, q_2, q_3\}$-shape coordinates and K-coordinate) integrated together, until scission or ER formation

Dissipation for shape coordinates

$\beta = \gamma / m$ (dissipation rate)



The one-body dissipations
(large mean free path)

collisions of independent particles with moving potential well. Wall and Wall-and-Window formulas.
J. Blocki et al, Ann Phys (1978)

k_s – scaling factor (k_s=0.27)

J.R. Nix and A.J. Sierk, Proc JINR (1987)

$k_s(\mathbf{q})$ – scaling factor found on the basis of the “chaos-weighted wall” formula
G. Chaudhuri and S. Pal, PRC (2001)

One-body mechanism is expected to dominate. Due to Pauli blocking principle two-body interactions are less probable

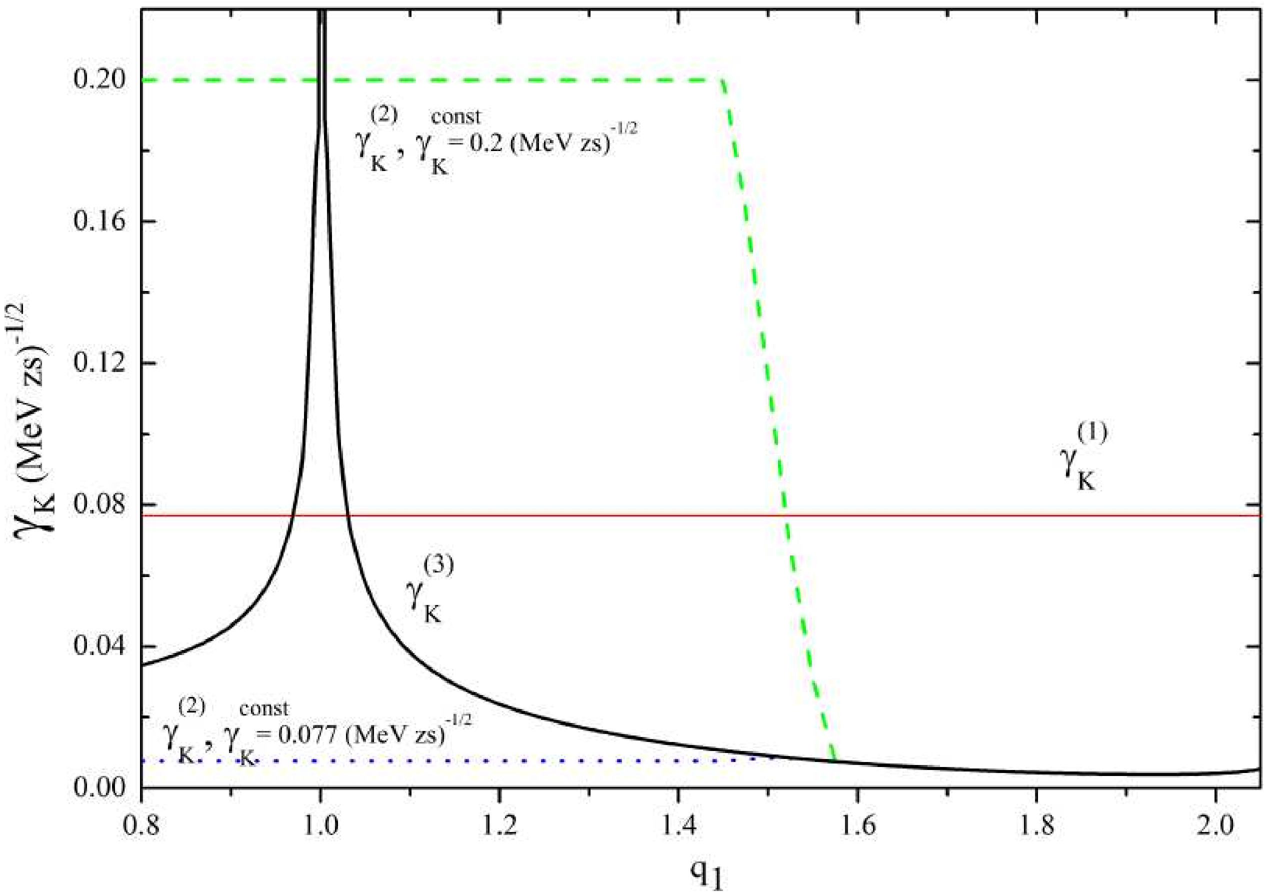
Dissipation coefficient for the K-coordinate

Estimation of dissipation coefficient γ_K:

$$\gamma_K = \frac{1}{R_N R_{c.m.} \sqrt{2\pi^3 n_0}} \sqrt{\frac{J_\parallel | J_{\text{eff}} | J_R}{J_\perp^3}}$$

(in case of a dinucleus)

J. P. Lestone et al.



1) Constant value
$\gamma_K = 0.077$ (MeV zs)$^{-1/2}$
J.P. Lestone et al (1999)

2) Constant value for compact shapes
$\gamma_K^{const} + \gamma_K$ (**q**)
for dinucleus

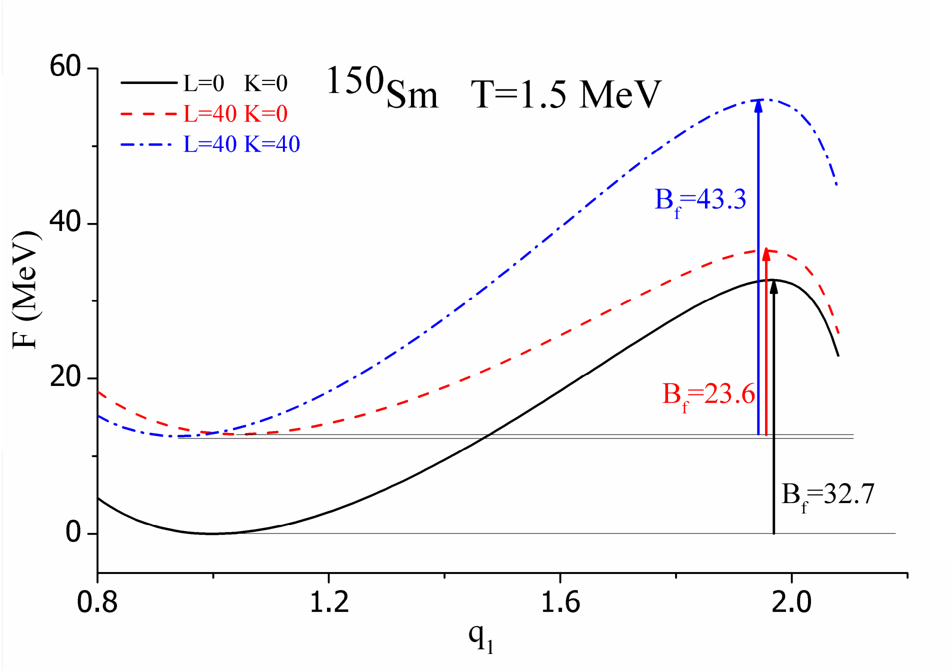
3) γ_K (**q**) for dinucleus extrapolated for compact shapes

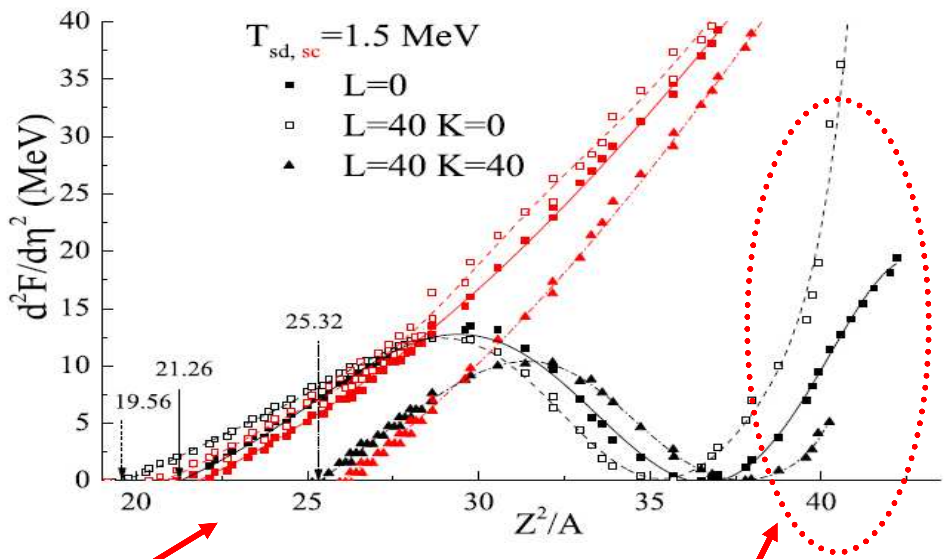
Orientation degree of freedom

Influence on

fission barrier height

mass asymetry coordinate

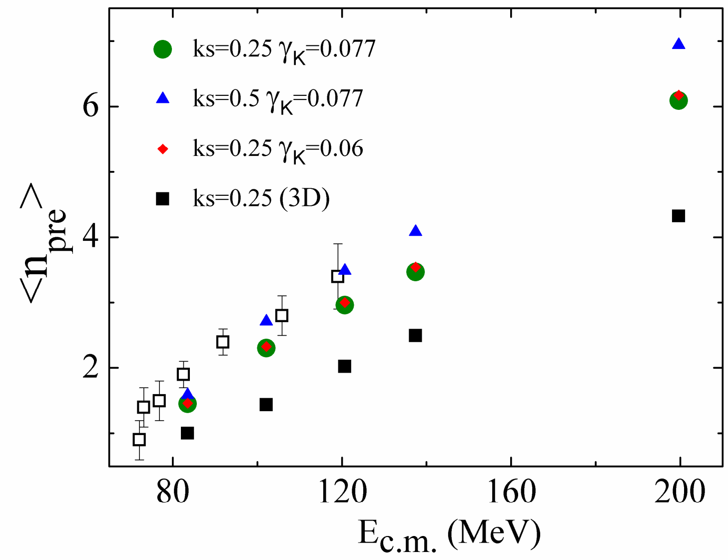


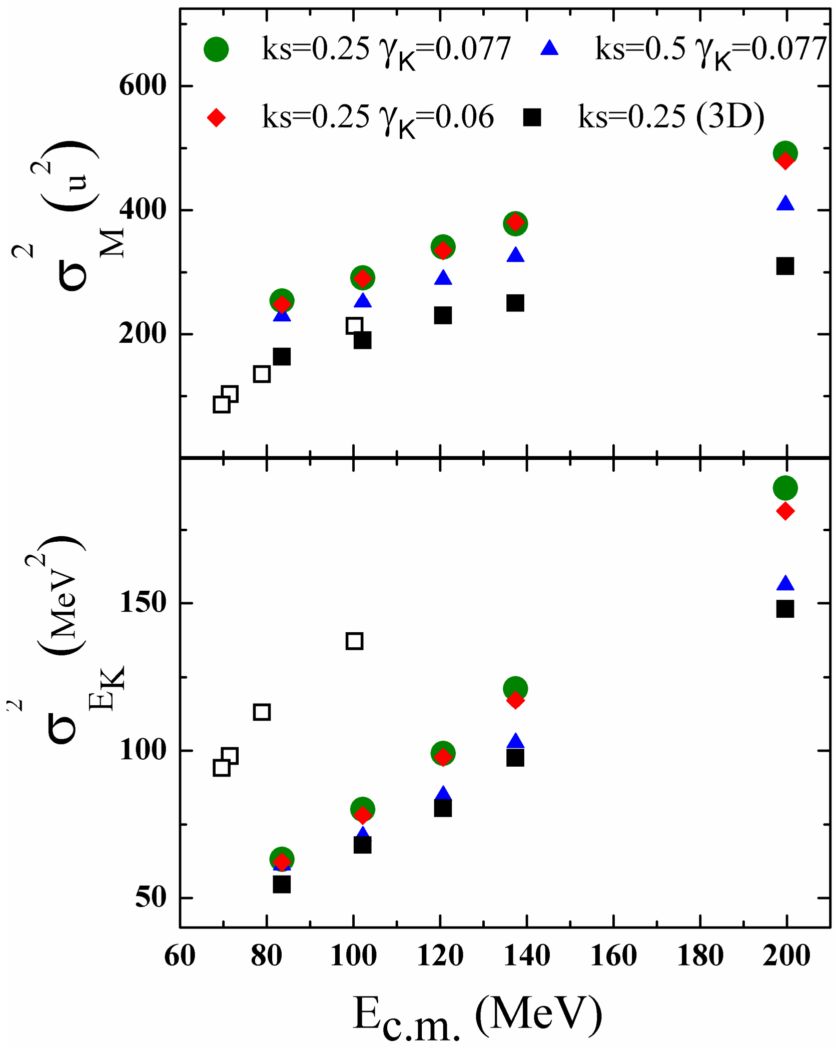


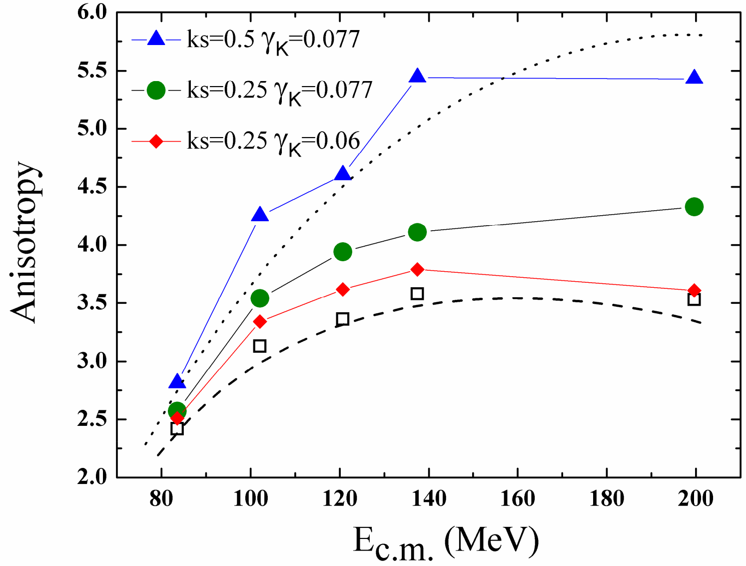
Shift of Businaro-Gallone point towards larger Z^2/A

Substantial decrease of potential energy stiffness with respect to mass-asymmetry for heavy nuclei

$^{16}O + ^{208}Pb \rightarrow ^{224}Th$ (Elab = 90, 110, 130, 148 and 215 MeV)

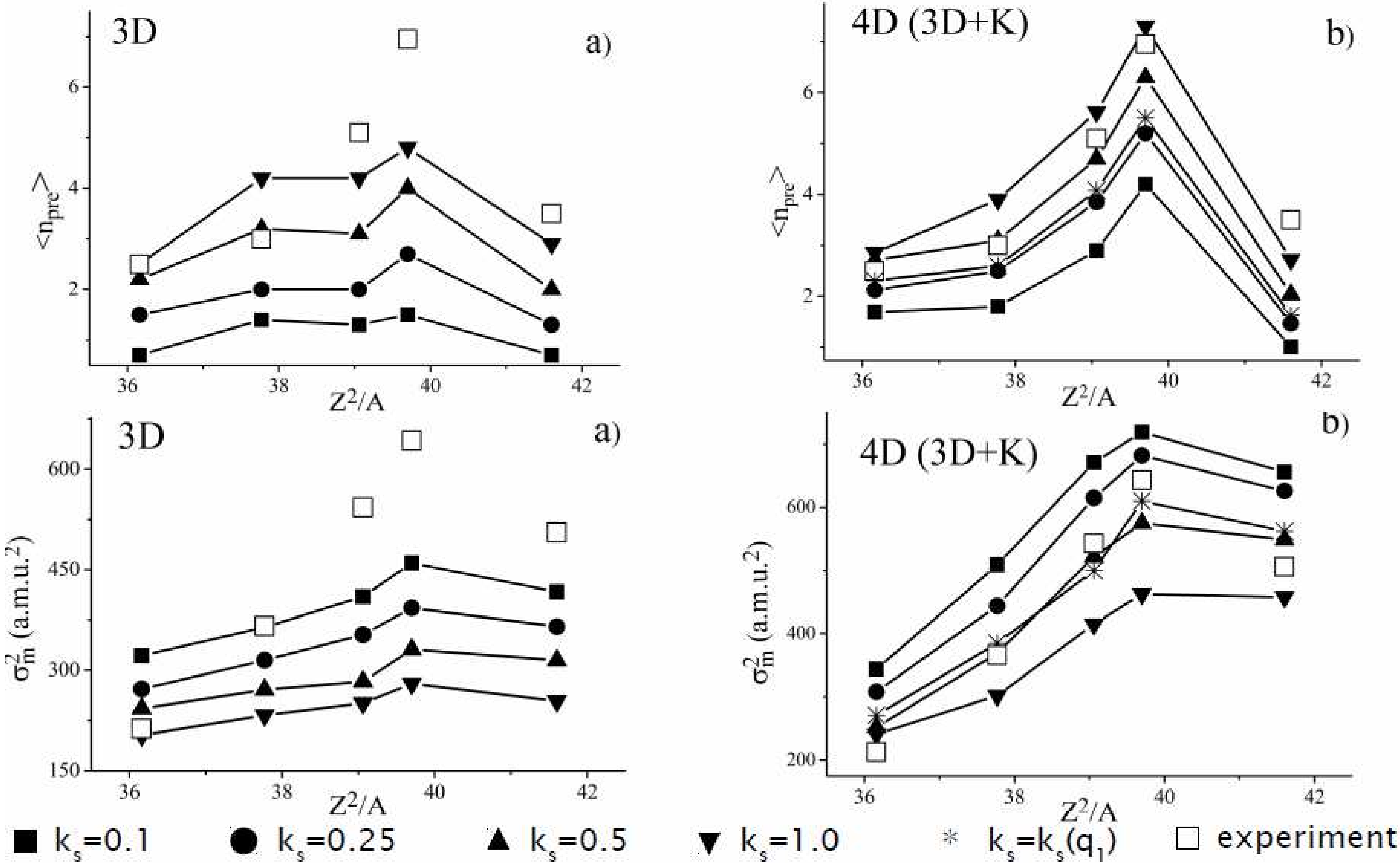






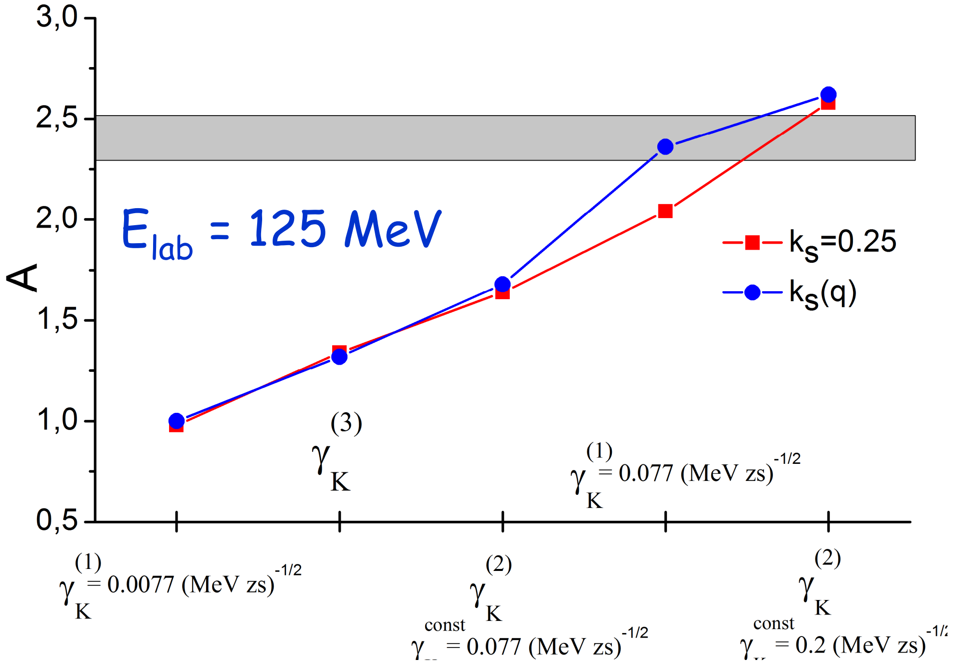
4D calculations with k_s=0.25 and γ_K = 0.06 (MeV zs)$^{-1/2}$ allow to describe σ_M^2 consistently with <n_{pre}> and anisotropy

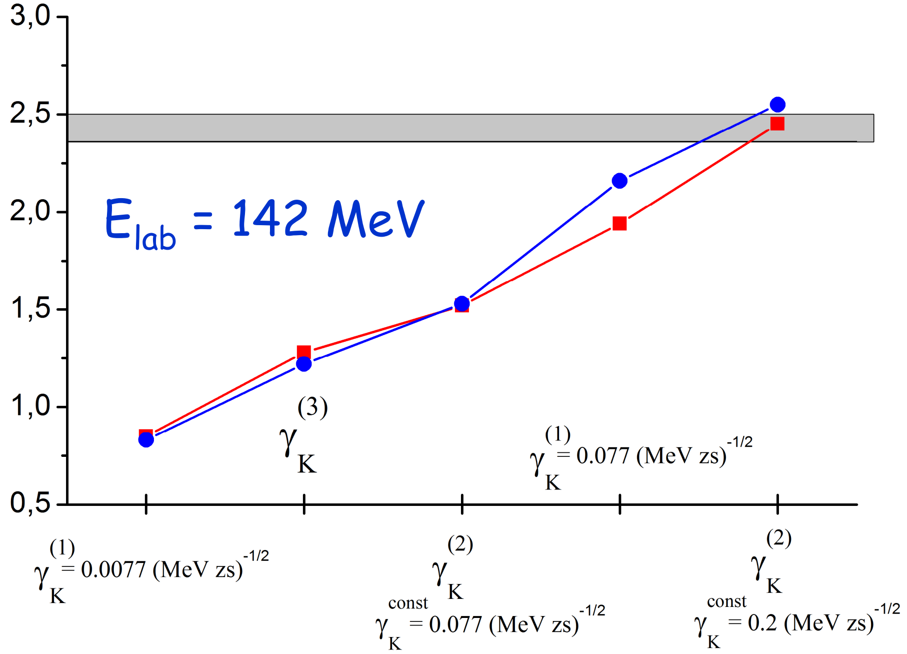
Influence of orientation degree of freedom

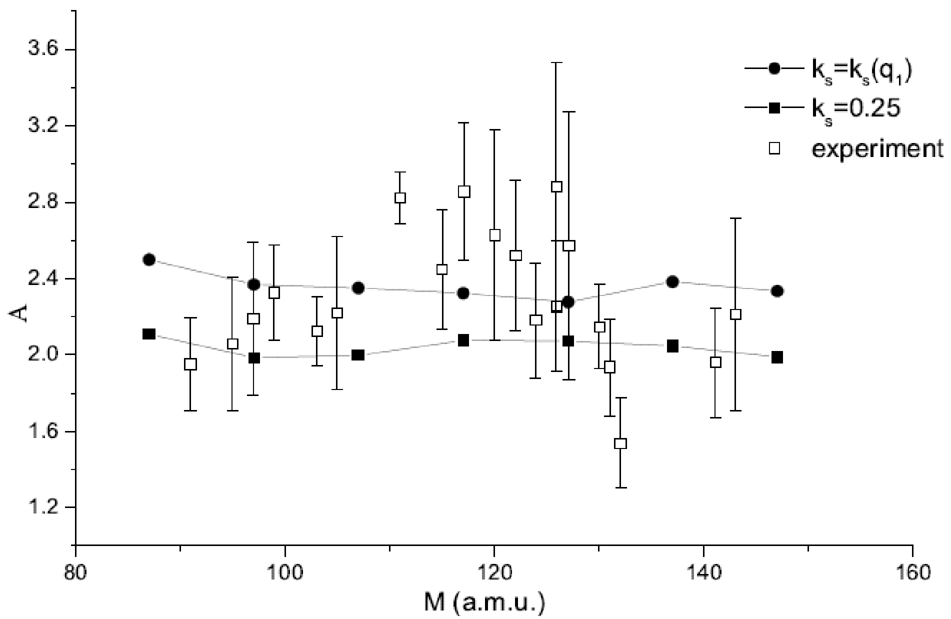


4D calculations provide more appropriate description of exp. data

Calculated anisotropy for ^{252}Fm







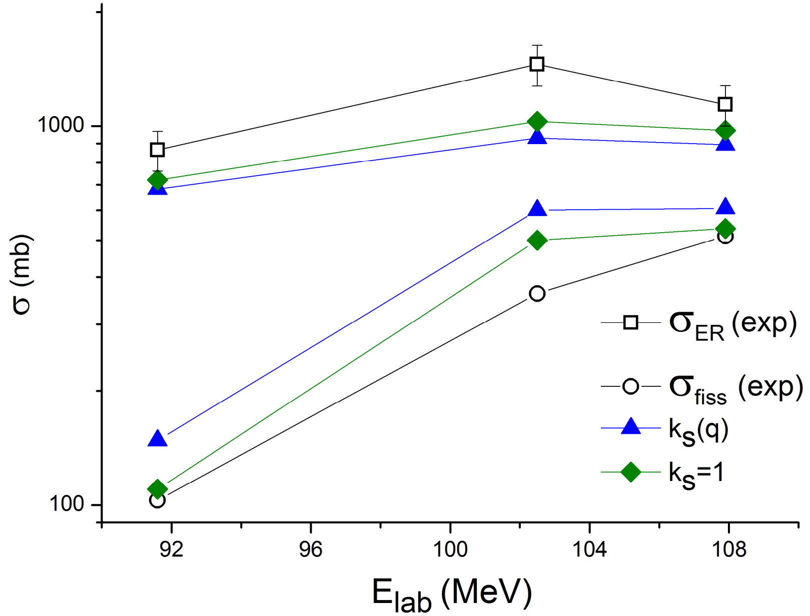
4D calculations with:

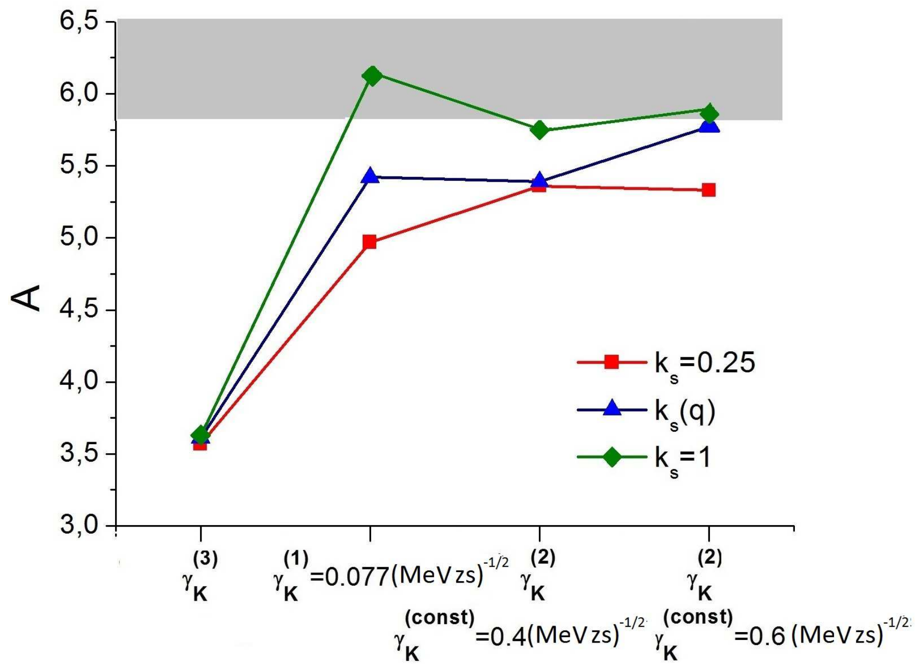
1) const. $\gamma_K \approx 0.08$ (MeV zs)$^{-1/2}$ or

2) const. $\gamma_K \approx 0.2$ (MeV zs)$^{-1/2}$ (for compact shapes) + $\gamma_K(q)$ (for dinucleus)

allow to describe exp. data

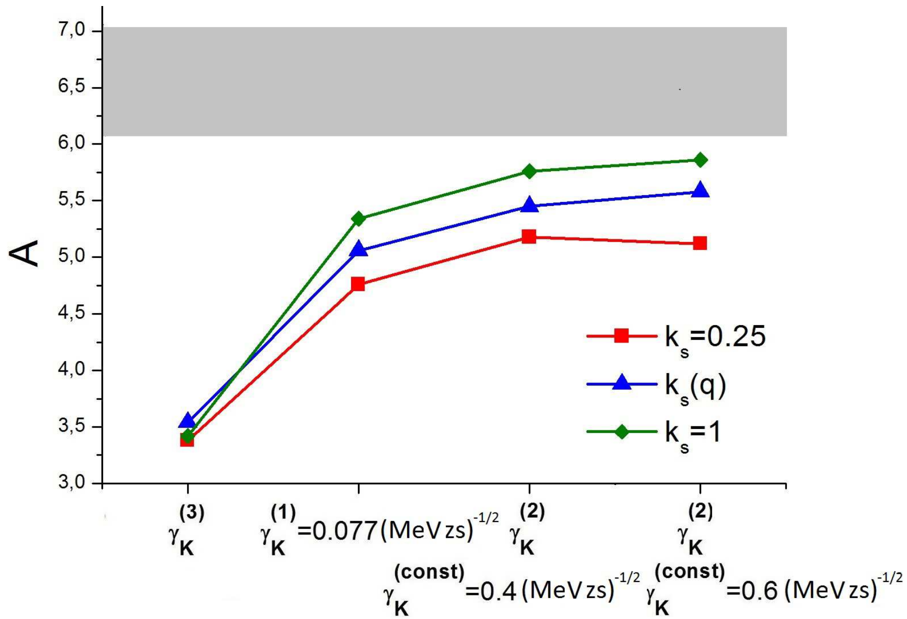
Calculated results for ^{200}Pb





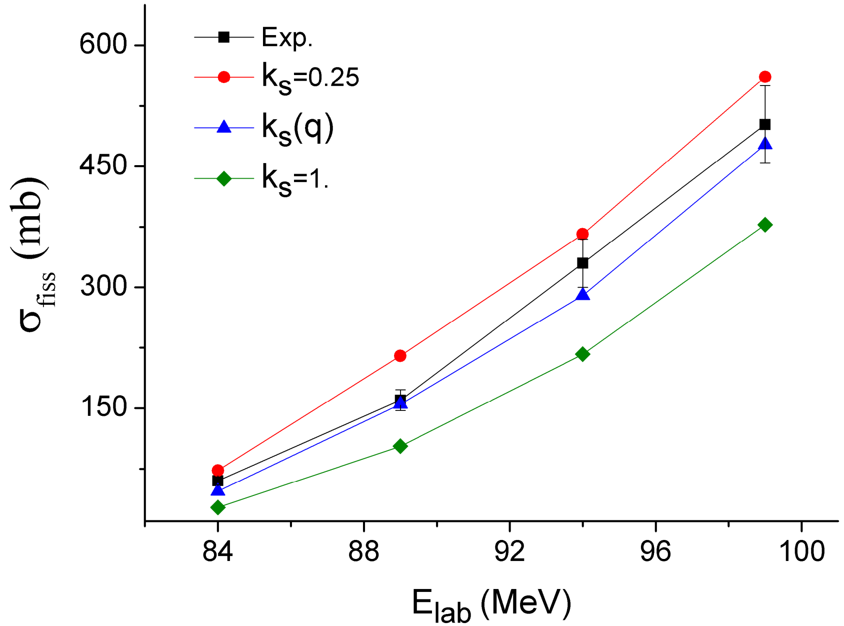
Ks = 1.0 reproduce the exp. data on fission and ER cross sections

independently on γ_K used.

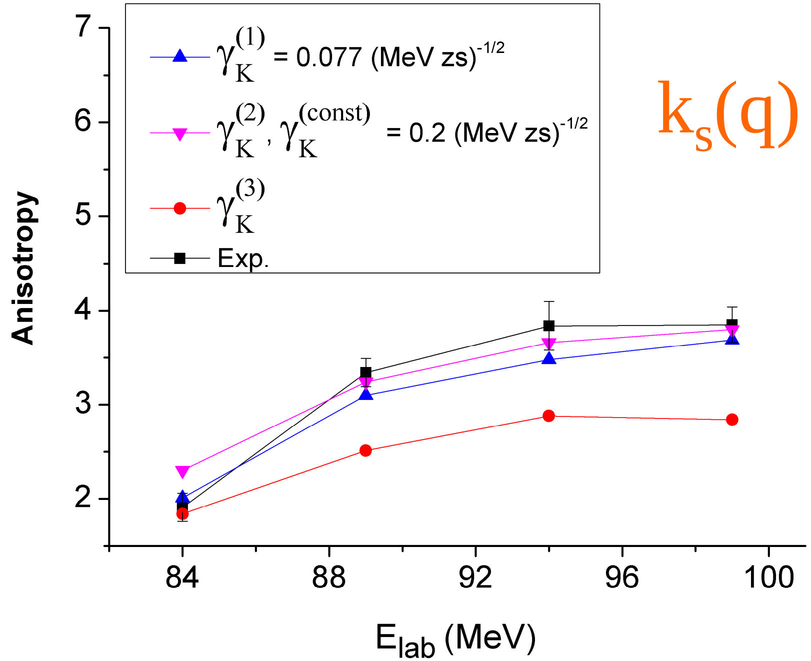


Anisotropy is reproduced with const.
$\gamma_K \approx 0.08$ (MeV zs)$^{-1/2}$ or const.
$\gamma_K \approx 0.4$ (for compact shapes) + γ_K(q) (for dinucleus)

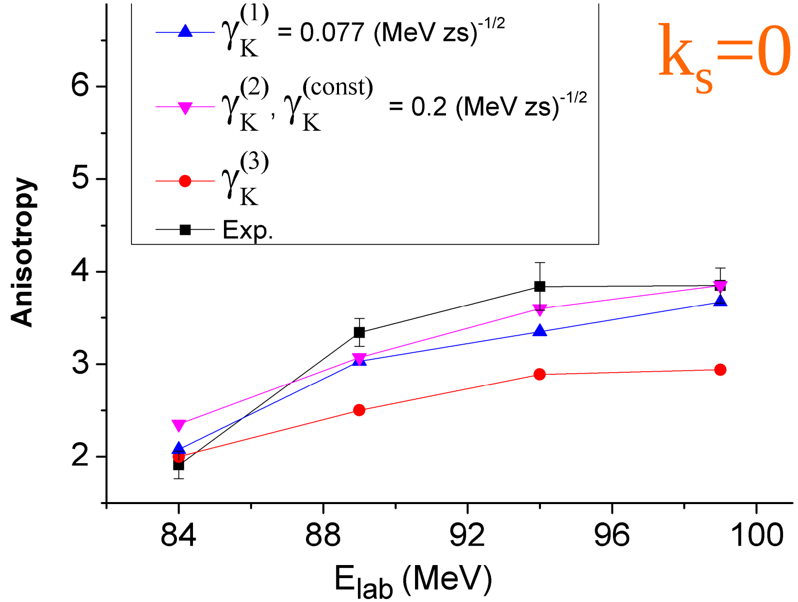
Calculated results for ^{204}Po



$k_s(q)$



$k_s=0.5$



Ks = 0.5 or Ks(q) reproduce well the exp. data on fission cross sections independently on γ_K used.

Anisotropy is reproduced with const. $\gamma_K \approx 0.08$ (MeV zs)$^{-1/2}$ or const.
$\gamma_K \approx 0.2$ (for compact shapes) + $\gamma_K(q)$ (for dinucleus)

Conclusions

- The 4D calculations for heavy nuclei allow consistent description of MED parameters and prescission particles multiplicities, which is impossible in 3D.

- The estimation of constant γ_K=0.077 (MeV zs)$^{-1/2}$ is good for fissioning nuclei from ^{200}Pb to ^{248}Cf.

- It is possible to use the deformation dependent γ_K coefficient, calculated according to Lestone et. al, for the shapes featuring a neck, which predicts quite a small values of γ_K=0.0077 (MeV zs)$^{-1/2}$, however in order to reproduce experimental data on the anisotropy it is necessary to increase the γ_K coefficient up to 0.2-0.4 (MeV zs)$^{-1/2}$ for compact shapes featuring no neck.