Theoretical studies of possible toroidal high-spin isomers in the light-mass region

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C.Y. Wong'72 "Toroidal nuclei", the radially displaced HO **(RDHO)**

$$\boxed{H = -\frac{\hbar^2}{2\mu}\nabla^2 + V_o(\rho, z) - \frac{2\kappa\hbar}{m\mathring{\omega}_o}\, \boldsymbol{s}\cdot(\nabla V_o \times \boldsymbol{p})} \qquad (1)$$

where $\boxed{V_o(\rho, z) = \frac{1}{2}m\omega_T^2(\rho - R)^2 + \frac{1}{2}m\omega_T^2 z^2}$ Here, κ is a dimensionless number for which Nilsson et al. gave the value of 0.06 [8] and R is the major radius. The quantum energy for oscillation in the minor radius $\hbar\omega_T$ is related to the energy quantum $\hbar\mathring{\omega}_o$ for the spherical nucleus with the same volume by

$$\hbar\omega_T = \hbar\mathring{\omega}_o(3\pi R/2d)^{\frac{1}{3}}$$

where d is the minor radius.

We choose a set of basis states:

$$|n_\rho n_z \Lambda\Omega\rangle = R_{n_\rho\Lambda\Sigma}(\rho) Z_{n_z}(z)\Phi_\Lambda(\varphi)\chi^{\frac{1}{2}}_{\Sigma}$$

where n_ρ and n_z are the number of nodes of the eigenfunctions $R_{n_\rho\Lambda\Omega}$ and Z_{n_z}, respectively, Λ is the z-component of the orbital angular momentum, Σ is the z-component of the spinor and $\Omega = \Lambda + \Sigma$. One neglects the non-diagonal matrix elements of the spin-orbit term in (1) and obtains a separable potential. The eigenvalues can then be calculated by the JWKB method.

The result for the single-particle states of a toroidal nucleus is shown in fig. 1 as a function of the breathing deformation parameter $\cosh\eta_o$ which equals the ratio of major to minor radius. Each level is labeled by the quantum numbers (n_ρ, n_z, $|\Lambda|$ and $|\Omega|$). States with different values of $n_\rho + n_z$ are separated by energies in units of $\hbar\omega_T$ which is of the order of 2 to 2.5 $\hbar\mathring{\omega}_o$. The quantum number $n_\rho + n_z$ corresponds to the number of oscillator quanta in the minor radius. States with definite n_ρ and n_z quantum numbers but different Λ quantum numbers form a band with energies increasing approximately as Λ^2. They correspond to single-particle rotational motion about the symmetry axis. Because of time reversal invariance, for every state with angular momentum pointing in one direction about the symmetry axis, there is another state of the same energy with the angular momentum pointing in the opposite direction. The degeneracy of each level

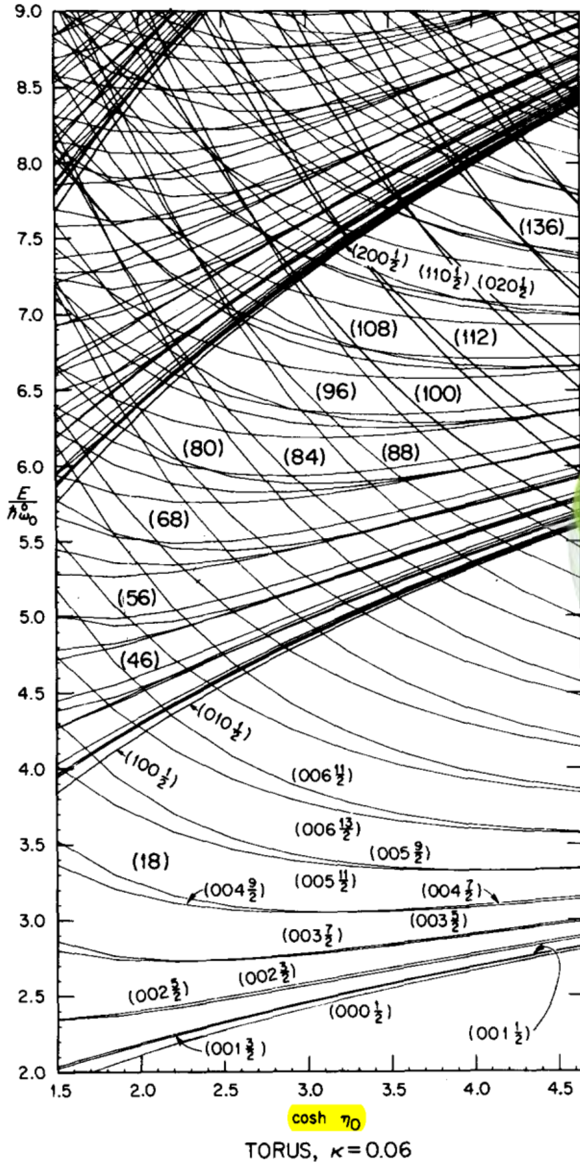
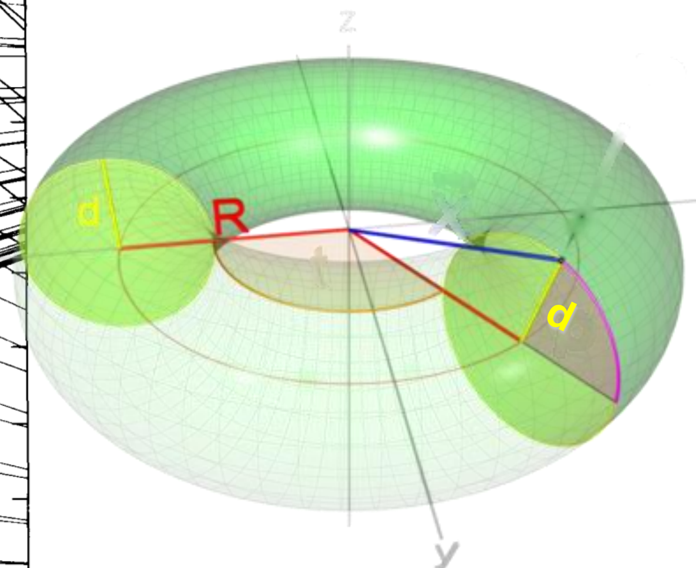


Fig. 1. Single-particle scheme for a toroidal nucleus in the breathing degree of freedom. The breathing deformation parameter $\cosh\eta_o$ is equal to the ratio of major to minor radius R/d. The levels are labeled by the quantum numbers (n_ρ, n_z, $|\Lambda|$, $|\Omega|$). Each level has a degeneracy of two. Some occupation numbers are also indicated.



High-K toroidal isomers from the α-cluster rings
(as the initial configurations)

T. Ichikawa, J.A. Maruhn, N. Itagaki, K. Matsuyanagi, P.-G. Reinhard, and S. Ohkubo, Phys. Rev. Lett. **109**, 232503 (2012).

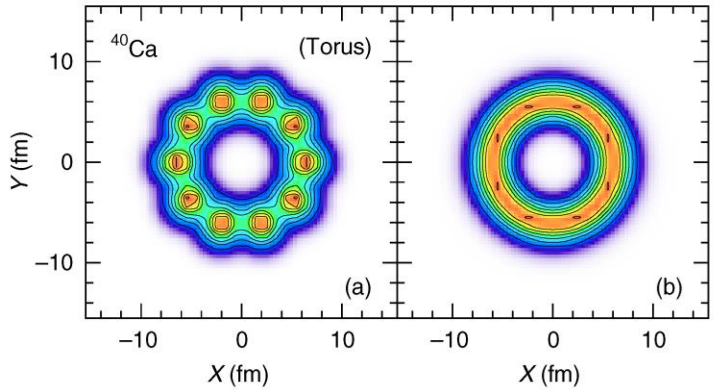


FIG. 1 (color online). Total density for (a) the initial condition of the HF iterations and (b) the calculated result with $\hbar\omega = 1.5$ MeV at the 15 000 HF iterations. The density is integrated in the z direction. The contours correspond to multiple steps of 0.05 fm^{-2}. The color (gray scale) is normalized by the largest density in each plot.

T. Ichikawa, K. Matsuyanagi, J.A. Maruhn, and N. Itagaki, Phys. Rev. C **89**, 011305(R) (2014).
T. Ichikawa, K. Matsuyanagi, J.A. Maruhn, and N. Itagaki, Phys. Rev. C **90**, 034314 (2014).

Model

We use a three-step method to study the occurrence of the high-spin toroidal isomeric states:

First, using the quadrupole moment constrained Skyrme-HFB (or -HF+BCS) model we look for the oblate configurations with toroidal density distributions without rotation.

Next, we take these toroidal configurations as the starting points in Q_{20}-constrained and cranked Skyrme-HF calculations to gain the stabilizing effect for these toroidal configurations due to a non-collective rotation around symmetry z-axis.

In the last step, when we locate a local minimum for each quantized value of angular momentum $I=I_z$, we repeat the unconstrained and symmetry-unrestricted cranked Skyrme-HF calculations to find the high-spin toroidal isomeric states.

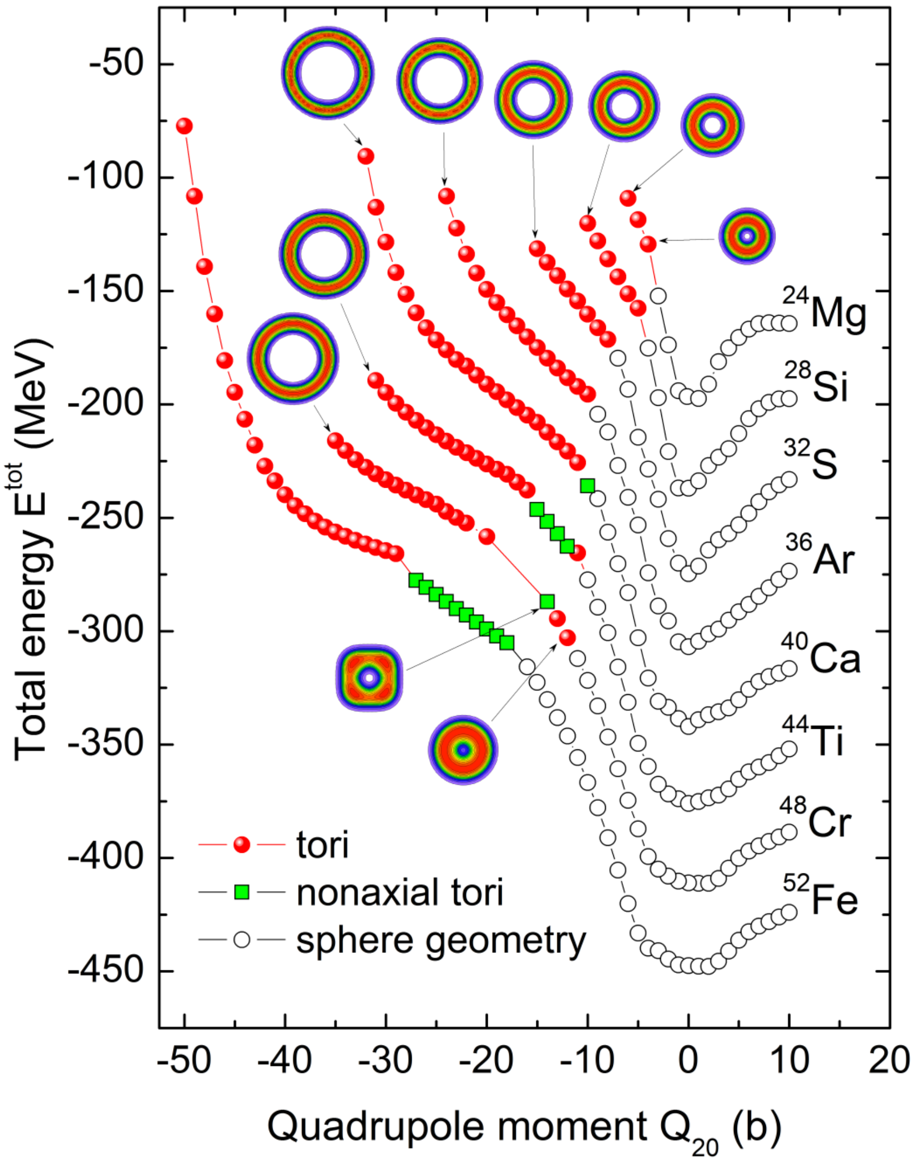
The symmetry unrestricted code HFODD [1] and an augmented Lagrangian method [2] were used to solve constrained HFB (or HF+BCS) equations with SkM* Skyrme force [3] in the p-h channel and a density dependent mixed pairing [4, 5] interaction in the p-p channel.

The stretched harmonic oscillator basis of HFODD was composed of states having not more than N_0 = 26 quanta in either of the Cartesian directions, and not more than 1140 states in total.

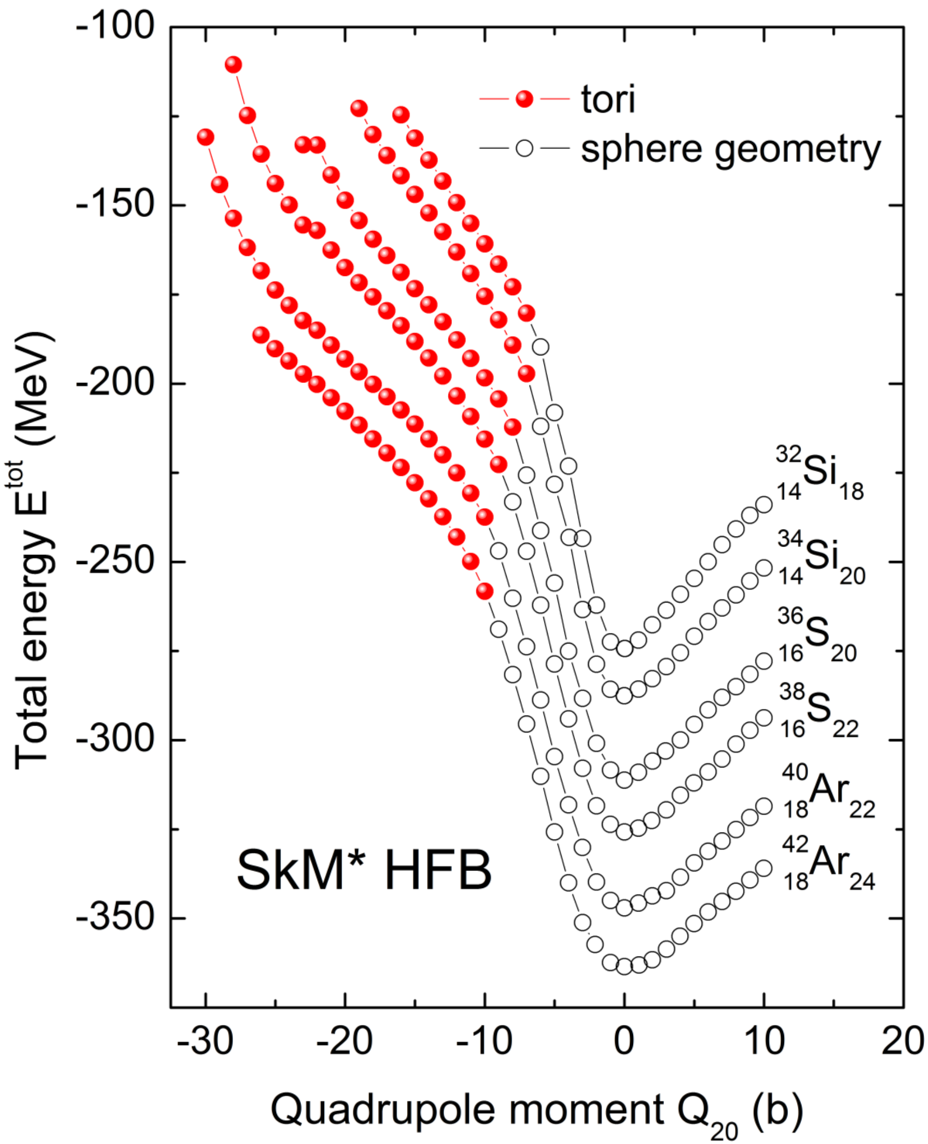
[1] N. Schunck *et al.*, **183**, 166 (2012).
[2] A. Staszczak, M. Stoitsov, A. Baran, and W. Nazarewicz, Eur. J. Phys. A **46**, 85 (2010).
[3] J. Bartel *et al.*, Nucl. Phys. A **386**, 79 (1982).
[4] J. Dobaczewski, W. Nazarewicz, and M. V. Stoitsov, Eur. J. Phys. A **15**, 21 (2002).
[5] A. Staszczak, A. Baran, J. Dobaczewski, and W. Nazarewicz, Phys. Rev. C **80**, 014309 (2009).

Light *N*=*Z* toroidal nuclei in constrained SkM*-HFB model

A. Staszczak, C. Y. Wong, Phys. Lett. B **738** (2014) 401,
Acta Phys. Pol. B **46** (2015) 675.

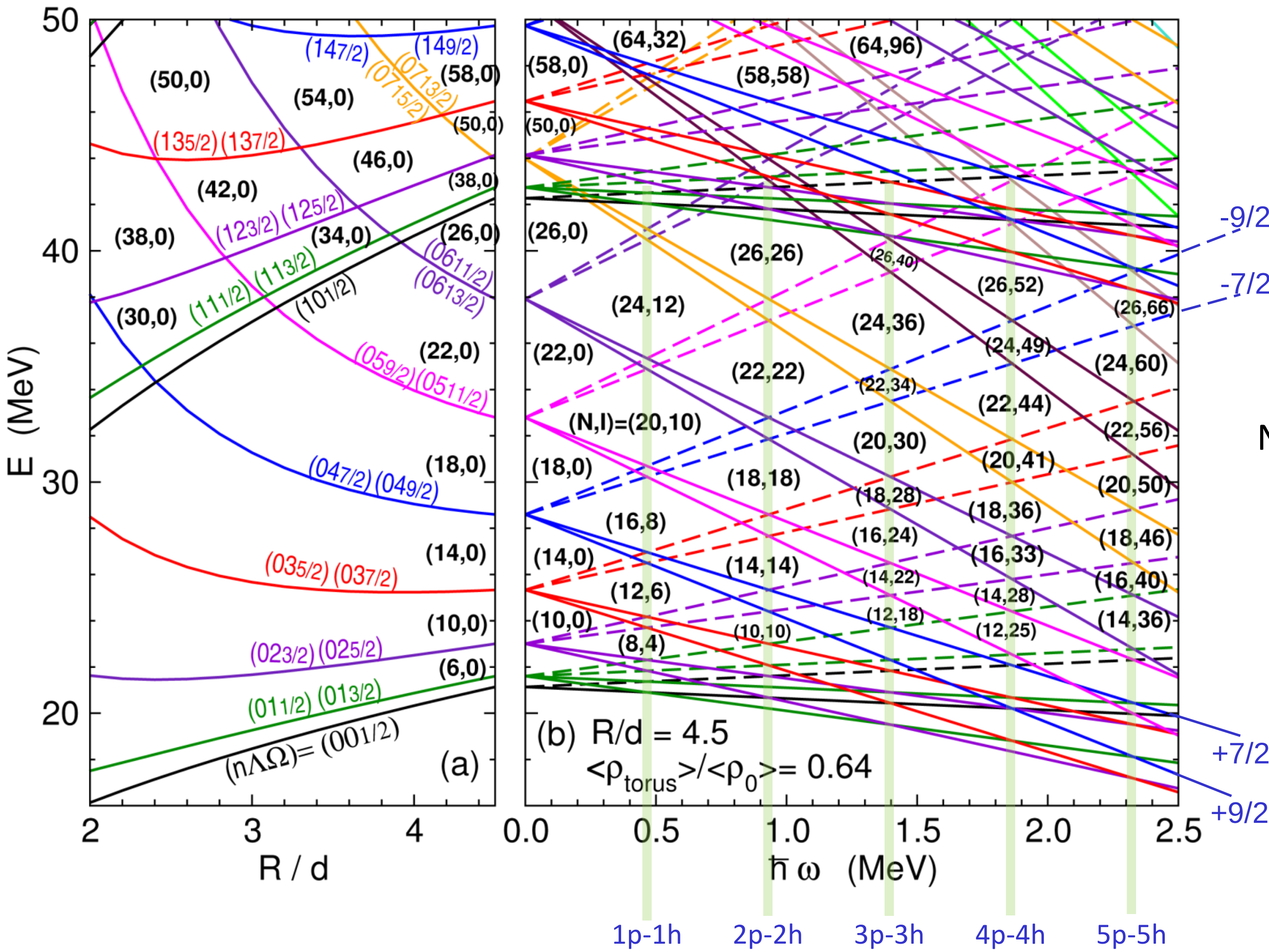


Light $N \neq Z$ toroidal nuclei in constrained SkM*-HFB model



A. Staszczak, C. Y. Wong, submitted to: Phys. Scr.

RDHO (a) and cranked RDHO (b) s.p. states of a toroidal nucleus as a function of R/d and ω



Labels: **(n Λ Ω)**, where

$n=(n_z+n_\rho)$

n_z- azimuthal nodal q.n.,

n_ρ- radial nodal q.n.,

$\pm\Lambda$- *z*-component of orbital angular momentum,

$$\Omega = \left|\Lambda \pm \tfrac{1}{2}\right|, \quad \Omega_z = \pm\Omega.$$

Note that the low-lying states have $n_z = n_\rho = 0$.

The slope of the Routhians:

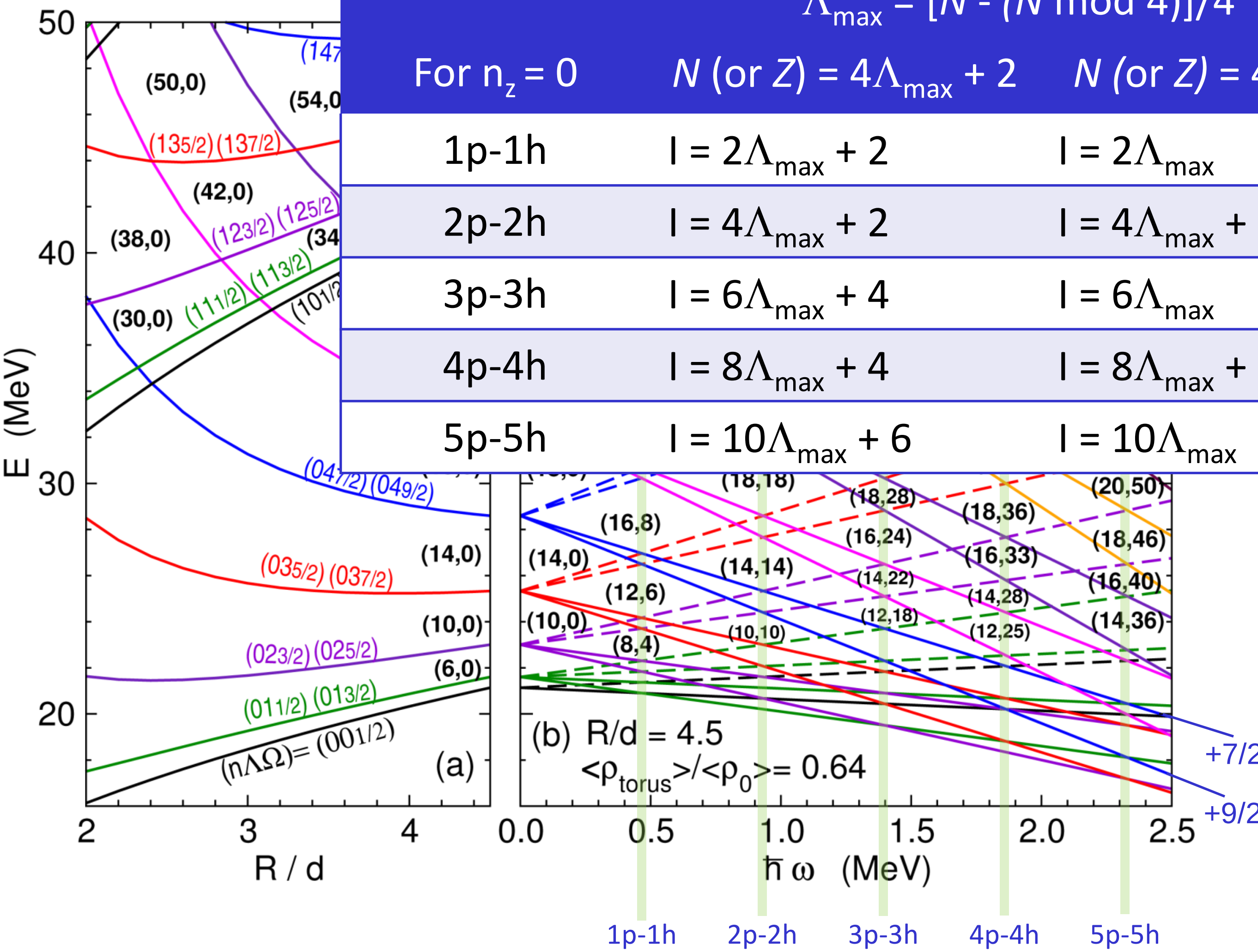
$$\frac{\partial E_i^{\omega}}{\partial \omega} = -\Omega_{z_i}.$$

The spin is given by

$$I = \sum_{exci} (\Omega_{z_i}^{part} - \Omega_{z_i}^{hole}).$$

RDHO (a) and cranked RDHO (b) s.p. states of a toroidal nucleus as a function of R/d and ω

For $n_z = 0$	$\Lambda_{max} = [N - (N \bmod 4)]/4$ N (or Z) $= 4\Lambda_{max} + 2$	N (or Z) $= 4\Lambda_{max}$
1p-1h	$I = 2\Lambda_{max} + 2$	$I = 2\Lambda_{max}$
2p-2h	$I = 4\Lambda_{max} + 2$	$I = 4\Lambda_{max} + 1$
3p-3h	$I = 6\Lambda_{max} + 4$	$I = 6\Lambda_{max}$
4p-4h	$I = 8\Lambda_{max} + 4$	$I = 8\Lambda_{max} + 1$
5p-5h	$I = 10\Lambda_{max} + 6$	$I = 10\Lambda_{max}$



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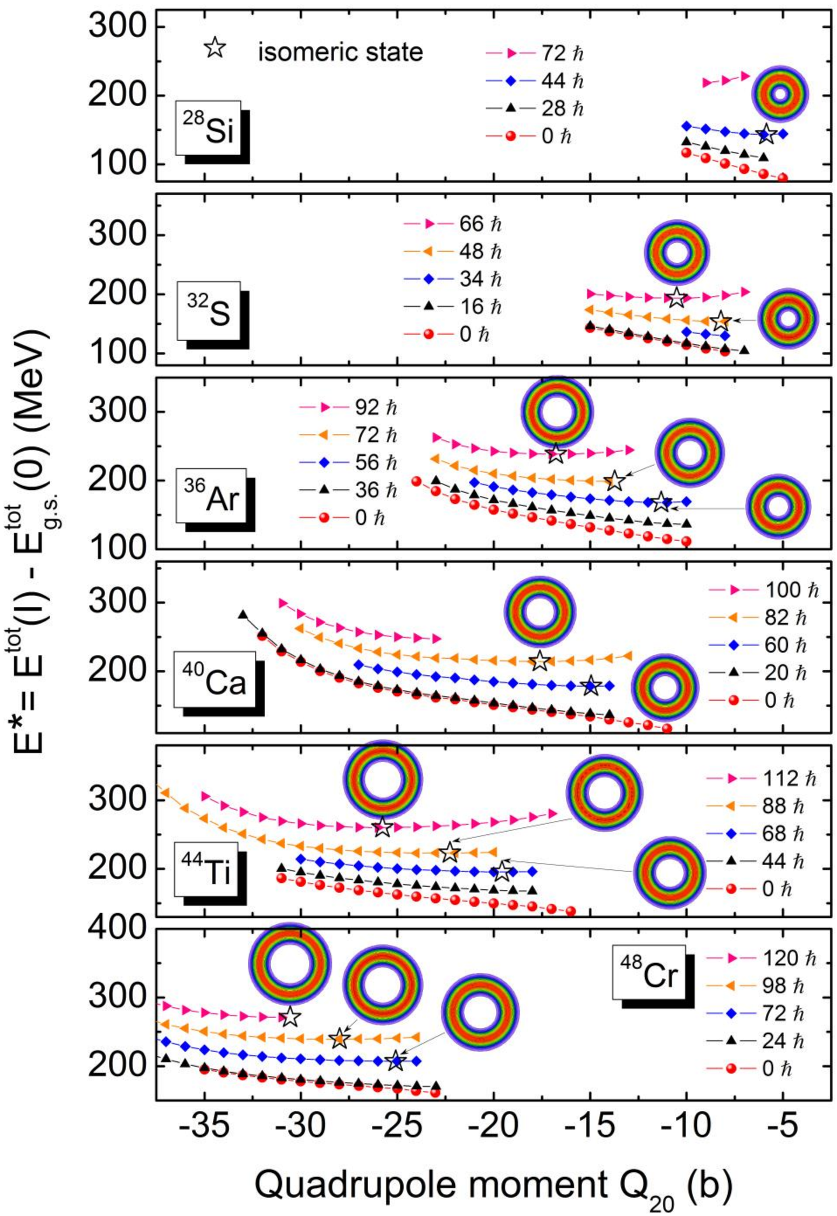
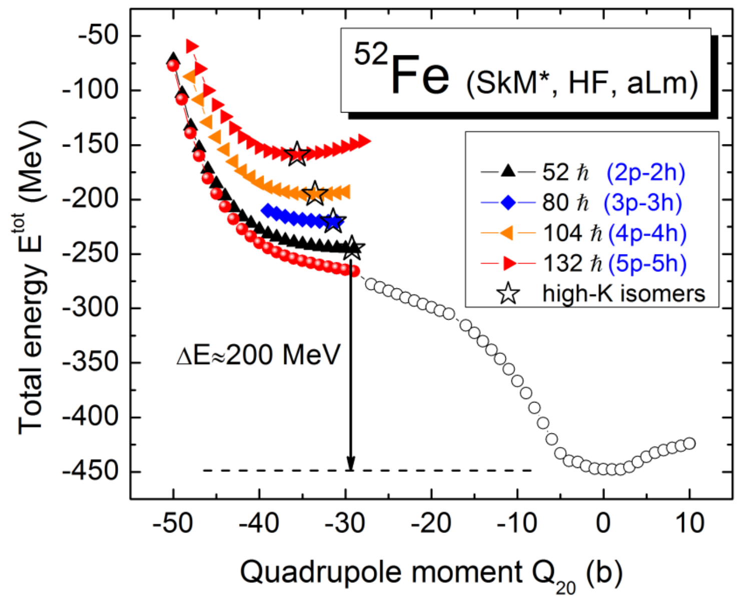
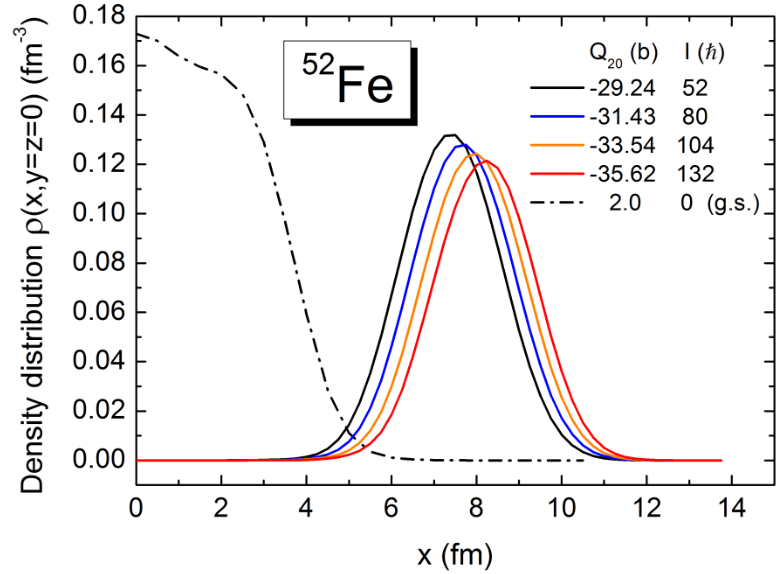
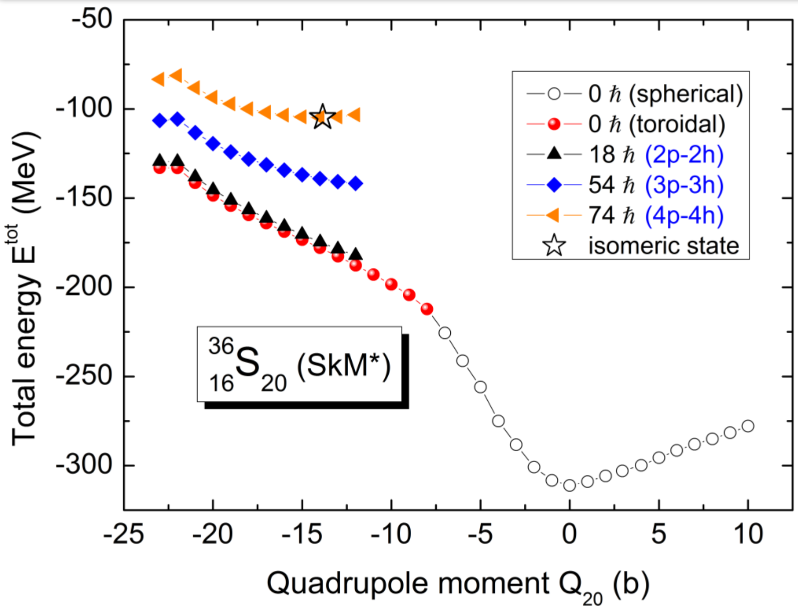
Toroidal high-spin isomers (*N*=*Z*)

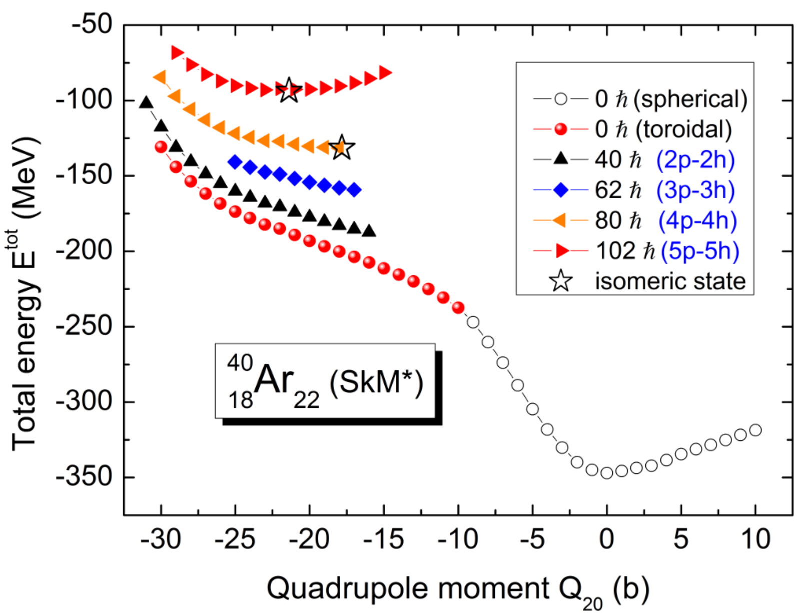


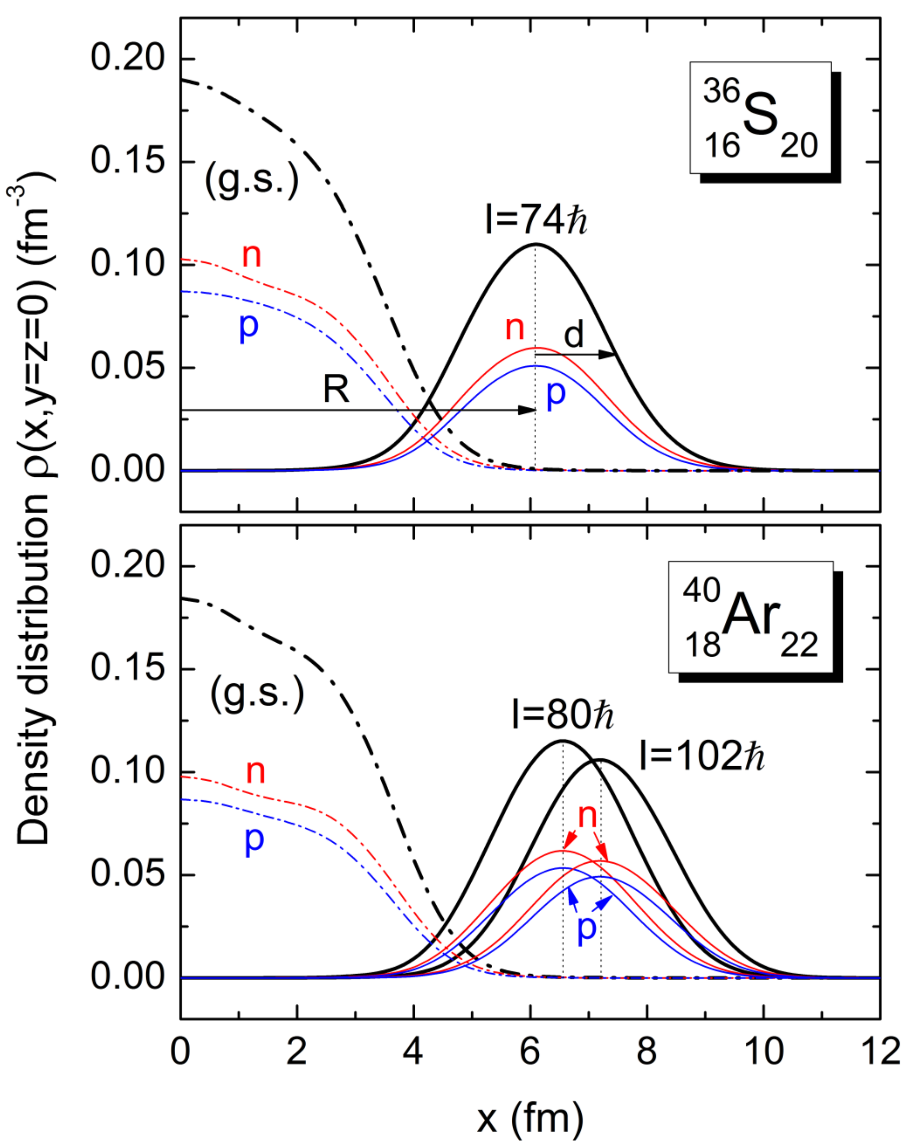




Toroidal high-spin isomers ($N \neq Z$)



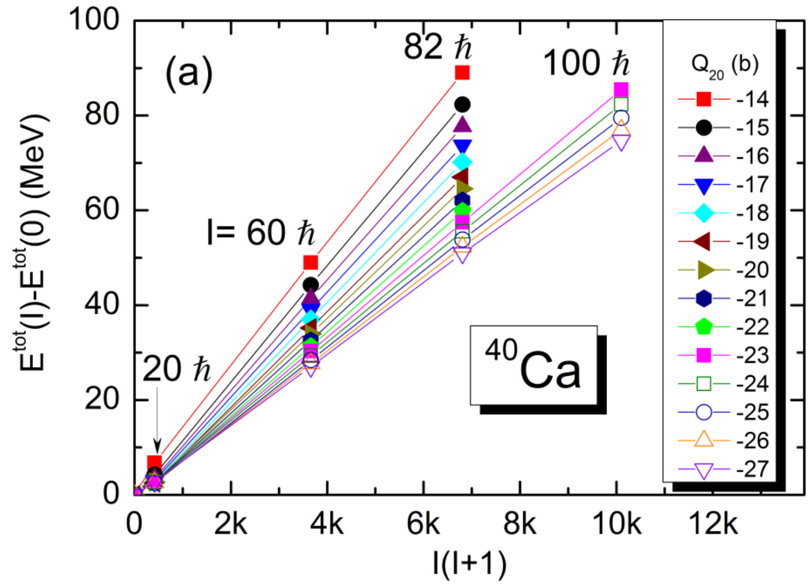




Effective moment of inertia

An effective moment of inertia $\Im_{eff}$:

$$E^{tot}(I) = E^{tot}(0) + \frac{\hbar^2}{2\Im_{eff}} I(I+1).$$



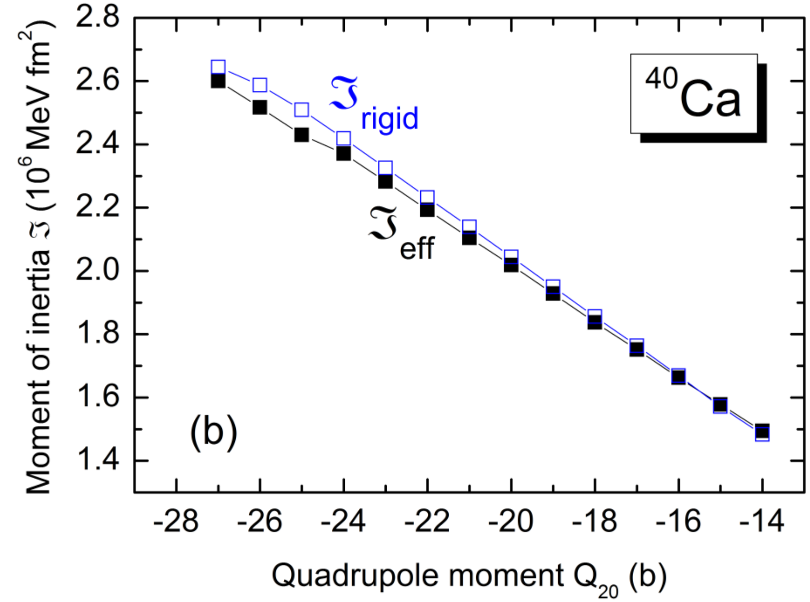
The toroidal density can be parameterized as

$$\rho(r,z) = \rho_{\max} \exp\{-[(r-R)^2 + z^2]/\sigma^2\},$$

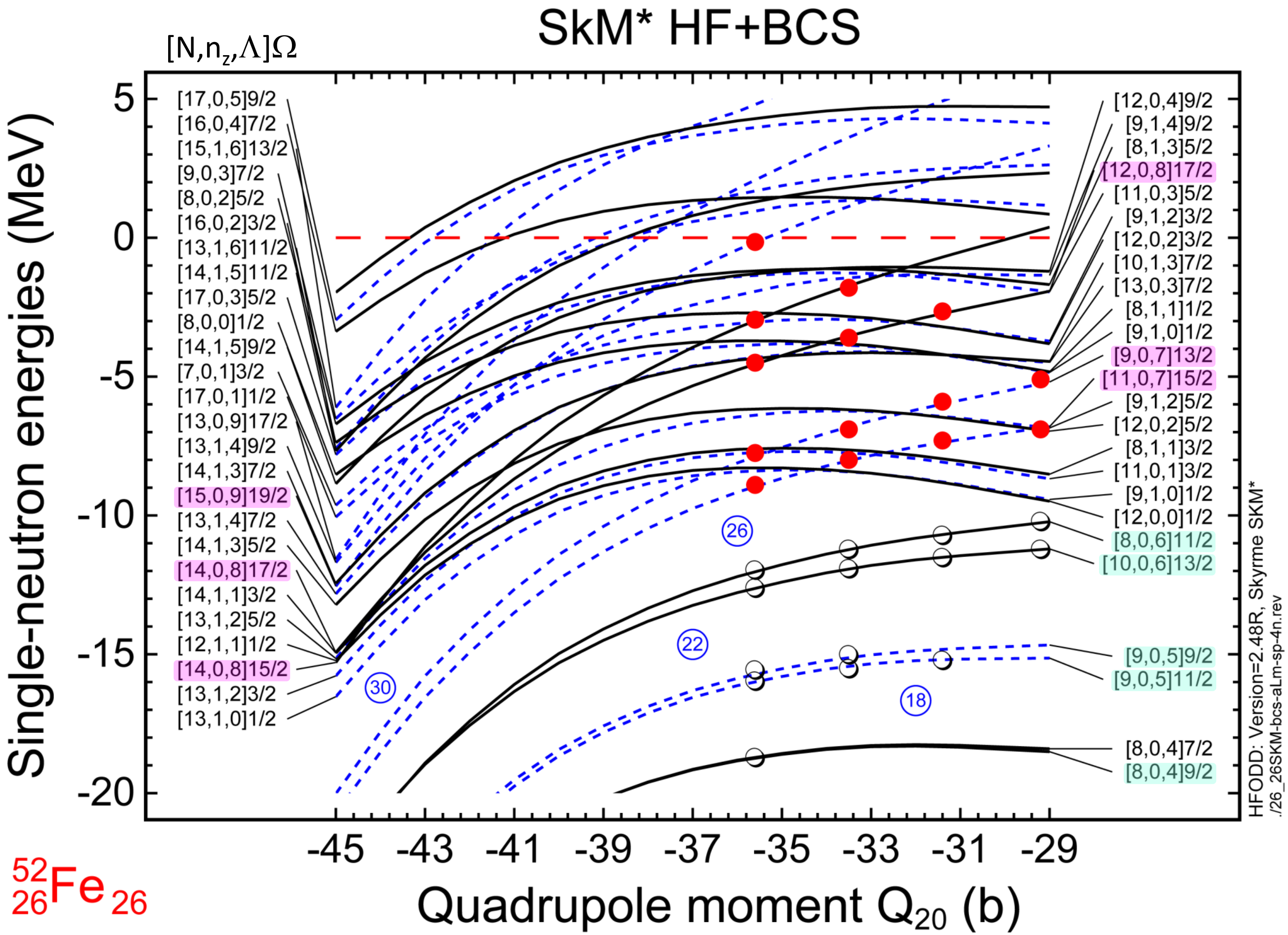
$$\sigma = d/\sqrt{\ln 2}, \quad d \equiv \tfrac{1}{2}\,\mathrm{FWHM},$$

and the rigid-body moment of inertia is

$$\Im_{rigid} = m_N 2\pi^2 R^2 \sigma^2 \rho_{\max} (R^2 + \tfrac{3}{2}\sigma^2).$$

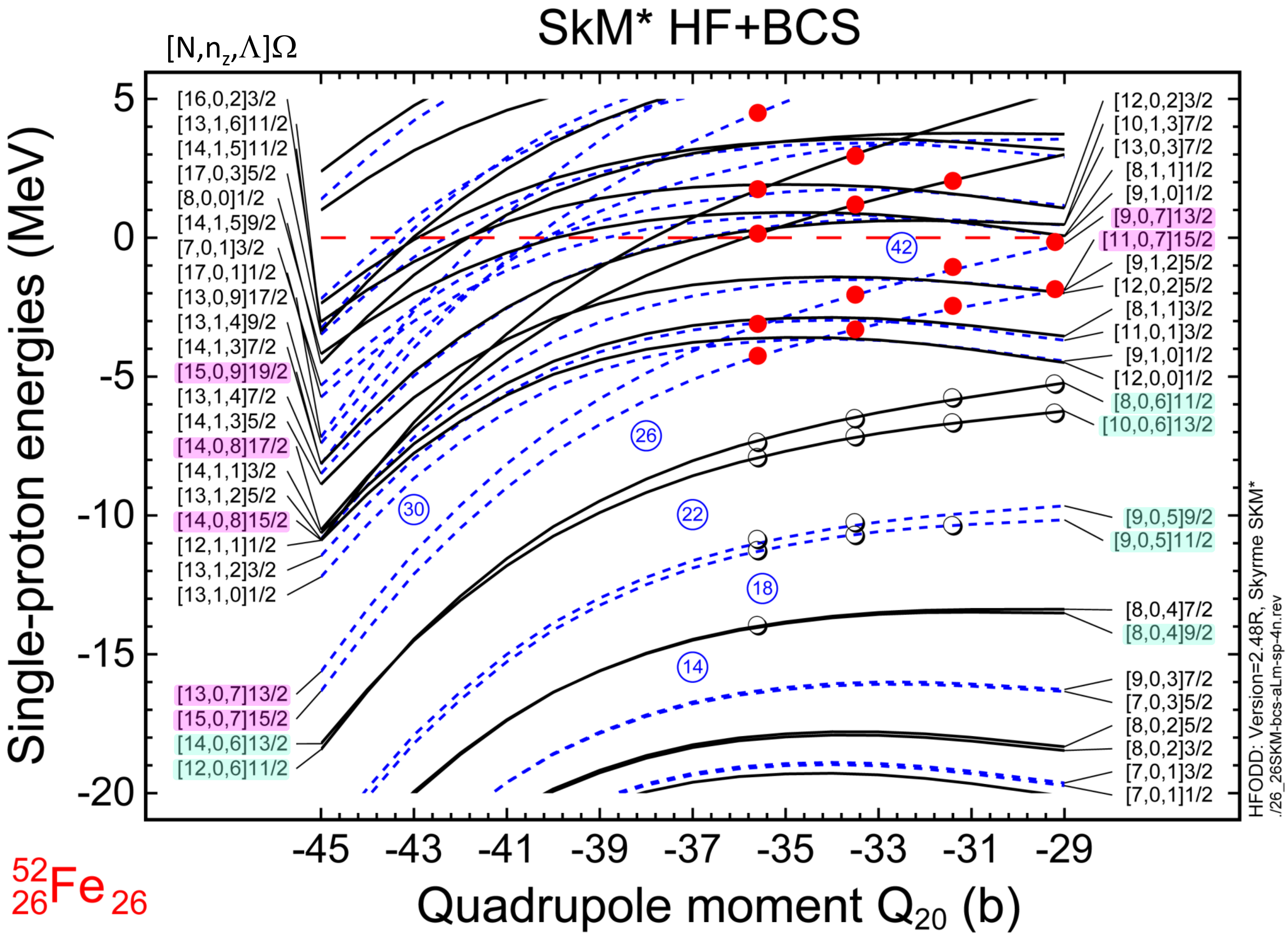


Neutron-quasiparticle energies for toroidal ^{52}Fe with I=0



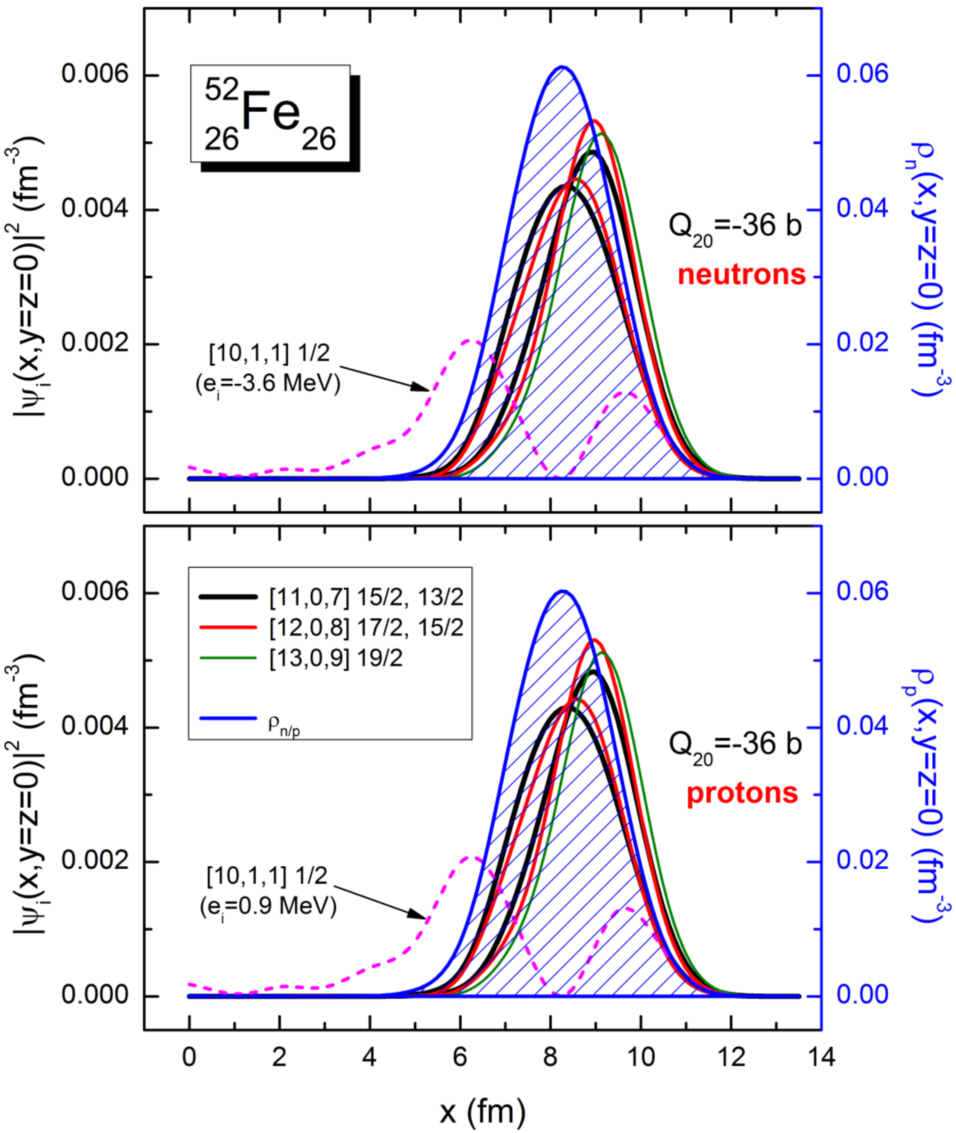
$^{52}_{26}Fe_{26}$

Proton-quasiparticle energies for toroidal ^{52}Fe with I=0



$^{52}_{26}Fe_{26}$

Stability of toroidal high-spin isomers against nucleon emission

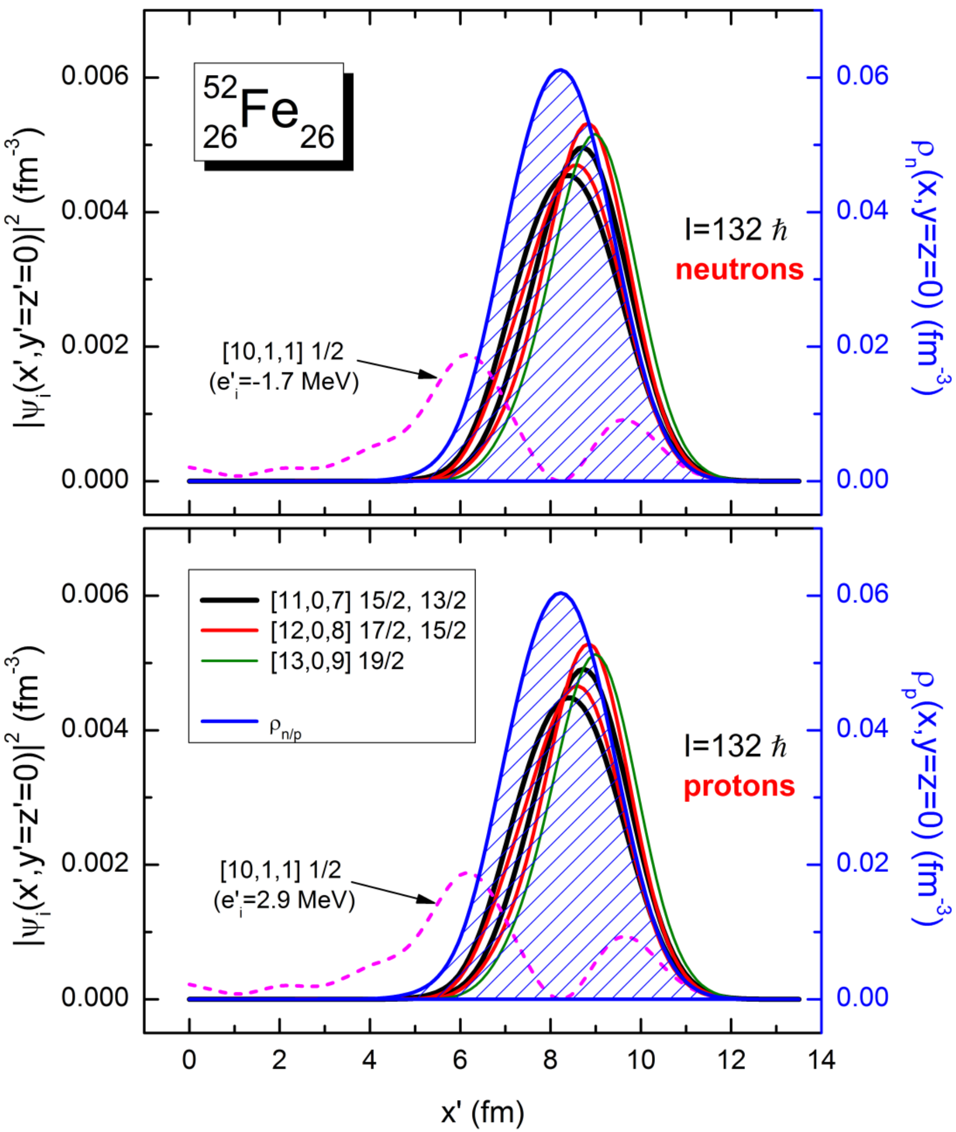


The modulus squared of the wave functions $[N,n_z=0,\Lambda=7,8,9]\Omega$, representing the particle states in the *5*p.-*5*h excit. of ^{52}Fe.

The wave functions and the density distributions ρ were calculated in the **SkM*-HF+BCS model** with a constrain on the quadrupole moment, Q_{20}=-36 b, which is close to the deformation of the toroidal ^{52}Fe(132$\hbar$) isomer.

The n_z=0 and Λ=7,8,9 wave functions do not exhibit the unbound characteristics of leakage and oscillation beyond the single-particle potential. They are well localized in the toroidal region of the attractive mean-field potential. These states seem analogous to the bound states in the continuum (BIC) first suggested by von Neumann and Wigner in 1929.

Stability of toroidal high-spin isomers against nucleon emission



The wave functions and the density distributions ρ were calculated in the **cranked SkM*-HF model** (in the intrinsic frame) for the equilibrium configuration of the ^{52}Fe(132ℏ) isomer.

The n_z=0 and Λ=7,8,9 wave functions do not exhibit the unbound characteristics of leakage and oscillation beyond the single-particle potential. They are well localized in the toroidal region of the attractive mean-field potential. These states seem analogous to the bound states in the continuum (BIC) first suggested by von Neumann and Wigner in 1929

.

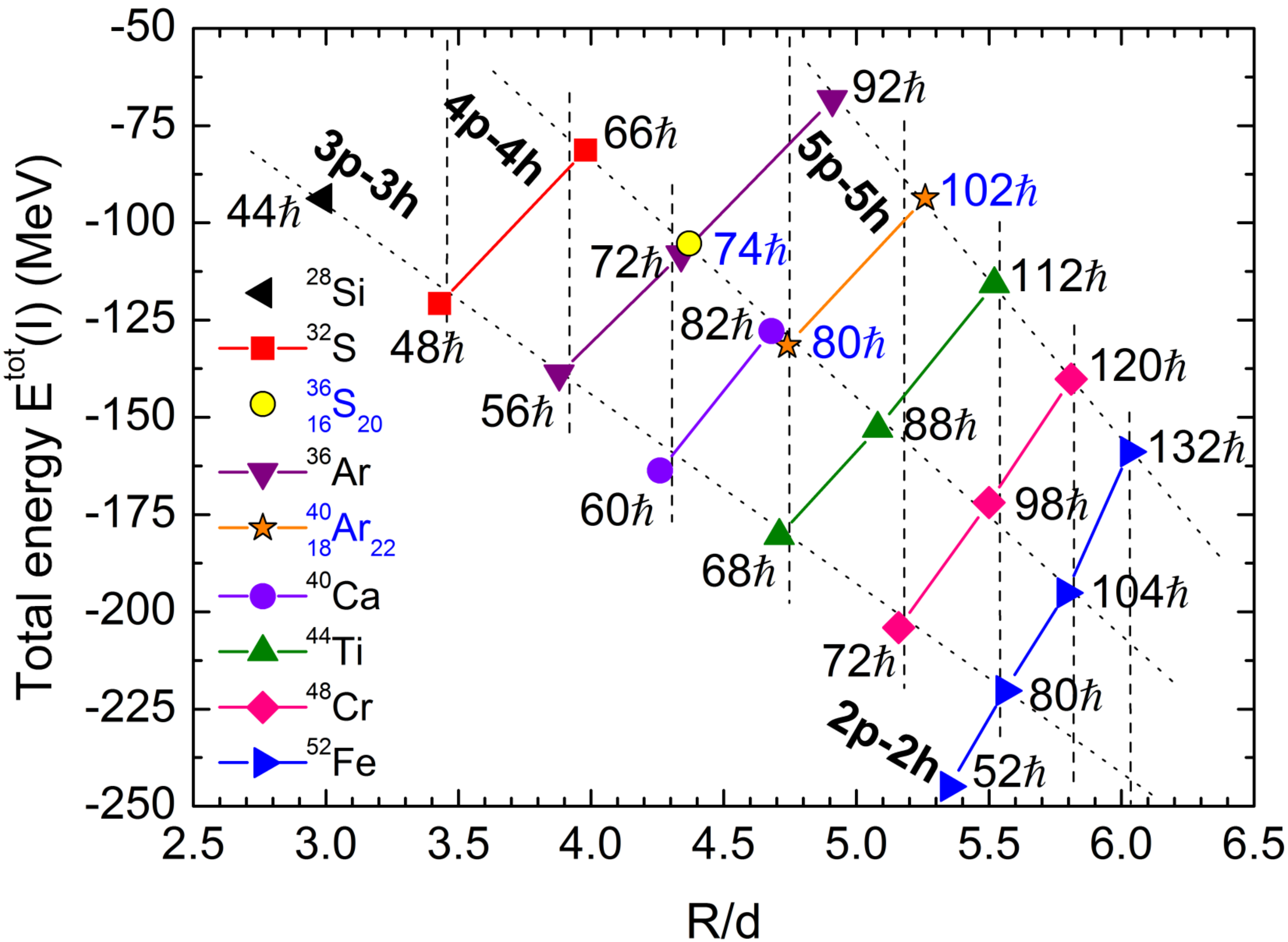
In contrast, the wave functions of the [10,1,1]1/2 state that is not used for the construction of the toroidal isomer, has a wave function extending outside the single-particle potential.

Properties of high-spin toroidal isomers with 28⩽A⩽52

Z	N	E*	I_z	$\hbar\omega$	Q_{20}	$R=x_c$	d	R/d	$\rho_{max}=A$	x_c	w	A
-	-	MeV	$\hbar$	MeV	b	fm	fm	-	fm^{-3}	fm	fm	fm^{-3}
14	14	143.18	44	2.8	-5.86	4.33	1.45	2.99	0.12	4.33	1.23	0.119
16	16	153.87	48	1.9	-8.22	4.87	1.42	3.43	0.12	4.87	1.21	0.122
		193.35	66	2.2	-10.51	5.57	1.40	3.98	0.11	5.57	1.19	0.108
16	20	206.49	74	1.8	-13.85	6.06	1.39	4.36	0.11	6.06	1.18	0.112
18	18	168.03	56	1.7	-11.31	5.44	1.40	3.88	0.12	5.44	1.19	0.125
		198.63	72	1.85	-13.73	6.04	1.39	4.34	0.11	6.04	1.18	0.113
		238.56	92	2.0	-16.78	6.73	1.37	4.91	0.10	6.73	1.17	0.103
18	22	215.49	80	1.65	-17.83	6.56	1.38	4.75	0.12	6.56	1.18	0.116
		253.42	102	1.85	-21.37	7.21	1.37	5.26	0.11	7.21	1.17	0.107
20	20	178.36	60	1.5	-14.96	5.97	1.40	4.26	0.13	5.97	1.19	0.126
		214.23	82	1.9	-17.61	6.51	1.39	4.68	0.12	6.51	1.18	0.117
22	22	195.46	68	1.2	-19.57	6.55	1.39	4.71	0.13	6.55	1.18	0.128
		223.09	88	1.4	-22.27	7.01	1.38	5.08	0.12	7.01	1.17	0.120
		260.24	112	1.6	-25.76	7.56	1.37	5.52	0.11	7.56	1.16	0.113
24	24	207.12	72	1.2	-25.08	7.12	1.38	5.16	0.13	7.12	1.17	0.128
		239.26	98	1.4	-28.00	7.54	1.37	5.50	0.12	7.54	1.17	0.122
		271.02	120	1.43	-30.55	7.90	1.36	5.81	0.12	7.90	1.16	0.118
26	26	202.86	52	0.8	-29.24	7.39	1.38	5.35	0.13	7.39	1.17	0.134
		227.54	80	0.95	-31.43	7.68	1.38	5.56	0.13	7.68	1.17	0.130
		252.65	104	1.3	-33.54	7.94	1.37	5.79	0.13	7.94	1.16	0.126
		288.91	132	1.5	-35.62	8.20	1.36	6.03	0.12	8.20	1.16	0.123

$$y(x) = Ae^{-\frac{(x-x_c)^2}{2w^2}}, \quad R = x_c, \quad d \equiv \tfrac{1}{2}\,\mathrm{FWHW} = \sqrt{2\ln 2}\,w$$

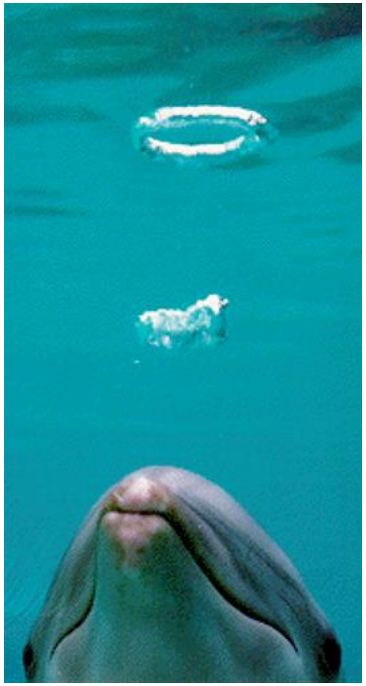
High-spin toroidal isomers in 28⩽A⩽52



Conclusions

- ✓ Light nuclei under non-collective rotation about the symmetry axis can assume a toroidal shape (toroidal high-spin isomers).
- ✓ Our investigation into the region of nuclei with $N \neq Z$ indicates that just as the $N=Z$ nuclei, toroidal high-spin nuclei are also commonly present in the mass region $28 \leq A \leq 52$.
- ✓ In the light high-spin toroidal isomers all occupied single-particle states have the same quantum number $n_z=0$.
- ✓ All these states, even with the positive energies, do not exhibit the unbound characteristics. They are not only localized in the toroidal region of the attractive mean-field potential but also square integrable. These states seem analogous to the bound states in the continuum (BIC) first suggested by von Neumann and Wigner in 1929 [1,2,3].

[1] J. von Neumann and E. Wigner, *Über merkwürdige diskrete Eigenwerte*, Z. Phys. **30** (1929) 465-467.
[2] F.H. Stillinger and D.R. Herrick, Phys. Rev. A **11** (1975) 446.
[3] A. Staszczak and C.Y. Wong, submitted to: Phys. Scr.



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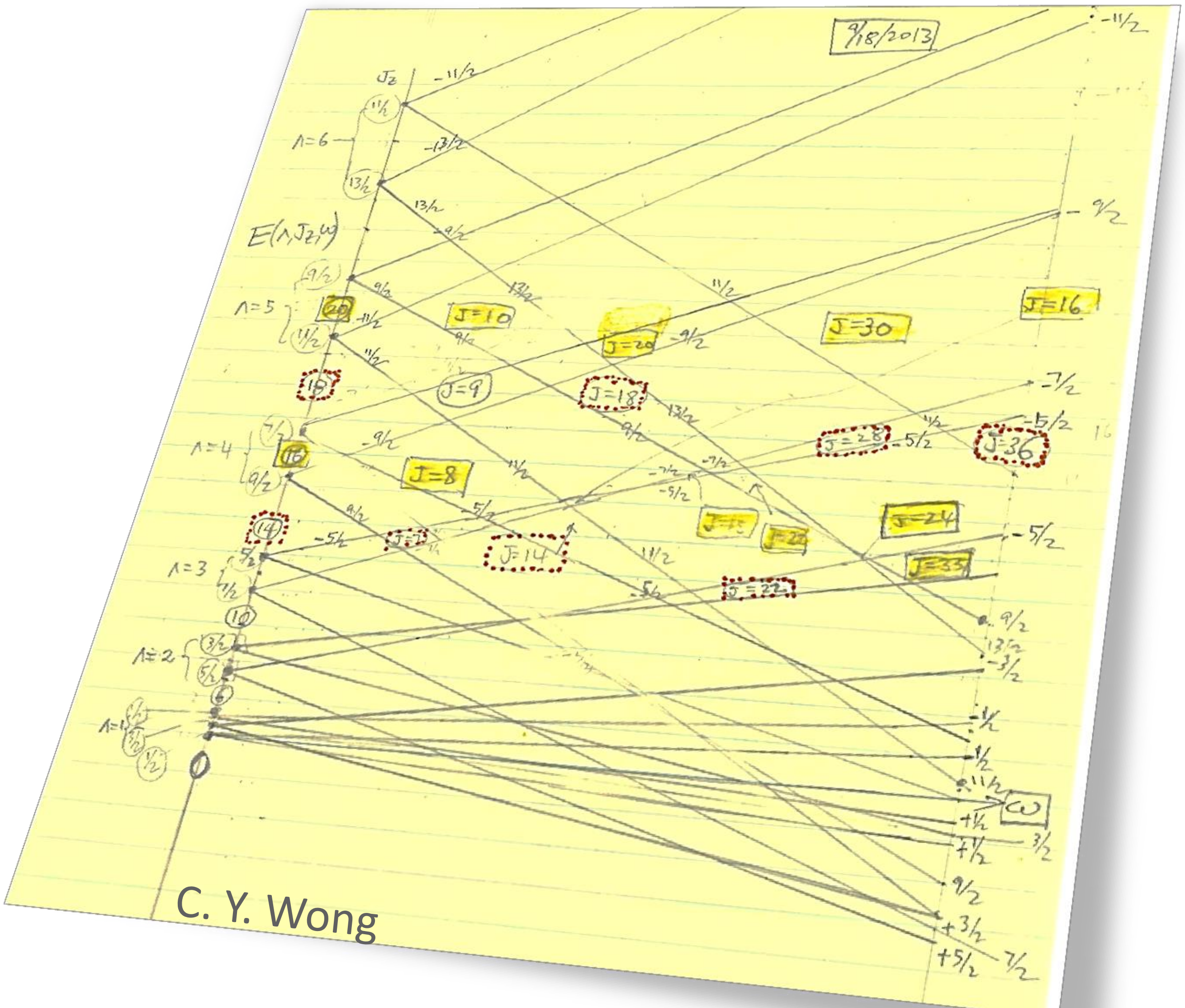
Non-collective *rotation* about symmetry axis



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