Impact of the In-Medium Conservation of Energy on the $\pi^-/\pi^+$ Multiplicity Ratio

Dan Cozma
IFIN-HH
Magurele/Bucharest, Romania
dan.cozma@theory.nipne.ro

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Elliptic Flow vs. Pion ratios

\[ \frac{dN}{d\phi} \sim 1 + 2v_1 \cos \phi + 2v_2 \cos 2\phi \]

**Isobar model** (no symmetry potential)

\[ \frac{\pi^-}{\pi^+} = \frac{(5N^2 + NZ)}{(5Z^2 + NZ)} \]

**UrQMD** - Y. Wang et al. PRC 89, 044603 (2014)

**TuQMD** - linear/moderately stiff
M.D. Cozma et al. PRC 88, 044912 (2013)

**UrQMD** – linear
P. Russotto et al. PLB 697, 491 (2011)

**IBUU** - linear/moderately stiff
G.-C. Yong private communication

**ImQMD** – stiff
Z.-Q. Feng et al., PLB 683, 140 (2010)

**Boltzmann-Langevin** – super-soft
W.-J. Xie et al., PLB 718, 1510 (2013)

**TuQMD (VEC)** – super-soft
M.D. Cozma (arXiv:1409.3110)

**pBUU** – no sensitivity to SE
J. Hong et al. PRC 90, 024605 (2014)
Energy Conservation - The Smoking Gun

80's transport models – total energy conserved (potentials dependent only on density)

Collective phenomena – momentum dependence
Isospin effects – isospin asymmetry dependence

Violation of total energy conservation

Determination of final state kinematics of 2-body collisions neglects medium effects

HA: Hartnack, Aichelin, PRC 49, 2901 (1994)
Transport Model

Quantum Molecular Dynamics (TuQMD):

Monte Carlo cascade + Mean field + Pauli-blocking+ in medium cross section

all 4* resonances below 2 GeV - 10 Δ* and 11 N*

baryon-baryon collisions:

all elastic channels

inelastic channels \( NN \rightarrow NN^* , NN \rightarrow N\Delta, NN \rightarrow \Delta N^* , NN \rightarrow \Delta\Delta^* , NR \rightarrow NR' \)

pion-absorption ⇄ resonance-decay channels: \( \Delta \leftrightarrow N\pi , \Delta^* \leftrightarrow \Delta\pi , N^* \leftrightarrow N\pi \)

meson production/absorption: \( \eta(547), \rho(770), \omega(782), \eta'(958), f_0(980), a_0(980), \Phi(1020) \)

previously applied to study:

- dilepton emission in HIC: K.Shekter, PRC 68, 014904 (2003); D. Cozma, PLB640,170 (2006); E.Santini PRC78,03410
- In-medium effects and HIC dynamics: C. Fuchs, NPA 626,987 (1997); U. Maheswari NPA 628,669 (1998)

upgrades implemented in Bucharest:

- various parametrizations for the EoS: optical potential, symmetry energy(power-law, Gogny)
- threshold effects for baryon resonance reaction emission absorption, \( \pi \) emission/absorption
- in-medium pion potential
- clusterization algorithms (MST, SACA): promising preliminary results
- planned: account for threshold effects for reactions involving strangeness degrees of freedom
Pion production

two step process:
- resonance excitation in baryon-baryon collisions
  parametrization of the OBE model of S.Huber et al., NPA 573, 587 (1994)
- resonance decay:
  Breit-Wigner shape of the resonance spectral function;

decay channels:
- pion absorption:
  - resonance model (all 4* resonances below 2 GeV)
  \[ R \to N \pi, \quad R \to N \pi \pi \]
  \[ R \to \Delta(1232) \pi, \quad R \to N(1440) \pi \]

pion absorption:
- resonance model (all 4* resonances below 2 GeV)

additional channels:
vector meson production/absorption
\[ V + B \to \pi + B', \pi + \pi + B' \]
\[ \pi + B \to V + B' \]

vector meson decay
\[ \rho \to \pi + \pi \]
\[ \omega \to 3\pi, \ 2\pi^0 \]

pion annihilation
\[ \pi + \pi \to \rho \]
Isospin dependence of EoS

a) momentum dependent – generalization of the Gogny interaction:


\[ U(\rho, \beta, p, \tau, x) = A_u(x) \frac{\rho_\tau}{\rho_0} + A_l(x) \frac{\rho_\tau}{\rho_0} + B(\rho/\rho_0)^\sigma (1 - x/\beta^2) - 8\tau x \frac{B}{\sigma + 1} \frac{\rho^{\sigma-1}}{\rho_0^\sigma} \beta \rho_\tau \]

\[ + \frac{2C_{\tau \tau}}{\rho_0} \int d^3 p' \frac{f_\tau(\vec{p}, \vec{p}')}{1 + (\vec{p} - \vec{p}')^2/\Lambda^2} + \frac{2C_{\tau \tau}}{\rho_0} \int d^3 p' \frac{f_{\tau'}(\vec{p}, \vec{p}')}{1 + (\vec{p} - \vec{p}')^2/\Lambda^2} \]

\[
S(\rho) = S(\rho_0) + \frac{L_{\text{sym}}}{3} \frac{\rho - \rho_0}{\rho_0} + K_{\text{sym}} \frac{(\rho - \rho_0)^2}{18} \rho_0^2
\]

<table>
<thead>
<tr>
<th>x</th>
<th>$L_{\text{sym}}$ [MeV]</th>
<th>$K_{\text{sym}}$ [MeV]</th>
</tr>
</thead>
<tbody>
<tr>
<td>-2</td>
<td>152</td>
<td>418</td>
</tr>
<tr>
<td>-1</td>
<td>106</td>
<td>127</td>
</tr>
<tr>
<td>0</td>
<td>61</td>
<td>-163</td>
</tr>
<tr>
<td>1</td>
<td>15</td>
<td>-454</td>
</tr>
<tr>
<td>2</td>
<td>-31</td>
<td>-745</td>
</tr>
</tbody>
</table>

b) momentum dependent – power law

\[ U_{\text{sym}}(\rho, \beta) = \begin{cases} \quad S_0(\rho/\rho_0)\gamma - \text{linear, stiff} \\ a + (18.5 - a)(\rho/\rho_0)\gamma - \text{soft, supersoft} \end{cases} \]
Energy conservation (in-medium)

\[ \sqrt{p_1^2 + m_1^2} + U(p_1) + \sqrt{p_2^2 + m_2^2} + U(p_2) = \sqrt{p_1'^2 + m_1^2} + U(p_1') + \sqrt{p_2'^2 + m_2^2} + U(p_2') \]

- rarely considered in transport models below 1 AGeV, with a few exceptions:
  G. Ferini et al. PRL 97, 202301 (2006), C.Fuchs et al. PRC 55, 411 (1997),
  T. Song, C.M. Ko PRC 91, 014901 (2015)

- Ansatz for the isospin 3/2 resonance potential motivated by decay channel
  → see also S.A. Bass et al., PRC 51, 3343 (1995)

- imposed in the CM of the colliding nuclei (not in the Eckart frame)

- reactions: NN ↔ NR, R ↔ Nπ (R ↔ Nππ not corrected)

<table>
<thead>
<tr>
<th>Reaction</th>
<th>ΔU=U^f-U^i</th>
<th>Effect</th>
</tr>
</thead>
<tbody>
<tr>
<td>nn → pΔ^-</td>
<td>U^p-U^n&lt;0</td>
<td>more π^-</td>
</tr>
<tr>
<td>nn → nΔ^0</td>
<td>1/3(U^p-U^n)&lt;0</td>
<td>more π^-, π^0</td>
</tr>
<tr>
<td>np → pΔ^0</td>
<td>1/3(U^p-U^n)&lt;0</td>
<td>more π^-, π^0</td>
</tr>
<tr>
<td>np → nΔ^+</td>
<td>1/3(U^n-U^p)&gt;0</td>
<td>less π^+, π^0</td>
</tr>
<tr>
<td>pp → pΔ^+</td>
<td>1/3(U^n-U^p)&gt;0</td>
<td>less π^+, π^0</td>
</tr>
<tr>
<td>pp → nΔ^++</td>
<td>U^n-U^p&gt;0</td>
<td>less π^+</td>
</tr>
</tbody>
</table>

B.-A. Li, NPA 708 (365) 2002
Approximations

**Elastic scattering:**
\[ \sqrt{s_f} \approx \sqrt{s_i} \]
\[ \sqrt{s^*} = 0.5(\sqrt{s_f} + \sqrt{s_i}) \]

**Resonance excitation:**
\[ \sqrt{s_f} - \sqrt{s_i} \approx 25 \text{ MeV} \]
\[ \sqrt{s^*} = \sqrt{s_f} \]

**Resonance absorption:** detailed balance

P. Danielewicz et al., NPA 533, 712 (1992)

\[
\frac{d\sigma^{NR\rightarrow NN}}{d\Omega} = \frac{1}{4} \frac{m_r p_{NN}^2}{p_{NR}} \frac{d\sigma^{NN\rightarrow NR}}{d\Omega} \times \left[ \frac{1}{2\pi} \int_{m_N + m_r}^{\sqrt{s_i} - m_N} dM \frac{p_{NR} A_R(M)}{M} \right]^{-1}
\]

\[
\Delta E = \sqrt{s_f} - \sqrt{s_i}
\]

[Diagrams of initial, final state, and rescattering processes]
Different “scenarios”

VEC – vacuum energy conservation constraint

LEC - “local” energy conservation – limited impact on multiplicities and ratios

GEC - “global” energy conservation – conserve energy of the entire system

-in-medium cross-sections for the inelastic channels

\[
\sigma^{*\,(12\rightarrow 23)} = \left[ \frac{m_1^* m_2^* m_3^* m_4^*}{m_1 m_2 m_3 m_4} \right]^{1/2} \sigma^{(12\rightarrow 34)}
\]

Experimental data: W. Reisdorf et al. (FOPI) NPA 848, 366 (2010)

AuAu@400 MeV, b=0 fm
**Multiplicity Ratio**

**Energy Conservation Scenario**

![Graph showing multiplicity ratio for Au+Au at 400 AMeV with different optical potential models.](image)

**Optical Potential Dependence**

![Graph showing multiplicity ratio for Au+Au at 400 AMeV with different optical potential models.](image)
Pion ratio vs Elliptic Flow

Consistency between pion ratio and elliptic flow constraints can be achieved.

Conservation of the total energy of the system:

finer details:
medium modified cross-sections,
delta isobar potential

Impact of the $\Delta(1232)$ potential

**Phenomenology** – inclusive electron nucleus scattering (He,C,Fe) attractive
- $\Delta$-nucleus potential deeper than the nucleon-nucleus potential
  O’Connel, Sealock PRC 42, 2290 (1990)

**Ab initio calculations** – Argonne $v_{28}$ interaction (BBG) $\Rightarrow$ repulsive $\Delta$ potential

Baldo, Ferreira NPA 569, 645 (1994)  - 3D reduction of Bethe-Salpeter equation similar (DB)
Malfliet, de Jong PRC 46, 2567 (1992)  - strong dominant repulsive contribution from $T=2$ sector

\[ V(\Delta^{++}) = V_s + \frac{3}{2} V_v \]
\[ V(\Delta^+) = V_s + \frac{1}{2} V_v \]
\[ V(\Delta^0) = V_s - \frac{1}{2} V_v \]
\[ V(\Delta-) = V_s - \frac{3}{2} V_v \]
\[ V_s = \frac{1}{2} (V_n + V_p) \]
\[ \delta = \frac{1}{3} (V_n - V_p) \]


softer
Energy dependence

close or below threshold

Samurai (TPC+Nebula Collaboration)
- pion production, flow (including neutrons)
- energies 285-350 MeV

^{132}Sn+^{124}Sn, ^{105}Sn+^{112}Sn


above threshold (FOPI/GSI)
Conclusions

**Conservation of Energy:** important impact on pion multiplicities in heavy-ion collisions at a few hundred MeV impact energy

\[ \pi^-/\pi^+ : \] confirmed as sensitive to the stiffness of asy-EoS
- increased sensitivity below production threshold
- dependence on in-medium cross-sections and optical potential choice is modest

However:
- isovector part of the in-medium \( \Delta \) potential has a large/decisive impact

**Good news:** consistency between pion ratio and elliptic flow constraints can be achieved (GEC scenario only!)

**Differences between different models** – most likely due to different choices for the isovector \( \Delta(1232) \) and pion potentials

**To do list:**
- retardation and relativistic corrections
- in-medium baryon potentials (particularly \( \Delta(1232) \))
- pion potentials (recently included)

**Worst case scenario:** test of our understanding of hadronic interaction in the few hundred MeV energy domain
Approximations

final state phase space in NN->NR

\[ \sqrt{s} = \sqrt{m_N^2 + k^2} + \sqrt{m_R^2 + k^2} \]

**VEC**

\[ \sqrt{s} = \sqrt{m_N^2 + k^2} + \sqrt{m_R^2 + k^2} + V_N(p_N) + V_R(p_R) \]

**LEC, GEC**

\[ m_{\text{th}} \leq m_R \leq m_{\text{max}} \]
\[ 0 \leq \theta \leq \pi \]
\[ 0 \leq \Phi \leq 2\pi \]

consistent with \( \frac{d\sigma}{d\Omega} \)

\[ m_{\text{th}} \leq m_R \leq m_{\text{max}} \]
\[ \theta_{\text{min}} \leq \theta \leq \theta_{\text{max}} \]
\[ \Phi_{\text{min}} \leq \Phi \leq \Phi_{\text{max}} \]

approximate to vacuum case