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THE $q\bar{q}$ S-WAVE AXIAL-VECTOR MESONS IN THE COVARIANT $\widetilde{U}(12)$-SCHEME

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Abstract

We study the properties of axial vector mesons a_1 and b_1 as relativistic S-wave states which are predicted in the $\widetilde{U}(12)$-scheme, through the analyses of their radiative and pionic decays. Specifically, partial widths of the strong $a_1(b_1) \to \rho(\omega)\pi$ processes, their D/S-wave amplitude ratios, and radiative transition widths of $a_1(b_1) \to \pi\gamma$ processes are calculated by using a simple decay interaction model, and made a comparison with the respective experimental values.

1 Introduction

In recent years we have proposed the $\widetilde{U}(12)$-scheme [1, 2], a relativistically covariant level-classification scheme of hadrons. In this scheme, the ground state (GS) of light $q\bar{q}$ meson system is assigned as $\mathbf{12} \times \mathbf{12}^* = \mathbf{144}$- representation of the $U(12)_{SF}$-group at their rest frame. The $U(12)_{SF}$-group includes, in addition to the conventional non-relativistic $SU(6)_{SF}$-group, the new symmetry $SU(2)_\rho$ [1], which corresponds to the degree of freedom associated with negative energy Dirac spinor solutions of confined quarks inside hadrons. By inclusion of this extra $SU(2)$ spin freedom, it leads to the possible existence of some extra multiples, called *chiral states*, which do not exist in the ordinary non-relativistic quark model (NRQM). As an example, the light scalar $f_0(600)/\sigma$ meson, a controversial particle for long time, is identified as S-wave chiral state as well as π meson, and they play mutually the role of chiral partners in the $\widetilde{U}(12)$-scheme. As is well known, in conventional level classification scheme based on NRQM, lowest scalar meson is obliged to be assigned as orbital P-wave excited state. As an another example, the a_1 meson, possibly to be identified as the $q\bar{q}$ S-wave axial-vector mesons in the $\widetilde{U}(12)$-scheme. They form a linear representation of chiral symmetry with the S-wave ρ meson. Here it is notable that these σ and a_1 mesons are expected to have the light mass compared with the conventional case of the P-wave states. Furthermore, **144**-representation includes the another axial-vector meson state with $J^{PC} = 1^{+-}$, to be identified with the b_1 meson.

In this work, we try to elucidate the properties of our new-type S-wave axial-vector mesons, a_1 and b_1, whose existence are predicted in the $\widetilde{U}(12)$-scheme, through the analyses of their radiative and pionic decay. In the actual analyses, we identify our chiral S-wave a_1 and b_1 mesons as the experimentally well-known states, $a_1(1260)$ and $b_1(1235)$, respectively. Then, by using a simple decay interaction, their partial widths of the strong a_1 $(b_1)\rightarrow \rho$ $(\omega)\pi$ decays (with D/S-wave amplitude ratios,) and radiative transition widths of a_1 (b_1) $\rightarrow \pi\gamma$ processes are calculated in comparison with the respective experimental values.

2 Wave functions of the a_1 and b_1 mesons as S-wave chiral states

In this section we collect the concrete expressions of meson wave function (WF) in our scheme necessary for the relevant applications [2].

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[1] The new degree of freedom corresponding to the $SU(2)_\rho$-symmetry is called the ρ-spin, after the well-known $\rho \otimes \sigma$-decomposition of Dirac matrices.

[2] In more detail, see Ref. 1, 2, 4)

Basic framework of our level-classification scheme is what is called the boosted LS-coupling (bLS) scheme. In this scheme, the WF of $q\bar{q}$ GS mesons are given by the following (bi-local Klein Gordon) field with one each upper and lower indices [3],

$$\Phi(X,x)_A^{(+)B} = Ne^{+iP\cdot X}\ W(v)_{\alpha,a}^{(+)\beta,b}\ f_G(v,x). \tag{1}$$

Where $A=(\alpha,a)$ $(B=(\beta,b))$ denotes Dirac spinor and flavor indices respectively, X_μ (x_μ) represents the center of mass (CM) (relative) coordinate of the composite meson. The P_μ ($v_\mu = P_\mu/M$, M being the mass of meson; $v_\mu^2=-1$, $v_0=+1$) denotes 4-momentum (4-velicity) of the relevant mesons. In the bLS scheme, respective spin ($W(v)_A{}^B$) and space-time [4] ($f_G(v,x)$) parts of WF are, separately, made covariant by boosting from the corresponding parts of NR ones.

Important feature of $\widetilde{U}(12)$-scheme is that the spin WF contains extra $SU(2)$ spin degree of freedom, called ρ-spin. As expansion bases of spinor WF, we use the Dirac spinor with hadron on-shell 4-velocity,

$$\{u_+(v), u_-(v)\}. \quad (\rho_3\ u_\pm = \pm u_\pm) \tag{2}$$

Here, u_+ corresponds to conventional constituent quark degree of freedom, while u_- is indispensable for covariant description of confined quarks [5] . Accordingly, expansion basis of $q\bar{q}$ meson WF is given by direct product of the respective spinor WF corresponding to the relevant constituent quark. They consist of totally 16 members in $\tilde{U}(4)_S$-space as,

$$W(v)_\alpha{}^\beta = u_r(v)_\alpha \bar{v}_{r'}(v)^\beta. \quad (r,r') = (\rho_3,\bar{\rho}_3) \tag{3}$$

We show the specific form of spin WF for the respective members of $q\bar{q}$ S-wave mesons, appeared in the relevant applications, in Table 1. Here it should be noted that, in the actual application, being based on its success [6)] with $SU(6)_{SF}$-description for $\rho(770)$-nonet, it seems that its WF should be taken as the form containing only positive ρ_3-states. This is made by taking the equal-weight superposition of two spin WF which belongs to the different chiral representation, respectively.

[3] For simplicity, the only positive frequency part of WF is shown here.

[4] We have been adopted a definite metric type 4-dimensional oscillator function as $f_G(v,x)$ [7)].

[5] They form the chiral partner in basic representation of the chiral group.

Table 1: *Spin wave functions of S-wave mesons applying for in this work, at their rest frame. Note that the physical ρ meson WF is given by as a sum of the following two vector WF, the one only with $(\rho_3, \bar{\rho}_3) = (+,+)$.* (i=1,2,3)

Mesons	J^{PC}	$W(v=0)^{(+)}$	$SU(2)_L \otimes SU(2)_R$	$(\rho_3, \bar{\rho}_3)$
$a_1(1260)$	1^{++}	$\frac{\gamma_5 \gamma_i}{2}$	$(1_L, 0_R) \oplus (0_L, 1_R)$	$\frac{(-,+)+(+,-)}{\sqrt{2}}$
$b_1(1235)$	1^{+-}	$\frac{i\gamma_5 \sigma_{i4}}{2}$	$(\frac{1}{2}{}_L, \frac{1}{2}{}_R)$	$\frac{i((-,+)+(+,-))}{\sqrt{2}}$
$\rho(770)$	1^{--}	$\frac{i\gamma_i}{2}$	$(1_L, 0_R) \oplus (0_L, 1_R)$	$\frac{(+,+)+(-,-)}{\sqrt{2}}$
$\rho(1250)$ [9]	1^{--}	$\frac{\sigma_{i4}}{2}$	$(\frac{1}{2}{}_L, \frac{1}{2}{}_R)$	$\frac{(+,+)-(-,-)}{\sqrt{2}}$
$\pi(140)$	0^{-+}	$\frac{i\gamma_5}{2}$	$(\frac{1}{2}{}_L, \frac{1}{2}{}_R)$	$\frac{(+,+)+(-,-)}{\sqrt{2}}$

3 Radiative decays of the a_1 and b_1 mesons

At first, we will consider the radiative decays of a_1 and b_1 mesons. In this work, we focus on the radiative transitions among the GS mesons. Therefore we are able to adopt simply the effective spin-type interaction,

$$H = \bar{q}\, \sigma_{\mu\nu} F_{\mu\nu} ((iv\gamma)g + g')\, q\ . \tag{4}$$

Here we introduced two independent coupling parameters g and $g^{'}$. The g term contributes to only quark chirality conserving transitions, while the $g^{'}$ term does to chirality non-conserving ones. By applying the quark-photon interaction (4), the effective meson current is given by the following formulas,

$$J_\mu(P, P^{'}) = J_{1,\mu}(P, P^{'}) + J_{2,\mu}(P, P^{'}). \tag{5}$$

Here, subscript 1 (2) represents the coupling of the emitted single photon with the relevant meson system through constituent quark (anti-quark). The specific form of the current is represented by

$$J_{1,\mu}(P, P^{'}) = e_q I_G^{(\gamma)} \ \langle \bar{W}^{(-)}(v^{'})[2gi\sigma_{\mu\nu}q_\nu] iv\gamma W^{(+)}(v) iv\gamma \rangle\ , \tag{6}$$

$$J_{2,\mu}(P, P^{'}) = e_{\bar{q}} I_G^{(\gamma)} \ \langle iv\gamma W^{(+)}(v) iv\gamma [-2g(-i\sigma_{\mu\nu}q_\nu)] \bar{W}^{(-)}(v^{'}) \rangle \tag{7}$$

for the case of the chirality conserving transition; and similarly

$$J^{'}_{1,\mu}(P, P^{'}) = e_q I_G^{(\gamma)} \ \langle \bar{W}^{(-)}(v^{'})[2g^{'} i\sigma_{\mu\nu}q_\nu] W^{(+)}(v) iv\gamma \rangle\ , \tag{8}$$

$$J^{'}_{2,\mu}(P, P^{'}) = e_{\bar{q}} I_G^{(\gamma)} \ \langle iv\gamma W^{(+)}(v)[-2g^{'}(-i\sigma_{\mu\nu}q_\nu)] \bar{W}^{(-)}(v^{'}) \rangle\ , \tag{9}$$

for the case of the chirality non-conserving transition. Here $q_\mu = P_\mu - P'_\mu$ denotes the 4-momentum of emitted photon, $I_G^{(\gamma)}$ is overlapping integral (OI) of space-time oscillator function, which gives a Lorentz invariant transition form factor as

$$I_G^{(\gamma)} = \int d^4x f_G^*(v',x) f_G(v,x) e^{-i\frac{1}{2}q_\mu x_\mu} \tag{10}$$

$$= \left(\frac{2MM'}{M^2+M'^2}\right)\exp\left[-\frac{1}{2\Omega}\frac{(M^2-M'^2)^2}{M^2+M'^2}\right], \tag{11}$$

where we introduce the parameter Ω corresponding to the Regge slope inverse. In our scheme the relativistic covariance of the spin current, due to the inclusion of Dirac spinor with negative ρ_3-value, plays an important role in some radiative transition processes. To clarify this point, we rewrite the spin current vertex operator as

$$\sigma_{\mu\nu} i q_\nu A_\mu = \sigma_{\mu\nu} F_{\mu\nu} = \boldsymbol{\sigma}\cdot\mathbf{B} - i\rho_1 \boldsymbol{\sigma}\cdot\mathbf{E} \quad . \tag{12}$$

In the cases of transition between both positive (negative) ρ_3 Dirac spinors, as is well known, the main contribution comes from the magnetic interaction. On the other hand, in the case of transitions between Dirac spinors with positive and negative ρ_3-values, the electric interaction, coming from the $\sigma_{i4} i q_i A_4$-term, becomes a dominant contribution. As a results, this *intrinsic electric dipole* [5] transition gives an important role for the transition accompanied with their parity change, such as $a_1(b_1) \to \pi\gamma$ processes.
In this work, we take the following values of parameters in our scheme.

- (g, g')=(2.59, 1.40) from $\Gamma_{\rm EXP}(b_1^+ \to \pi^+\gamma)$ and $\Gamma_{\rm EXP}(\rho^+ \to \pi^+\gamma)$
- $\Omega_{n\bar{n}} = 1.13$ GeV^2 from $\Omega = M(^3P_2)^2 - M(^3S_1)^2 = M(a_2(1320))^2 - M(\rho(770))^2$

The masses of the respective mesons are taken from PDG [3], except for the one of the pion in the form factor with $M_\pi = 0.78$ GeV. The estimated widths are in comparison with experiment in Table 2. Results for this calculation are consistent with experiments.

Table 2: *Radiative decay widths in comparison with experiment. Experimental data are taken from PDG* [3]. (KeV)

Process	Our results	Experimental values
$\rho(770) \to \pi\gamma$	68 (input)	68±7
$b_1(1235) \to \pi\gamma$	230 (input)	230±60
$a_1(1260) \to \pi\gamma$	604	640±246

4 Pion emissions of a_1 and b_1 mesons

Next we consider the strong decays with one pion emission. We adopt simply the following two types of effective quark-pion interactions;

$$L_{ps} = g_{ps}\ \bar{q}(-i\gamma_5)q\ \pi, \tag{13}$$
$$L_{pv} = g_{pv}\ \bar{q}(-i\gamma_5\gamma_\mu)q\ \partial_\mu\pi. \tag{14}$$

Note that here, π (and σ) meson is treated as an external local-field. Resultant matrix elements are given as a sum of two terms;

$$T = T_{ps} + T_{pv}\ , \tag{15}$$
$$T_{ps} = g_{ps} I_G^{(\pi)}\ \langle W(v^{'})(-i\gamma_5\pi)W(v) iv\gamma\rangle + c.c.\ \ , \tag{16}$$
$$T_{pv} = g_{pv} I_G^{(\pi)}\ \langle W(v^{'})(-\gamma_5\gamma_\mu q_\mu\pi)W(v) iv\gamma\rangle + c.c.\ \ . \tag{17}$$

In the above case, the OI of the space-time WF is given by

$$I_G^{(\pi)} = \int d^4x f_G^*(v^{'},x) f_G(v,x) e^{-i\frac{1}{2}q_\mu x_\mu} \tag{18}$$
$$= (\frac{2MM^{'}}{M^2+M^{'2}-m_\pi^2})\exp[-\frac{(M^2-M^{'2})^2-m_\pi^2(M^2+M^{'2})}{2\Omega\,(M^2+M^{'2}-m_\pi^2)}], \tag{19}$$

where $q^2 = -m_\pi^2$, $q_\mu = P_\mu - P_\mu^{'}$ being the 4-momentum of emitted pion. The relevant decay amplitude is

$$T = f_1\ \epsilon_\mu(v')\epsilon_\mu(v)\ +\ f_2\ (q_\mu\epsilon_\mu(v'))(q_\nu\epsilon_\nu(v)). \tag{20}$$

The explicit forms of f_1 and f_2 are shown in Table 3. It may be worthwhile to note that at least two coupling types (expressed f_1 and f_2 in the above) are required to reproduce the experimental data on D/S-wave amplitude ratios.

Our decay interaction contains two independent coupling parameters, g_{ps} and g_{pv}, which will be commonly applied to all quark-pion vertices [6]. These are determined from the experimental data of D/S-wave amplitude ratio and total width of b_1 meson as,

- $\frac{g_{ps}}{g_{pv}} = 0.149$ GeV from $T_D/T_S|_{\rm EXP}(b_1^+ \to \omega\pi^+) = +0.277$
- $g_{pv} = 14.0$ from $\Gamma_{\rm EXP}(b_1^+ \to \omega\pi^+) \approx \Gamma_{\rm EXP}(b_1^+{}_{\rm total}) = 142$ MeV.

The masses of the relevant mesons are taken from PDG [3)]. The numerical results are shown in Table 3.

Table 3: *Coefficients of decay amplitude (20) for $a_1 \to \rho\pi$ and $b_1 \to \omega\pi$ process.*

	$b_1 \to \omega\pi$	$a_1 \to \rho\pi$
f_1	$I_G \times (-g_{ps} + (\omega M - M^{'})g_{pv})$	$I_G \times (-g_{ps}\omega + (M - \omega M^{'})g_{pv})$
f_2	$I_G \times (-g_{pv}\frac{1}{M^{'}})$	$I_G \times (g_{ps}\frac{1}{MM^{'}} + g_{pv}\frac{1}{M})$

Table 4: *Numerical results for pion emissions of a_1 and b_1 mesons. Experimental data are taken from PDG [3)].*

	T_D/T_S		Width (MeV)	
process	Our results	Experimental values	$\Gamma^{\rm theor.}_{\rm partial}$	$\Gamma^{\rm Exp.}_{\rm total}$
$b_1 \to \omega\pi$	0.277(input)	0.277± 0.027	142(input)	142±9
$a_1 \to \rho\pi$	-0.344	-0.108± 0.016	191	$250 \sim 600$

5 Concluding remarks

In this work, we investigate the decay properties of $q\bar{q}$ S-wave a_1 and b_1 mesons in the $\tilde{U}(12)$-scheme, by assigning them with $a_1(1260)$ and $b_1(1235)$ mesons, respectively.

At first, it is shown that the radiative decay widths of $(a_1, b_1, \rho) \to \pi\gamma$ processes are consistently reproduced by using the simple spin-type quark-photon effective interaction in the framework of the $\widetilde{U}(12)$-scheme.

[6]As an example, it is applied to the study of 'extra'- κ meson [8)].

Secondarily, for the strong one-pion emission decays, assuming the ps- and pv-type quark-pion effective interactions, the D/S-wave amplitude ratios and partial widths of $a_1(b_1) \to \rho(\omega)\pi$ decays are evaluated. As a results, by inputting the data for the b_1 meson, the sign of D/S-wave amplitude ratio for the $a_1 \to \rho\pi$ decay agrees with the experiments, but its absolute value is about three time larger than experiment. Partial width of $a_1 \to \rho\pi$ is predicted with $\Gamma(a_1 \to \rho\pi) \sim 200$MeV.
The interaction adopted in this work for the radiative/strong decays should be tested by applying for other various decay processes.

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