Structure of Mass Gap Between Two Spin Multiplets

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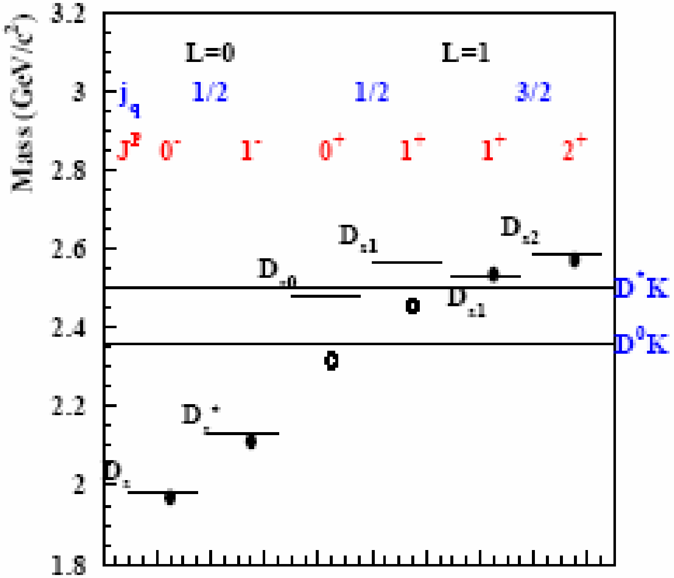
with T. Morii (Kobe U.) and K. Sudoh (KEK)

Based on

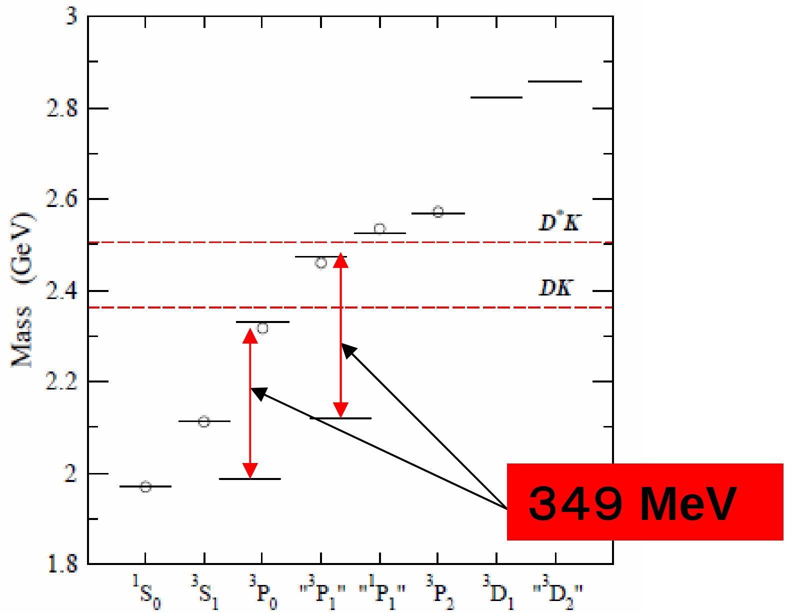
- **Structure of mass gap between two spin multiplets,**
  hep-ph/0710.0325
- **Radial Excitations of Heavy-Light System,**
  Eur. Phys. J. C31(2007) 701
- **New Heavy-Light Mesons Qqbar,**
  Prog. Theor. Phys. 51 (2007) 1077 (hep-ph/0605019).
- . **0^+ and 1^+ States of B and B_s Mesons,**
  Phys. Lett. B606 (2005) 329 (hep-ph/0411034).
- **Spectroscopy of heavy mesons expanded in $1/m_Q$,**
  Phys. Rev. D56 (1997) 5646.

Motivation1

- Successful prediction/reproduction of ***D_s*** mass spectra using our semi-relativistic potential model
  - Lowering 0^+ and 1^+ of $D_0^*(2308)$ and $D_1'(2427)$ compared with other potential models



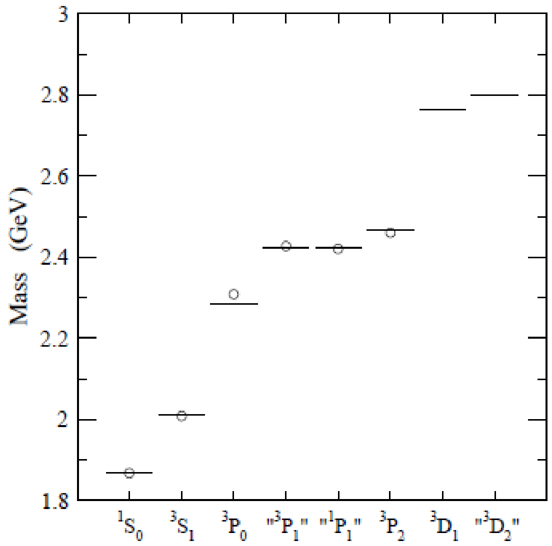
prediction by conventional potential model (Godfrey & Isgur, PRD43, 1679 (1991))



prediction by our semi-relativisitic potential model (Prog. Theor. Phys.117 (2007) 1077)

Other Mass Spectra

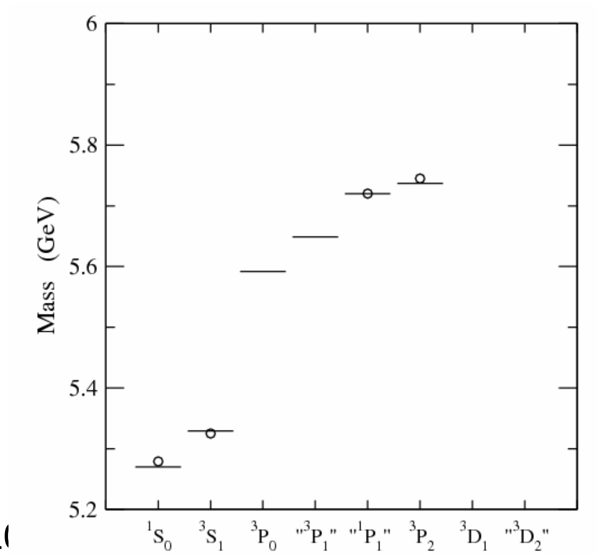
D



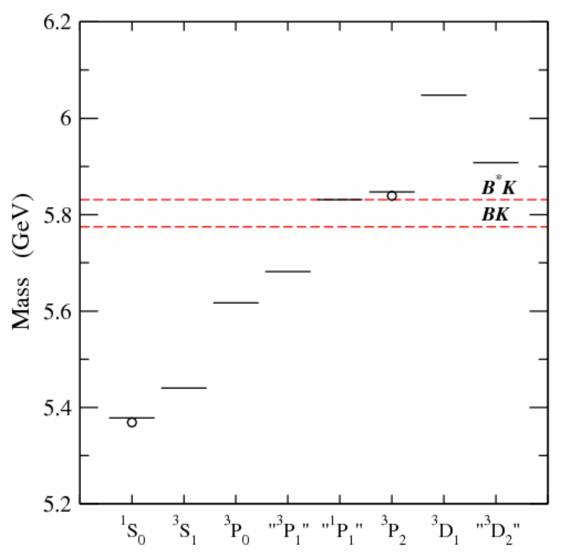
Successful reproduction of the following spectra

- D_0*(2308) and D_1'(2427) by Belle
- D_{s0}(2860) and D_s^*(2715) by BaBar & Belle (n=2; 0^+ and 1^- states of D_s)
- B_1(5720) and B_2*(5745) by D0 (1+ and 2+ states of B)
- B_{s2}*(5839) by D0 (2^+ state of B_s)

B



B_s



Motivation2

- Successful prediction of the mass gap using an effective Lagrangian approach (Bardeen & Hill)
  - $\Delta M(m_c) = 338\ \mathrm{MeV}$
  - Later corrected to the experimentally observed value by B+Eichten+H using ***S(3)***
  - **Wonderful agreement** with the experiments (349 MeV) !
  - Is this prediction?
  - Is this describing the underling physics?
  - Is an effective theory constructed from a four-fermi theory a **true theory**?
  - Their prediction has no light quark flavor dependency.
  - BEH tried to predict the **mass gap for D as 255 MeV** but the experiment gives more than 400 MeV.

    $D(0^+)$-$D(0^-)$=2308-1867=441MeV

    $D(1^+)$-$D(1^-)$=2427-2008=419MeV
  - Hence there is no way to distinguish their prediction with that for heavy D, B, or Bs mesons
- If finding the mass gap eq. depending on the light quark mass, there might be a hint to get to the true underlying physics.

D mesons

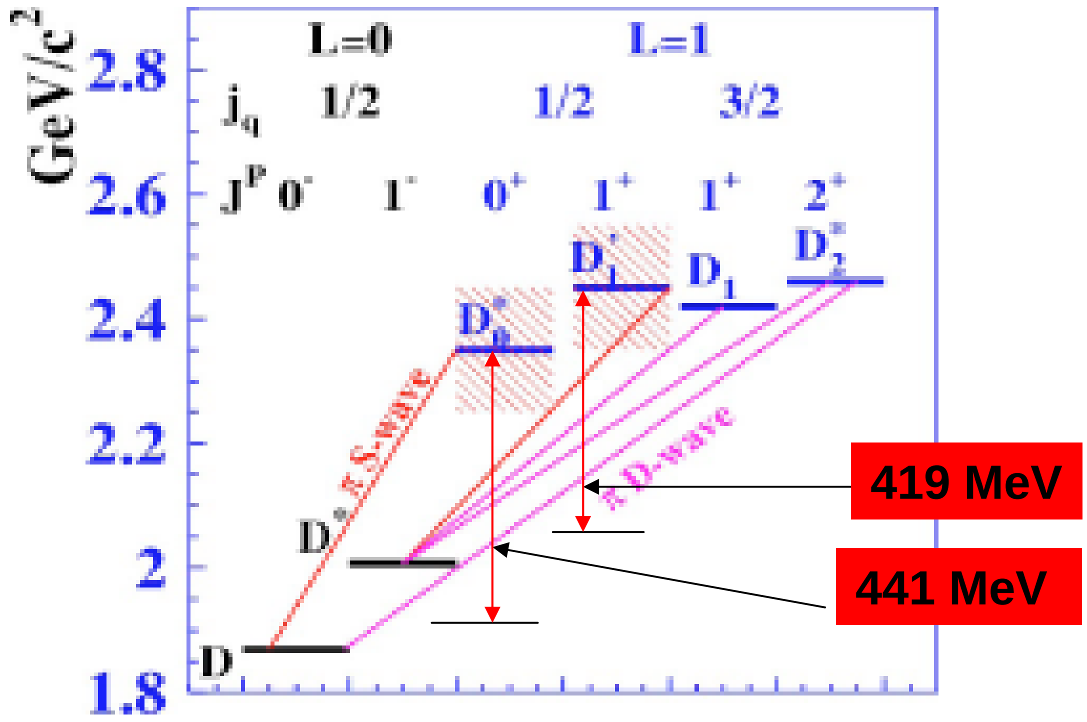


FIG. 1. Spectroscopy of *D*-meson excitations. The lines show possible single pion transitions.

Belle collab. PR D69 (2004) 112002

Our Model Hamiltonian for $Q\bar{q}$ System

- Start with an effective Hamiltonian for 2-body bound states

$$H = (\vec{\alpha}_q \cdot \vec{p}_q + \beta_q m_q) + (\vec{\alpha}_Q \cdot \vec{p}_Q + \beta_Q m_Q) + \beta_q \beta_Q S$$
$$+ \left[1 - \tfrac{1}{2} \left\{ \vec{\alpha}_q \cdot \vec{\alpha}_Q + (\vec{\alpha}_q \cdot \vec{n})(\vec{\alpha}_Q \cdot \vec{n}) \right\} \right] V$$

$$S(r) = \frac{r}{a^2} + b, \quad V(r) = -\frac{4}{3}\frac{\alpha_s}{r}$$

Semi-relativistic approach to heavy meson

→ Foldy-Wouthuysen-Tani (FWT) transformation to the heavy quark ~1/m_Q expansion

- Heavy Meson System (eigenvalue equation): mass of a heavy meson= $m_Q + E^l$

$$H\psi_l = E^l \psi_l \qquad H = H_{\mathrm{FWT}} - m_Q = m_Q H_{-1} + H_0 + \frac{1}{m_Q} H_1 + \frac{1}{m_Q^2} H_2 + \cdots$$

$$E^l = E_0^l + \frac{1}{m_Q} E_1^l + \frac{1}{m_Q^2} E_2^l + \cdots$$

$$\psi_l = \psi_{l0} + \frac{1}{m_Q} \psi_{l1} + \frac{1}{m_Q^2} \psi_{l2} + \cdots$$

Effective Hamiltonian Expanded in $1/m_Q$

$$(H_{\rm FWT} - m_Q) \otimes \psi_{\rm FWT} = \tilde{E}\psi_{\rm FWT}$$

spin-spin interaction

$$H_{\rm FWT} - m_Q = H_{-1} + H_0 + H_1 + H_2$$

$$H_{-1} = -(1+\beta_Q)m_Q$$

$$H_0 = \vec{\alpha}_q \cdot \vec{p} + \beta_q m_q - \beta_q \beta_Q S + \left\{1 + \tfrac{1}{2}[\vec{\alpha}_q \cdot \vec{\alpha}_Q + (\vec{\alpha}_q \cdot \vec{n})(\vec{\alpha}_Q \cdot \vec{n})]\right\} V$$

$$H_1 = -\frac{1}{2m_Q}\beta_Q \vec{p}^{\,2} + \frac{1}{m_Q}\beta_q \vec{\alpha}_Q \cdot \left(\vec{p} + \tfrac{1}{2}\vec{q}\right) S + \frac{1}{2m_Q}\vec{\gamma}_Q \cdot \vec{q} V$$

$$-\frac{1}{2m_Q}\left[\beta_Q\left(\vec{p} + \tfrac{1}{2}\vec{q}\right) + i\vec{q} \times \beta_Q \vec{\Sigma}_Q\right] \cdot \left[\vec{\alpha}_q + (\vec{\alpha}_q \cdot \vec{n})\vec{n}\right] V$$

LS force

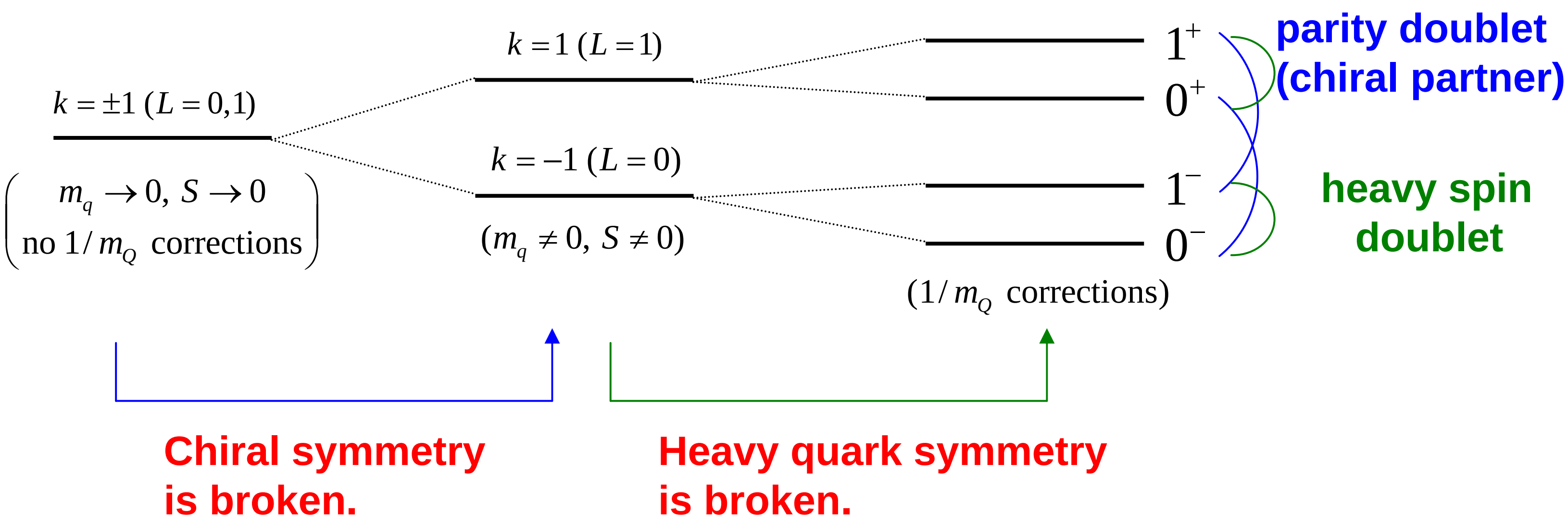
$$H_2 = \frac{1}{2m_Q}\beta_q \beta_Q \left(\vec{p} + \tfrac{1}{2}\vec{q}\right)^2 S - \frac{i}{4m_Q^2}\vec{q} \times \vec{p} \cdot \beta_q \beta_Q \vec{\Sigma}_Q S - \frac{1}{8m_Q^2}\vec{q}^{\,2} V - \frac{i}{4m_Q^2}\vec{q} \times \vec{p} \cdot \vec{\Sigma}_Q V$$

$$-\frac{1}{8m_Q^2}\left\{(\vec{p} + \vec{q})(\vec{\alpha}_Q \cdot \vec{p}) + \vec{p}[\vec{\alpha}_Q \cdot (\vec{p} + \vec{q})] + i\vec{q} \times \vec{p}\gamma_Q^5\right\} \cdot \left[\vec{\alpha}_q + (\vec{\alpha}_q \cdot \vec{n})\vec{n}\right] V$$

$$\because\ \vec{p} = \vec{p}_q = -\vec{p}_Q, \quad \vec{p}\,' = \vec{p}\,'_q = -\vec{p}\,'_Q, \quad \vec{q} = \vec{p}\,' - \vec{p}$$

Patterns of Symmetry Breaking

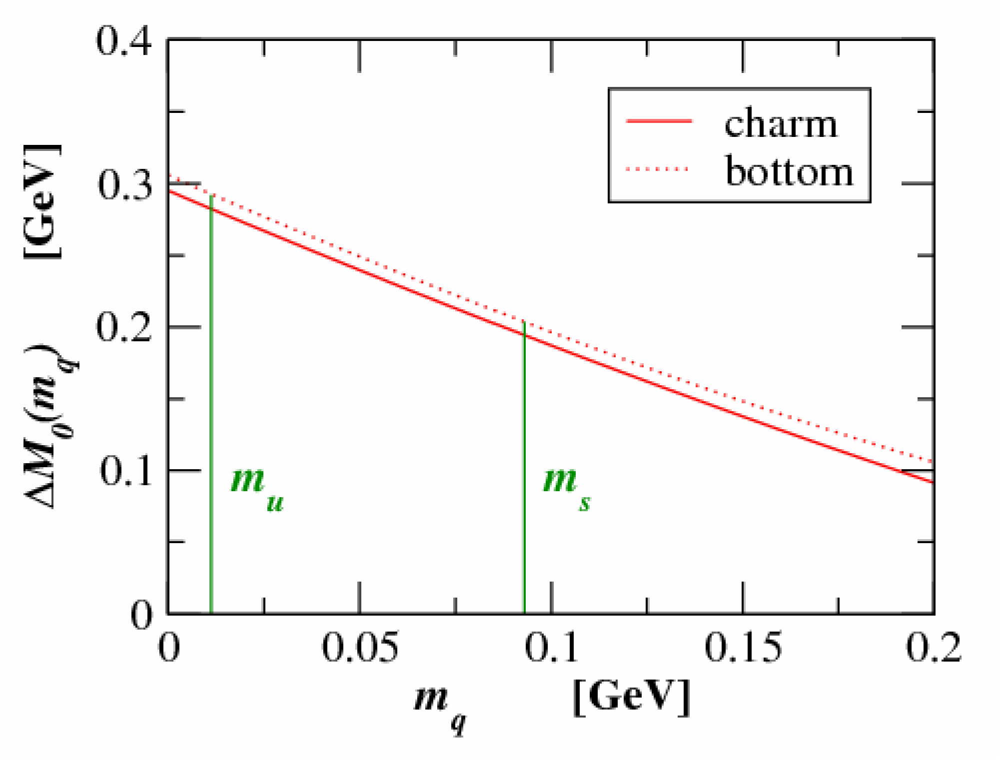
- mass gap between degenerate masses
  - In the heavy quark limit, masses for 0^- and 1^-, and also 0^+ and 1^+ are degenerate
  - In the chiral limit, all these four states are degenerate (S has mass dimension)



Numerical Results of ΔM

(without $1/m_Q$)

- mass gap between degenerate masses (figure at the lowest order, i.e., without $1/m_Q$ corrections)
  - In the heavy quark limit, masses for 0^- and 1^-, and also 0^+ and 1^+ are degenerate
  - Light quark mass dependence of $\Delta M(m_q) = M(0^+) - M(0^-) = M(1^+) - M(1^-)$



Numerical Formula for ΔM

- First derive numerical mass gap between two spin (degenerate) multiplets

$$\Delta M_0 = M(0^+) - M(0^-) = M(1^+) - M(1^-) \approx \Lambda_Q - m_q$$

$$\Lambda_Q = 300 \text{ MeV} \quad \text{and } m_q \text{ light quark mass}$$

  - More precisely

$$\Delta M_0 = M(0^+) - M(0^-) = M(1^+) - M(1^-) = g_0 \Lambda_Q - g_1 m_q$$

$$g_0 = 0.984, \ g_1 = 1.080 \quad \text{for } D/D_s$$

$$g_0 = 1.017, \ g_1 = 1.089 \quad \text{for } B/B_s$$

- From these equations, the mass gap for a degenerate system is essentially given by (Note negative dependency on m_q)

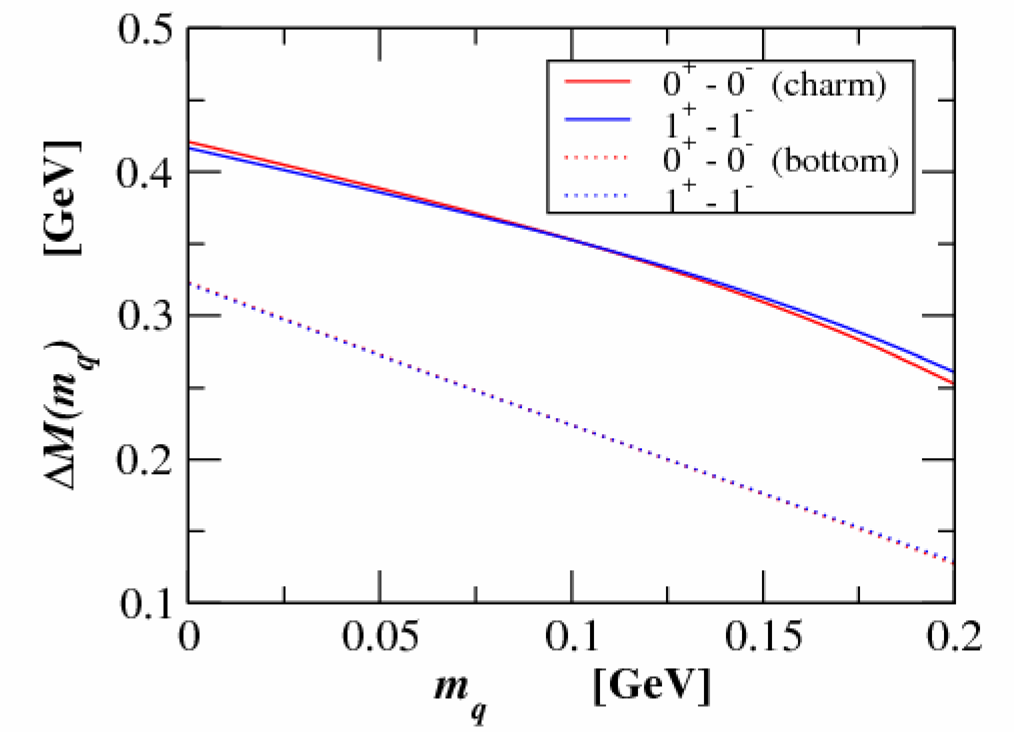
$$\Delta M_0 = \Lambda_Q - m_q$$

Numerical Results of ΔM

(first order)

- mass gap between masses of spin multiplets (figure at the next order, i.e., with 1/m_Q corrections included)
  - Light quark mass dependence is given by the equation

$$\Delta M(m_q) = M(0^+) - M(0^-) \simeq M(1^+) - M(1^-) \simeq g_0\Lambda_Q - g_1 m_q + \frac{c + d \bullet m_q}{m_Q}$$



Prediction

- mass gap between masses of spin multiplets
  - Light quark mass dependence is given by the equation

$$\Delta M(m_q) = M(0^+) - M(0^-) \simeq M(1^+) - M(1^-) \simeq g_0 \Lambda_Q - g_1 m_q + \frac{c + d \bullet m_q}{m_Q}$$

  - where

$$c = 1.28 \times 10^5 \text{ MeV}^2$$

$$d = 4.26 \times 10^2 \text{ MeV}$$

this predicts

$$B(0^+) - B(0^-) \approx B(1^+) - B(1^-) \approx 322 \text{ MeV}$$

$$B_s(0^+) - B_s(0^-) \approx B_s(1^+) - B_s(1^-) \approx 240 \text{ MeV}$$

Numerical Values1

TABLE I: Optimal values of parameters.

Parameters	α_s^c	α_s^b	a (GeV^{-1})	b (GeV)
	0.261±0.001	0.393±0.003	1.939±0.002	0.0749±0.0020
	$m_{u,d}$ (GeV)	m_s (GeV)	m_c (GeV)	m_b (GeV)
	0.0112±0.0019	0.0929±0.0021	1.032±0.005	4.639±0.005
	# of data	# of parameter	total χ^2/d.o.f	
	18	8	107.55	

TABLE II: Degenerate masses of model calculations and their mass gap between $0^+(1^+)$ and $0^-(1^-)$ for $n = 1$.

	$M_0(D)$	$M_0(D_s)$	$M_0(B)$	$M_0(B_s)$
$0^-/1^-$	1784	1900	5277	5394
$0^+/1^+$	2067	2095	5570	5598
$0^+(1^+) - 0^-(1^-)$	283	195	293	204

Decreasing as
D -> Ds
(u -> s)

TABLE III: Model calculations of the mass gap. Values in brackets are taken from the experiments. Units are MeV.

Mass gap ($n = 1$)	$\Delta M(D)$	$\Delta M(D_s)$	$\Delta M(B)$	$\Delta M(B_s)$
$0^+ - 0^-$	414 (441)	358 (348)	322	239
$1^+ - 1^-$	410 (419)	357 (348)	320	242

($n = 2$)	$\Delta M(D)$	$\Delta M(D_s)$	$\Delta M(B)$	$\Delta M(B_s)$
$0^+ - 0^-$	308	274	206	160
$1^+ - 1^-$	350	327	216	171

Numerical Values2

TABLE IV: D/D_s meson mass spectra for both the calculated and experimentally observed ones. Units are MeV.

$^{2s+1}L_J(J^P)$	$M_{calc}(D)$	$M_{obs}(D)$	$M_{calc}(D_s)$	$M_{obs}(D_s)$
$^1S_0(0^-)$	1869	1867	1967	1969
$^3S_1(1^-)$	2011	2008	2110	2112
$^3P_0(0^+)$	2283	2308	2325	2317
"$^3P_1$"(1^+)	2421	2427	2467	2460

TABLE V: B/B_s meson mass spectra for both the calculated and experimentally observed ones. Units are MeV.

$^{2s+1}L_J(J^P)$	$M_{calc}(B)$	$M_{obs}(B)$	$M_{calc}(B_s)$	$M_{obs}(B_s)$
$^1S_0(0^-)$	5270	5279	5378	5369
$^3S_1(1^-)$	5329	5325	5440	–
$^3P_0(0^+)$	5592	–	5617	–
"$^3P_1$"(1^+)	5649	–	5682	–

recent experiments find

$$B(1^+) = 5720\,(5720),\ B(2^+) = 5737\,(5745)$$

$$B_s(2^+) = 5847\,(5839) \quad \text{MeV}$$

numbers in brackets are from expt.

Explanation due to Effective Theory

- Lets try to explain the light quark dependency of ΔM by using the chiral effective theory with heavy quark symmetry

(Bardeen et al., PRD68 (2003) 054024)

- mass gap is given by

$$\text{Parity doublets}: \mathcal{H} = (0^-, 1^-), \mathcal{H}' = (0^+, 1^+)$$

$$L_{mass} = \frac{g_\pi}{4}\left[\mathrm{Tr}\left(\bar{\mathcal{H}}'\tilde{\sigma}\mathcal{H}'\right) - \mathrm{Tr}\left(\bar{\mathcal{H}}\tilde{\sigma}\mathcal{H}\right)\right]$$

$$\langle\tilde{\sigma}\rangle = f_\pi I_3 \qquad \tilde{\sigma} = \sigma + m_q$$

Plus sign instead of **minus** sign

- Goldberger-Treiman relation

$$\Delta M(m_q) = M_X(1^+) - M_X(1^-) = M_X(0^+) - M_X(0^-) = g_\pi f_\pi$$

$$\Delta M(m_c) = 349, \Delta M(\infty) = 338 \text{ for } D_s$$

349 MeV is an input and cannot be identified to be DsJ

Discussions and Summary

- Globa Flavor SU(3) Recovery (Dmitrasinovic, PRL 94 (2005) 162002)
  - He tried to explain by tetraquark.

$$M(D(0^+)) = 2308, \quad M(D_s(0^+)) = 2317$$

$$M(D(1^+)) = 2427, \quad M(D_s(1^+)) = 2460$$

  - This can be nicely explained by our mass gap expression, i.e., monotonously decreasing behavior of the mass gap between two spin multiplets.
- Mass gap of Heavy Baryons
  - $QQ = 3^* \rightarrow QQq = 3^* \times 3$ (like a heavy meson)
  - May apply our formula. For instance,

$$\Delta M = M'(ccq) - M(ccq) = \Lambda_Q - m_q + \frac{c + d \bullet m_q}{m_{Q1} + m_{Q2}}$$

- We have derived an empirical formula for the mass gap between two spin multiplets, i.e., $(0^-, 1^-)$ and $(0^+, 1^+)$, which is given above.
- May give us a hint to find the underlying (true?) physics.

Backups

- Have time, then show these

Intuitive Explanation of ΔM

m_q dependence of the mass gap:

$$\Delta M = M^1(0^+) - M^{-1}(0^-) = M^1(1^+) - M^{-1}(1^-)$$

$$= \int \frac{d^3x}{4\pi r^2}\left\{\Phi_1^\dagger(r)\begin{pmatrix} m_q+S+V & -\partial_r+\frac{1}{r} \\ \partial_r+\frac{1}{r} & -m_q-S+V\end{pmatrix}\Phi_1(r) - \Phi_{-1}^\dagger(r)\begin{pmatrix} m_q+S+V & -\partial_r-\frac{1}{r} \\ \partial_r-\frac{1}{r} & -m_q-S+V\end{pmatrix}\Phi_{-1}(r)\right\}$$

$$= \int dr\left[\Phi_1^\dagger(r)K_1\Phi_1(r) - \Phi_{-1}^\dagger(r)K_{-1}\Phi_{-1}(r)\right] + m_q\int dr\left[\Phi_1^\dagger(r)\beta\Phi_1(r) - \Phi_{-1}^\dagger(r)\beta\Phi_{-1}(r)\right].$$

where

$$\Phi_k(r) = \begin{pmatrix} u_k(r) \\ v_k(r)\end{pmatrix}, \quad K_k = \begin{pmatrix} S(r)+V(r) & -\partial_r+\frac{k}{r} \\ \partial_r+\frac{k}{r} & -S(r)+V(r)\end{pmatrix}$$

k=-1(+1) corresponds to L=0(L=1) states, then k=+1 is more relativisitc than k=-1, which means a lower component $v_1(r)$ is larger than $v_{-1}(r)$. Hence $(u_1)^2-(v_1)^2 = \Phi_1^\dagger(r)\beta\Phi_1(r)$ becomes smaller than $\Phi_{-1}^\dagger(r)\beta\Phi_{-1}(r)$.

Thus, the coefficient of m_q becomes negative.

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$0^-/1^-$	1784	1900	5277	5394
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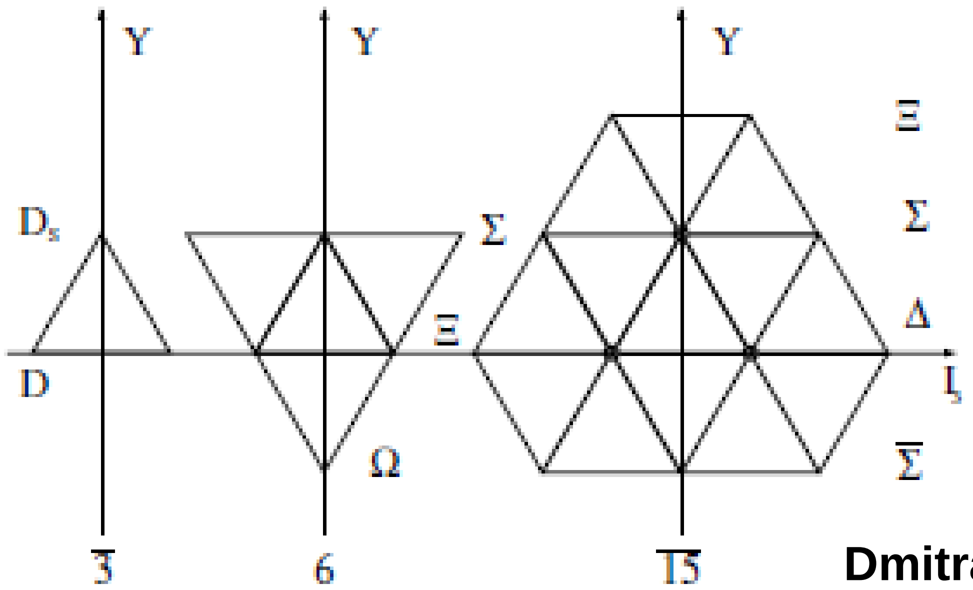
Tetra-quark model

$$|D^0 \subset \bar{\mathbf{3}}_A\rangle = \frac{1}{2}|c(s(\bar{u}\,\bar{s} - \bar{s}\,\bar{u}) - d(\bar{d}\,\bar{u} - \bar{u}\,\bar{d}))\rangle,$$

$$|D^+ \subset \bar{\mathbf{3}}_A\rangle = \frac{1}{2}|c(s(\bar{d}\,\bar{s} - \bar{s}\,\bar{d}) - u(\bar{d}\,\bar{u} - \bar{u}\,\bar{d}))\rangle,$$

$$|D_s^+ \subset \bar{\mathbf{3}}_A\rangle = \frac{1}{2}|c(u(\bar{u}\,\bar{s} - \bar{s}\,\bar{u}) - d(\bar{d}\,\bar{s} - \bar{s}\,\bar{d}))\rangle,$$



Dmitrasinovic, PRL, 94 (2005) 162002