
Analysis of Rare Radiative Decays of ϕ and Mixing Between Low and High Mass Scalar Mesons

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1. Introduction

- To reveal the structure of the light scalar mesons $a_0(980)$ and $f_0(980)$, we studied the recent rare radiative decays $\phi \rightarrow \eta\pi^0\gamma$ and $\phi \rightarrow \pi^0\pi^0\gamma$ assuming that these decays proceed through the intermediate $a_0\gamma$ and $f_0\gamma$ states, respectively.
- Fitting the experimental data of the $\eta\pi^0$ and $\pi^0\pi^0$ invariant mass spectrum¹⁾, we have found that the processes $\phi \rightarrow a_0\gamma$ and $\phi \rightarrow f_0\gamma$ are dominated by the K^+K^- loop interaction rather than the point like $\phi a_0(f_0)\gamma$ interaction, both for the non-derivative and derivative *SPP* coupling.

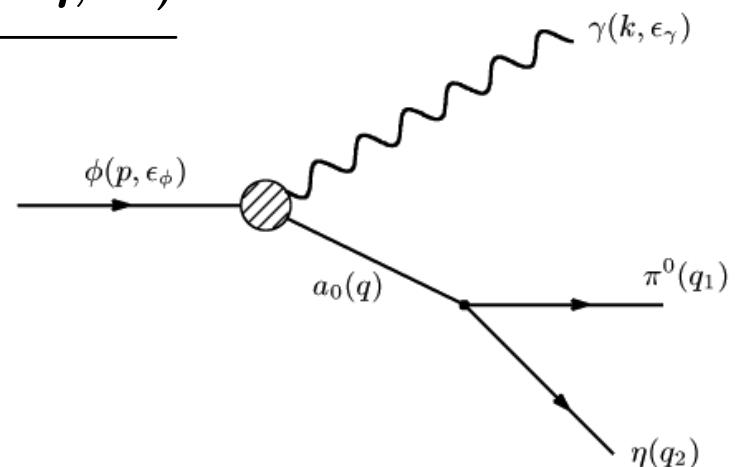
- 1) SND Collaboration data: Phys. Lett. B **479**, 53(2000)
CMD-2 Collaboration data: Phys. Lett. B **462**, 371(1999); **462**, 380(1999)
KLEO Collaboration data: Phys. Lett. B **536**, 209(2002); **537**, 21(2002)

- We also studied the decays $a_0 \rightarrow \pi\eta$, $K\bar{K}$, $K_0^* \rightarrow K\pi$ $f_0 \rightarrow \pi\pi$, $K\bar{K}$ and analyzed the *SPP* coupling strengths of $g_{f_0 K\bar{K}}$ and $g_{a_0 K\bar{K}}$ considering the effects of mixing between low mass scalar octet $qq\bar{q}\bar{q}$ states containing $a_0(980)$ and $f_0(980)$ and high mass scalar octet $q\bar{q}$ states and glueball.
- Comparing the strengths of $g_{f_0 K\bar{K}}$ and $g_{a_0 K\bar{K}}$ and the result $g_{f_0 K\bar{K}}/g_{a_0 K\bar{K}} \approx 2$, we predict that the mixing are rather large and non-derivative *SPP* coupling is favored.

2. Analysis of the $\phi \rightarrow \pi^0 \eta \gamma$ and $\phi \rightarrow \pi^0 \pi^0 \gamma$ decays

- Invariant mass distribution of the branching ratio for $\phi \rightarrow a_0(980) \gamma \rightarrow \pi^0 \eta \gamma$ decay

$$\frac{dBR(\phi \rightarrow a_0 \gamma \rightarrow \pi^0 \eta \gamma)}{dm} = \frac{2m^2}{\pi} \frac{1}{\Gamma_\phi} \frac{\Gamma(\phi \rightarrow a_0 \gamma; m) \Gamma(a_0 \rightarrow \pi^0 \eta; m)}{|D_{a_0}(m^2)|^2}$$



- $\Gamma(a_0 \rightarrow \pi^0 \eta; m)$ is the decay width **on the virtual mass of intermediate a_0 state**

$$\Gamma(a_0 \rightarrow \pi^0 \eta; m) = \frac{g_{a_0 \pi \eta}^2}{8\pi m^2} \frac{\sqrt{(m^2 - (m_\pi + m_\eta)^2)(m^2 - (m_\pi - m_\eta)^2)}}{2m}$$

$$\times \begin{cases} 1 & \text{for non-derivative coupling} \\ \left(\frac{m^2 - m_\pi^2 - m_\eta^2}{2}\right)^2 & \text{for derivative coupling} \end{cases}$$

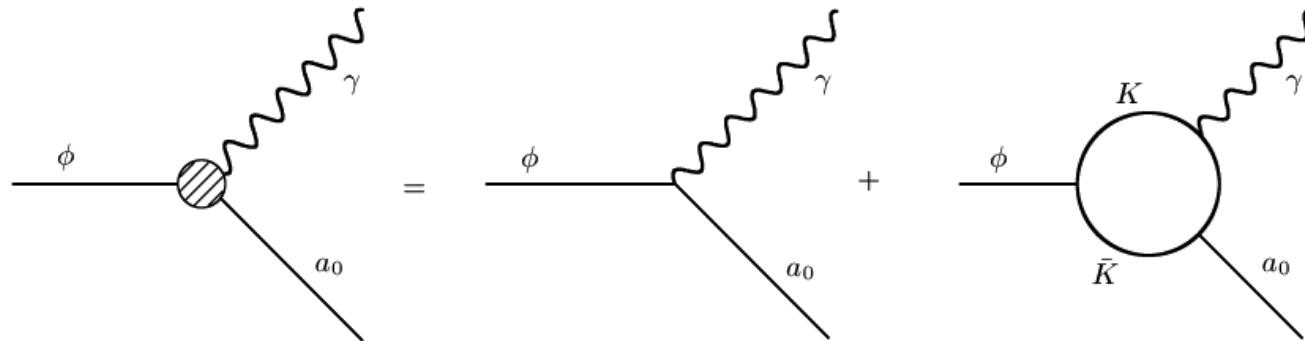
$$M(a_0(q) \rightarrow \pi^0(q_1) + \eta(q_2)) = g_{a_0 \pi \eta} \times \begin{cases} 1 & \text{for non-derivative coupling} \\ q_1 \cdot q_2 & \text{for derivative coupling} \end{cases}$$

- $\Gamma(\phi \rightarrow a_0 \gamma; m)$ is the decay width on the virtual a_0 state mass m

$$\Gamma(\phi \rightarrow a_0 \gamma; m) = \frac{\alpha}{3} g_{\phi a_0 \gamma}^2(m) \left(\frac{m_\phi^2 - m^2}{2m_\phi} \right)^3$$

$$M(\phi(p, \varepsilon_\phi) \rightarrow a_0(q) + \gamma(k, \varepsilon_\gamma)) = e g_{\phi a_0 \gamma}(m) (p \cdot k \varepsilon_\phi \cdot \varepsilon_\gamma - p \cdot \varepsilon_\gamma k \cdot \varepsilon_\phi)$$

- For the $\phi a_0 \gamma$ coupling, we consider the **point like interaction** and **K^+K^- loop interaction**



$$g_{\phi a_0 \gamma}(m) = g_{\phi a_0 \gamma}^{\text{pointlike}} + g_{\phi a_0 \gamma}^{K\bar{K} \text{ loop}}(m),$$

$$g_{\phi a_0 \gamma}^{K\bar{K} \text{ loop}}(m) = \frac{g_{\phi K\bar{K}} g_{a_0 K\bar{K}}}{2\pi^2 i m_K^2} \left[\frac{2m_K^2 - m^2}{2} \right] I(a, b),$$

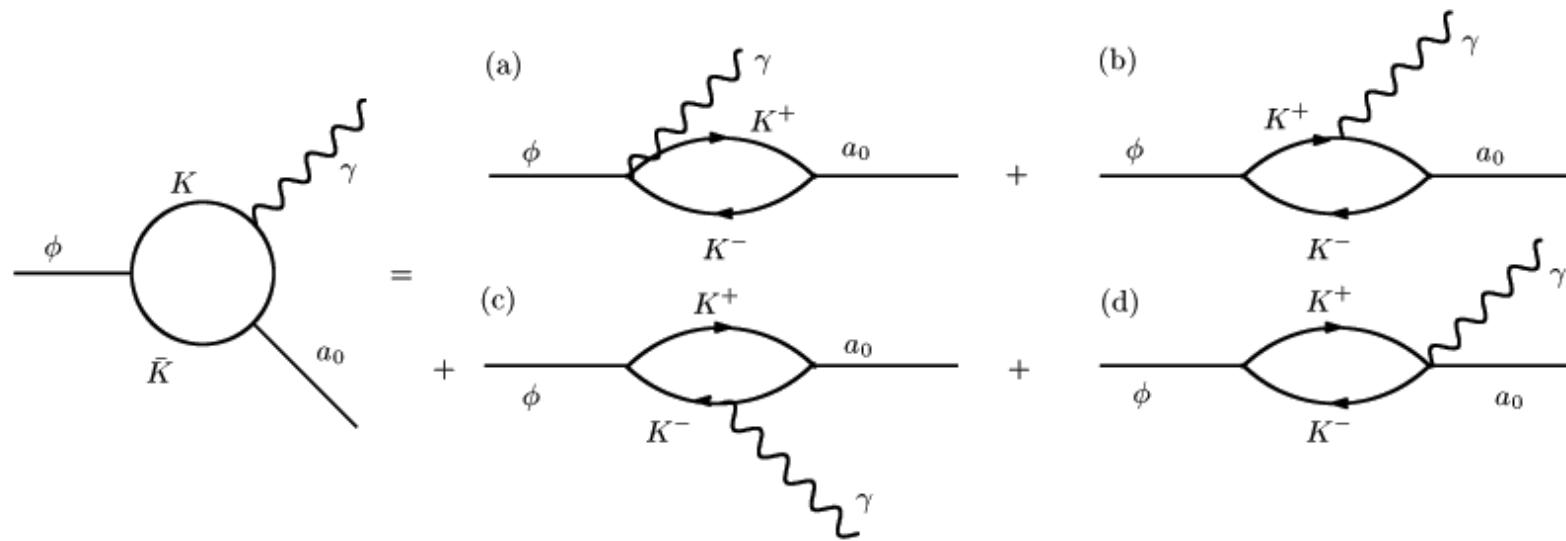
[] term corresponds to the derivative coupling

$$I(a, b) = \frac{1}{2(a-b)} - \frac{2}{(a-b)^2} \left\{ f\left(\frac{1}{b}\right) - f\left(\frac{1}{a}\right) \right\} + \frac{a}{(a-b)^2} \left\{ g\left(\frac{1}{b}\right) - g\left(\frac{1}{a}\right) \right\}$$

where $a = \frac{m_\phi^2}{m_K^2}, \quad b = \frac{m}{m_K^2}$

$$f(x) = \begin{cases} -\left(\sin^{-1}\left(\frac{1}{2\sqrt{x}}\right)\right)^2, & x > \frac{1}{4} \\ \frac{1}{4}\left[\log\frac{\eta_+}{\eta_-} - i\pi\right]^2, & x < \frac{1}{4} \end{cases}, \quad g(x) = \begin{cases} \sqrt{4x-1}\left(\sin^{-1}\left(\frac{1}{2\sqrt{x}}\right)\right), & x > \frac{1}{4} \\ \frac{1}{2}\sqrt{1-4x}\left[\log\frac{\eta_+}{\eta_-} - i\pi\right], & x < \frac{1}{4} \end{cases}$$

$$\eta_\pm = \frac{1}{2x}(1 \pm \sqrt{1-4x})$$



N. N. Achasov and V. N. Ivanchenco, Nucl. Phys. B315, 465(1989).

F. E. Close and N. Isgur and S. Kumano, Nucl. Phys. B389(1993).

D. Black, M. Harada and J. Schechter, Phys. Rev. D73, 054017(2006).

- Invariant mass distributions are parametrised by only two parameters G_1 and G_2

$$\frac{dBR(\phi \rightarrow a_0\gamma \rightarrow \pi^0\eta\gamma)}{dm} = G_1 \frac{\left| G_2 + \frac{1}{i} \left[\frac{2m_K^2 - m^2}{2} \right] I(a, b) \right|^2}{\left| G_2 + \frac{1}{i} \left[\frac{2m_K^2 - m_a^2}{2} \right] I(a, b_0) \right|^2} \left(\frac{m_\phi^2 - m^2}{m_\phi^2 - m_a^2} \right)^3$$

$$\times \frac{m_a}{m} \frac{m_a^2 \Gamma_a^2}{(m^2 - m_a^2)^2 + m_a^2 \Gamma_a^2} \sqrt{\frac{(m^2 - (m_\eta + m_\pi)^2)(m^2 - (m_\eta - m_\pi)^2)}{(m_a^2 - (m_\eta + m_\pi)^2)(m_a^2 - (m_\eta - m_\pi)^2)}}$$

$$G_1 = \frac{2}{\pi \Gamma_\phi \Gamma_a^2} \Gamma(\phi \rightarrow a_0\gamma : m_a) \Gamma(a_0 \rightarrow \pi^0\eta : m_a)$$

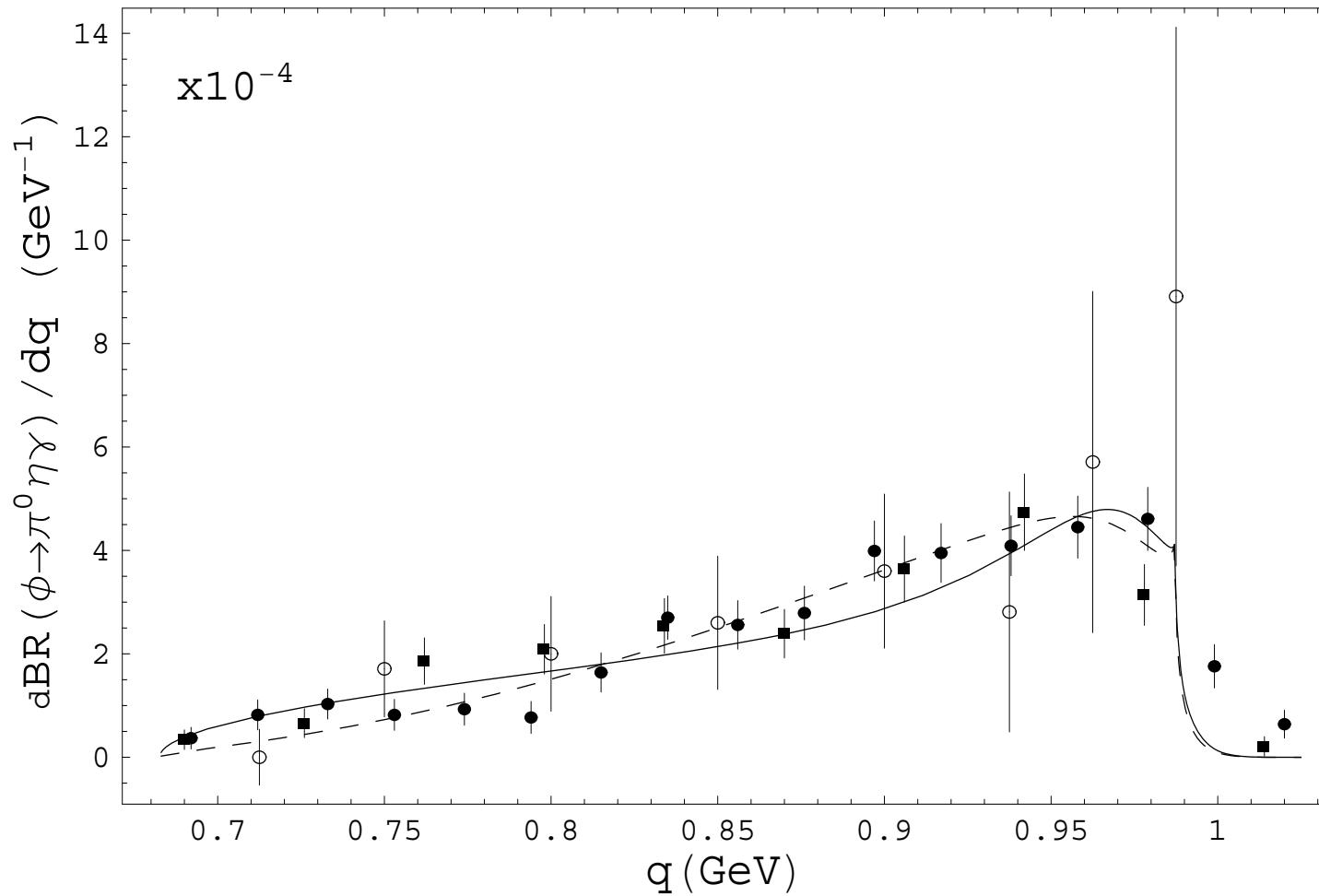
$$G_2 = g_{\phi\gamma a}^{\text{pointlike}} / \left(\frac{g_{\phi K\bar{K}} g_{a_0 K\bar{K}}}{2\pi^2 m_K^2} \right), \quad b_0 = \frac{m_a^2}{m_K^2}$$

Experimental data

M. N. Achasov et al.[SND Collaboration]: Phys. Lett. B479, 53(2000)

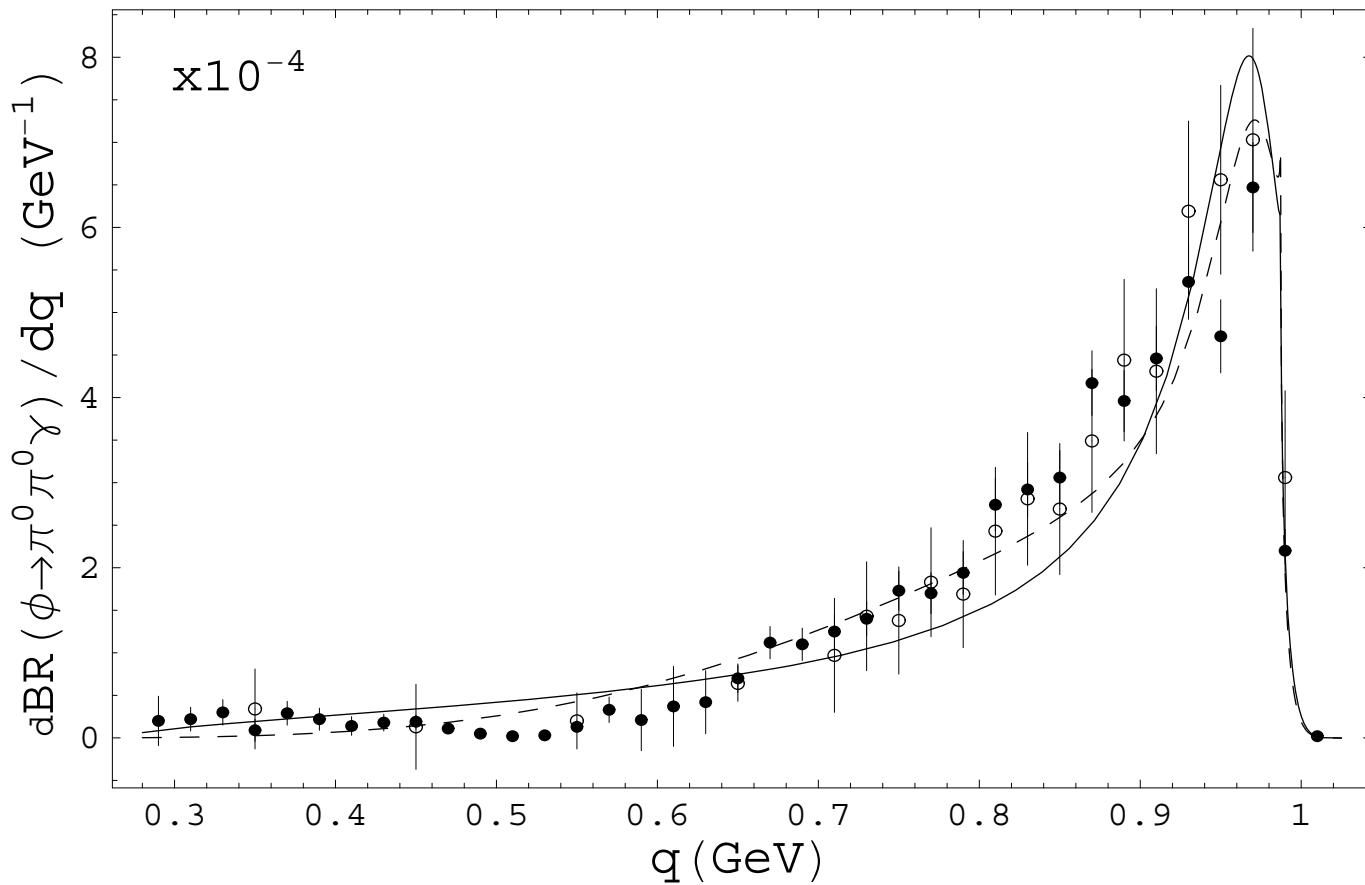
R. R. R. Akhmetshin et al.[CMD-2 Collaboration]: Phys. Lett. B462, 371, 380(1999)

A. Aloisio et al.[KLEO Collaboration]: Phys. Lett. B536, 209(2002); Phys. Lett. B537, 21(2002)



$\phi \rightarrow \pi^0 \eta \gamma$ decay invariant mass distribution. Circles are SND collab. data, and filled circles and filled squares are KLEO collab. data. Solid line shows the best fitted curve for the non-derivative *SPP* coupling and dashed line shows that for the derivative *SPP* coupling.

■ For $\phi \rightarrow \pi^0 \pi^0 \gamma$ decay



$\phi \rightarrow \pi^0 \pi^0 \gamma$ decay invariant mass distribution. Circles are SND collab. data, and filled circles are KLEO collab. data. Solid line shows the best fitted curve for the non-derivative *SPP* coupling and dashed line shows that for the derivative *SPP* coupling.

$$G_1 = 4.1 \times 10^{-4} \text{ GeV}^{-1}, G_2 = -0.16, BR(\phi \rightarrow \pi^0 \eta \gamma) = 7.03 \times 10^{-5}$$

for non-derivative coupling

$$G_1 = 3.9 \times 10^{-4} \text{ GeV}^{-1}, G_2 = 0.08, BR(\phi \rightarrow \pi^0 \eta \gamma) = 7.12 \times 10^{-5}$$

for derivative coupling

cf. $BR^{\exp}(\phi \rightarrow \pi^0 \eta \gamma) = (8.3 \pm 0.5) \times 10^{-5}$,

$$|I(a, b_0)| = 0.902 \text{ for } m_a = 0.985 \text{ GeV}$$

$$G_1 = 7.1 \times 10^{-4} \text{ GeV}^{-1}, G_2 = 0.001, BR(\phi \rightarrow \pi^0 \pi^0 \gamma) = 1.06 \times 10^{-4}$$

for non-derivative coupling

$$G_1 = 6.9 \times 10^{-4} \text{ GeV}^{-1}, G_2 = 0.055, BR(\phi \rightarrow \pi^0 \pi^0 \gamma) = 1.08 \times 10^{-4}$$

for derivative coupling

cf. $BR(\phi \rightarrow \pi^0 \pi^0 \gamma) = (1.09 \pm 0.06) \times 10^{-4}$,

$$|I(a, b_0)| = 0.783 \text{ for } m_f = 0.980 \text{ GeV}$$

For the $\phi \rightarrow a_0\gamma$, $\phi \rightarrow f_0\gamma$ decays, point like diagram contribution

(G_2 contribution) are assumed to be negligible, then the following results

are obtained from the decay widths of $\phi \rightarrow a_0\gamma$, $\phi \rightarrow f_0\gamma$ decays,

$$g_{a_0 K\bar{K}} = \begin{cases} 2.18 \pm 0.12 \text{ GeV}, & \text{for non-derivative coupling} \\ 9.04 \pm 0.50 \text{ GeV}^{-1}, & \text{for derivative coupling} \end{cases}$$

$$g_{a_0 \pi\eta} = \begin{cases} 1.89 \pm 0.75 \text{ GeV}, & \text{for non-derivative coupling} \\ 5.79 \pm 2.32 \text{ GeV}^{-1}, & \text{for derivative coupling} \end{cases}$$

$$g_{f_0 K\bar{K}} = \begin{cases} 4.72 \pm 0.82 \text{ GeV}, & \text{for non-derivative coupling} \\ 20.0 \pm 0.50 \text{ GeV}^{-1}, & \text{for derivative coupling} \end{cases}$$

$$g_{f_0 \pi\eta} = \begin{cases} 1.12 \pm 0.69 \text{ GeV}, & \text{for non-derivative coupling} \\ 2.43 \pm 1.50 \text{ GeV}^{-1}, & \text{for derivative coupling} \end{cases}$$

$g_{f_0 K\bar{K}} / g_{a_0 K\bar{K}} \approx 2.2$ suggests that the a_0 and f_0 scalar mesons are

not the pure $qq\bar{q}\bar{q}$ states. This is explained in next section.

3. Mixing between Low and High Mass Scalar Mesons

Low mass scalar nonet: $S^a_b \sim \varepsilon^{acd} q_c q_d \varepsilon_{bef} \bar{q}^e \bar{q}^f$

$$a_0^+, a_0^0, a_0^- \Leftrightarrow \bar{d}\bar{s}s u, \frac{1}{\sqrt{2}}(\bar{d}\bar{s}ds - \bar{s}\bar{u}s u), \bar{s}uds$$

$$K_0^{*+}, K_0^{*0}, \bar{K}_0^{*0}, K_0^{*-} \Leftrightarrow \bar{d}\bar{s}ud, \bar{s}uud, \bar{u}\bar{d}su, \bar{u}\bar{d}su$$

$$f_0(980) \sim f_{NS} \Leftrightarrow \frac{1}{\sqrt{2}}(\bar{d}\bar{s}ds + \bar{s}\bar{u}s u)$$

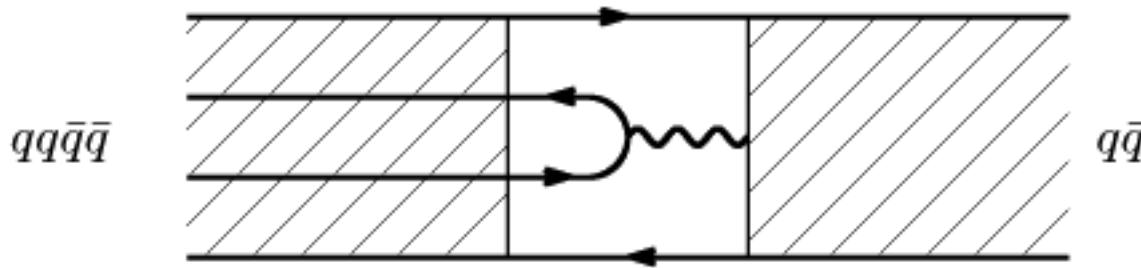
$$f_0(600) \sim f_{NN} \Leftrightarrow \bar{u}\bar{d}ud$$

High mass scalar + glueball:

$$S'^a_b \sim \bar{q}^a q_b \quad G \sim gg$$

Mixing between $qq\bar{q}\bar{q}$ mesons and $q\bar{q}$ mesons

$$\begin{aligned}
 L_{\text{int}} = & \lambda_{01} [a_0^+ a'_0^- + a_0^- a'^+_0 + a_0^0 a'^0_0 \\
 & + K_0^{*+} K'^{* -}_0 + K_0^{*-} K'^{*+}_0 + K_0^{*0} K'^{*0}_0 + \bar{K}_0^{*0} \bar{K}'^{*0}_0 \\
 & + \sqrt{2} f_{NN} f'^-_N + f_{NS} f'^+_N + \sqrt{2} f_{NS} f'^-_S]
 \end{aligned}$$



Mixing between $I = 1$ $qq\bar{q}\bar{q}$ mesons $\Leftrightarrow q\bar{q}$ mesons

$$\begin{pmatrix} m_{\overline{a_0(980)}}^2 & \lambda_{01} \\ \lambda_{01} & m_{\overline{a_0(1450)}}^2 \end{pmatrix},$$

$$\begin{cases} a_0(980) = \cos \theta_a \overline{a_0(980)} - \sin \theta_a \overline{a_0(1450)} \\ a_0(1450) = \sin \theta_a \overline{a_0(980)} + \cos \theta_a \overline{a_0(1450)} \end{cases}$$

$$m_{\overline{a_0(1450)}} = 1.474 \pm 0.019 \text{ GeV}, m_{\overline{a_0(980)}}^2 = 0.9848 \pm 0.0012 \text{ GeV}$$

$$\varepsilon_a = \frac{m_{\overline{a_0(1450)}}^2 - m_{\overline{a_0(980)}}^2}{2} - \sqrt{\left(\frac{m_{\overline{a_0(1450)}}^2 - m_{\overline{a_0(980)}}^2}{2}\right)^2 - \lambda_{01}}$$

$$\theta_a = \tan^{-1} \frac{\varepsilon_a}{\lambda_{01}}$$

$$m_{\overline{a_0(1450)}} = \sqrt{m_{\overline{a_0(1450)}}^2 + \varepsilon_a}, \quad m_{\overline{a_0(980)}} = \sqrt{m_{\overline{a_0(980)}}^2 - \varepsilon_a}$$

Mixing between $I = 1/2$ $qq\bar{q}\bar{q}$ mesons $\Leftrightarrow q\bar{q}$ mesons

$$\begin{pmatrix} m_{K_0^*(800)}^2 & \lambda_{01} \\ \lambda_{01} & m_{K_0^*(1430)}^2 \end{pmatrix},$$

$$\begin{cases} K_0^*(800) = \cos \theta_K \overline{K_0^*(800)} - \sin \theta_K \overline{K_0^*(1430)} \\ K_0^*(1430) = \sin \theta_K \overline{K_0^*(800)} + \cos \theta_K \overline{K_0^*(1430)} \end{cases}$$

$$m_{K_0^*(1430)} = 1.414 \pm 0.006 \text{ GeV}, m_{K_0^*(800)}^2 = 0.841 \pm 0.030 \text{ GeV}$$

$$\varepsilon_K = \frac{m_{K_0^*(1430)}^2 - m_{K_0^*(800)}^2}{2} - \sqrt{\left(\frac{m_{K_0^*(1430)}^2 - m_{K_0^*(800)}^2}{2}\right)^2 - \lambda_{01}}$$

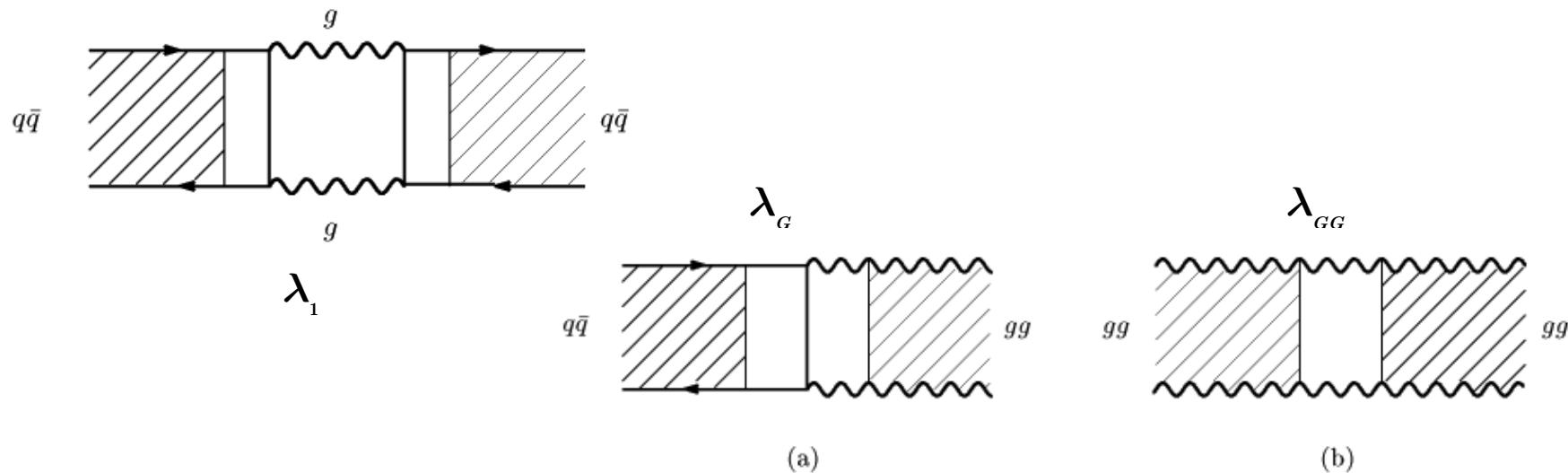
$$\theta_K = \tan^{-1} \frac{\varepsilon_K}{\lambda_{01}}$$

$$m_{\overline{K_0^*(1450)}} = \sqrt{m_{K_0^*(1430)}^2 + \varepsilon_K}, \quad m_{\overline{K_0^*(800)}} = \sqrt{m_{K_0^*(800)}^2 - \varepsilon_K}$$

Intra mixing among $I = 0$ $q\bar{q}$ mesons + gg (glueball)

$$\begin{pmatrix} m_{N'}^2 + 2\lambda_1 & \sqrt{2}\lambda_1 & \sqrt{2}\lambda_G \\ \sqrt{2}\lambda_1 & m_{S'}^2 + 2\lambda_1 & \lambda_G \\ \sqrt{2}\lambda_G & \lambda_G & \lambda_{GG} \end{pmatrix}$$

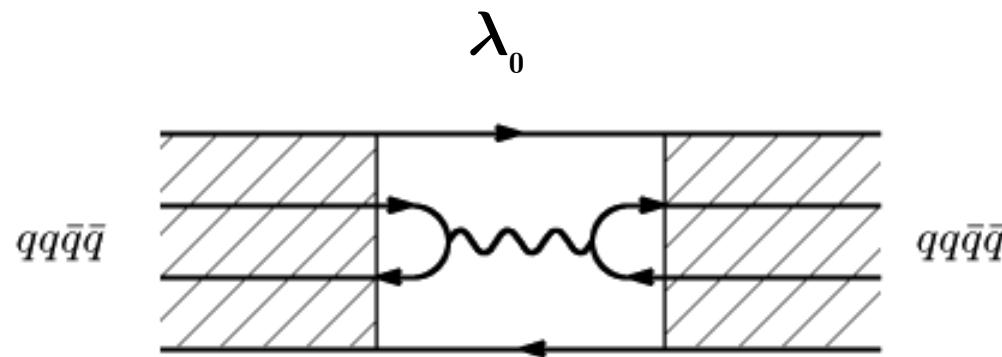
$$m_{N'}^2 = m^2 \frac{1}{a_0(1450)}, \quad m_{S'}^2 = 2m^2 \frac{1}{K_0^*(1430)} - m^2 \frac{1}{a_0(1450)}$$



Intra mixing among $I = 0$ $qq\bar{q}\bar{q}$ mesons

$$\begin{pmatrix} m^2_{NN} + \lambda_0 & \sqrt{2}\lambda_0 \\ \sqrt{2}\lambda_0 & m^2_{NS} + 2\lambda_0 \end{pmatrix}$$

$$m^2_{NN} = 2m^2_{\overline{K}_0^*(800)} - m^2_{\overline{a_0}(980)}, \quad m^2_{NS} = m^2_{\overline{a_0}(980)}$$



$I = 0 \quad qq\bar{q}\bar{q} \Leftrightarrow q\bar{q} + gg$ overall (intra+inter) mixing

$$\begin{pmatrix} m_{NN}^2 + \lambda_0 & \sqrt{2}\lambda_0 & \sqrt{2}\lambda_{01} & 0 & 0 \\ \sqrt{2}\lambda_0 & m_{NS}^2 + 2\lambda_0 & \lambda_{01} & \sqrt{2}\lambda_{01} & 0 \\ \sqrt{2}\lambda_{01} & \lambda_{01} & m_{N'}^2 + 2\lambda_1 & \sqrt{2}\lambda_1 & \sqrt{2}\lambda_G \\ 0 & \sqrt{2}\lambda_{01} & \sqrt{2}\lambda_1 & m_{S'}^2 + 2\lambda_1 & \lambda_G \\ 0 & 0 & \sqrt{2}\lambda_G & \lambda_G & \lambda_{GG} \end{pmatrix}$$

$m_{f_0(600)} = 0.80 \pm 0.40 \text{GeV}$, $m_{f_0(980)} = 0.980 \pm 0.010 \text{GeV}$, $m_{f_0(1370)} = 1.350 \pm 0.150 \text{GeV}$,
 $m_{f_0(1500)} = 1.507 \pm 0.005 \text{GeV}$, $m_{f_0(600)} = 1.718 \pm 0.006 \text{GeV}$,

$$\begin{pmatrix} f_0(600) \\ f_0(980) \\ f_0(1370) \\ f_0(1500) \\ f_0(1710) \end{pmatrix} = \begin{pmatrix} R_{f_0(600)NN} & R_{f_0(600)NS} & R_{f_0(600)N'} & R_{f_0(600)S'} & R_{f_0(600)G} \\ R_{f_0(980)NN} & R_{f_0(980)NS} & R_{f_0(980)N'} & R_{f_0(980)S'} & R_{f_0(980)G} \\ R_{f_0(1370)NN} & R_{f_0(1370)NS} & R_{f_0(1370)N'} & R_{f_0(1370)S'} & R_{f_0(1370)G} \\ R_{f_0(1500)NN} & R_{f_0(1500)NS} & R_{f_0(1500)N'} & R_{f_0(1500)S'} & R_{f_0(1500)G} \\ R_{f_0(1710)NN} & R_{f_0(1710)NS} & R_{f_0(1710)N'} & R_{f_0(1710)S'} & R_{f_0(1710)G} \end{pmatrix} \begin{pmatrix} f_{NN} \\ f_{NS} \\ f_{N'} \\ f_{S'} \\ f_G \end{pmatrix}$$

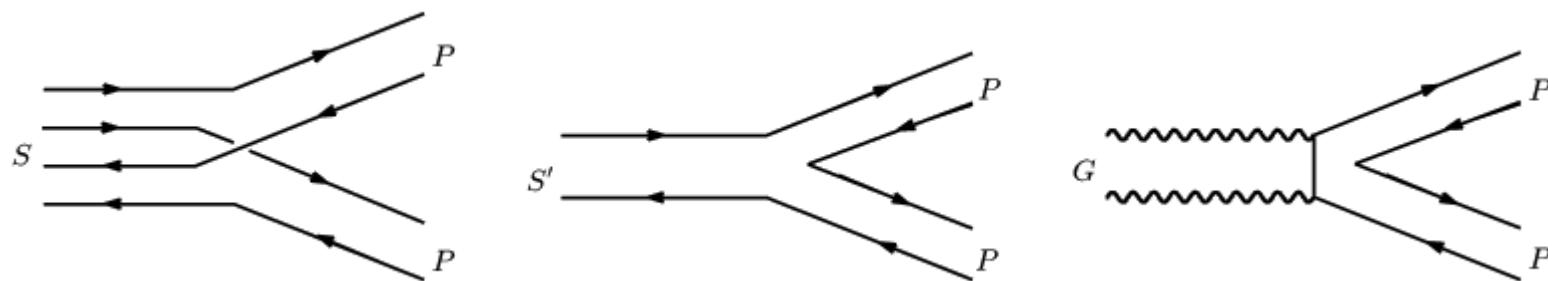
$\lambda_{01}(\text{GeV}^2)$	$\theta_a(\text{°})$	$\theta_K(\text{°})$	$\lambda_0(\text{GeV}^2)$	$\lambda_1(\text{GeV}^2)$	$\lambda_G(\text{GeV}^2)$	$\lambda_{GG}(\text{GeV}^2)$
	$R_{f_0(980)NN}$	$R_{f_0(980)NS}$	$R_{f_0(980)N'}$	$R_{f_0(980)S'}$	$R_{f_0(980)G}$	
0.20	9.7 ± 0.5	9.0 ± 0.5	0.018 ± 0.009	0.275 ± 0.007	0.04 ± 0.04	$(1.152 \pm 0.008)^2$
	-0.023 ± 0.014	-0.972 ± 0.002	0.065 ± 0.006	0.226 ± 0.004	-0.010 ± 0.010	
0.25	12.3 ± 0.6	11.4 ± 0.6	0.032 ± 0.010	0.264 ± 0.008	0.05 ± 0.05	$(1.512 \pm 0.007)^2$
	-0.027 ± 0.026	-0.954 ± 0.003	0.086 ± 0.008	0.284 ± 0.005	-0.016 ± 0.016	
0.30	15.0 ± 0.8	13.8 ± 0.8	0.050 ± 0.009	0.252 ± 0.009	0.04 ± 0.04	$(1.512 \pm 0.008)^2$
	-0.046 ± 0.024	-0.932 ± 0.004	0.110 ± 0.009	0.341 ± 0.006	-0.016 ± 0.016	
0.35	17.8 ± 1.0	16.4 ± 1.0	0.072 ± 0.012	0.233 ± 0.008	0.05 ± 0.05	$(1.511 \pm 0.008)^2$
	-0.065 ± 0.025	-0.902 ± 0.007	0.140 ± 0.012	0.401 ± 0.007	-0.024 ± 0.024	
0.40	20.8 ± 1.2	19.1 ± 1.2	0.104 ± 0.012	0.213 ± 0.009	0.05 ± 0.05	$(1.509 \pm 0.006)^2$
	-0.094 ± 0.021	-0.864 ± 0.010	0.178 ± 0.014	0.461 ± 0.007	-0.028 ± 0.028	
0.45	24.2 ± 1.6	22.1 ± 1.5	0.146 ± 0.014	0.178 ± 0.007	0.04 ± 0.04	$(1.506 \pm 0.002)^2$
	-0.116 ± 0.021	-0.813 ± 0.011	0.226 ± 0.015	0.523 ± 0.006	-0.014 ± 0.014	

The values of transition parameters, mixing angles and mixing parameters for various values of λ_{01} .

4. Coupling constant g_{SPP} and mixing between $qq\bar{q}\bar{q}$ and $q\bar{q}$ scalar mesons

SPP interaction

$$\begin{aligned}
 L_I &= A \epsilon^{abc} \epsilon_{def} S_a^d [\partial^\mu] P_b^e [\partial_\mu] P_c^f + A' S_a^b \{ [\partial^\mu] P_b^c, [\partial_\mu] P_c^a \} + A'' G \{ [\partial^\mu] P_a^b, [\partial_\mu] P_b^a \} \\
 &= g_{a_0 K\bar{K}} [\partial^\mu] \bar{K} \vec{\tau} \cdot \overrightarrow{a_0} [\partial_\mu] K + g_{a_0' K\bar{K}} [\partial^\mu] \bar{K} \vec{\tau} \cdot \overrightarrow{a_0'} [\partial_\mu] K + g_{a_0 \pi\eta} \overrightarrow{a_0} \cdot [\partial^\mu] \vec{\pi} [\partial_\mu] \eta \\
 &\quad + g_{a_0' \pi\eta} \overrightarrow{a_0'} \cdot [\partial^\mu] \vec{\pi} [\partial_\mu] \eta + \dots + g_{K_0^* K\pi} ([\partial^\mu] \bar{K} \vec{\tau} \cdot [\partial_\mu] \vec{\pi} K_0^* + H.C.) \\
 &\quad + g_{K_0^* K\pi} ([\partial^\mu] \bar{K} \vec{\tau} \cdot [\partial_\mu] \vec{\pi} K_0^* + H.C.) + \dots + g_{f_0(M)\pi\pi} \frac{1}{2} f_0(M) [\partial^\mu] \vec{\pi} \cdot [\partial_\mu] \vec{\pi} \\
 &\quad + g_{f_0(M)K\bar{K}} f_0(M) [\partial^\mu] K \cdot [\partial_\mu] \bar{K} + \dots
 \end{aligned}$$



$$g_{a_0(980)K\bar{K}} = \sqrt{2}(A \cos \theta_a - A' \sin \theta_a)$$

$$g_{a_0(1450)K\bar{K}} = \sqrt{2}(A \sin \theta_a + A' \cos \theta_a)$$

$$g_{a_0(980)\pi\eta} = 2(A \cos \theta_a \sin \theta_P - \sqrt{2}A' \sin \theta_a \cos \theta_P)$$

$$g_{a_0(1450)\pi\eta} = 2(A \sin \theta_a \sin \theta_P + \sqrt{2}A' \cos \theta_a \cos \theta_P)$$

$$g_{K_0^*(800)\pi K} = \sqrt{2}(A \cos \theta_K - A' \sin \theta_K)$$

$$g_{K_0^*(1430)\pi K} = \sqrt{2}(A \sin \theta_K + A' \cos \theta_K)$$

$$g_{f_0(M)\pi\pi} = 2(-A R_{f_0(M)NN} + \sqrt{2}A' R_{f_0(M)N'} + 2A'' R_{f_0(M)G})$$

$$g_{f_0(M)K\bar{K}} = \sqrt{2}(-A R_{f_0(M)NS} + A' R_{f_0(M)N'} + \sqrt{2}A' R_{f_0(M)S'} + 2\sqrt{2}A'' R_{f_0(M)G})$$

$$g_{f_0(M)\eta\eta} = 2(-A R_{f_0(M)NS} \cos \theta_P \sin \theta_P + \frac{1}{2} A R_{f_0(M)NN} \cos^2 \theta_P$$

$$+ \frac{1}{\sqrt{2}} A' R_{f_0(M)N'} \sin 2\theta_P - A' R_{f_0(M)S'} \sin \theta_P)$$

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- We have analyzed the following processes

W.-M.Yao et al. (Particle Data Group), J. Phys. G. 33, 1(2006)

$$\Gamma(a_0(980) \rightarrow \pi\eta + K\bar{K}) = 75 \pm 25 \text{ MeV}$$

$$\Gamma(a_0(1450) \rightarrow \pi\eta + \pi\eta' + K\bar{K}) = 265 \pm 13 \text{ MeV}$$

$$\Gamma(K_0^*(1430) \rightarrow \pi K) = 270 \pm 43 \text{ MeV}$$

$$\Gamma(f_0(980) \rightarrow \pi\pi + K\bar{K}) = 70 \pm 30 \text{ MeV}$$

$$\Gamma(f_0(1370) \rightarrow \pi\pi + K\bar{K} + \eta\eta) = 214 \pm 120 \text{ MeV}$$

$$\Gamma(f_0(1500) \rightarrow \pi\pi + K\bar{K} + \eta\eta + \eta\eta') = 55 \pm 9 \text{ MeV}$$

$$\Gamma(f_0(1710) \rightarrow \pi\pi + K\bar{K} + \eta\eta) = 137 \pm 8 \text{ MeV}$$

$\lambda_{01}(\text{GeV}^2)$	A	A'	A''	θ_P	$\Gamma_{a_0(980) \rightarrow \pi\eta + K\bar{K}}$
	GeV	GeV	GeV	degree($^\circ$)	$0.075 \pm 0.025 \text{ GeV}$
0.30	2.3	1.2	-0.24	36.7	0.097
0.35	1.9	1.3	-0.24	50.2	0.081
0.40	1.7	1.3	-0.24	59.2	0.072
	$\Gamma_{a_0(1450) \rightarrow \pi\eta + \pi\eta' + K\bar{K}}$	$\Gamma_{K_0^*(1430) \rightarrow \pi K}$	$\Gamma_{f_0(980) \rightarrow \pi\pi + K\bar{K}}$	$\Gamma_{f_0(1370) \rightarrow \pi\pi + K\bar{K} + \eta\eta}$	$\Gamma_{f_0(1500) \rightarrow \pi\pi + K\bar{K} + \eta\eta'}$
	$0.265 \pm 0.013 \text{ GeV}$	$0.270 \pm 0.043 \text{ GeV}$	$0.070 \pm 0.030 \text{ GeV}$	$0.214 \pm 0.120 \text{ GeV}$	$0.055 \pm 0.009 \text{ GeV}$
0.30	0.258	0.214	0.104	0.029	0.055
0.35	0.273	0.232	0.098	0.034	0.058
0.40	0.263	0.233	0.098	0.040	0.054
	$\Gamma_{f_0(1710) \rightarrow \pi\pi + K\bar{K} + \eta\eta}$	$g_{a_0(980) K\bar{K}}$	$g_{a_0(980) \pi\eta}$	$g_{f_0(980) K\bar{K}}$	$g_{f_0(980) \pi\pi}$
	$0.137 \pm 0.008 \text{ GeV}$	$2.18 \pm 0.12 \text{ GeV}$	$1.89 \pm 0.75 \text{ GeV}$	$4.72 \pm 0.82 \text{ GeV}$	$1.12 \pm 0.69 \text{ GeV}$
0.30	0.140	2.70	1.95	4.05	0.61
0.35	0.151	2.00	2.06	3.74	0.78
0.40	0.129	1.59	2.06	3.63	1.00

The results of the best fit analyses for non-derivative coupling case.

$\lambda_{01}(\text{GeV}^2)$	A	A'	A''	θ_P	$\Gamma_{a_0(980) \rightarrow \pi\eta + K\bar{K}}$
	GeV^{-1}	GeV^{-1}	GeV^{-1}	degree($^\circ$)	$0.075 \pm 0.025 \text{ GeV}$
0.30	7.2	0.51	-0.33	36.7	0.124
0.35	6.0	0.54	-0.35	36.7	0.068
0.40	5.1	0.57	-0.36	36.7	0.054
	$\Gamma_{a_0(1450) \rightarrow \pi\eta + \pi\eta' + K\bar{K}}$	$\Gamma_{K_0^*(1430) \rightarrow \pi K}$	$\Gamma_{f_0(980) \rightarrow \pi\pi + K\bar{K}}$	$\Gamma_{f_0(1370) \rightarrow \pi\pi + K\bar{K}}$	$\Gamma_{f_0(1500) \rightarrow \pi\pi + K\bar{K}}$
	$0.265 \pm 0.013 \text{ GeV}$	$0.270 \pm 0.043 \text{ GeV}$	$0.070 \pm 0.030 \text{ GeV}$	$0.214 \pm 0.120 \text{ GeV}$	$0.055 \pm 0.009 \text{ GeV}$
0.30	0.279	0.267	0.036	0.099	0.053
0.35	0.279	0.266	0.028	0.090	0.055
0.40	0.279	0.266	0.025	0.082	0.059
	$\Gamma_{f_0(1710) \rightarrow \pi\pi + K\bar{K}}$	$g_{a_0(980) K\bar{K}}$	$g_{a_0(980) \pi\eta}$	$g_{f_0(980) K\bar{K}}$	$g_{f_0(980) \pi\pi}$
	$0.137 \pm 0.008 \text{ GeV}$	$9.04 \pm 0.50 \text{ GeV}^{-1}$	$5.79 \pm 2.32 \text{ GeV}^{-1}$	$20.0 \pm 3.48 \text{ GeV}^{-1}$	$2.43 \pm 1.50 \text{ GeV}^{-1}$
0.30	0.152	9.65	8.02	9.94	0.87
0.35	0.152	7.85	6.45	8.23	1.03
0.40	0.152	6.45	5.24	6.94	1.28

The results of the best fit analyses for derivative coupling.

The values for $g_{a_0(980)\pi\eta}$, $g_{a_0(980)K\bar{K}}$, $g_{f_0(980)\pi\eta}$, $g_{f_0(980)K\bar{K}}$ obtained when $\lambda_{01} = 0.30 \sim 0.35(\text{GeV}^2)$ are nearest to the values obtained in the analyses of $\phi \rightarrow a_0\gamma / \eta\pi^0\gamma$, $\phi \rightarrow f_0\gamma / \pi^0\pi^0\gamma$.

The result of the non-derivative coupling is more reasonable than the result of the derivative coupling.

Mixing angles and parameters for the mixing strength

$$\lambda_{01} = 0.30 \sim 0.35(\text{GeV}^2)$$

are

$$\theta_a = (15.0 \sim 17.8)^\circ, \quad \theta_K = (13.8 \sim 16.4)^\circ$$

	f_{NN}	f_{NS}	$f_{N'}$	$f_{S'}$	f_G
$f_0(600)$	$-0.98 \leftrightarrow -0.97$	$0.05 \leftrightarrow 0.07$	$0.20 \leftrightarrow 0.23$	$-0.06 \leftrightarrow -0.08$	$-0.00 \leftrightarrow -0.01$
$f_0(980)$	$-0.05 \leftrightarrow -0.07$	$-0.93 \leftrightarrow -0.90$	$0.11 \leftrightarrow 0.14$	$0.34 \leftrightarrow 0.40$	~ -0.02
$f_0(1370)$	$0.13 \leftrightarrow 0.16$	$-0.25 \leftrightarrow -0.29$	$0.48 \leftrightarrow 0.49$	$-0.83 \leftrightarrow -0.80$	~ 0.02
$f_0(1500)$	$-0.02 \leftrightarrow -0.03$	$-0.03 \leftrightarrow -0.05$	$-0.09 \leftrightarrow -0.10$	$-0.02 \leftrightarrow -0.03$	~ 0.99
$f_0(1710)$	$-0.16 \leftrightarrow -0.19$	$-0.25 \leftrightarrow -0.30$	$-0.85 \leftrightarrow -0.82$	$-0.44 \leftrightarrow -0.43$	$-0.10 \leftrightarrow -0.12$

4. Conclusions

1. Invariant mass distributin for $\phi \rightarrow a_0(980)\gamma \rightarrow \eta\pi^0\gamma, \phi \rightarrow f_0(980)\gamma \rightarrow \pi^0\pi^0\gamma,$ suggests that **the K^+K^- loop diagram contributions are dominant.**
2. Applying this picture to the decays $\phi \rightarrow a_0(980)\gamma, \phi \rightarrow f_0(980)\gamma,$ $g_{f_0 K\bar{K}} / g_{a_0 K\bar{K}} \approx 2.2$ is obtained.
3. In our model considering the mixing between $qq\bar{q}\bar{q}$ scalar mesons and $q\bar{q}$ scalar mesons + glueball, the *SPP* coupling constants $g_{f_0 K\bar{K}}, g_{a_0 K\bar{K}}$ etc. at the mixing strength $\lambda_{01} = 0.30 \sim 0.35 \text{GeV}^2$ are nearest to the values $g_{f_0 K\bar{K}} / g_{a_0 K\bar{K}} \approx 2.2.$
4. In *SPP* interaction, **non-derivative coupling is more reasonable** than the derivative one.
5. In the present analysis, **$f_0(1500)$ meson is the most reasonable scalar glueball candidate.**