Electromagnetic decays of vector mesons

A phenomenological model developed by:

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October 9^{th}, 2007

Outline

- Electromagnetic decays of vector mesons
- Light-Front dynamics
- The model
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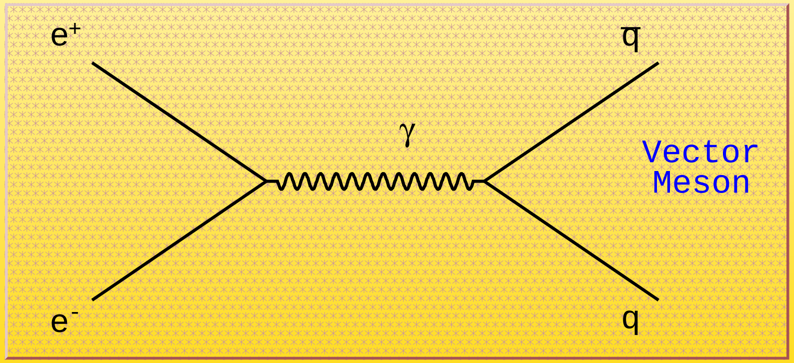
  - Phenomenological Ansatz for the vector meson Bethe-Salpeter amplitude
  - q-γ vertex
- Tested wave functions
- Results
- Summary and Outlook

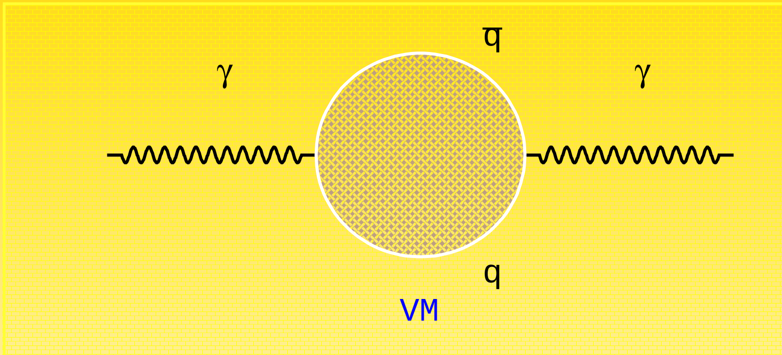
Electromagnetic decays of vector mesons

VM $\to e^+e^-$: the $q\bar{q}$ pair composing the vector meson annihilates in a virtual photon, that produces a lepton pair.



The $q\bar{q}$ vertex is strategic in the Vector Meson Dominance description of the photon propagation:

...where the photon is described through its hadronic component $q\bar{q}$

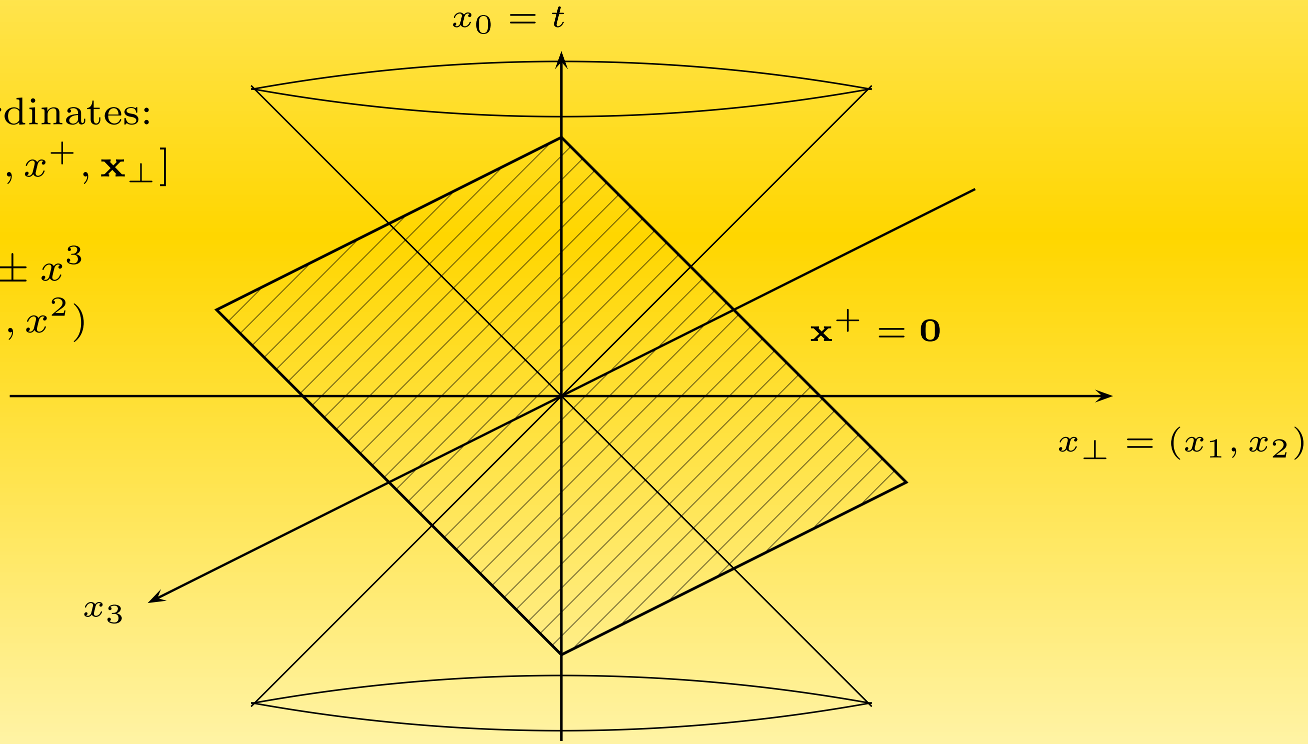


Useful to study electromagnetic form factors!

Theoretical framework: Light-Front dynamics

Three forms of relativistic dynamics: the Instant-Form, the Point-Form and the Front-Form. *We will use the Front-Form dynamics.*

New coordinates:
$x^\mu = [x^-, x^+, \mathbf{x}_\perp]$
where
$x^\pm = x^0 \pm x^3$
$\mathbf{x}_\perp = (x^1, x^2)$



Why using the Light Front?

- largest number of kinematical generators
- pair-production mechanisms are not allowed, and the *vacuum* of the theory is an exact eigenstate of the full Hamiltonian.
- ...it open a unique possibility to study the hadronic state, both in the valence and in the non-valence sector

$$|meson\rangle = \; |q\bar{q}\rangle + |q\bar{q}q\bar{q}\rangle + |q\bar{q}\; g\rangle.....$$

$$|baryon\rangle = \underbrace{|qqq\rangle}_{\text{valence}} + \underbrace{|qqq\; q\bar{q}\rangle + |qqq\; g\rangle.....}_{\text{non-valence}}$$

since the Fock expansion is meaningful within LF framework.

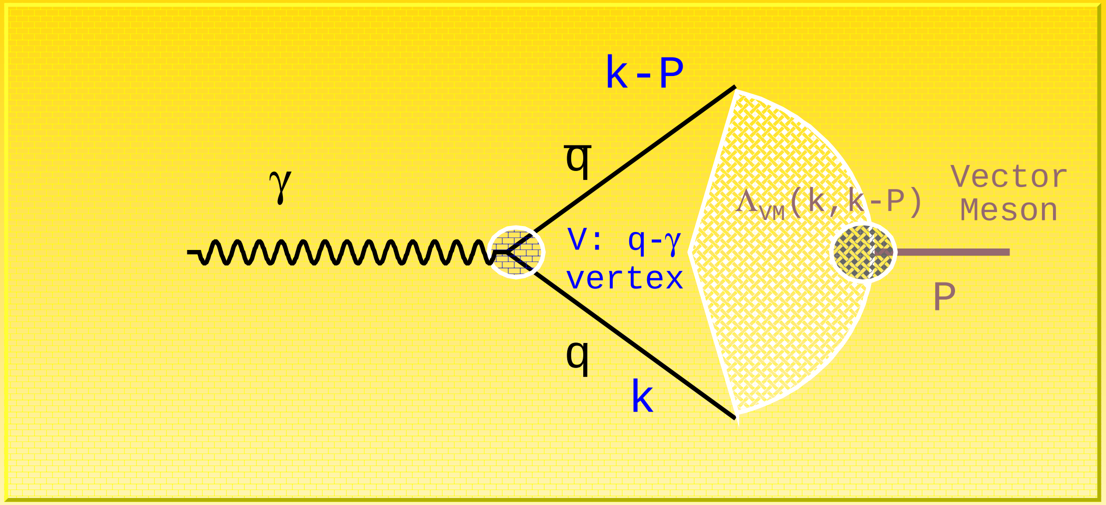
Mandelstam Formula for decay constant

The decay constant and the width are defined as:

$$\langle 0|J^\mu(0)|P,\lambda\rangle = \imath\sqrt{2} f_V \epsilon^\mu_\lambda \equiv \mathcal{V}^\mu_\lambda \quad \rightarrow \quad \Gamma_{e^+e^-} = \frac{8\pi\alpha^2}{3} \frac{|f_V|^2}{M_V^3} \qquad (1)$$

It can be approximated *á la* Mandelstam through:

$$\mathcal{V}^\mu_\lambda = \mathcal{F}_V \frac{N_c}{(2\pi)^4} \int d^4k \frac{\Lambda_V(k, k-P, m_1, m_2)}{(k^2 - m_1^2 + \imath\epsilon)\ [(P-k)^2 - m_2^2 + \imath\epsilon]} \times$$
$$\mathrm{Tr}[\ \epsilon_\lambda(P) \cdot V\ (\not{k} - \not{P} + m_2)\gamma^\mu(\not{k} + m_1)] \qquad (2)$$



Main ingredients: Bethe-Salpeter amplitude for the VM

$$\Lambda_V(k, k-P) = \mathcal{N}\ \left[k^2 - m_1^2 + (P-k)^2 - m_2^2\right] \times$$
$$\frac{1}{\left[k^2 - m_{R_1}^2 + \imath\epsilon\right]\left[(P-k)^2 - m_{R_1}^2 + \imath\epsilon\right]} \times$$
$$\frac{1}{\left[k^2 - m_{R_2}^2 + \imath\epsilon\right]\left[(P-k)^2 - m_{R_2}^2 + \imath\epsilon\right]} \times$$
$$\frac{1}{\left[k^2 - m_{R_3}^2 + \imath\epsilon\right]} \frac{1}{\left[(P-k)^2 - m_{R_3}^2 + \imath\epsilon\right]} \tag{3}$$

m_{Ri} **are free parameters**

chosen in order to:

- *fulfill the right symmetry properties in order to respect the* **covariance** *(symmetry with respect to the quark momenta exchange);*
- **regularize** *the integrations.*

Main ingredients: $q - \gamma$ vertex

The following vertex function is adopted in the work:

$$V^\mu = \frac{M}{M + m_1 + m_2} \left[\gamma^\mu + \imath \frac{1}{M} \, \sigma^{\mu\nu} P_\nu \right] \tag{4}$$

In the matrix element the operator V^μ is multiplied by the polarization vector $\epsilon^\mu_\lambda(P)$, orthogonal to P^μ, so that:

$$\begin{aligned} \epsilon_\lambda(P) \cdot V &= \frac{M}{M + m_1 + m_2} \left[\not\epsilon_\lambda(P) - \frac{1}{M} \not\epsilon_\lambda(P) \not P \right] = \\ &= \frac{1}{M + m_1 + m_2} \not\epsilon_\lambda(P) \; [M - \not P] \end{aligned} \tag{5}$$

The valence component of the BSA

The valence component of the Bethe-Salpeter amplitude for a vector meson is defined as:

$$\Psi_\lambda(k,P) = \frac{\not k + m_1}{k^2 - m_1^2 + \imath\epsilon}\ [\epsilon_\lambda(P)\cdot V(k,k-P)] \\ \times\ \Lambda_V(k,k-P)\ \frac{\not k - \not P + m_2}{(k-P)^2 - m_2^2 + \imath\epsilon} \tag{6}$$

where the Dirac structure is chosen in order to reproduce the Melosh rotations for a $^3S_1$ meson, i.e.

$$(\not k + m_1)\ [\epsilon_\lambda(P)\cdot V(k,(k-P))]\ [(\not k - \not P) + m_2] \tag{7}$$

By:

* taking out the Dirac structure from $\Psi_\lambda(k,P)$
* integrating over k^- with the constraint $P^+ > k^+ > 0$
* multiplying by the factor $k^+\ (P-k)^+$

one has the definition of the momentum component of the valence wave function...

The momentum component looks like...

$$
\begin{aligned}
\phi(k^+, \mathbf{k}_\perp; M, \vec{0}) &= \imath \int \frac{dk^-}{2\pi} \frac{\Lambda_V(k, k-P)}{(k^- - k^-_{(1)})\left(k^-_{(2)} - k^-\right)} = \\
&= \left.\frac{\Lambda_V(k, k-P)}{(k^-_{(2)} - k^-)}\right|_{k^-=k^-_{(1)}} + \frac{1}{(k^-_{R_1} - k^-_{(1)})} \left.\left[\frac{(k^- - k^-_{R_1})\Lambda_V(k, k-P)}{(k^-_{(2)} - k^-)}\right]\right|_{k^-=k^-_{R_1}} \\
&+ \frac{1}{(k^-_{R_2} - k^-_{(1)})} \left.\left[\frac{(k^- - k^-_{R_2})\Lambda_V(k, k-P)}{(k^-_{(2)} - k^-)}\right]\right|_{k^-=k^-_{R_2}} + \\
&+ \frac{1}{(k^-_{R_3} - k^-_{(1)})} \left.\left[\frac{(k^- - k^-_{R_3})\Lambda_V(k, k-P)}{(k^-_{(2)} - k^-)}\right]\right|_{k^-=k^-_{R_3}} = \\
&= -\mathcal{N}\,\Phi(\xi, \mathbf{k}_\perp; P^+, \mathbf{P}_\perp = \mathbf{0}, R_1, R_2, R_3)
\end{aligned}
\tag{8}
$$

$\mathcal{N}$ is determined by charge normalization

Valence component probability

The probability $P_{q\bar{q}}$ of the valence component reads

$$P_{q\bar{q}} = N_c \; I_V \; \mathcal{N}^2 \; \frac{1}{(2\pi)^3} \; \frac{1}{[P^+]^2} \; \int_0^1 \frac{d\xi}{\xi \; (1-\xi)}$$

$$\int d\mathbf{k}_\perp \; \left\{ M_0^2 \; + [\epsilon_\lambda(P) \; \cdot (k-P) - \epsilon_\lambda(P) \; \cdot k]^2 \right\} \times$$

$$|\Phi(\xi, \mathbf{k}_\perp; P^+, \mathbf{P}_\perp = \mathbf{0}, R_1, R_2, R_3)|^2 \tag{9}$$

and depends on $\Phi(\xi, \mathbf{k}_\perp; P^+, \mathbf{P}_\perp = \mathbf{0}, R_1, R_2, R_3)$.

Tested models

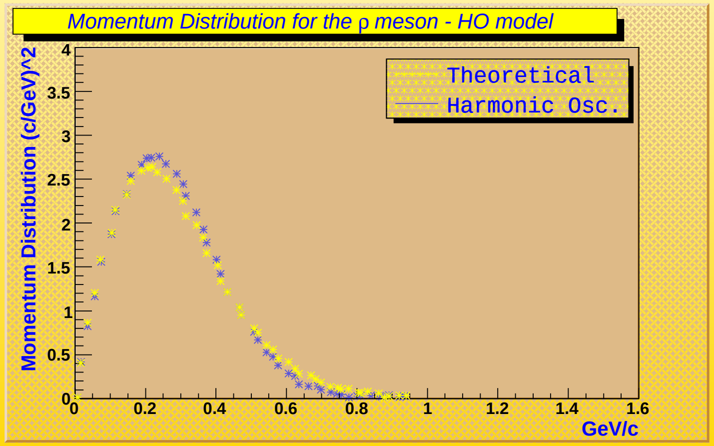
The momentum distributions of three different models are investigated, in particular:

- A `Harmonic-Oscillator-like` potential
- A `Godfrey-Isgur` potential (*PRD* **32**, *189 (1985)*)
- An `effective light-front QCD-inspired dynamical model` regulated at short-distances (*EPJA* **27** *213 (2006)*)

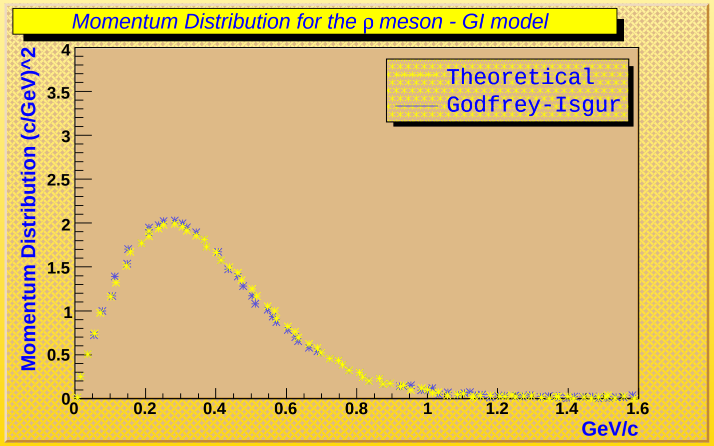
The momentum distribution obtained within our model is fitted to these distributions, and the free parameters m_{Ri} are fixed.

Then, the electromagnetic widths are predicted!!

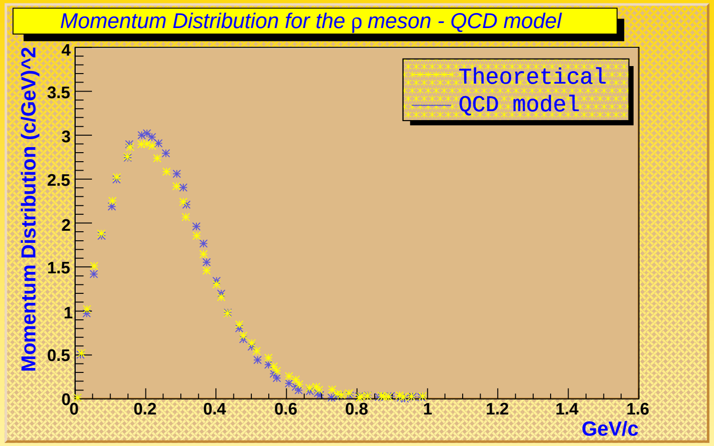
Momentum distribution for the ρ meson



(a) **model vs. harmonic oscillator**

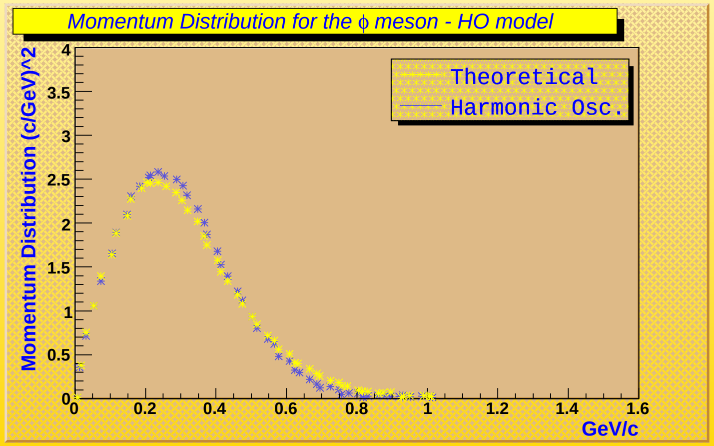


(b) **...vs. Godfrey-Isgur**

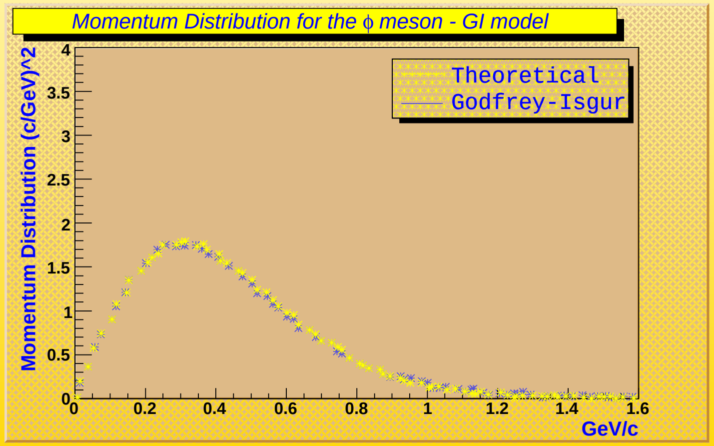


(c) **...and vs. a QCD inspired model**

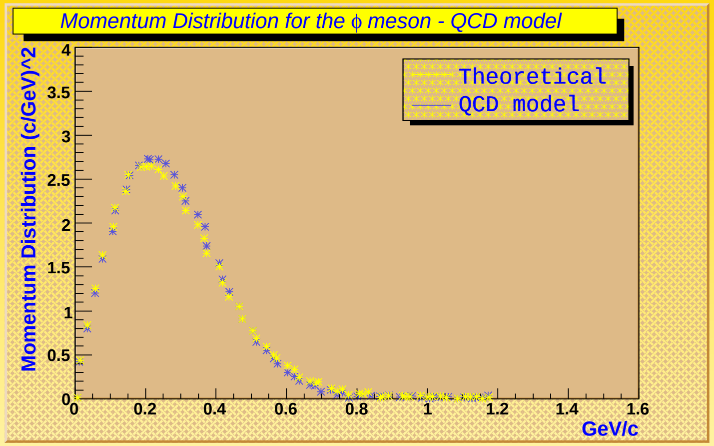
Momentum distribution for the ϕ meson



(d) **model vs. harmonic oscillator**

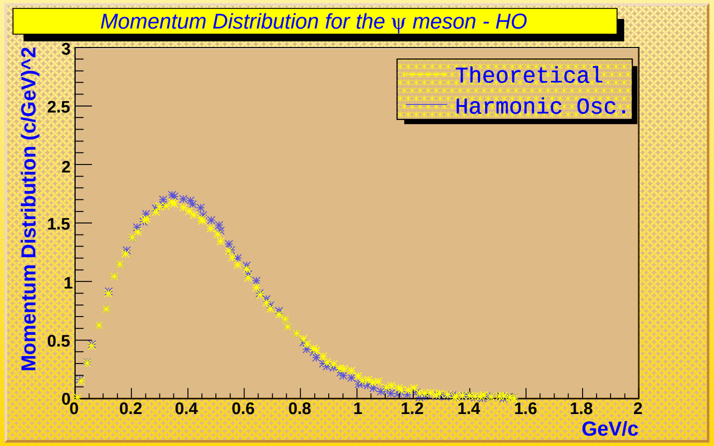


(e) **...vs. Godfrey-Isgur**

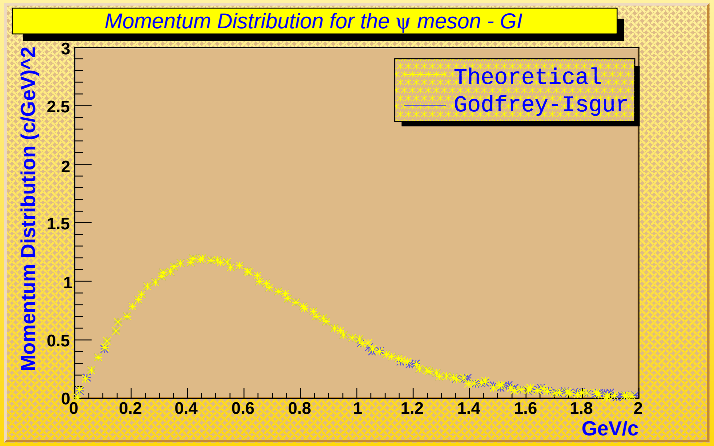


(f) **...and vs. a QCD inspired model**

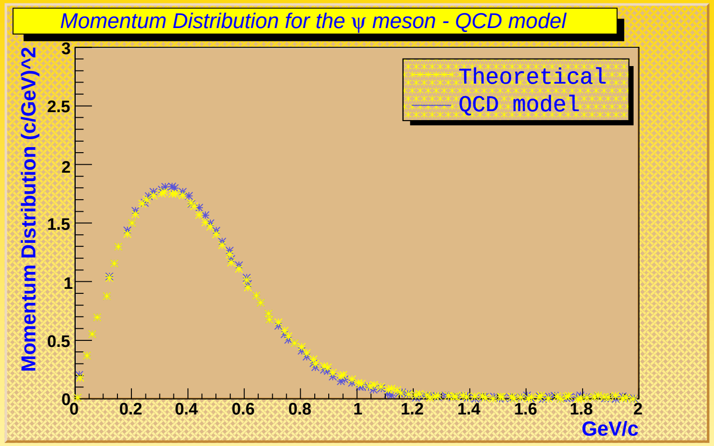
Momentum distribution for the ψ meson



(g) **model vs. harmonic oscillator**



(h) **...vs. Godfrey-Isgur**



(i) **...and vs. a QCD inspired model**

Electromagnetic Widths and Probabilities 1: harmonic oscillator

VM	m_{VM} (MeV)	$P_{q\bar{q}}$	$\Gamma^{th}_{e^+e^-}$ (keV)	$\Gamma^{exp}_{e^+e^-}$ (keV)
ρ	775.5 ± 0.4	0.884	10.328	7.02 ± 0.11
ϕ	1019.460 ± 0.019	0.961	1.582	1.32 ± 0.06
J/ψ	3096.916 ± 0.011	0.787	1.572	$5.55 \pm 0.14 \pm 0.02$

It reproduces quite well the light sector ρ, ϕ, but a reasonable description of heavy mesons is still lacking!

Electromagnetic Widths and Probabilities 2: Godfrey-Isgur

VM	m_{VM} (MeV)	$P_{q\bar{q}}$	$\Gamma^{th}_{e^+e^-}$ (KeV)	$\Gamma^{exp}_{e^+e^-}$ (keV)
ρ	775.5 ± 0.4	0.411	18.098	7.02 ± 0.11
ϕ	1019.460 ± 0.019	0.906	3.733	1.32 ± 0.06
J/ψ	3096.916 ± 0.011	0.908	5.911	$5.55 \pm 0.14 \pm 0.02$

Differently from the Harmonic Oscillator case, within the Godfrey-Isgur model the electromagnetic properties for the light sector are overestimated, while a better description of the heavy mesons is obtained.

Electromagnetic Widths and Probabilities 3: QCD model

VM	m_{VM} (MeV)	$P_{q\bar{q}}$	$\Gamma^{th}_{e^+e^-}$ (keV)	$\Gamma^{exp}_{e^+e^-}$ (keV)
ρ	775.5 ± 0.4	0.913	7.548	7.02 ± 0.11
ϕ	1019.460 ± 0.019	0.995	1.294	1.32 ± 0.06
J/ψ	3096.916 ± 0.011	0.726	1.250	$5.55 \pm 0.14 \pm 0.02$

The light sector is reproduced very well, but the heavy sector is absolutely not described.

Summary and Outlook

- An Ansatz for VM the Bethe-Salpeter amplitude has been tested...
- ...together with a q-γ vertex;
- three different models have been investigated, fitting to them the momentum distribution obtained from the valence component of the BSA;
- the light sector is successfully described with an harmonic oscillator model, while a Godfrey-Isgur model seems more appropriate for the heavy sector.
- a new, QCD-inspired model for the mass spectrum is under investigation, in order to mediate between these two regimes.