Spectrum and decays of pentaquarks, including $\text{SU}(3)_F$ breaking.

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1) Historical review: $\text{SU}(6)_{CS}$
2) Form of the spectrum and determination of the parameters
3) Conclusions
Historical review.

More than 30 years ago De Rujula, Georgi and Glashow, short time after QCD was proposed as the theory of strong interactions, have shown that the chromo-magnetic interaction

\[ H_{CM} = \frac{\bar{\sigma}\lambda_a (1) \cdot \bar{\sigma}\lambda_a (2)}{r} \]

accounts for the mass splittings within the SU(6)$_{FS}$ multiplets:

\[ M(\Delta) - M(N) = C_{qq} [C_6(70) - C_6(20) + 1] = 4 \ C_{qq} \]

\[ M(\Sigma) - M(\Lambda) = 8/3 \ (C_{qq} - C_{qs}) \]

They also predict:

\[ M(\Xi^*) - M(Y^*) = M(\Xi) - M(\Sigma) \]

By applying the same concepts to the mesons one gets:

\[ M(\rho) - M(\pi^0) = C_{q-q\bar{q}}[C_6(35) - 2/3] = 16/3 \ C_{q-q\bar{q}} \]
One has

\[ C_{q\bar{q}} = \frac{3}{16} [M(K^*) - M(K)] = \frac{1}{4} [M(\Delta) - M(N)] = C_{qq} \]

Jaffe proposed to consider mesons consisting of 2q and 2qbar with the spectrum given by the chromo-magnetic interaction.

Pauli principle relates the SU(3)_F and SU(6)_{CS} transformation properties of 2q and 2qbar:

\[ 2q : \quad 6_F \times 15_{CS} + 3_F \times 21_{CS} \]

To get light states one should have high Casimir for 2q and 2qbar and small for the (2q2qbar) mesons. From the tensor products:

\[ 21 \times 21 = 1 + 35 + 405 \]
\[ 15 \times 15 = 1 + 35 + 189 \]
\[ 21 \times 1\bar{5} = 280 + 35 \]
\[ 15 \times 21 = 280 + 35 \]
And the values of the SU(6)_{CS} Casimir:

\[
\begin{array}{cccccccc}
1 & 15 & 21 & 35 & 189 & 280 & (280\text{bar}) & 405 \\
1 & 14/3 & 20/3 & 6 & 10 & 12 & 14
\end{array}
\]

The spin singlets (0^+ scalars) are contained in the 1, 189 and 405 representations, the spin triplets (1^+ axials) in the 35, 280 and \(\overline{280}\) and the spin quintets (2^+ tensors) in the 189 and 405.

So we expect as lower state \((21 \times \overline{21})_1\) a scalar nonet \((1_F + 8_F)\).
Another nonet of scalar mesons and a 36-plet \((1_F + 8_F + 27_F)\) are and several axial states are expected with a higher mass value. Another 36-plet of scalars is at a still higher mass together with some axials and two tensor states.

Up to now there is evidence for the two scalar nonets with the isovector degenerate with the heavier isoscalar (which is a sort of "smoking gun" for the presence of "hidden strangness") (Maiani, Piccinini, Polosa and Riquer) and the \(\phi \omega\) resonance seen at BES may belong to the heavier scalar 36-plet.

Jaffe observed that some of these mesons may decay just from separation of the constituents and called these states "open door" and one expects large couplings to these final states: in fact the existence of the lightest scalar has been for a long time controversial for its very large width.

Recently a Group theoretical criterion has been found to select the possible "open door" channels.
It is the selection rule coming from SU(6)$_{CS}$ conservation and from the transformation properties of the pseudoscalar (P) and vector (V) mesons, as a singlet (1) and an adjoint (35) representation, respectively. Therefore only the scalars, which are SU(6)$_{CS}$ singlets and the axials, which transform as a 35, may decay into PP or PV, respectively.

In conclusion the lightest particles, which transform as small SU(6)$_{CS}$ transformations, are the ones with “open door” decay into PP and PV, while the heaviest ones, transforming as the 189, 280, 280 and 405 representations, have “open door” decays only into VV.

Similar considerations may be applied to the pentaquarks, keeping into account that the baryon octet (decuplet) transforms as a 70 (20) representation of SU(6)$_{CS}$: therefore only the spin 1/2 (3/2) states transforming as a 70 (20) may have “open door” decay into N(Δ) P channels.

To construct a pentaquark one has to combine 4q in a 3$_C$ in order to combine with the q̄b into a colour singlet. Pauli principle relates the SU(6)$_{CS}$ and SU(3)$_F$ properties.
Fermion statistics relates the SU(3)$_F$ and SU(6)$_{CS}$ transformation properties to get antisymmetric wave function

<table>
<thead>
<tr>
<th>4$q$</th>
<th>4$q$ $\bar{s}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>SU(3)$_F$</td>
<td>SU(6)$_{CS}$</td>
</tr>
<tr>
<td>6</td>
<td>105</td>
</tr>
<tr>
<td>15</td>
<td>105</td>
</tr>
<tr>
<td>15</td>
<td>15</td>
</tr>
</tbody>
</table>

Which shows that it is impossible to get 210$_{CS}$ Y=2 states and that:

$$M(I=2) > M(I=1) > M(I=0)$$

which compares well with the D waves found

$$M(D15) = (2074-2150) > M(D03) = 1865$$

For positive parity states the presence of the orbital momentum changes the relationship between the flavour and the SU(6)$_{CS}$ transformation properties.
The J = ½ S-wave states may have very large couplings into NK and consequently very broad to be detected, instead the D0 3/2 is expected to be lighter than the D1 5/2 and we have the spectrum:

I=0 : 1561 S ½ and 1881 D 3/2 in good agreement with the 1865 found experimentally

I=1 : 1780 and 2050 S ½ 1690 and 2090 D 3/2 and 2100 D 5/2 in good agreement with the range 2074-2150 found experimentally

I=2 : 2370 S ½ and 2120 D 3/2

For the positive parity states built with 4q light in P-wave and the sbar in S-wave with respect to them one has for the states behaving almost as a 70 with “open door” decay into KN:

I=0 : 1540 P ½ and 1610 3/2

I=1 : 1725 P 1/2 and 1780 3/2 to be compared with 1720 and 1780

I=2 : 1855 P ½ and 1910 3/2
Conclusions

1) The present experimental situation on pentaquarks is the one expected, if chromo- magnetism and spin-orbit, which describe the splittings of ordinary hadrons and tetraquarks, gives their mass splittings.

2) Selection rules, which imply too large widths for S-wave decays account for the fact that the first negative parity states to be discovered are the D03 and D15 just with the predicted mass difference. For the positive parity states only few of them are expected to be found in KN scattering and the lowest I=0 and 1 states have been found with the mass, they are expected to have. It is difficult to find \( \Delta K \), since they have no common “open door” channel with KN.

3) The evaluation of mass-spectrum with symmetry breaking may be applied to the study of multiquark states containing heavy quarks.
Long time before the controversial $\Theta^+(1540)$ $Y=2$ states have been found in partial wave analysis in $K^+p(d)$ scattering and put in the review of particle physics [1985 and 1992]

$$P_{11}(1720) \quad P_{13}(1780) \quad D_{03}(1865) \quad D_{15}(2074 \text{ or } 2150)$$

Also a $\Xi^{-}\pi$ resonance has been seen by NA49 at 1872 with a $\Xi^*(1520) \pi$ decay at the same mass.

These states may be interpreted within the constituent quark model as $qqqq\bar{q}$ states.

To build a pentaquark one should have as many constituents already present at the beginning of the reaction
The best:
\[ K^+n \]
\[(u\bar{s})(u\bar{d}d) \rightarrow uud\bar{d}\bar{s}\]

The second possibility: deep inelastic on s d or \( \bar{d} \) parton:
\[ e^- + p(uud\bar{s}s) \rightarrow e^- + s\bar{d} + d(uud\bar{s}) \]
\[ \bar{\nu}_\mu + p(uudd\bar{d}) \rightarrow \mu^+ + \bar{u}s + \bar{s}(uudd) \]
\[ e^- + p(uudd\bar{d}) \rightarrow e^- + d\bar{s} + \bar{s}(uudd) \]
\[ \nu_\mu + p(uuds\bar{s}) \rightarrow \mu^- + c\bar{d} + d(uuds) \]

Third possibility: photo-production
\[ \gamma + p(uud) \rightarrow d\bar{s} + d\bar{s}(uud) \]

Difficult:
\[ e^- + p(uud) \rightarrow e^- + u(\bar{u}\bar{d}s) + (u\bar{d}s)(ud) \]

Still more difficult:
\[ e^+ + e^- \rightarrow s(\bar{u}\bar{u}\bar{d}\bar{d}) + \bar{s}(uudd) \]
Some skepticism motivated by:

1) Why so low mass and a P-wave state?

2) Why so narrow?

3) \( m_{\Xi^-} (1864) - m_{\Theta^+} (1540) = 314 > m_s - m_u \)
   
   too large to be in the same multiplet

4) Where are all the states one one can build with 4q and a \( \bar{q} \)?
We consider 4q in P-wave and the $\bar{q}$ in S-wave with respect to them. Moreover, since the chromo-magnetic interaction is at short-range one considers only the q pairs in S-wave for which one has

<table>
<thead>
<tr>
<th>$SU(6)_{cs}$</th>
<th>$SU(3)_c \times SU(2)_s$</th>
<th>$\frac{\Delta m_{qq}}{C_{qq}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>21</td>
<td>(3, 1)</td>
<td>$-2$</td>
</tr>
<tr>
<td>21</td>
<td>(6, 3)</td>
<td>$-\frac{1}{3}$</td>
</tr>
<tr>
<td>15</td>
<td>(3, 3)</td>
<td>$+\frac{2}{3}$</td>
</tr>
<tr>
<td>15</td>
<td>(6, 1)</td>
<td>$+1$</td>
</tr>
</tbody>
</table>

which gives lower mass to the states built with two 21’s

Including the interaction with the $\bar{s}$, the spin-orbit term and the orbital kinetic energy gives the mass formula:

$$m(p) = \sum_{i=1}^{4} m_{q_i} + m_{\bar{q}} + \Delta m_{qq}^1 + \Delta m_{qq}^2 + C_{4q,q} \left[ C_6(p) - C_6(t) - \frac{1}{3} C_2(p) + \frac{1}{3} C_2(t) - \frac{4}{3} \right] + a \vec{L} \cdot \vec{S}_{\bar{q}} + K_1$$
Let us consider a negative parity pentaquark with all the constituents in S-wave gives the mass formula

\[ m^- = \sum_{i=1}^{4} m_{q_i} + m_{\bar{q}} - C_{qq} \left[ C_6(t) - \frac{1}{3} C_2(t) - \frac{26}{3} \right] \]
\[ + C_{q_{4\bar{q}}} \left[ C_6(p) - C_6(t) - \frac{1}{3} C_2(p) + \frac{1}{3} C_2(t) - \frac{4}{3} \right] \]

This implies that to get a low masses one has to look for high SU(6)_{CS} Casimir representations for t and low Casimir for p.