The K-matrix analysis of the meson spectrum: scalar and tensor states below 1.9 GeV

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Combined Partial Wave Analysis

1. **The two pseudoscalar particle final states from pion induced reactions reveal very limited number of states. e.g. CERN-Munich:**

   $\rho(770)$, $f_2(1270)$, $f_0(980)$, $\rho_3(1690)$, $f_4(2050)$

   $\rho(1450)$?, $\rho(1700)$?, $f_0(1500)$, $f_0(1750)$?

   $f_0(1370)$, $f_2(1560)$, $f_0(1710)$, $\rho(1900)$, $f_2(1950)$, $f_2(2010)$, $f_0(2020)$

2. **A large set of the resonances had been discovered in the analysis of the proton-antiproton annihilation and in πN interaction into multi-body final states.**

3. **In many cases only combined analysis of the reactions with multi-body and two body final states can produce an unambiguous result.**

Problem:

The pion induced reactions are not a two particle collision reactions. They are $\pi N \to \pi\pi N$ reactions which is much more complicated process.

Fitted data: Two body reactions:

Reaction	Experiment	Reaction	Experiment
$\pi^+\pi^- \to \pi^+\pi^-$ (all waves)	CERN-Münich		
$\pi\pi \to \pi^0\pi^0$ (S-wave)	GAMS	$\pi\pi \to \pi^0\pi^0$ (S,D,G-waves)	E852
$\pi\pi \to \eta\eta$ (S-wave)	GAMS	$\pi\pi \to \eta\eta'$ (S-wave)	GAMS
$\pi\pi \to K\bar{K}$ (S-wave)	BNL	$K^-\pi^+ \to K^-\pi^+$ (S-wave)	LASS

Fitted data: Three body reactions from Crystal Barrel: (L-liquid, G-gaseous targets).

Reaction	Target	Reaction	Target	Reaction	Target
$\bar{p}p \to \pi^0\pi^0\pi^0$	(L) H_2	$\bar{p}p \to \pi^+\pi^0\pi^-$	(L) H_2	$\bar{p}p \to K_S K_S \pi^0$	(L) H_2
$\bar{p}p \to \pi^0\eta\eta$	(L) H_2	$\bar{p}n \to \pi^0\pi^0\pi^-$	(L) D_2	$\bar{p}p \to K^+K^-\pi^0$	(L) H_2
$\bar{p}p \to \pi^0\pi^0\eta$	(L) H_2	$\bar{p}n \to \pi^-\pi^-\pi^+$	(L) D_2	$\bar{p}p \to K_L K^\pm \pi^\mp$	(L) H_2
$\bar{p}p \to \pi^0\pi^0\pi^0$	(G) H_2			$\bar{p}n \to K_S K_S \pi^-$	(L) D_2
$\bar{p}p \to \pi^0\eta\eta$	(G) H_2			$\bar{p}n \to K_S K^- \pi^0$	(L) D_2
$\bar{p}p \to \pi^0\pi^0\eta$	(G) H_2				

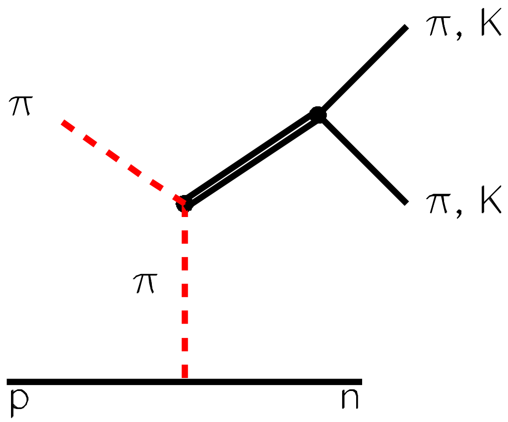
The data are available from: PWA.HISKP.UNI-BONN.DE

Problems in the analysis of the $\pi N \to XN$ reactions

The πN reaction with large energy of initial pion should be described by t-exchanges. However:

1. **There is no analysis of the data based on the particle exchanges: there are only models.**
2. **There is no a solid analysis which preserves unitarity and includes all known states in P and D-waves. This is should be important at small t where the π exchange is a dominant one and data are close to the unitarity limit.**
3. **Most of the models have problems at large t where exchanges of particles with large spin play a significant role.**

Cross section for the reactions $\pi N \to \pi\pi N, KKN, \eta\eta N$



$$d\sigma = \frac{(2\pi)^4|A|^2}{8\sqrt{s_{\pi N}}|\vec{p_2}|} d\Phi(p_1+p_2,k_1,k_2,k_3)$$

$$d\Phi(p_1+p_2,k_1,k_2,k_3) = (2\pi)^3 d\Phi(P,k_1,k_2)\, d\Phi(p_1+p_2,P,k_3)\, ds\ ,$$

Assuming that amplitude depends only on t and s:

$$d\Phi(p_1+p_2,P,k_3) \quad = \quad \frac{1}{(2\pi)^5}\frac{dt}{8|\vec{p_2}|\sqrt{s_{\pi N}}} \qquad t=(k_3-p_2)^2$$

and

$$d\Phi(P,k_1,k_2) = \frac{1}{(2\pi)^5}\rho(s)d\Omega \qquad\qquad \rho(s) = \frac{1}{16\pi}\frac{2|\vec{k}_1|}{\sqrt{s}},$$

Then:

$$d\sigma \quad = \quad \frac{(2\pi)^4 |A|^2 (2\pi)^3}{8|\vec{p_2}|\sqrt{s_{\pi N}}} \frac{1}{(2\pi)^5} \frac{dt 2M\, dM\, d\Phi(P,k_1,k_2)}{8|\vec{p_2}|\sqrt{s_{\pi N}}} = \frac{(M|A|^2\rho) dt\, dM\, d\Omega}{(2\pi)^3 32 |\vec{p_2}|^2\, s_{\pi N}}$$

Unitarity relation:

$$Im A = \rho(s)|A|^2$$

And the cross section can be expressed in the terms of spherical functions:

$$\frac{d^4\sigma}{dt\, dM\, d\Omega} = N \sum_l \left(< Y_l^0 > Y_l^0(\Omega) + \sum_{m=0}^{l} 2 < Y_l^m > Re\, Y_l^m(\Omega) \right)$$

CERN-Munich approach

The CERN-Munich model was developed for the analysis of the data on $\pi^- p \to \pi^+\pi^- n$ reaction and based partly on the absorbtion model but mostly on the phenomenological observations.

$$|A|^2 = |\sum_{J=0} A^0_J Y^0_J + \sum_{J=1} A^-_J ReY^1_J|^2 + |\sum_{J=1} A^+_J ReY^1_J|^2$$

Additional assumptions:

1) helicity 1 amplitudes are equal for natural and unnatural exchanges:

$$A^{(-)}_J = A^{(+)}_J$$

2) The ratio of the $A^{(-)}_J$ and the A^0_J amplitudes is a polynomial over mass of the two pion system which does not depend on J up to total normalization.

$$A^{(-)}_J = \frac{A^0_J}{C_J \sum\limits_{n=0}^{3} b_n M^n},$$

The amplitude squared can be rewritten via density matrices

$$\rho_{00}^{nm} = A_n^0 A_m^{0*} \qquad \rho_{01}^{nm} = A_n^0 A_m^{(-)*}, \qquad \rho_{11}^{nm} = 2A_n^{(-)} A_m^{(-)},$$

as:

$$|A|^2 = \sum_{J=0} Y_J^0 \left(\sum_{n,m} d_{n,m,J}^{0,0,0} \rho_{00}^{nm} + d_{n,m,J}^{1,1,0} \rho_{11}^{nm} \right) + \sum_{J=0} ReY_J^1 \left(\sum_{n,m} d_{n,m,J}^{1,0,1} \rho_{10}^{nm} + d_{n,m,J}^{0,1,1} \rho$$

where

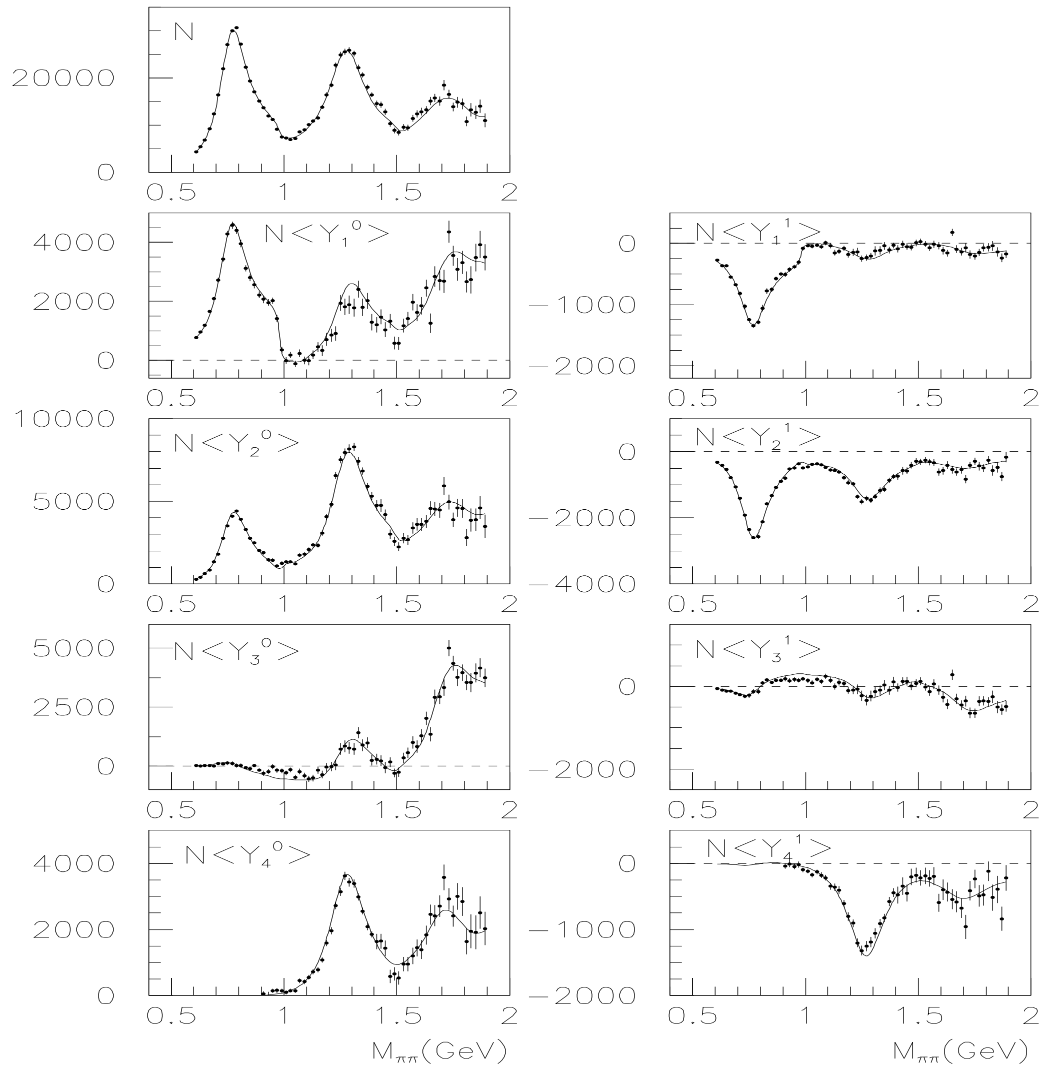
$$d_{n,m,J}^{i,k,l} = \frac{\int d\Omega\, ReY_n^i(\Theta,\varphi)\, ReY_m^k(\Theta,\varphi)\, ReY_J^l(\Theta,\varphi)}{\int d\Omega\, ReY_J^l(\Theta,\varphi)\, ReY_J^l(\Theta,\varphi)}$$

Substituting such amplitude into cross section one can directly fit the moments $< Y_J^m >$.

Description of the CERN-Munich data

$$\pi^- p \rightarrow \pi^+ \pi^- n$$

$$A_{1j} = K_{1m}(I - i\hat{\rho}(s)\hat{K})^{-1}_{mj}$$



GAMS, VES and BNL approach

The Cern-Munich approach does not work for large t and does not work for many other final states.

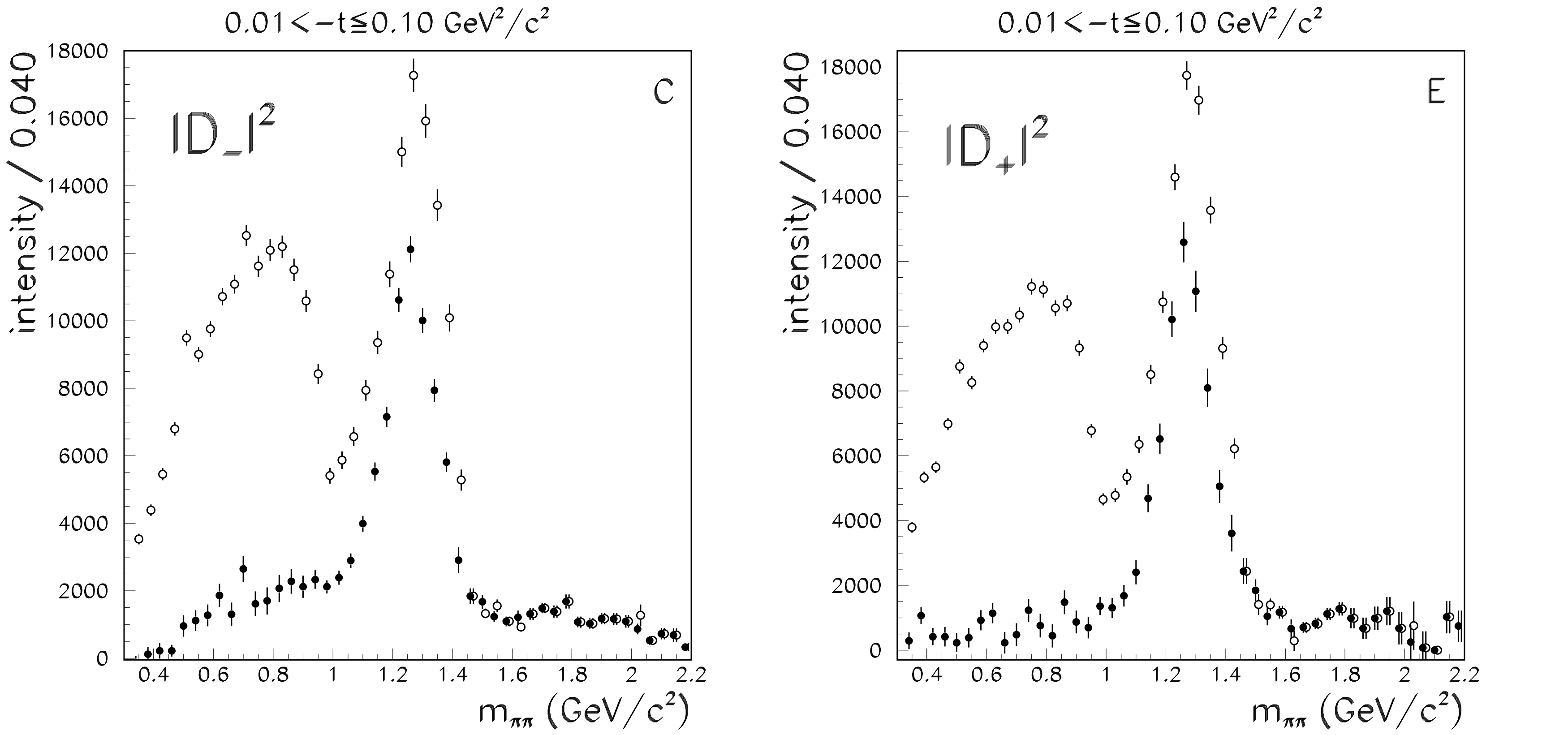
The πN data are decomposed as a sum of amplitudes with angular dependence defined by spherical functions:

$$|A^2| = |\sum_{J=0} A^0_J Y^0_J + \sum_{J=1} A^-_J \sqrt{2}\, Re\, Y^1_J|^2 + |\sum_{J=1} A^+_J \sqrt{2}\, Im\, Y^1_J|^2$$

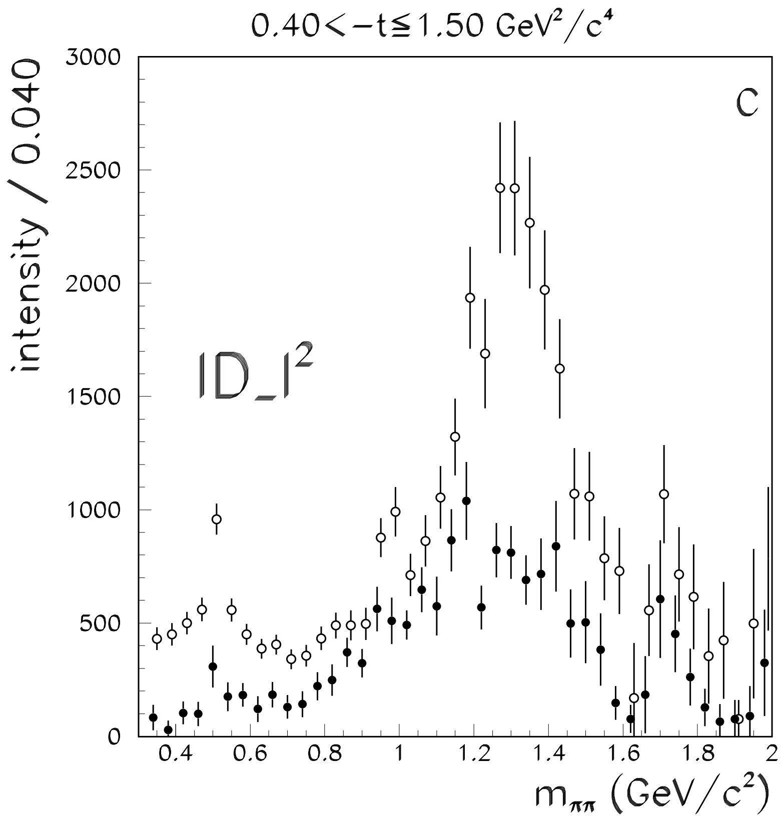
Here the A^0_J functions are called $S_0, P_0, D_0, F_0 \ldots$, the A^-_J functions defined as $P_-, D_-, F_-, \ldots$ and the A^+_J functions as $P_+, D_+, F_+, \ldots$.

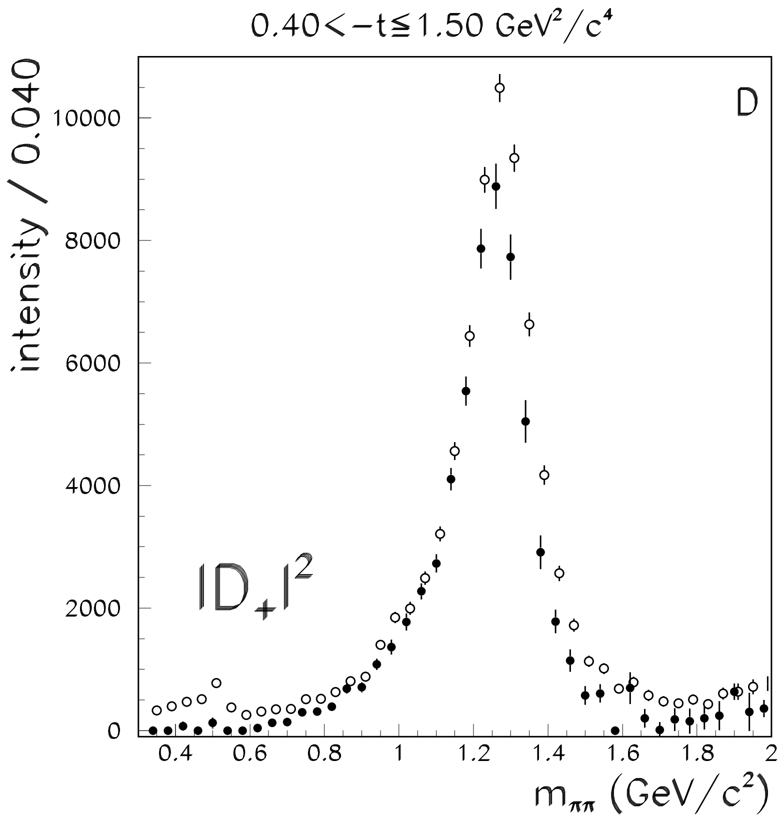
No assumptions that helicity 1 amplitudes with natural and unnatural exchanges are equal each to another.

At small t: $|t| < 0.1$:

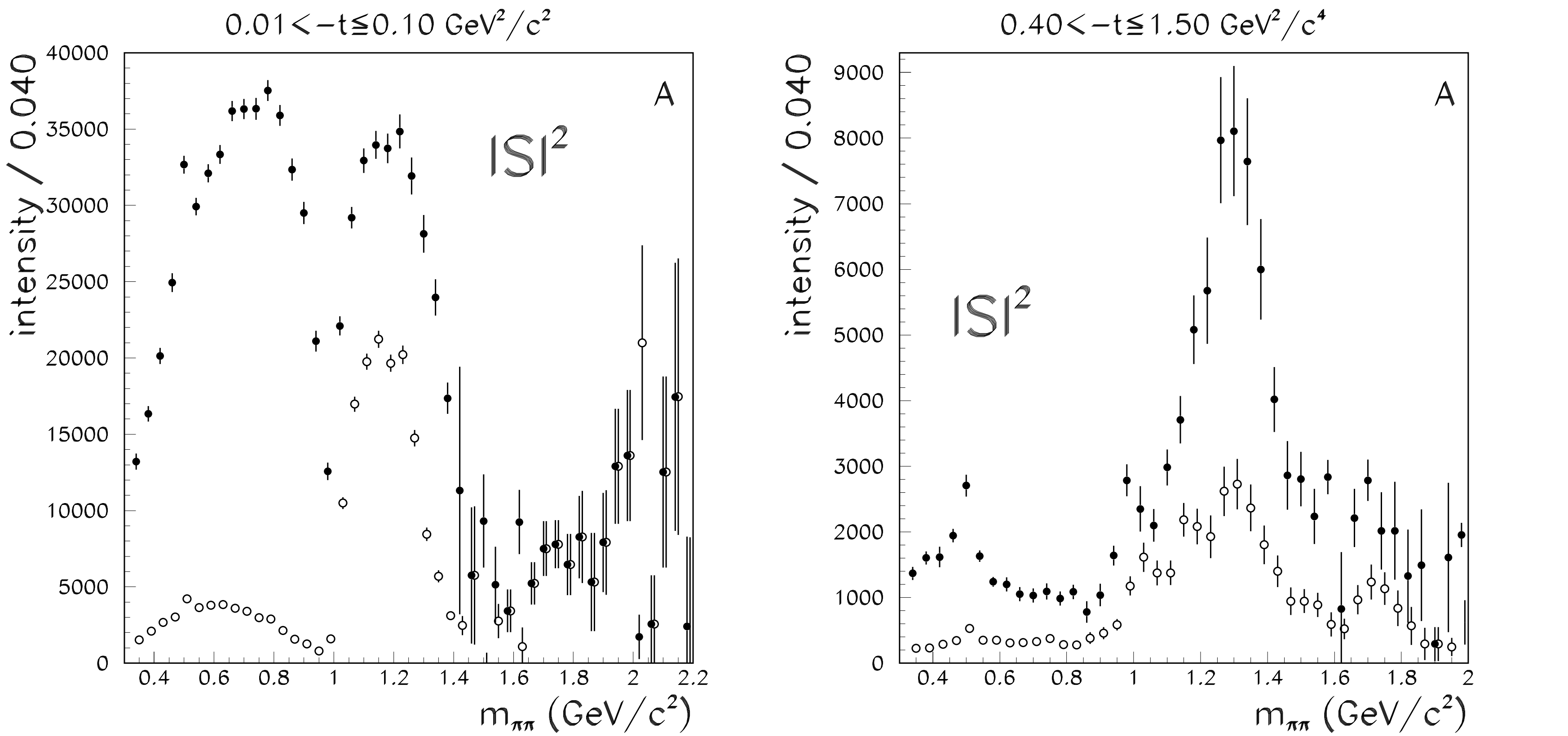


At large t: $|t| > 0.4$:

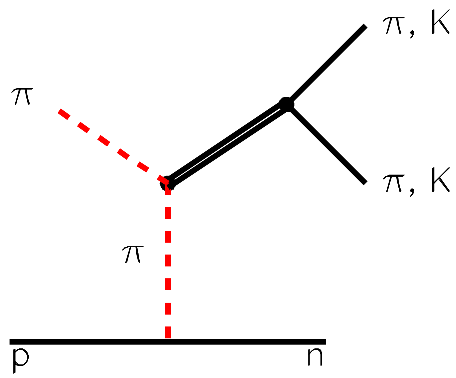


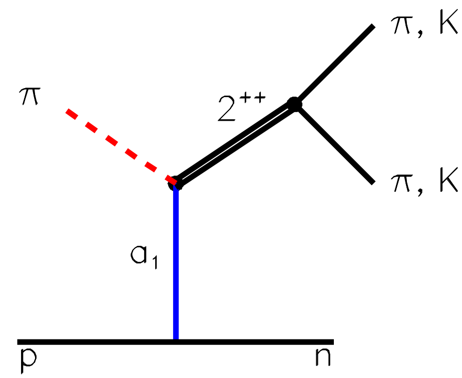


The S-wave has a very prominent structure at large $|t|$.



Reggezied exchanges:





$$A^{(\text{pion trajectories})}_{\pi p\to\pi\pi n} = \sum_{\pi_j} A(\pi\pi_j \to \pi\pi) R_{\pi_j}(s_{\pi N}, q^2) \;\; \varphi^+_n(\vec\sigma \vec p_\perp)\varphi_p \;\; g^{(\pi_j)}_{pn} \,.$$

$$A^{(a_1-\text{trajectories})}_{\pi p\to\pi\pi n} = \sum_{a_1^{(j)}} A(\pi a_1^{(j)} \to \pi\pi) R_{a_1^{(j)}}(s_{\pi N}, q^2) \;\; \varphi^+_n(\vec\sigma \vec n_z)\varphi_p \;\; g^{(a_1 j)}_{pn} \,.$$

$$R_{\pi_j}(s_{\pi N}, q^2) = \exp \;\; -i\frac{\pi}{2}\alpha^{(j)}_\pi(q^2) \;\; \frac{(s_{\pi N}/s_{\pi N0})^{\alpha^{(j)}_\pi(q^2)}}{\sin \;\; \frac{\pi}{2}\alpha^{(j)}_\pi(q^2) \;\; \Gamma \;\; \frac{1}{2}\alpha^{(j)}_\pi(q^2)+1}$$

$$R_{a_1^{(j)}}(s_{\pi N}, q^2) = i\exp \;\; -i\frac{\pi}{2}\alpha^{(j)}_{a_1}(q^2) \;\; \frac{(s_{\pi N}/s_{\pi N0})^{\alpha^{(j)}_{a_1}(q^2)}}{\cos \;\; \frac{\pi}{2}\alpha^{(j)}_{a_1}(q^2) \;\; \Gamma \;\; \frac{1}{2}\alpha^{(j)}_{a_1}(q^2)+\frac{1}{2}}$$

The π and a_1 exchanges do not interfere if the polarization of nucleons is not detected. Indeed πN coupling is described by γ_5 (singlet) while a_1 coupling is proportional to γ_μ (triplet).

$$Sp\Big[(m_N + \hat{k}_3)\gamma_5(m_N + \hat{p}_2)\gamma_\mu\Big] = 0$$

This is true for π and a_2 exchanges.

It means that

$$|S|^2 = |S_\pi|^2 + |S_{a_1}|^2$$

And D_p-wave could only interfere with S_{a_1}.

Already a_1 and π exchanges produced a more complicated structure.

Nice features of reggezied a_1 exchange:

$$A(\pi a_1^{(j)} \to \pi\pi) = \sum_J \epsilon_\beta^{(-)} \left[A^{(J+)}_{\pi a_1^{(j)} \to \pi\pi} X^{(J+1)}_{\beta\mu_1\ldots\mu_J} + A^{(J-)}_{\pi a_1^{(j)} \to \pi\pi} Z^{\beta}_{\mu_1\ldots\mu_J} \right] X^{(J)}_{\nu_1\ldots\nu_J} ,$$

$$A(\pi a_1^{(k)} \to \pi\pi) = \sum_J \alpha_J |\vec{p}|^{J-1} |\vec{k}|^J \left(W_0^{(J)} Y_J^0(\Theta,\varphi) + W_1^{(J)} Re Y_J^1(\Theta,\varphi) \right)$$

where:

$$\begin{aligned} W_{0k}^{(J)} &= -N_{J0} \left(k_{3z} - \frac{|\vec{p}|}{2} \right) \left(|\vec{p}|^2 A^{(J+)}_{\pi a_1^{(k)} \to \pi\pi} - A^{(J-)}_{\pi a_1^{(k)} \to \pi\pi} \right) \\ W_{1k}^{(J)} &= -\frac{N_{J1}}{J(J+1)} k_{3x} \left(|\vec{p}|^2 J\, A^{(J+)}_{\pi a_1^{(k)} \to \pi\pi} + (J+1) A^{(J-)}_{\pi a_1^{(k)} \to \pi\pi} \right) \end{aligned} \qquad (1)$$

Then $< Y_J^2 >$ moments in the cross section are $(k_{3x}/k_{3z})^2$.

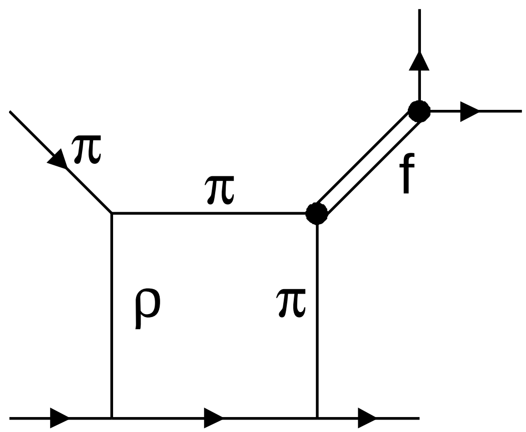
However the contribution to $< Y_J^0 >$ could be rather large already at small t.

The exchange which interferes with pion exchange is either π_2 or double meson exchange.

$$A^{(\pi_2)}_{\pi p\to\pi\pi n} = A_{\alpha\beta}(\pi\pi_2\to\pi\pi)\varepsilon^{(a)-}_{\alpha\beta} R_{\pi_2}(s_{\pi N},q^2)\varepsilon^{(a)+}_{\alpha'\beta'} X^{(2)}_{\alpha'\beta'}(p_2)\left(\varphi^+_n(\vec\sigma\vec p_\perp)\varphi_p\right) g^{(\pi_2)}_{pn}\,.$$

However leading contribution is taken into account by π-trajectory exchange.

Double exchange:



Interferes with π exchange and a_1 exchange but should be small at large —t—.

K-matrix for S and D-waves.

In the K-matrix form the unitarity condition is satisfied if:

$$A^J_{m\to n}(s) = \sum_i \hat{K}^J_{mi}(I - i\hat{\rho}^J(s)\hat{K}^J)^{-1}_{in}$$

where $\hat{\rho}$ is the diagonal matrix with elements:

$$\rho^J_{ii}(s) = \frac{2\sqrt{-k^2_{i\perp}}}{\sqrt{s}}(-k^2_{i\perp})^J \,.$$

In the present work we parameterized the elements of the K-matrix as follows:

$$K^J_{mn} = \sum_\alpha \frac{1}{B_J(k^2_{m\perp}, r_\alpha)}\left(\frac{g^{\alpha(J)}_m g^{\alpha(J)}_n}{M^2_\alpha - s}\right)\frac{1}{B_J(k^2_{n\perp}, r_\alpha)} + \frac{f^{(J)}_{mn}}{B_J(k^2_{m\perp}, r_0)B_J(k^2_{n\perp}, r_0)}$$

and P-vector as

$$P^J_m = \sum_\alpha \frac{1}{B_J(k^2_{m\perp}, r_\alpha)}\left(\frac{\Lambda_{\alpha(J)} g^{\alpha(J)}_n}{M^2_\alpha - s}\right)\frac{1}{B_J(k^2_{n\perp}, r_\alpha)} + \frac{F^{(J)}_n}{B_J(k^2_{m\perp}, r_0)B_J(k^2_{n\perp}, r_0)}$$

S-wave K-matrix: 5 poles 5 channels: $\pi\pi$ $K\bar{K}$**,** $\eta\eta$ $\eta\eta'$ **and** 4π
D-wave K-matrix 4 poles and 5 channels: $\pi\pi$**,** $K\bar{K}$**,** $\eta\eta$**,** 4π **and** $\omega\omega$**.**

Under 2 particle threshold we used:

1) analytical continuation of the phase volume

2)subtracted dispersion integral:

$$\rho(s<4m^2) = (s-4m^2)\int\limits_{4m^2}^{\infty} \frac{ds'}{\pi} \frac{\rho(s')}{B^2(k^2,r^2)(s'-4m^2)(s'-s)}$$

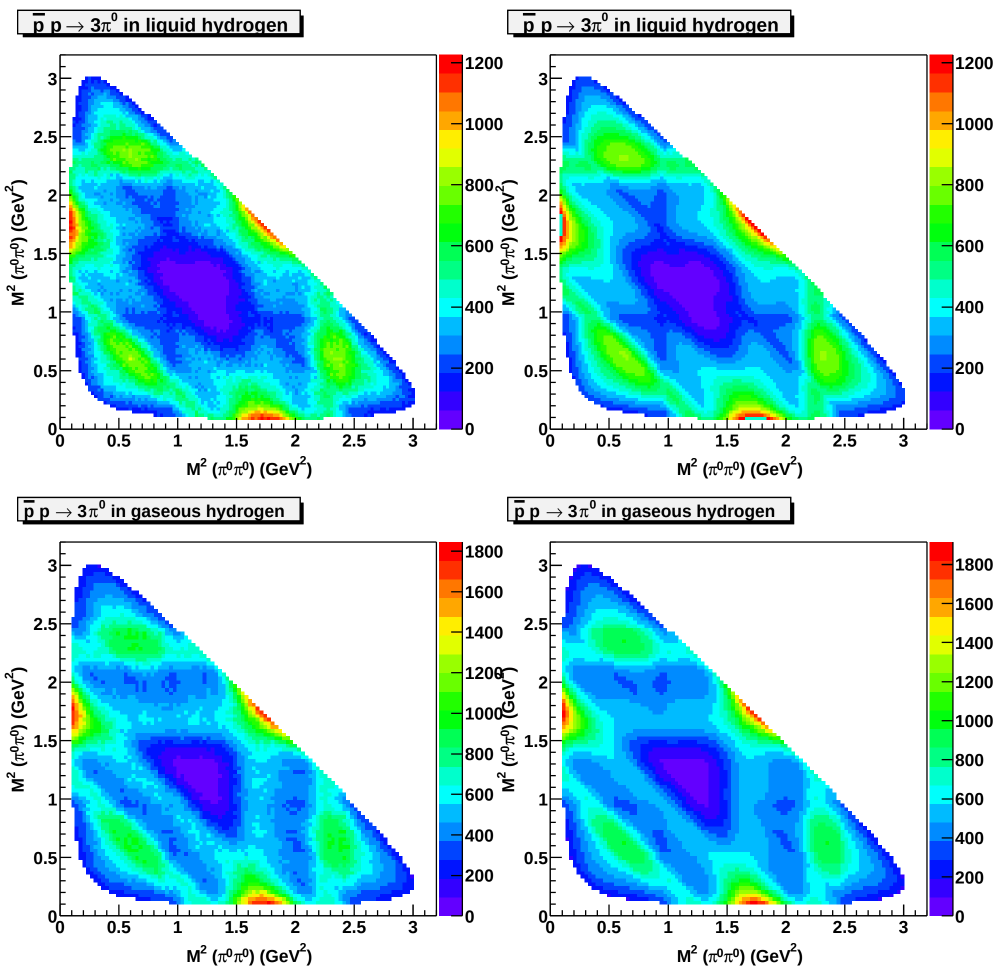
Masses of the S-wave K-matrix poles:

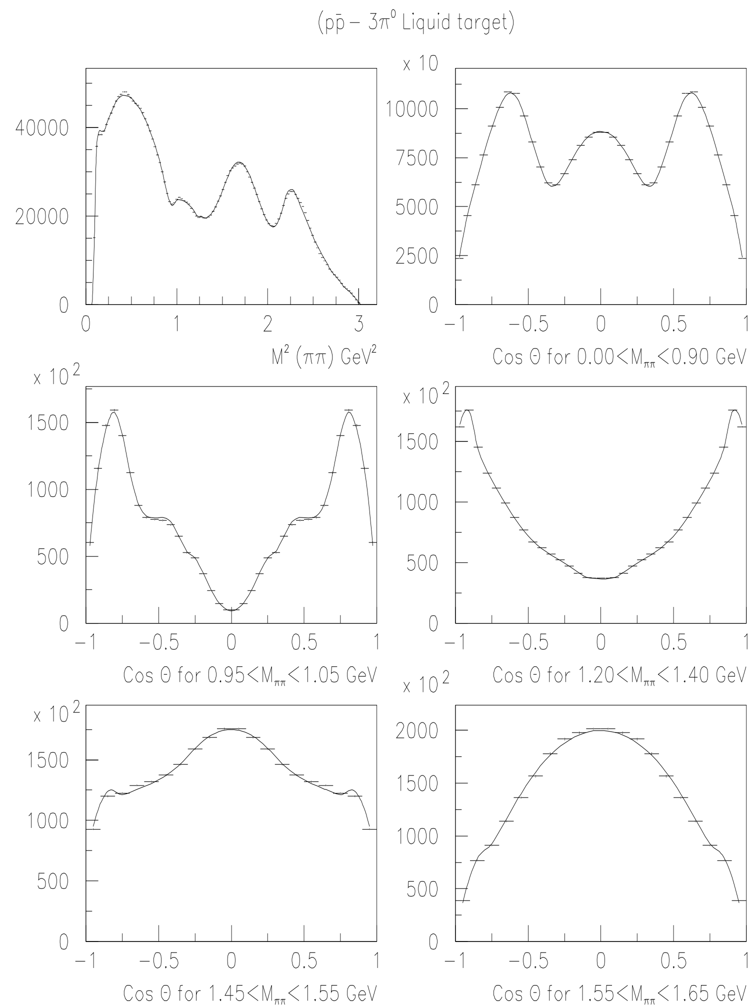
650, 1210 ($\pi\pi, K\bar{K}$**), 1275 (**$\pi\pi, 4\pi$**), 1650(**$\pi\pi, K\bar{K}, 4\pi$**), 1800** $(K\bar{K}, 4\pi)$ **MeV.**

Masses of the D-wave K-matrix poles:

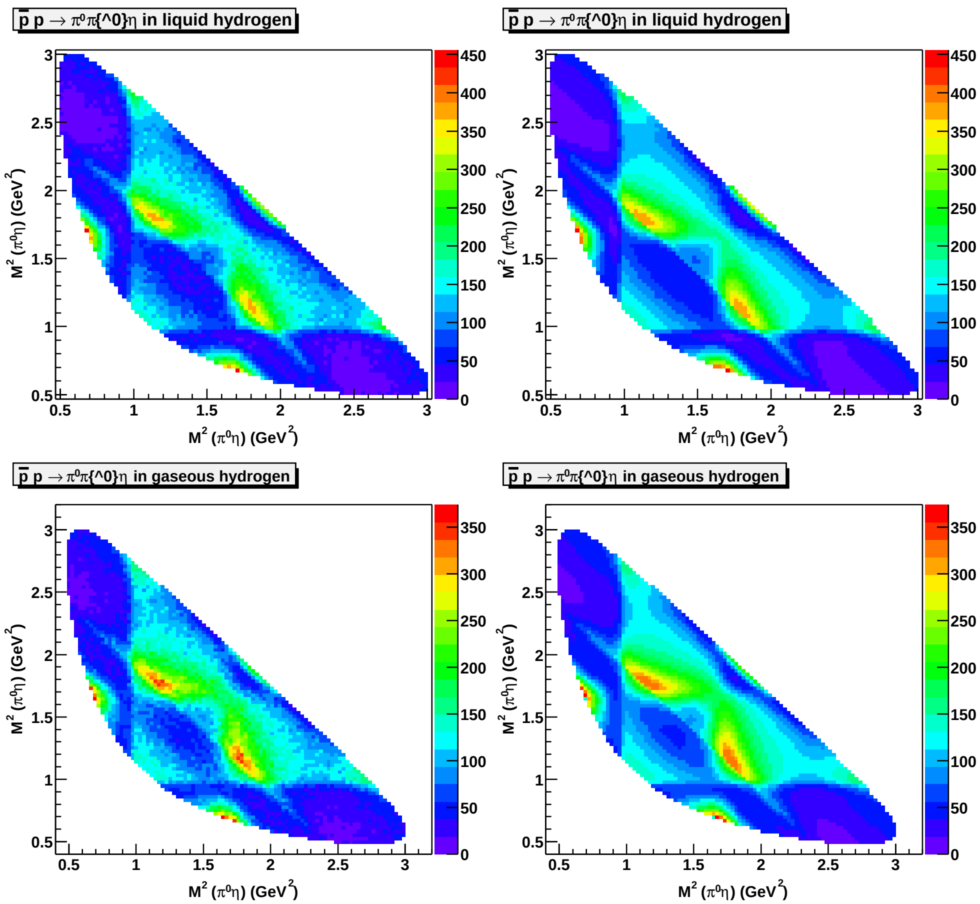
1280 ($\pi\pi$**), 1510 (**$K\bar{K}$**), 1560 (**$\omega\omega, 4\pi$**), 1920 (**$\pi\pi, \omega\omega, 4\pi$**)**

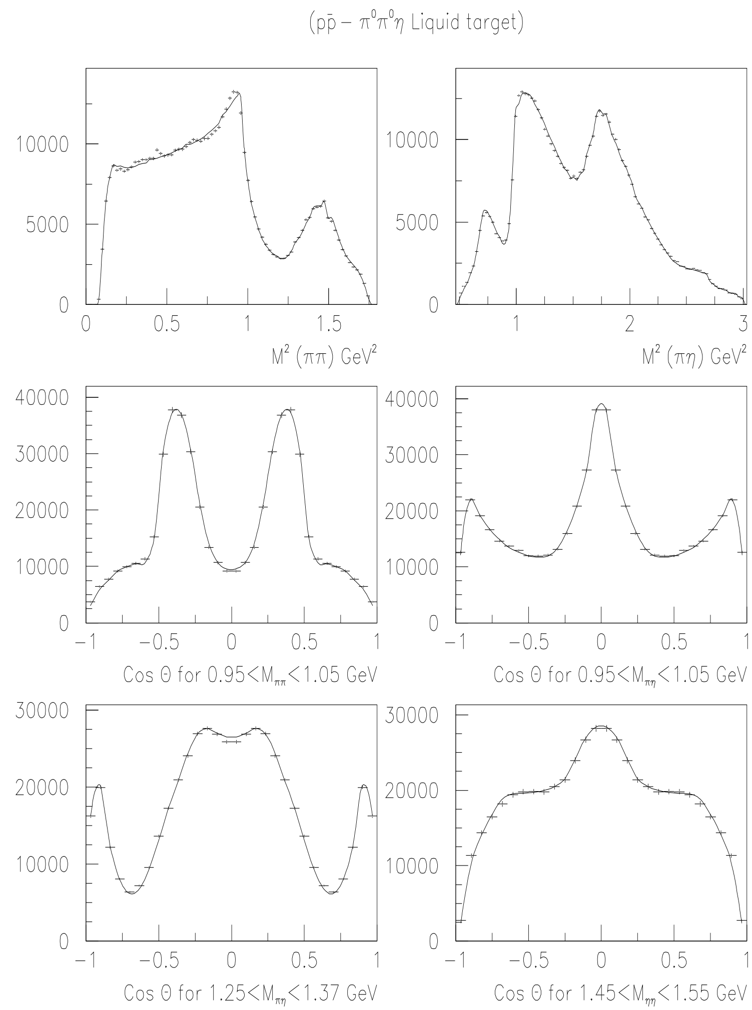
The description of $p\bar{p} \to 3\pi^0$ CB-LEAR data



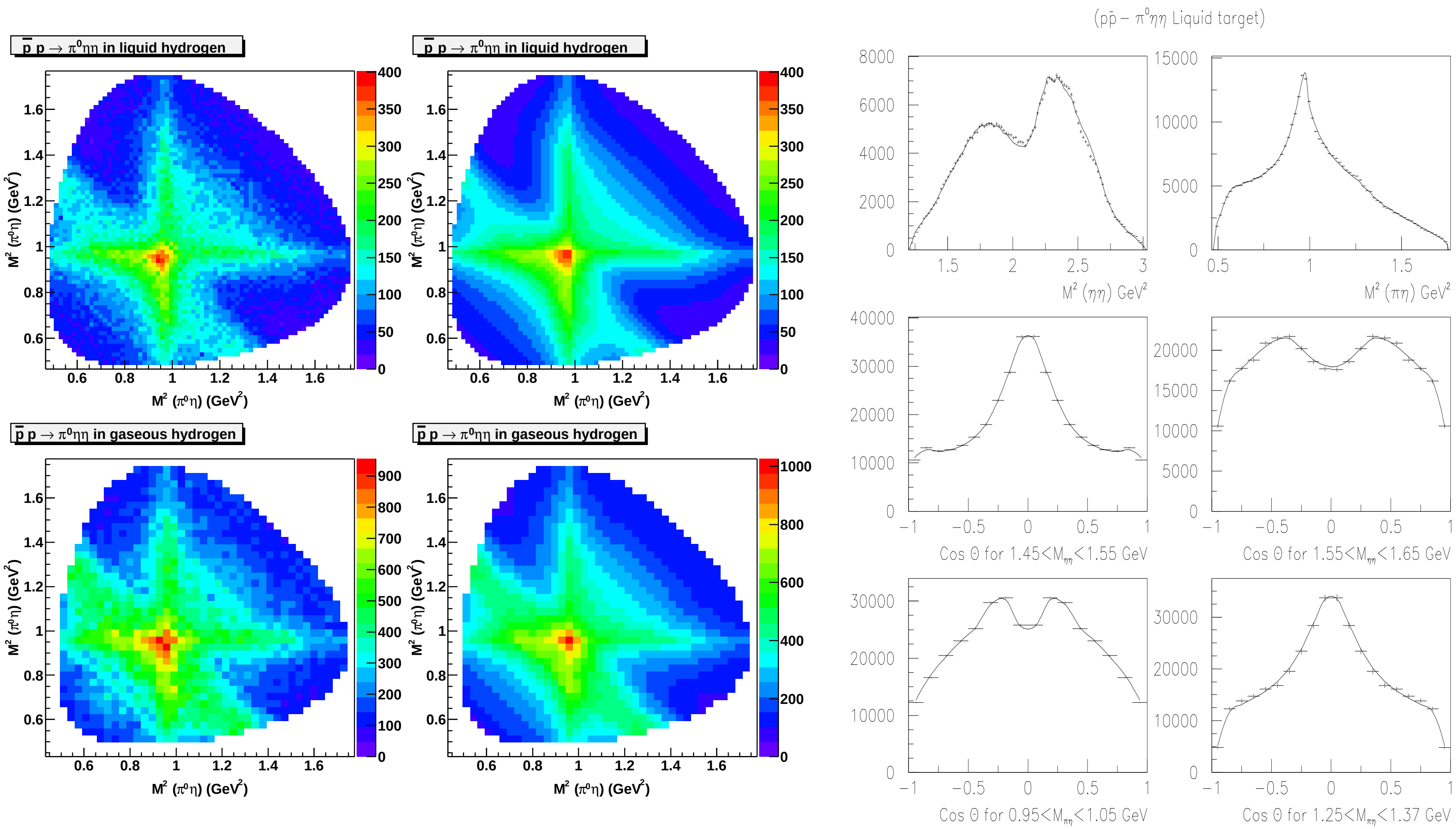


The description of $p\bar{p} \to \pi^0\pi^0\eta$ CB-LEAR data

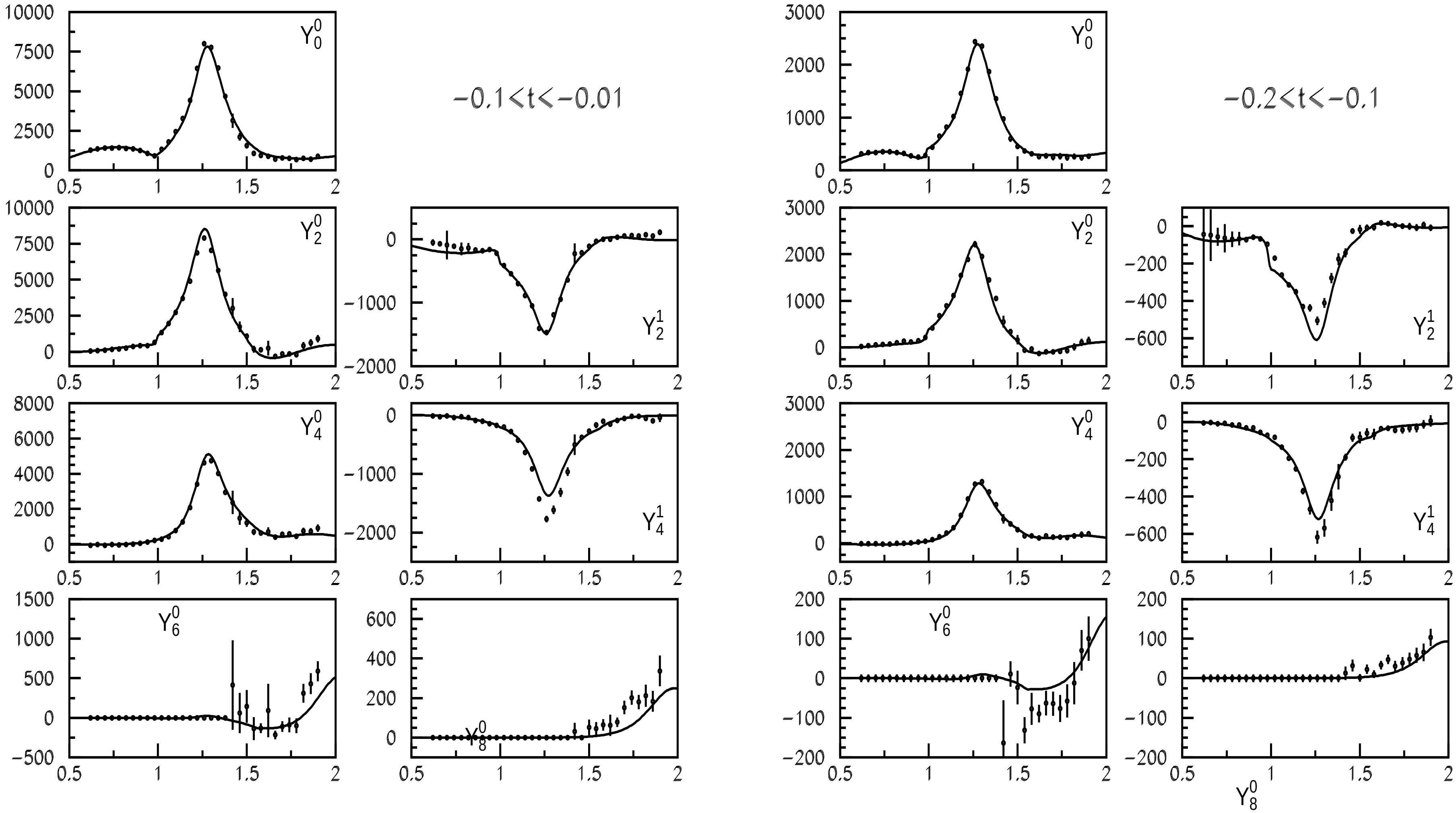




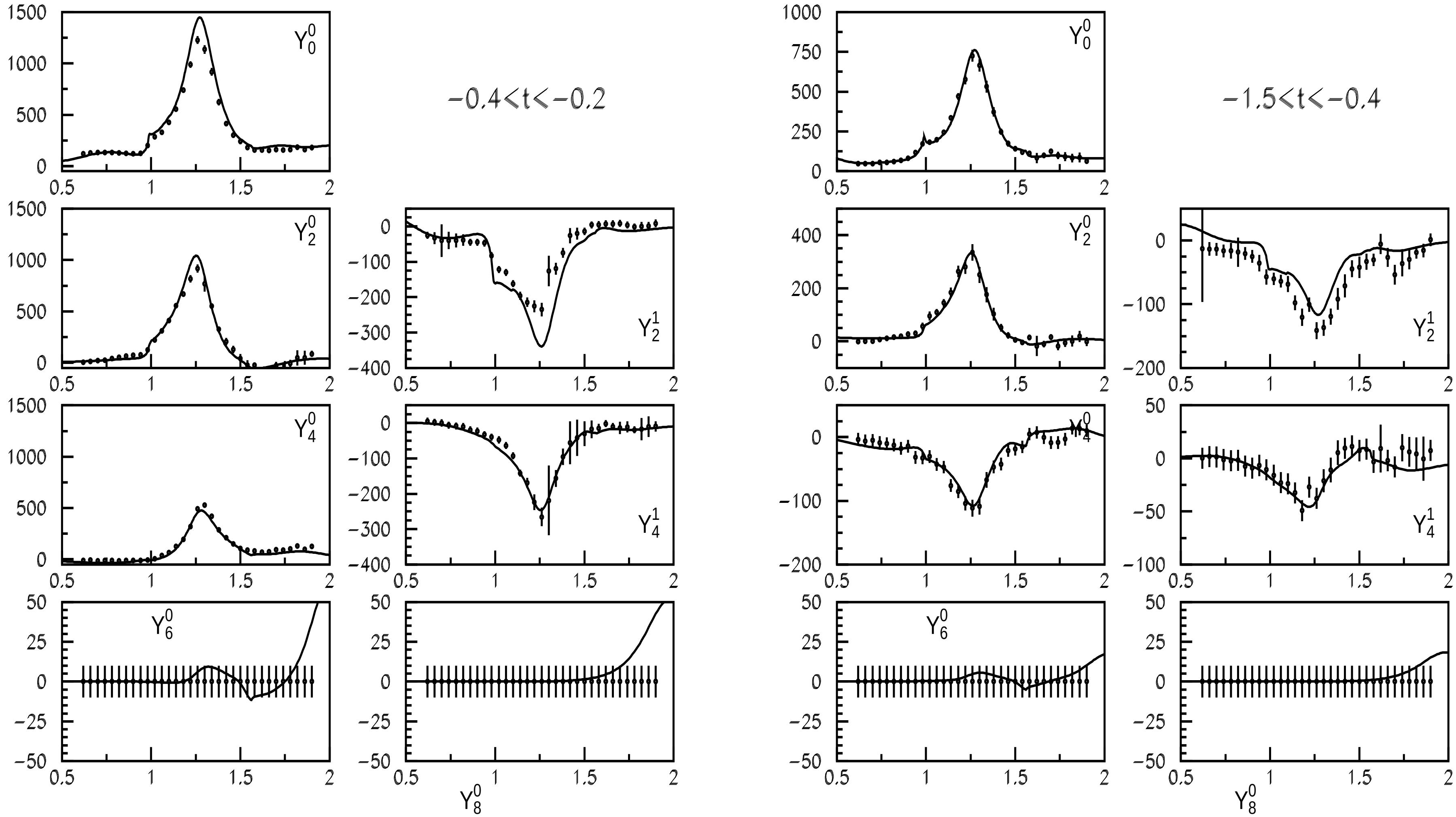
The description of $p\bar{p} \longrightarrow \pi^0\eta\eta$ CB-LEAR data



The description of $\pi N \to \pi^0\pi^0 N$ (E852)



The description of $\pi N \to \pi^0\pi^0 N$ (E852)



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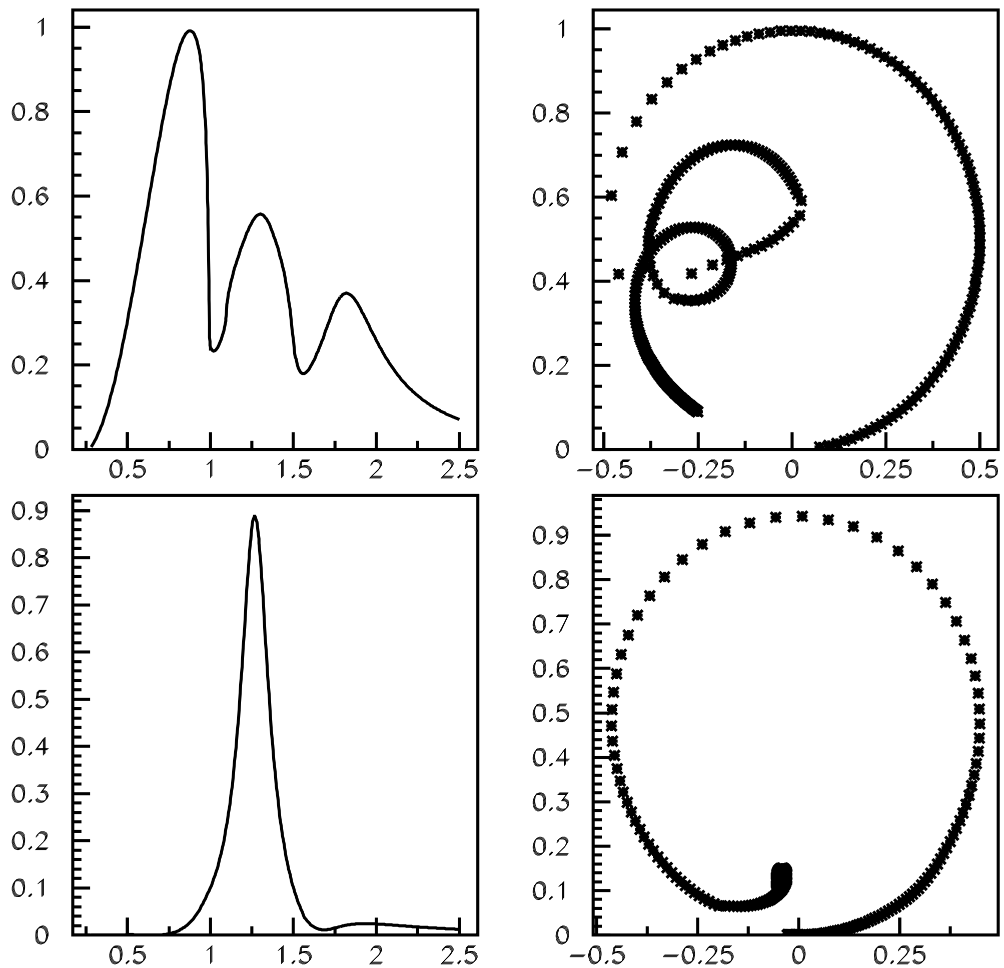
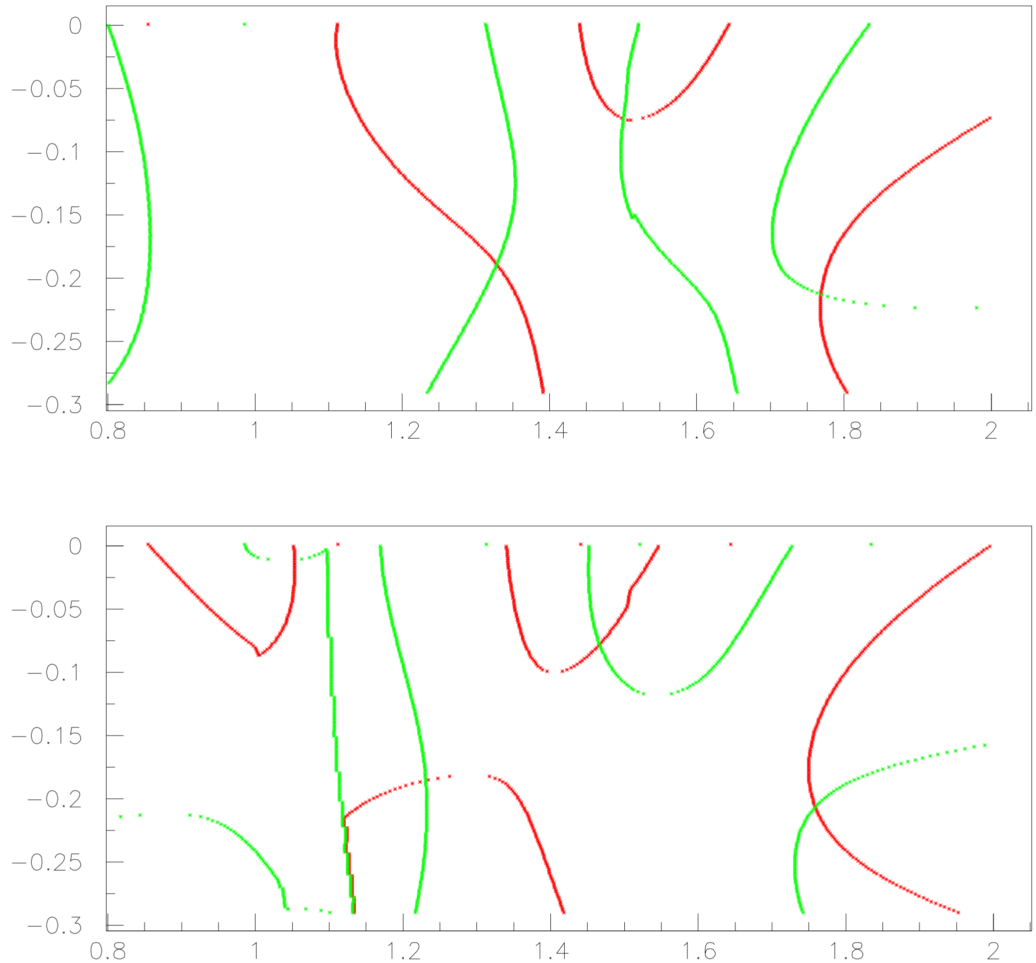
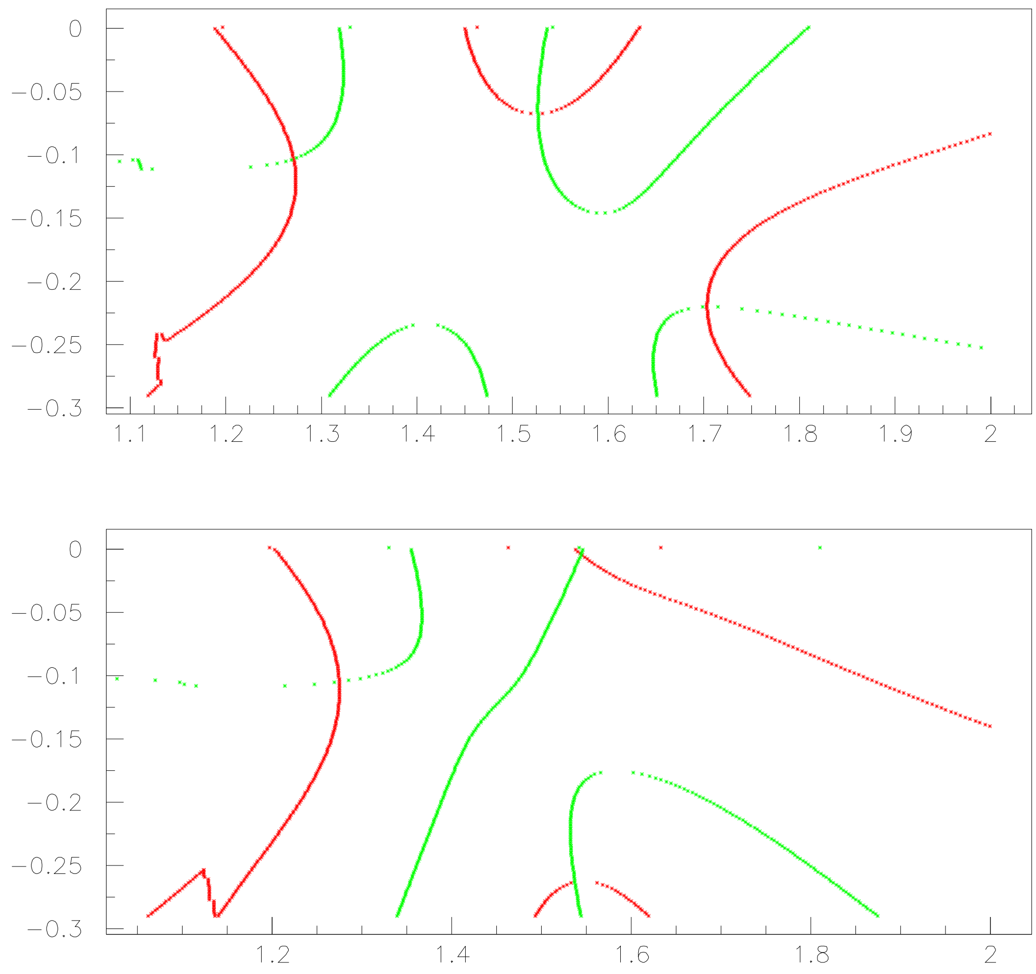


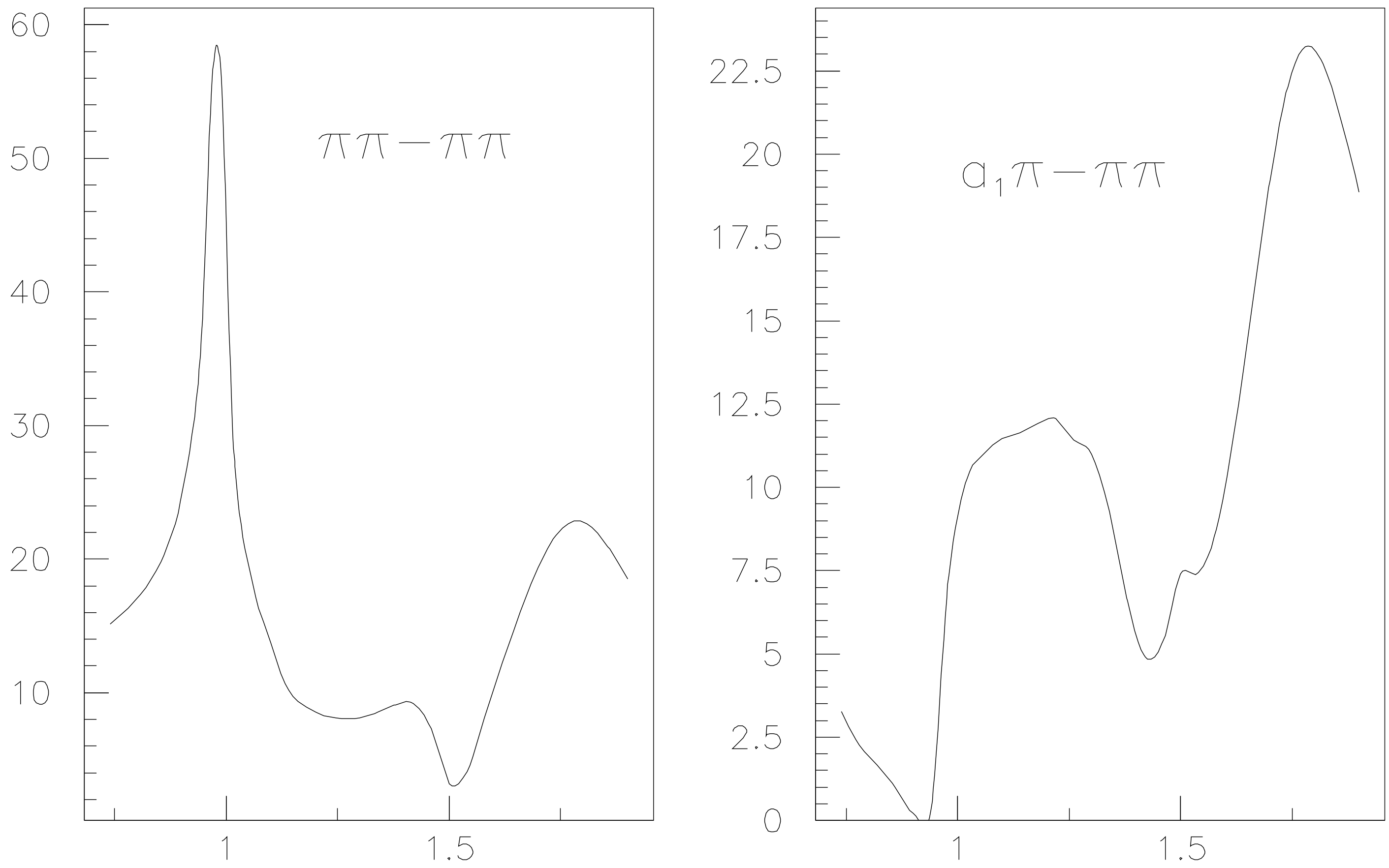
The S-wave pole position in the complex S-plane



The D-wave pole position in the complex S-plane



S-wave at large $|t|$ due to π and a_1 exchanges



CONCLUSION

1. **The combined K-matrix analysis of proton-antiproton annihilation into three mesons and the $\pi N \to \pi^0\pi^0 N$ data at different t-intervals shows a good compatibility.**
2. **The $f_0(1370)$, $f_2(1560)$ and $f_2(1980)$ are needed for the data description.**
3. **At large $|t|$ the signal from $f_0(1370)$ due to a_1-exchange is clearly seen. But systematic check of all possibilities is not done yet.**
4. **The πN interaction provide a very important information about t-channel exchanges. It is very pity that it is not explored systematically.**