KLOE Dalitz plot analysis of $\eta \rightarrow 3\pi$ decays

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for the KLOE Collaboration
• $e^+ e^- \text{ collider } @ \sqrt{s} = M_\phi = 1019.4 \text{ MeV}$
• Separate $e^+$, $e^-$ rings
• Crossing angle: 12.5 mrad ($p_x(\phi) \approx 13 \text{ MeV}$)
• Injection during data-taking
• $L_{\text{peak}} = 1.5 \times 10^{32} \text{ cm}^{-2}\text{s}^{-1}$
• $L_{\text{int}} (\text{KLOE}) = 2.7 \text{ fb}^{-1}$

**STATUS:**
March 2006: *end of KLOE data taking*

2500 pb$^{-1}$ on-peak $\Rightarrow 8 \times 10^9 \phi$ decays
200 pb$^{-1}$ off-peak (energy scan + 1 GeV run)
η / η' production rate at DAFNE

<table>
<thead>
<tr>
<th>Process</th>
<th>Cross section (µb)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$e^+e^- \rightarrow \phi$</td>
<td>$\approx 3$</td>
</tr>
<tr>
<td>$e^+e^- \rightarrow e^+e^-(\gamma)$</td>
<td>$6.2 \ (\theta &gt; 20^\circ)$</td>
</tr>
<tr>
<td>$e^+e^- \rightarrow \mu^+\mu^-(\gamma)$</td>
<td>0.1</td>
</tr>
<tr>
<td>$e^+e^- \rightarrow \pi^+\pi^-(\gamma)$</td>
<td>0.05</td>
</tr>
</tbody>
</table>

Collected:
- 100 millions η decays
- 500 thousands η’ decays
The KLOE detector

- $\sigma_{E/E} = 5.7\% /\sqrt{E(\text{GeV})}$
- $\sigma_t = 54 \text{ ps} /\sqrt{E(\text{GeV})} \oplus 100 \text{ ps}$
- $\sigma_{vtx(L\to\pi^0\pi^0)} \sim 1.5 \text{ cm}$

- $\sigma_{p/p} = 0.4\%$ (tracks with $\theta > 45^\circ$)
- $\sigma_{x/y} = 150 \mu\text{m}$, $\sigma_z = 2 \text{ mm}$
- $\sigma_{vtx} \sim 3 \text{ mm}$
- $\sigma(M_{Ks \to \pi^+\pi^-}) \sim 1 \text{ MeV}$
The decay $\eta \to 3\pi$ occurs primarily on account of the d-u quark mass differences and the result arising from lowest order chiral perturbation theory is well known:

$$A(s,t,u) = \frac{1}{Q^2} \frac{m_K^2}{m_\pi^2} (m_\pi^2 - m_K^2) \frac{M(s,t,u)}{3\sqrt{3} F_\pi^2}$$

With:

$$Q^2 \equiv \frac{m_s^2 - \hat{m}^2}{m_d^2 - m_u^2}$$

And, at l.o.

$$M(s,t,u) = \frac{3s - 4m_\pi^2}{m_\eta^2 - m_\pi^2}$$

A good understanding of $M(s,t,u)$ can in principle lead to a very accurate determination of $Q$:

$$\Gamma(\eta \to 3\pi) \propto |A|^2 \propto Q^{-4}$$

Need to check the description of the dynamics for both $\pi^+\pi^-\pi^0$ and $3\pi^0$ final states!
At KLOE $\eta$ is produced in the process $\phi \rightarrow \eta \gamma$. The final state for $\eta \rightarrow \pi^+ \pi^- \pi^0$ is thus $\pi^+ \pi^- \gamma \gamma \gamma$, and the final state for $\eta \rightarrow \pi^0 \pi^0 \pi^0$ is $7\gamma$, both with almost no physical background.

$\eta \rightarrow 3\pi$ at KLOE

**$\pi^+ \pi^- \pi^0$ selection:**
- 2 track vertex + 3 $\gamma$ candidates
- Kinematic fit
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**$\pi^+ \pi^- \pi^0$ selection:**
- 2 track vertex+3 $\gamma$ candidates
- Kinematic fit

**$\pi^0 \pi^0 \pi^0$ selection:**
- 7 $\gamma$ candidates
- Kinematic fit
$\pi^+\pi^-\pi^0$ : resolution and efficiency

\[ X = \sqrt{3} \frac{T_x - T_z}{Q_\eta} = \frac{\sqrt{3}}{2M_\eta Q_\eta} (u - t) \]

\[ Y = \frac{3T_0}{Q_\eta} - 1 = \frac{3}{2m_\eta Q_\eta} \left\{ (m_\eta - m_{\pi^0})^2 - s \right\} - 1 \]

Efficiency almost flat, and \( \approx 35\% \)

"core" \( \sigma_X = 0.018 \)

"core" \( \sigma_Y = 0.020 \)
Signal

\[ N_{\text{obs}} = (1.377 \pm 0.001) \times 10^6 \]

\[ \text{B/S} \approx 0.3\% \]
Results

$$|A(X,Y)|^2 = 1 + aY + bY^2 + cX + dX^2 + eXY + fY^3$$

\[
\begin{align*}
a &= -1.090 \pm 0.005 \text{(stat)}^{+0.008}_{-0.019} \text{(syst)} \\
b &= 0.124 \pm 0.006 \text{(stat)} \pm 0.010 \text{(syst)} \\
d &= 0.057 \pm 0.006 \text{(stat)}^{+0.007}_{-0.016} \text{(syst)} \\
f &= 0.14 \pm 0.01 \text{(stat)} \pm 0.02 \text{(syst)} \\
c &= 0.002 \pm 0.003 \text{(stat)} \pm 0.001 \text{(syst)} \\
e &= -0.006 \pm 0.007 \text{(stat)}^{+0.005}_{-0.003} \text{(syst)}
\end{align*}
\]

The fit has \(P(\chi^2) = 75\%\) for 149 dof
Asymmetries

\[ A_{L,R} = \frac{N_+ - N_-}{N_+ + N_-} \]
\[ A_Q = \frac{N_1 + N_3 - N_2 - N_4}{N_1 + N_2 + N_3 + N_4} \]
\[ A_S = \frac{N_1 + N_3 + N_5 - N_2 - N_4 - N_6}{N_1 + N_2 + N_3 + N_4 + N_5 - N_6} \]

\[ A_{PDG} = (0.09 \pm 0.17) \cdot 10^{-2} \]
\[ A_{PDG} = (-0.17 \pm 0.17) \cdot 10^{-2} \]
\[ A_{PDG} = (0.18 \pm 0.16) \cdot 10^{-2} \]
Asymmetries

\[ A_{LR} = (0.09 \pm 0.10^{+0.09}_{-0.14}) \cdot 10^{-2} \]
\[ A_Q = (-0.5 \pm 0.10^{+0.03}_{-0.05}) \cdot 10^{-2} \]
\[ A_S = (0.09 \pm 0.10^{+0.08}_{-0.13}) \cdot 10^{-2} \]

In agreement with the null value of the c and e parameters in our fit, we find no evidence for C violation in the \( \eta \rightarrow \pi^+\pi^-\pi^0 \) decay.
The Dalitz plot density for $\eta \rightarrow \pi^0 \pi^0 \pi^0$ ($|A|^2$) is specified by a single quadratic slope $\alpha$:

$$|A|^2 \propto 1 + 2\alpha z$$

with:

$$z = \frac{2}{3} \sum_{i=1}^{3} \left( \frac{3E_i - m_\eta}{m_\eta - 3m_{\pi^0}} \right)^2 = \frac{\rho^2}{\rho_{\text{max}}^2}$$

$Z \in [0, 1]$,

$E_i$ = Energy of the $i$-th pion in the $\eta$ rest frame.

$\rho$ = Distance to the center of Dalitz plot.

$\rho_{\text{max}}$ = Maximum value of $\rho$. 
Using this discriminant variable one can obtain samples with different purity-efficiency.

**Purity:** Fraction of events with all photons correctly matched to $\pi^0$'s

To check systematics we used samples with purity ranging from 75% to 97% corresponding to datasets going from 1.4 MeVts down to 0.2 MeVts.
The fit procedure is quite complex: it is unbinned and includes a correction for the fraction of mis-id pions and the full convolution with the resolution function. We have tested it on MC by generating samples with different values of the parameter and looking at the result of our fit for these samples.
The edge of the flat part of the phase space depends on the value of the eta mass. What if its value on data is larger (e.g. by 0.5 MeV) than the nominal one?
Importance of $M_\eta$

The edge of the flat part of the phase space depends on the value of the eta mass. What if its value on data is larger (e.g. by 0.5 MeV) than the nominal one?

![Graph showing $Z_{\text{data}}/Z_{\text{MC}}$ as a function of $z$]
We give the preliminary result for the slope parameter $\alpha$ in correspondence of a sample of about 650 Kevts with 92% purity. Fitting in the range [0,0.7] we get:

$$\alpha = -0.027 \pm 0.004_{\text{stat}} \pm 0.006_{\text{syst}}$$

This results superseeds our previous preliminary result (presented at Hadron '05) which used $M_{\eta} = 547.3$ in the simulation and was thus systematically biased by the kinematic effect shown before.
Conclusions

• New accurate results have been produced for both Dalitz plot slopes and asymmetries in $\eta \rightarrow \pi^+\pi^-\pi^0$

• An unexpectedly large effect of the discrepancy between the PDG $\eta$ mass used in our MC simulation and the “true” $\eta$ mass caused a systematic bias in our preliminary result for the $\eta \rightarrow 3\pi^0$ slope. A new result is presented here. It is compatible with Crystal Ball result and with chiral unitary calculations.

• The experimental scenario for $\eta \rightarrow 3\pi$ dynamics is now clear and solid.
Kinematic fit

As a general tool we employ a kinematic fit imposing energy momentum conservation: this allows dramatic improvement on photon energy resolution.
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Comparison Data-MC: $\pi^+$

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Comparison Data-MC: $\pi^-$
Fit quality

\[ |A(X,Y)|^2 = 1 + aY + bY^2 + cX + dX^2 + eXY + fY^3 \]

The fit has \( P(\chi^2) = 75\% \) for 149 dof
Comparison Data-MC: X & Y
We have tested the effect of radiative corrections to the Dalitz plot density.

The ratio of the two plots has been fitted with the usual expansion: corrections to parameters are compatible with zero.
$\eta \rightarrow \pi^+\pi^-\pi^0(\gamma)$

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The ratio of the two plots has been fitted with the usual expansion: corrections to parameters are compatible with zero.
Dashen theorem violation?

Using preliminary KLOE results

B.V. Martemyanov and V.S. Sopov (hep-ph/0502023) have extracted:

\[ |A(X,Y)|^2 = 1 - 1.072Y + 0.117Y^2 + 0.047X^2 + 0.13Y^3 \]

\[ Q = 22.8 \pm 0.4 \text{ against } Q_{\text{Dashen}} = 24.2 \]

Remember

\[ Q^2 \equiv \frac{m_s^2 - \hat{m}^2}{m_d^2 - m_u^2} \]

\[ Q^2_{\text{Dashen}} = \frac{m_K^2}{m_{\pi}^2} \frac{m_K^2 - m_{\pi}^2}{m_{K^0}^2 - m_{K^+}^2 + m_{\pi^+}^2 - m_{\pi^0}^2} \]
Check on cubic term

Fitted function is actually:

\[
\frac{\varepsilon_{\text{Real}}(X,Y)}{\varepsilon_{\text{MC}}(X,Y)} \cdot |A(X,Y)|^2
\]

Assume:

\[
\frac{\varepsilon_{\text{Real}}(X,Y)}{\varepsilon_{\text{MC}}(X,Y)} \approx 1 + \alpha Y + \beta X^2
\]

No cubic dependence in \(|A(X,Y)|^2\)

\[
|A(X,Y)|^2 = 1 + \alpha Y + bY^2 + dX^2 \quad \text{with} \quad \alpha \approx -1.
\]

Can evaluate \(\alpha\) such as to mimic the cubic term.
Check on cubic term (2)

Cubic term is not an artifact …
### Dalitz expansion: theory vs experiment

<table>
<thead>
<tr>
<th>Calculation</th>
<th>a</th>
<th>b</th>
<th>d</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tree</td>
<td>-1.00</td>
<td>0.25</td>
<td>0.00</td>
</tr>
<tr>
<td>One-loop[1]</td>
<td>-1.33</td>
<td>0.42</td>
<td>0.08</td>
</tr>
<tr>
<td>Dispersive[2]</td>
<td>-1.16</td>
<td>0.26</td>
<td>0.10</td>
</tr>
<tr>
<td>Tree dispersive</td>
<td>-1.10</td>
<td>0.31</td>
<td>0.001</td>
</tr>
<tr>
<td>Absolute dispersive</td>
<td>-1.21</td>
<td>0.33</td>
<td>0.04</td>
</tr>
</tbody>
</table>


<table>
<thead>
<tr>
<th>Measurement</th>
<th>N_n</th>
<th>a</th>
<th>b</th>
<th>d</th>
</tr>
</thead>
<tbody>
<tr>
<td>Layter</td>
<td>80884</td>
<td>-1.08 ± 0.14</td>
<td>0.034 ± 0.027</td>
<td>0.046 ± 0.031</td>
</tr>
<tr>
<td>Gormley</td>
<td>30000</td>
<td>-1.17 ± 0.02</td>
<td>0.21 ± 0.03</td>
<td>0.06 ± 0.04</td>
</tr>
<tr>
<td>Crystal Barrel</td>
<td>1077</td>
<td>-0.94 ± 0.15</td>
<td>0.11 ± 0.27</td>
<td></td>
</tr>
<tr>
<td>Crystal Barrel</td>
<td>3230</td>
<td>-1.22 ± 0.07</td>
<td>0.22 ± 0.11</td>
<td>0.06 fixed</td>
</tr>
</tbody>
</table>
### Dalitz expansion: theory vs experiment

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<thead>
<tr>
<th>Calculation</th>
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</tr>
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<tbody>
<tr>
<td>Tree</td>
<td>0.00</td>
</tr>
<tr>
<td>One-loop[1]</td>
<td>0.0015</td>
</tr>
<tr>
<td>Dispersive[2]</td>
<td>-0.007 - 0.014</td>
</tr>
<tr>
<td>Tree dispersive</td>
<td>-0.006</td>
</tr>
<tr>
<td>Absolute dispersive</td>
<td>-0.007</td>
</tr>
</tbody>
</table>


<table>
<thead>
<tr>
<th>Experiment</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Alde (1984)</td>
<td>-0.022 ± 0.023</td>
</tr>
<tr>
<td>Crystal Barrel (1998)</td>
<td>-0.052 ± 0.020</td>
</tr>
<tr>
<td>Crystal Ball (2001)</td>
<td>-0.031 ± 0.004</td>
</tr>
</tbody>
</table>
Results (II)
The cuts used to select: $\eta \rightarrow \pi^0 \pi^0 \pi^0$ are:

- 7 and only 7 prompt neutral clusters with $21^\circ < \theta_\gamma < 159^\circ$ and $E_\gamma > 10$ MeV
- opening angle between each couple of photons $> 18^\circ$
- Kinematic Fit with no mass constraint
- $P(\chi^2) > 0.01$
- $320$ MeV $< E_{\gamma_{\text{rad}}} < 400$ MeV (after kin fit)

With these cuts the expected contribution from events other than the signal is $<0.5\%$
Comparison Data-MC
Resolution and efficiency

Phase space

Efficiency

Resolution

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Resolution and efficiency

Reconstructed efficiency distribution:

- Entries: 24532
- Mean: 0.4511
- RMS: 0.2882

Resolution distribution:

- Mean: 0.0367
- RMS: 0.1635
Resolution and efficiency

Reconstructed

Efficiency

Resolution

Entries 245233
Mean 0.4511
RMS 0.2882
Resolution and efficiency

Reconstructed

Efficiency

Resolution

$M\pi^0$
The center of Dalitz plot correspond to 3 pions with the same energy \( E_i = M_\eta/3 = 182.4 \text{ MeV} \). A good check of the MC performance in evaluating the energy resolution of \( \pi^0 \) comes from the distribution of \( E_{\pi^0} - E_i \) for \( z = 0 \).
Resolution (III)

A further check can be done comparing the energies of the two photons in the pion rest frame as function of pion energy

\[ E^*_{\gamma_1} - E^*_{\gamma_2} \]
A further check can be done comparing the energies of the two photons in the pion rest frame as function of pion energy.
Checks of purity on data

Purity on data compatible with the one expected from MC for all different samples analyzed
Linearity of DATA/MC ratio

High purity sample (used for the old preliminary result) -> no evident effect is visible
Linearity of DATA/MC ratio (2)

We observed a relevant dependence of \( \alpha \) on the fitting range for low purity (=high statistics) sample…
To understand the effect we used a toy MC to generate 1200000 events with different eta masses:

Sample 1 : M = 547.30 MeV
Sample 2 : M = 547.822 MeV

We observe that when the input mass value is used to build the z variable the phase space shape does not change. But if one uses M = 547.30 MeV to build the z variable for sample 2 big deviations are observed...

\[ z = \frac{2}{3} \sum_{i=1}^{3} \left( \frac{3E_i - m_\eta}{m_\eta - 3m_{\pi^0}} \right)^2 = \frac{\rho^2}{\rho_{\text{max}}^2} \]
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Linearity

If the effect is given by the eta mass, correcting for it now all samples should exhibit good linearity for the ratio \(\frac{\text{DATA}}{\text{MC}_{\text{rec}}}\) (phase space)

\[
\chi^2 \div \text{ndf } 43.35 \div 23
\]
Linearity

If the effect is given by the eta mass, correcting for it now all sample should exhibit good linearity for the ratio $\text{DATA/MC}_{\text{rec}}$ (phase space)
Effect of mass constraint in fit

Why did this effect not pop up in other experiments analyses? The reason is the effect is much less evident if you constrain in a kinematic fit the $\eta$ mass and then use the value you have constrained to build $z$…