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# Light scalar mesons at KLOE

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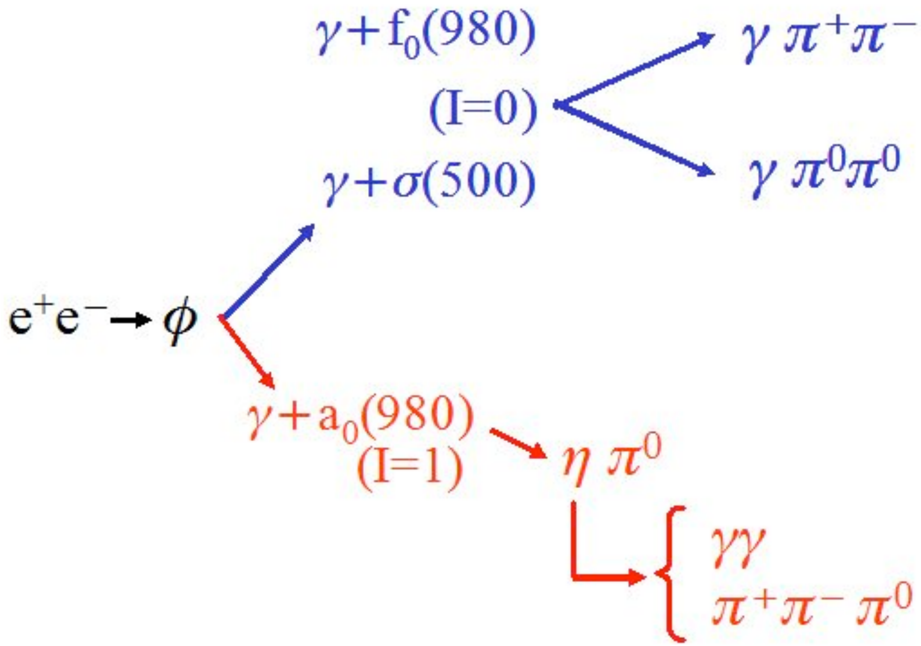
# Outline

- Motivations
- The spectrum from

  $f_0(980) \to \pi^+\pi^-$
- Dalitz plot from decay

  $f_0(980) \to \pi^0\pi^0$
- Analyses of the decay

  $a_0(980) \to \eta\pi^0$
- Conclusions and perspectives



# Motivations: open issues

- **Is σ(600) the lightest scalar meson?**
- **Do σ, $a_0$(980) and $f_0$(980) belong to the same $q\bar{q}$ $^3P_0$ nonet?**
- **If so, why is the mass spectrum inverted?**
- **If not, is their structure nonconventional?**

**Alternative hypotheses: $qq\bar{q}\bar{q}$ states (Jaffe, Achasov et al., Maiani et al.)**

**$K\bar{K}$ molecules (Weinstein-Isgur, Close et al., Kalashnikova et al.)**

**… … … … …**

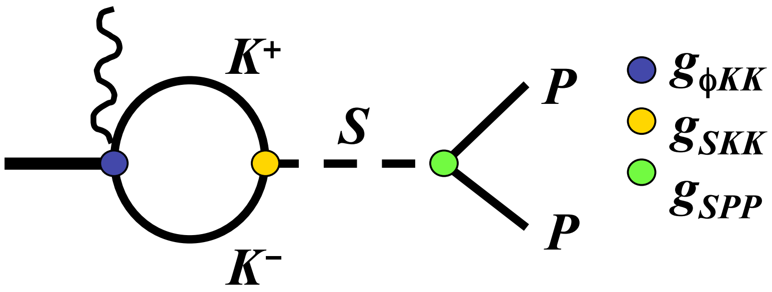
**ϕ-factory ⇒ study of $f_0$(980)[+σ(600)] and $a_0$(980)**

**from radiative ϕ decays**

# Models for the observed spectra

## 1) Kaon Loop [KL]

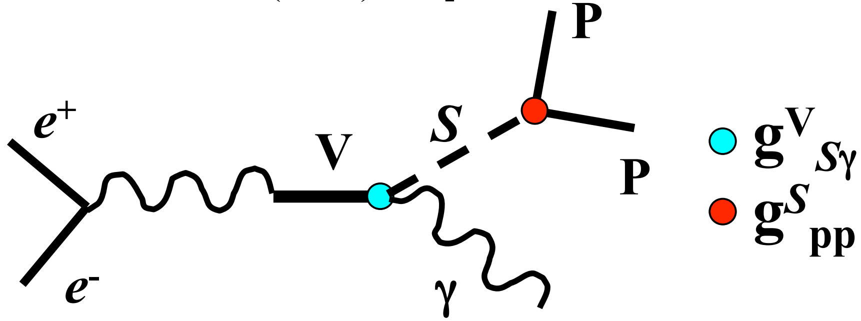
[N.N.Achasov, V.N.Ivanchenko, NPB315 (1989) 465]
[N.N.Achasov, V.V.Gubin, PRD 56 (1997) 4084]




dipole transition ($\propto E_\gamma^3$)
damped by loop function,
propagator accounts
for opening of KK channel

## 2) "No Structure" [NS]

[G.Isidori, L.Maiani, M.Nicolaci, S.Pacetti,
JHEP 0605 (2006) 049]




dipole transition ($\propto E_\gamma^3$)
damped by polynomial
terms, propagator
accounts for opening
of KK channel

$a_0$ ($f_0$) "couplings" to KK, $\eta\pi$ ($\pi\pi$)
and to $\phi \cong |ss\rangle$
is the $\sigma(600)$ needed to describe
the mass spectra?

# The $f_0(980) \to \pi^+\pi^-$ analysis

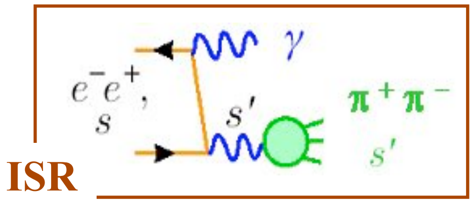
**published: PLB 634 (2006) 148**

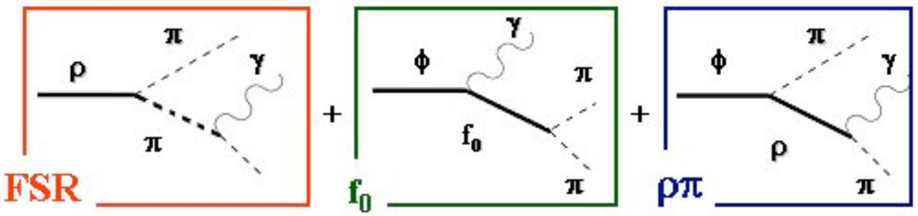
event selection:

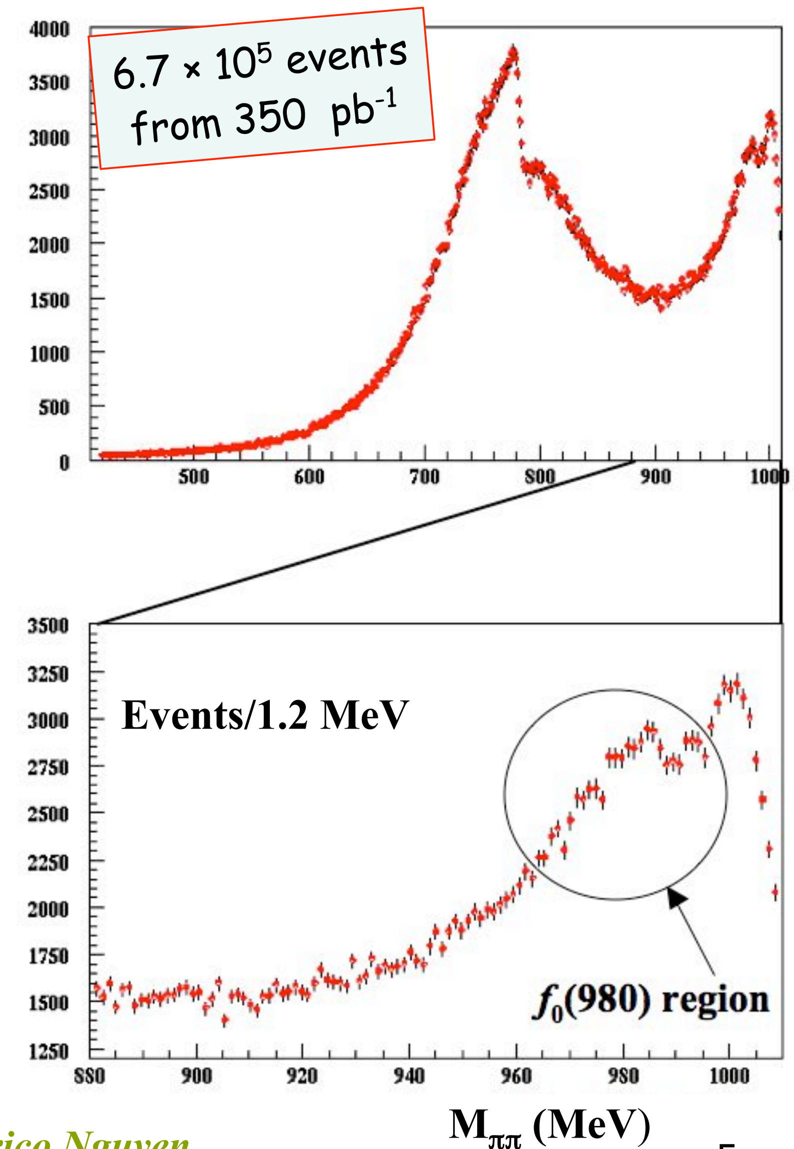
2 tracks with $\theta>45°$; missing momentum matching photon direction, with $\theta_\gamma>45°$;

$e^+e^-\gamma$ events rejection on calorimetric basis;

$\mu^+\mu^-\gamma$ and $\pi^+\pi^-\pi^0$ suppressed by means of kinematics;



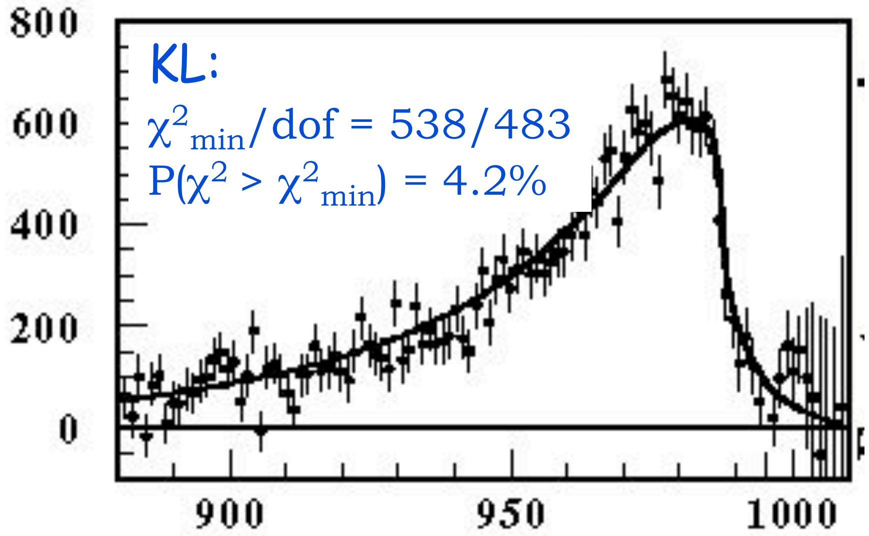



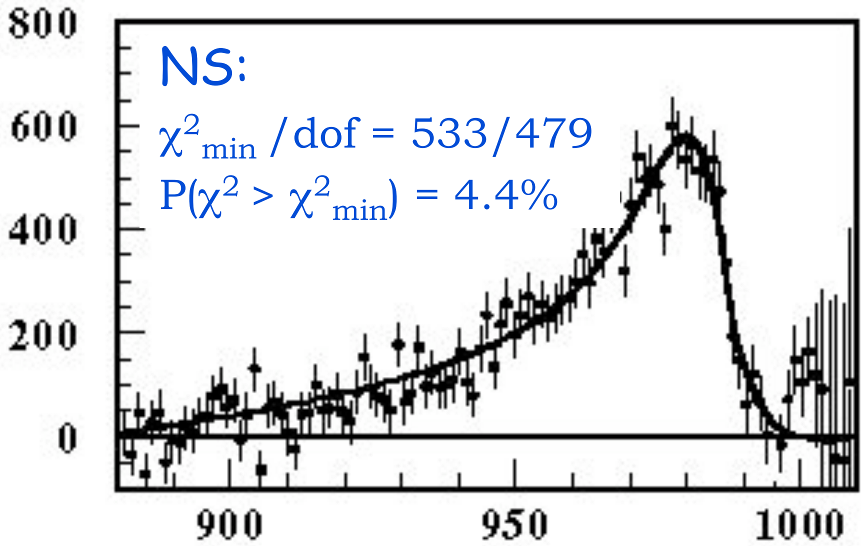



# Results for the $f_0(980) \to \pi^+\pi^-$ analysis

fitting function:

$$\frac{dN}{dm} = L_{\rm int}\epsilon(m)\left(\frac{d\sigma_{\rm ISR}}{dm} + \frac{d\sigma_{\rm FSR}}{dm} + \frac{d\sigma_{\rho\pi}}{dm} + \frac{d\sigma_{\rm scal}}{dm} \pm \frac{d\sigma^{\rm INT}_{\rm scal+FSR}}{dm}\right)$$




✓ peak at $M_{\pi\pi}$~ 980 MeV due to $\phi \to f_0(980)\gamma$, with negative interf. with FSR

✓ in both models **the $f_0(980)$ is strongly coupled to kaons and to the $\phi$**

✓ **introduction of the $\sigma(600)$ does not improve the fit**

both fits are satisfactory in describing the $f_0(980)$ structure

# The $f_0(980) \rightarrow \pi^0\pi^0$ analysis

published: EPJ C49 (2007) 473

a) 5 photons from the interaction point

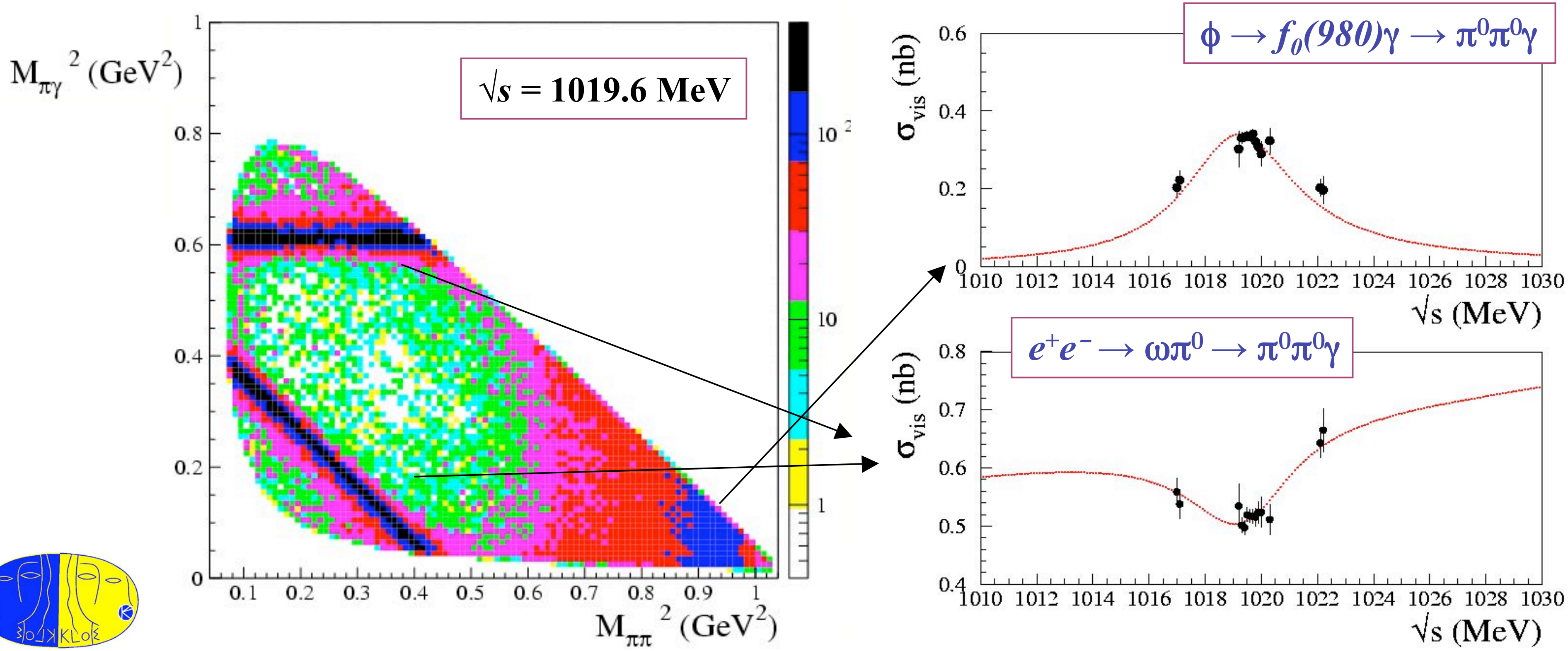
b) $1^{st}$ kinematic fit with 4-momentum conservation

c) $2^{nd}$ kinematic fit also with constraints on $\pi^0$ masses and cut $\chi^2$/dof < 5

**$M_{\pi\pi}$ vs. $M_{\pi\gamma}$** Dalitz Plot Fit,

bin widths: **10 MeV ($M_{\pi\pi}$) × 12.5 MeV ($M_{\pi\gamma}$)**

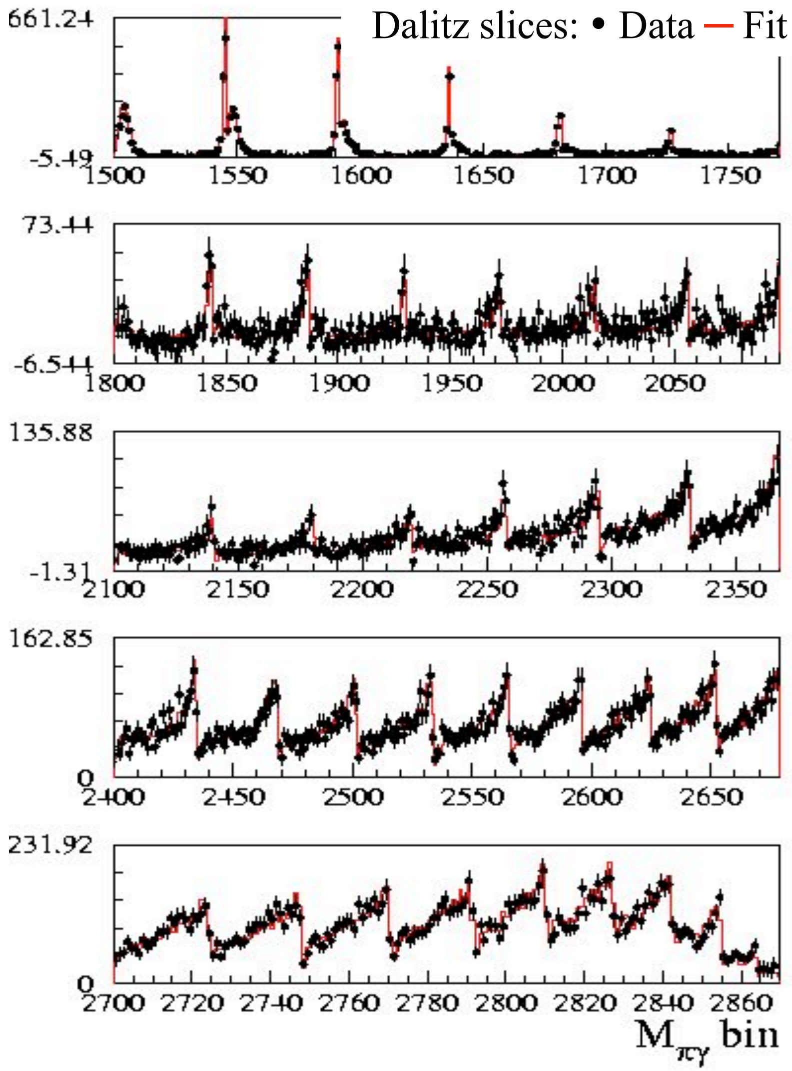
$4 \times 10^5$ events from 450 $pb^{-1}$



# Results for the $f_0(980) \to \pi^0\pi^0$ analysis

$\pi\pi$/KK scattering phases and $\sigma(600)$ fixed ($m \in [400,710]$ MeV, $\Gamma \in [240,490]$ MeV) from fits to published KLOE $\phi$ decays and to existing $\pi\pi$ scattering data [Achasov & Kiselev, PRD73(2006)054029] → 10 model variants

$$\frac{d\sigma^S}{dM_{\pi\pi}dM_{\pi\gamma}} + \frac{d\sigma^V}{dM_{\pi\pi}dM_{\pi\gamma}} \pm \frac{d\sigma^I}{dM_{\pi\pi}dM_{\pi\gamma}}$$




**KL:**

$\chi^2_{min}$/dof = 2754/2676

$P(\chi^2 > \chi^2_{min}) = 14.5\%$

**NS:**

$\chi^2_{min}$/dof = 2799/479

$P(\chi^2 > \chi^2_{min}) = 4.4\%$

KL (in particular):

- ✓the 6 fit variants with $P(\chi^2)>1\%$ are kept
- ✓best fit result → central value
- ✓maximum variation among fits → model dependence uncertainty

# Summary table and comparison

confidence intervals are given by exp. systematics,
except for KL in the $\pi^0\pi^0$ channel: model dependence

**KL fit results:**

**$\pi^0\pi^0$: $\sigma(600)$ [but with fixed values] needed to describe data,**

**$\pi^+\pi^-$: not sensitive to $\sigma(600)$**

**both channels: $f_0(980)$ strongly coupled to KK**

**NS fit results:**

**both channels: only $f_0(980)$ sufficient to describe data**

**$\pi^0\pi^0$ wrt $\pi^+\pi^-$: larger $g_{\phi f0\gamma}$ but weaker KK coupling**

**Kaon Loop**

| Parameter | $\pi^+\pi^-\gamma$ | $\pi^0\pi^0\gamma$ |
|---|---|---|
| $M_{f0}$ (MeV) | 980—987 | $976.8 \pm 0.3^{+0.9}_{-0.6}\ {}^{+10.1}$ |
| $g_{f0KK}$ (GeV) | 5.0—6.3 | $3.76 \pm 0.04^{+0.15}_{-0.08}\ {}^{+1.16}_{-0.48}$ |
| $g_{f0\pi\pi}$ (GeV) | 3.0—4.2 | $-1.43 \pm 0.01^{+0.01}_{-0.06}\ {}^{+0.03}_{-0.60}$ |
| $g^2_{f0KK} / g^2_{f0\pi\pi}$ | 2.2—2.8 | $6.9 \pm 0.1^{+0.2}_{-0.1}\ {}^{+0.3}_{-3.9}$ |

**No Structure**

| Parameter | $\pi^+\pi^-\gamma$ | $\pi^0\pi^0\gamma$ |
|---|---|---|
| $M_{f0}$ (MeV) | 973—981 | $984.7 \pm 0.4^{+2.4}_{-3.7}$ |
| $g_{f0KK}$ (GeV) | 1.6—2.3 | $0.40 \pm 0.04^{+0.62}_{-0.29}$ |
| $g_{f0\pi\pi}$ (GeV) | 0.9—1.1 | $1.31 \pm 0.01^{+0.09}_{-0.03}$ |
| $g^2_{f0KK} / g^2_{f0\pi\pi}$ | 2.6—4.4 | $0.09 \pm 0.02^{+0.44}_{-0.08}$ |
| $g_{\phi a0\gamma}$ (GeV$^{-1}$) | 1.2—2.0 | $2.61 \pm 0.02^{+0.31}_{-0.08}$ |

marginal agreement
between the 2 final states

# News for the $f_0(980) \rightarrow \pi^0 \pi^0$ analysis

2 changes in $\sigma$ couplings with respect to Achasov paper [PRD73(2006)054029]:

- $C_{f0\sigma} = -0.047$ GeV$^2$ (was $+0.047$ GeV$^2$) [private communication]
- $g_{\sigma\pi\pi} = 2.1$ GeV (was $-2.1$ GeV) [PRD 74 (2006) 059902(E)]

published result

$M_{f0} = (976.8 \pm 0.3_{fit} + 0.9/-0.6_{syst} + 10.1_{mod})$ MeV

$g_{f0K+K-} = (3.76 \pm 0.04_{fit} + 0.15/-0.08_{syst} + 1.16/-0.48_{mod})$ GeV

$g_{f0\pi+\pi-} = (-1.43 \pm 0.01_{fit} + 0.01/-0.06_{syst} + 0.03/-0.60_{mod})$ GeV

preliminary update

$M_{f0} = (984.7 \pm 1.9_{mod})$ MeV

$g_{f0K+K-} = (3.97 \pm 0.43_{mod})$ GeV

$g_{f0\pi+\pi-} = (-1.82 \pm 0.19_{mod})$ GeV

- ✓ 8 fit variants with $P(\chi^2) > 1\%$
- ✓ variants average → central value
- ✓ variants RMS → model dependence uncertainty

model uncertainty reduced of factor ~ 5 in $M_{f0}$ and of a factor ~ 3 in the couplings

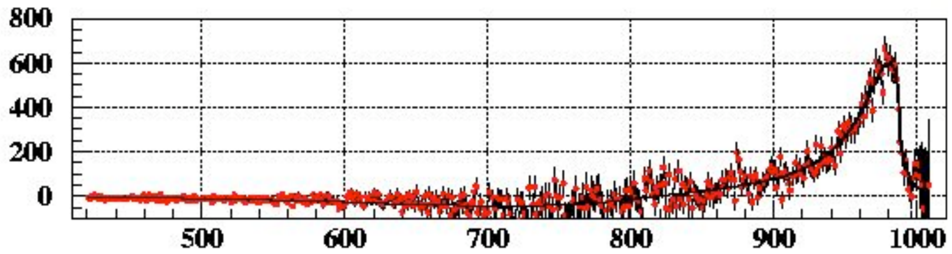
# News for the $f_0(980) \to \pi^+\pi^-$ analysis

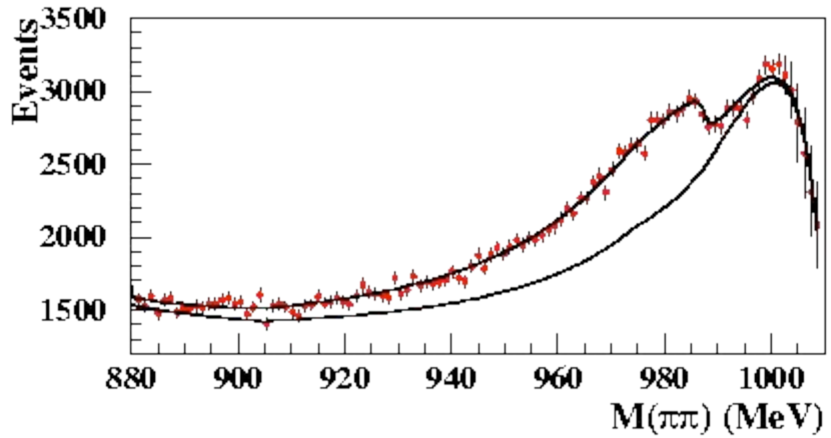
a) improved KL parametrization implemented also for $\pi^+\pi^-\gamma$ events

b) acceptable $\chi^2$ with reasonable ISR+FSR parameters

| $M_{f0}$ (MeV) | $g_{f0KK}$ (GeV) | $g_{f0\pi\pi}$ (GeV) | $M_\rho$ (MeV) | $\Gamma_\rho$ | $\alpha$ | $\beta$ | $A_{\rho\pi}$ | $P(\chi^2)$ |
|---|---|---|---|---|---|---|---|---|
| **983.7** | **4.74** | **-2.22** | 772.5 | 144.2 | 1.64 | −0.129 | 4.7 | 2.5% |

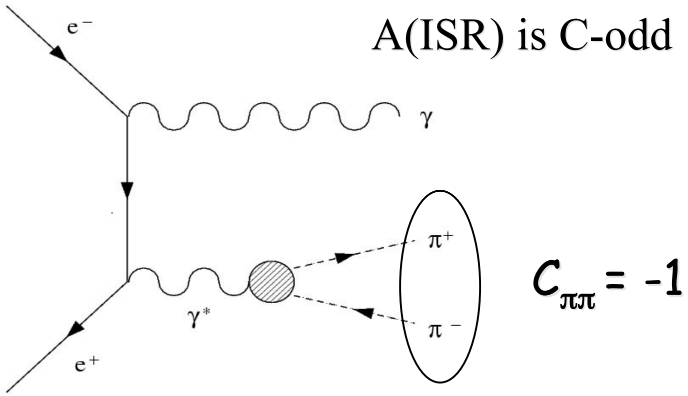
preliminary update

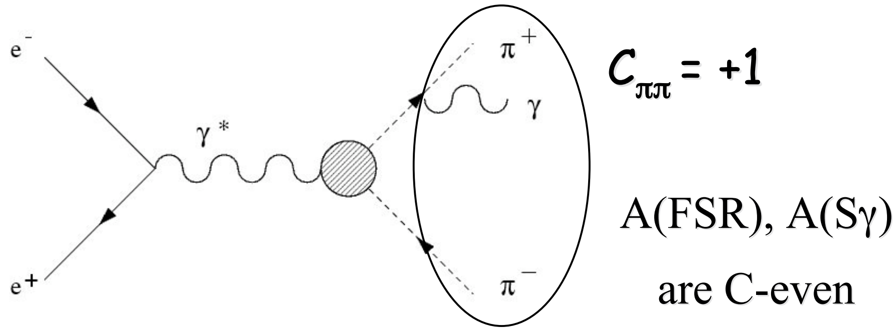



- results in better agreement between the 2 analyses
- $\phi \to \rho\pi$ interference with Scalar and FSR amplitudes to be considered

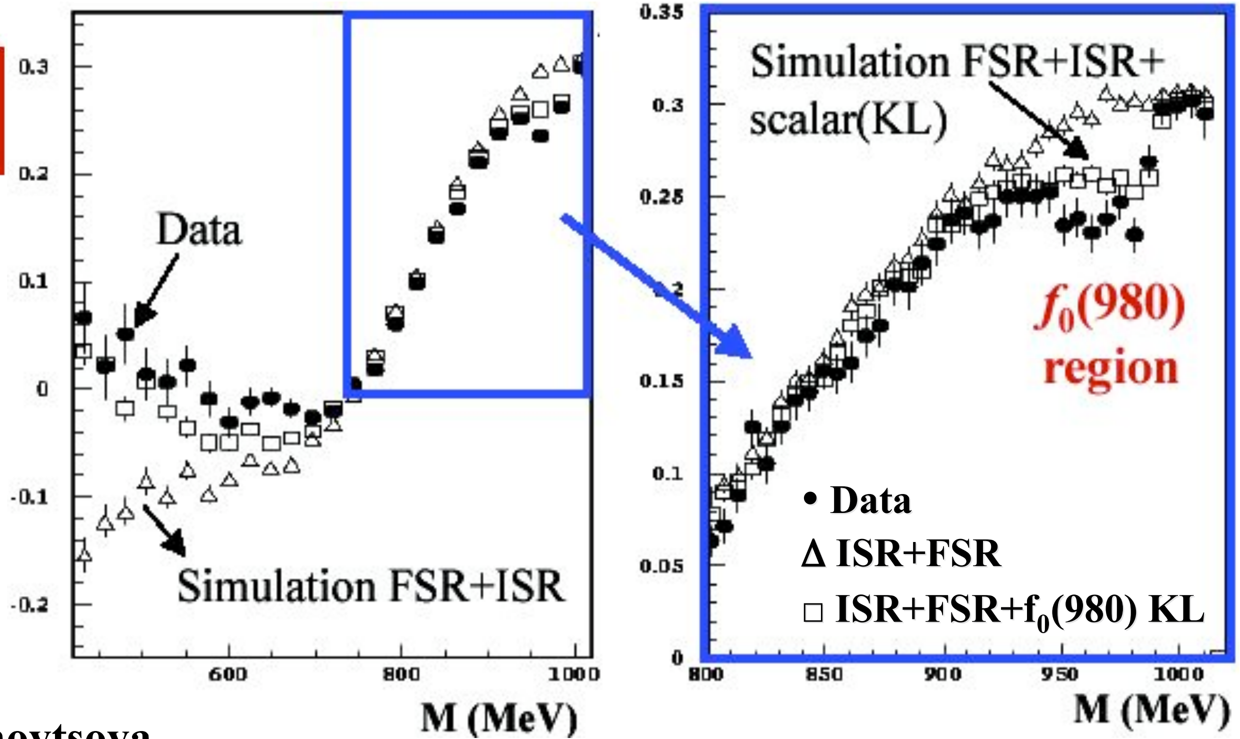
# Forward backward asymmetry




$$\mathcal{A}_{FB} = \frac{N(\theta_{\pi^+}>90°) - N(\theta_{\pi^+}<90°)}{N(\theta_{\pi^+}>90°) + N(\theta_{\pi^+}<90°)}$$

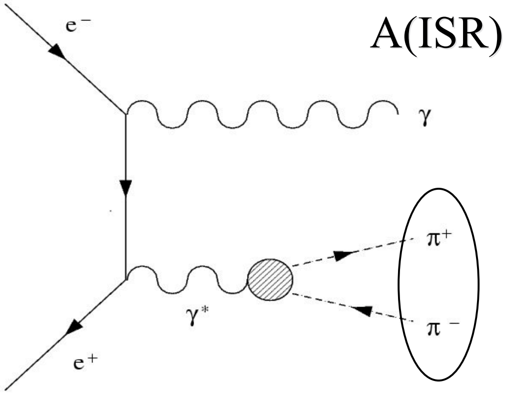
- ✓ Clear signal in the $f_0(980)$ mass region
- ✓ Discrepancy at low $M_{\pi\pi}$ disappears including $f_0(980)$



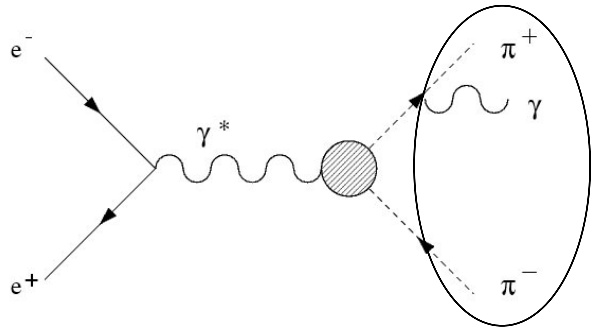

[G.Pancheri, O.Shekhovtsova, G.Venanzoni, PLB642 (2006) 342]

# Forward backward asymmetry - improved

A(ISR) is C-odd




$C_{\pi\pi} = -1$

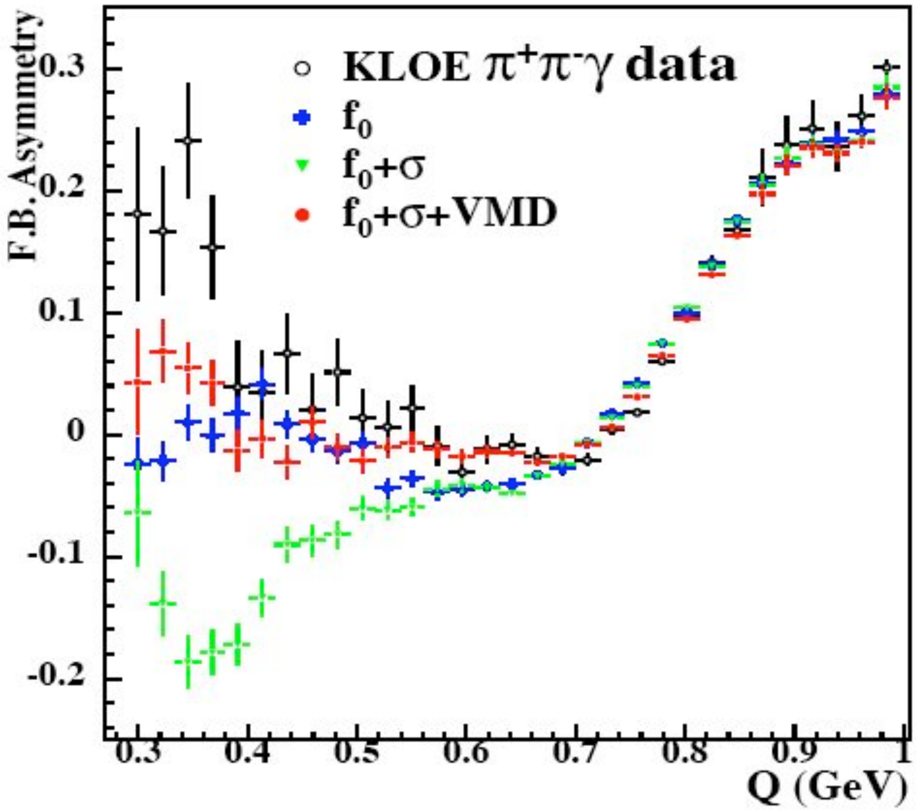



$C_{\pi\pi} = +1$

A(FSR), A(Sγ) are C-even

$$\mathcal{A}_{FB} = \frac{N(\theta_{\pi^+}>90°) - N(\theta_{\pi^+}<90°)}{N(\theta_{\pi^+}>90°) + N(\theta_{\pi^+}<90°)}$$

✓Including KL model with σ and $f_0$ parameters from the $\pi^0\pi^0$ analysis and VMD terms




**[G.Pancheri, O.Shekhovtsova, G.Venanzoni, 0706.3027]**

# The $a_0(980) \to \eta\pi^0$ analysis; $\eta \to \gamma\gamma$ final state

a) 5 photons from the interaction point

b) 1st kinematic fit with 4-mom. conservation

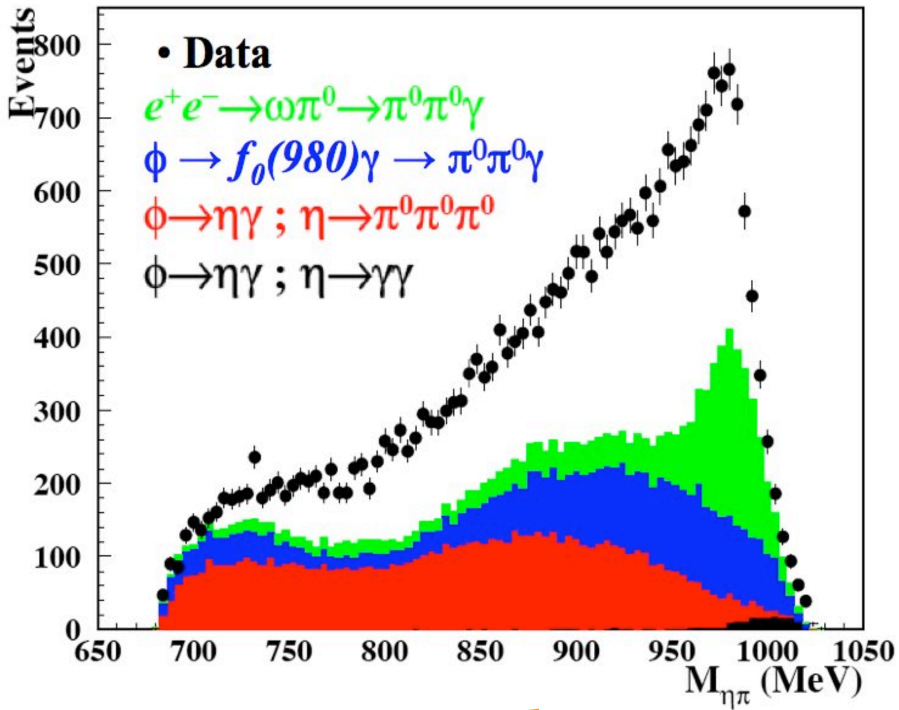
c) 2nd kinematic fit also with constraints

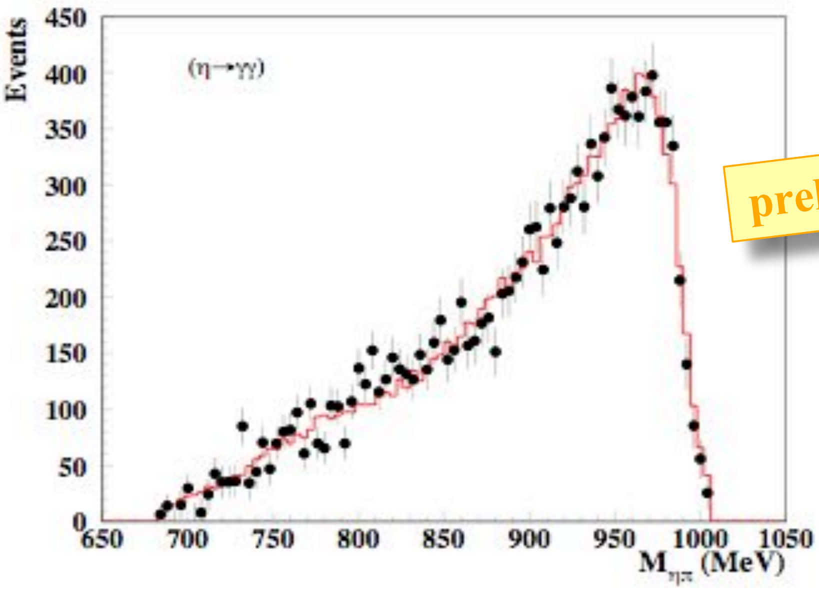
on $\eta, \pi^0$ masses $\to$ global $\varepsilon \sim 40\%$

d) $3 \times 10^4$ events from 414 $pb^{-1}$

e) large background ~ 55% from final

states with 5 or 7 $\gamma$




preliminary: ArXiv 0707.4609

from event counting

$$BR(\phi \to \eta\pi^0\gamma) =$$
$$(6.92 \pm 0.10_{stat} \pm 0.20_{sys}) \times 10^{-5}$$

# The $a_0(980) \to \eta\pi^0$ analysis; $\eta \to \pi^+\pi^-\pi^0$ final state

a) 2 tracks + 5 $\gamma$ from the interaction point

b) 1st kinematic fit with 4-mom. conservation

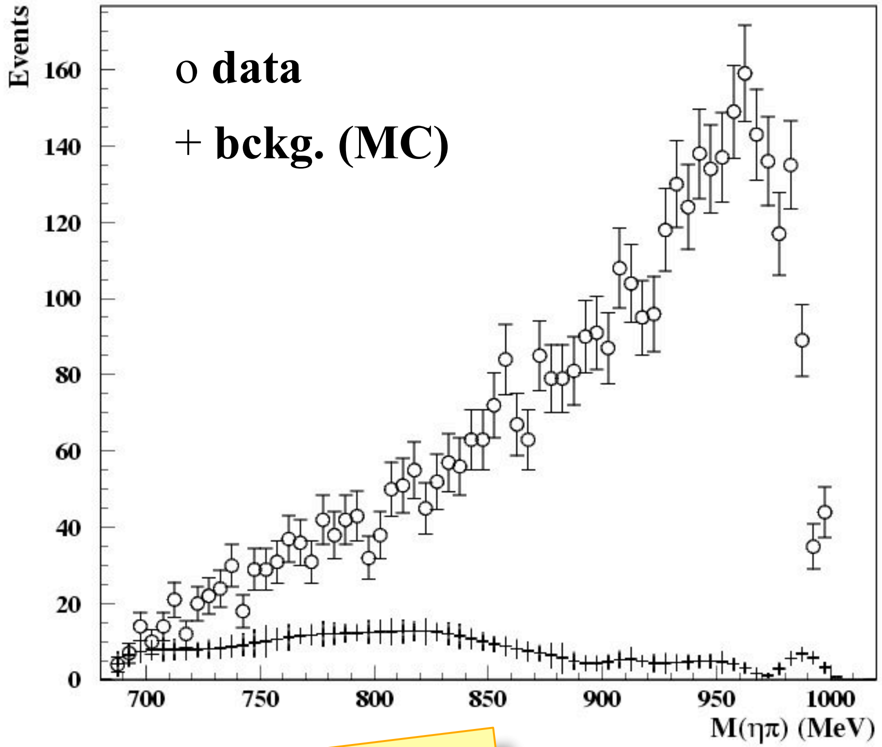
c) 2nd kinematic fit also with constraints

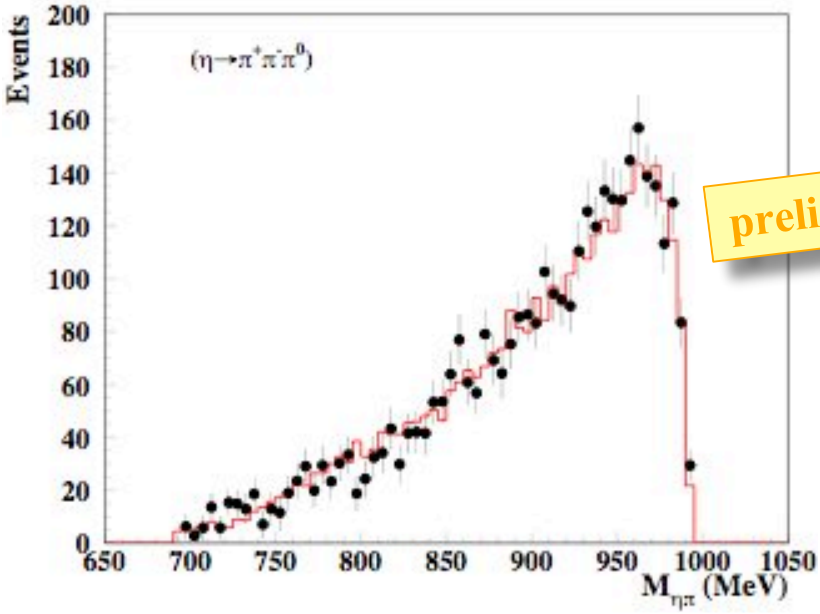
on $\eta$, $\pi^0$ masses → global $\varepsilon$ ~ 20%

d) $4.5 \times 10^3$ events from 414 $pb^{-1}$

e) small background ~ 15% from $\omega(\to\pi^+\pi^-\pi^0)\pi^0$

and $K_SK_L \to$ 2 tracks + 2-3 $\pi^0$




preliminary: ArXiv 0707.4609

from event counting

$$BR(\phi \to \eta\pi^0\gamma) =$$

$$(7.19 \pm 0.17_{stat} \pm 0.24_{sys}) \times 10^{-5}$$

# Results for the $a_0(980) \to \eta\pi^0$ analyses

$a_0$(980) parameters extracted from a simultaneous fit
to both $M_{\eta\pi}$ spectra (efficiency + resolution accounted)

free parameters:

**Ratio $BR_{\eta \to \gamma\gamma} / BR_{\eta \to \pi+\pi-\pi0}$**

**$BR(\phi \to \rho\pi^0 \to \eta\pi^0\gamma)$**

KL:

$M_{a0}$, $g_{a0KK}$ $g_{a0\eta\pi}$ couplings

NS:

$M_{a0}$, $g_{\phi a0\gamma}$ $g_{a0KK}$ $g_{a0\eta\pi}$ couplings

| Parameter | Kaon Loop | No Structure |
|---|---|---|
| $M_{a0}$ (MeV) | 983 ± 1 | 983 (fixed) |
| **$g_{a0KK}$ (GeV)** | **2.16 ± 0.04** | **1.57 ± 0.13** |
| $g_{a0\eta\pi}$ (GeV) | 2.8 ± 0.1 | 2.2 ± 0.1 |
| **$g_{\phi a0\gamma}$ (GeV$^{-1}$)** | — | **1.61 ± 0.05** |
| $BR(\phi\to\rho\pi\to\eta\pi\gamma)\times10^6$ | 0.9 ± 0.4 | 4.1 (fixed) |
| **$BR(\eta\to\gamma\gamma)/BR(\eta\to\pi\pi\pi)$** | **1.69 ± 0.04** | **1.69 ± 0.04** |
| **$\chi^2$/Ndf** | **156.6/136** | **146.8/134** |
| **$P(\chi^2)$** | **11%** | **21%** |

**preliminary: ArXiv 0707.4609**

# Comments on $a_0(980)$ results

✓ good consistency between the two analyses:

experimental systematics is under control

✓ small VMD contribution (when fitted)

✓ $BR(\eta \to \gamma\gamma)/BR(\eta \to \pi^+\pi^-\pi^0)$ consistent with the PDG

✓ $g_{a0KK} \sim 2.2$, $g_{a0KK}/g_{a0\eta\pi} \sim 0.8$ → in conflict with $qq\bar{q}\bar{q}$ hypothesis

(different from the $f_0(980)$)

✓ large $BR(\phi \to \eta\pi^0\gamma)$ and $g_{\phi a0\gamma}$ values (as for the $f_0(980)$)

$$g^2_{\phi M\gamma} = \frac{3}{\alpha}\left(\frac{2m_\phi}{m^2_\phi - m^2_M}\right)^3 \Gamma(\phi \to M\gamma)$$

| Meson | $g_{\phi M\gamma}$ ($GeV^{-1}$) |
|---|---|
| $\pi^0$ | 0.13 |
| $\eta$ | 0.71 |
| $\eta'$ | 0.75 |
| $a_0(980)$ | 1.6 |
| $f_0(980)$ | 1.2 – 2.7 |

increasing
*s* quark content

# Conclusions and outlook

- **Extensive study of $f_0(980)$ properties with 2001−2002 KLOE data:**
  - **First clear evidence of the $\phi \to f_0\gamma \to \pi^+\pi^-\gamma$ process both in $\pi^+\pi^-$ invariant mass and in the forward-backward asymmetry, not sensitive to $\sigma(600)$**
  - **The data sample allows for an accurate Dalitz plot analysis in the $\pi^0\pi^0$ final state for the first time: fixed $\sigma(600)$ parameters give better fits**
  - ***Combined fit coming soon*…, appetizers presented here:**
    - ✓ **Large reduction of theory spread in the neutral channel**
    - ✓ **Good agreement between the 2 channels using the same KL model**

- **Analyses of $\phi \to a_0(980)\gamma \to \eta\pi\gamma$ performed with different $\eta$ decay channels:**
  - **Good agreement btw the 2 analyses (different systematics) → combined fit**
  - **Preliminary results show sizeable s quark content for the $a_0$**

# Future prospects - KLOE2

- New scheme to increase the DAΦNE luminosity by a factor $O(5)$ has been proposed (crabbed waist collisions) $\Rightarrow$ test in Autumn 2007
- If successful, KLOE-2 data taking could start in 2009

(upgraded detector: Inner tracker, Tagger for $\gamma\gamma$ physics, Calorimeter read-out, ......)

- Observation of $\phi \to (f_0/a_0)\gamma \to K^0\overline{K}^0\gamma$
- Search for the $\sigma(500)$ in $e^+e^- \to e^+e^-\pi^0\pi^0$ ($\gamma\gamma \to \sigma \to \pi^0\pi^0$)
- If the high energy option (up to 2.5 GeV in the c.m.) will be exploited

$\Rightarrow$ measurement of $\Gamma(f_0 \to \gamma\gamma)$ and $\Gamma(a_0 \to \gamma\gamma)$ through $\gamma\gamma \to f_0, a_0$

# News for the $f_0(980) \to \pi^0\pi^0$ analysis

2 changes in $\sigma$ couplings with respect to Achasov paper [PRD73(2006)054029]:

- $C_{f0\sigma} = -0.047\ \mathrm{GeV}^2$ (was $+0.047\ \mathrm{GeV}^2$) [private communication]
- $g_{\sigma\pi\pi} = 2.1$ GeV (was $-2.1$ GeV) [PRD 74 (2006) 059902(E)]

| | $M_{f0}$(MeV) | $g_{f0K+K-}$(GeV) | $g_{f0\ \pi+\pi-}$(GeV) | $P(\chi^2)$ |
|---|---|---|---|---|
| old $C_{f0\sigma}$ | 976.8 ± 0.3 | 3.76 ± 0.04 | -1.43 ± 0.01 | 14.5% |
| old $g_{\sigma\pi\pi}$ | 984.5 ± 0.4 | 4.53 ± 0.05 | -1.90 ± 0.01 | 0.9% |
| new $C_{f0\sigma}$ | 984.5 ± 0.4 | 4.01 ± 0.10 | -1.79 ± 0.02 | 5.8% |
| new $g_{\sigma\pi\pi}$ | 982.1 ± 0.4 | 3.99 ± 0.09 | -1.79 ± 0.02 | 6.3% |

more stable $f_0$ parameters within different model assumptions

# News for the $f_0(980) \rightarrow \pi^0\pi^0$ analysis

published result

preliminary update

$M_{f0}$ = ( 976.8 ± 0.3$_{fit}$ + 0.9/−0.6$_{syst}$ + 10.1$_{mod}$ ) MeV

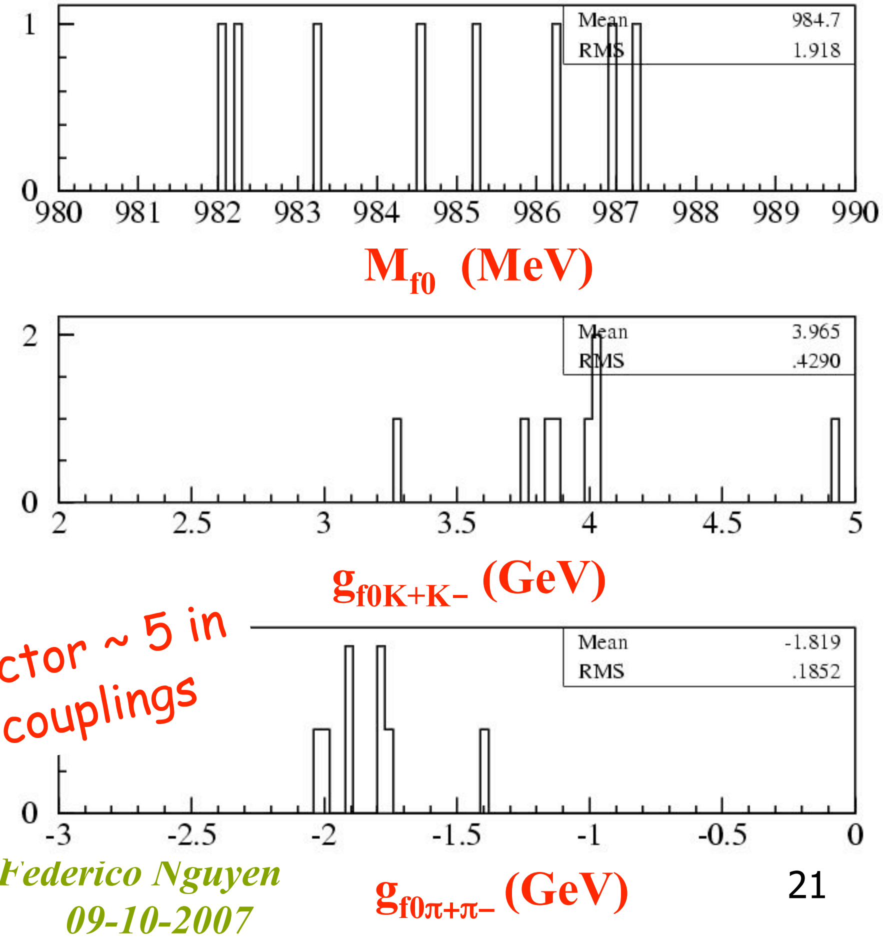
$g_{f0K+K-}$ = ( 3.76 ± 0.04$_{fit}$ + 0.15/−0.08$_{syst}$ + 1.16/−0.48$_{mod}$ ) GeV

$g_{f0\pi+\pi-}$ = ( −1.43 ± 0.01$_{fit}$ + 0.01/−0.06$_{syst}$ + 0.03/−0.60$_{mod}$ ) GeV

$M_{f0}$ = ( 984.7 ± 1.9$_{mod}$ ) MeV

$g_{f0K+K-}$ = ( 3.97 ± 0.43$_{mod}$ ) GeV

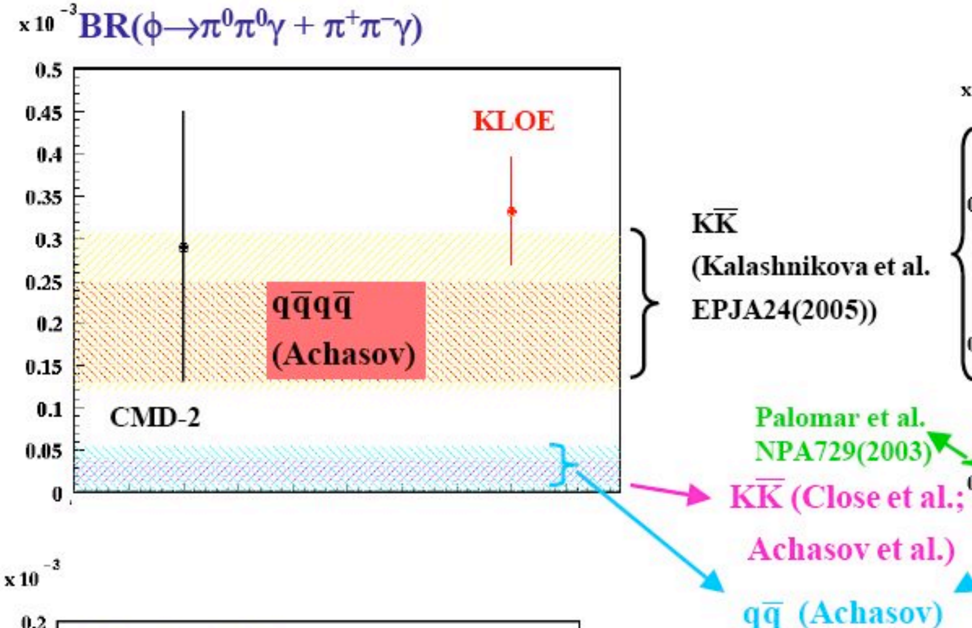
$g_{f0\pi+\pi-}$ = ( −1.82 ± 0.19$_{mod}$ ) GeV

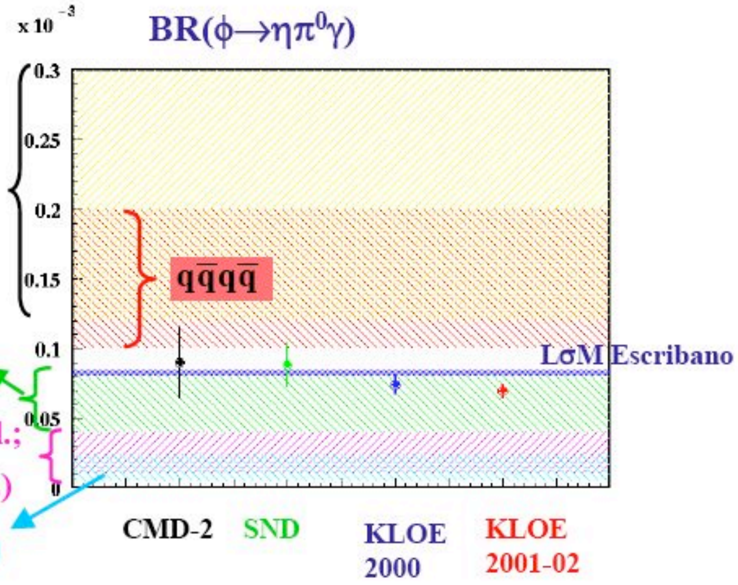


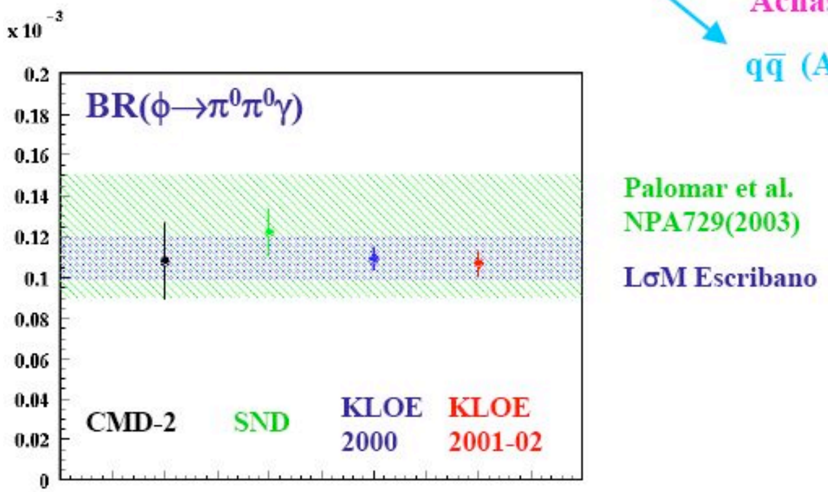

- ✓ 7 fit variants with $P(\chi^2) > 1\%$
- ✓ variants average → central value
- ✓ variants RMS → model dependence uncertainty

model uncertainty reduced of factor ~ 5 in $M_{f0}$ and of a factor ~ 3 in the couplings

# Branching ratios



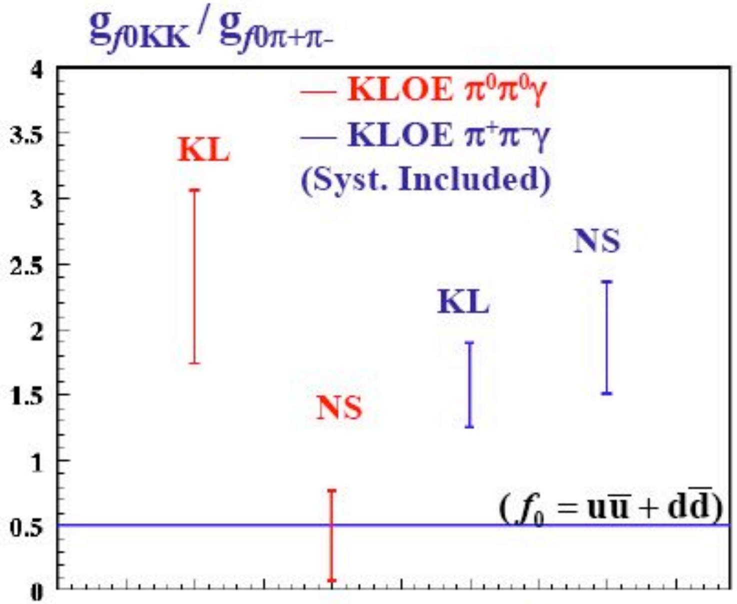




$$\boxed{\frac{BR(\phi \to f_0\gamma)}{BR(\phi \to a_0\gamma)} \approx 4.5}$$

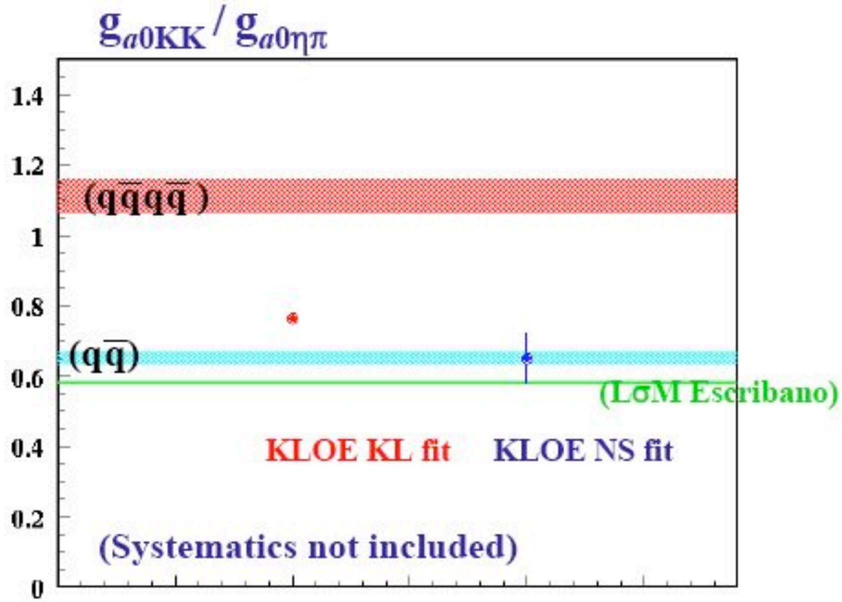
# Couplings

$$\boxed{g_{f_0 KK} > g_{f_0 \pi^+\pi^-}}$$

$$\boxed{g_{a_0 KK} < g_{a_0 \eta\pi}}$$



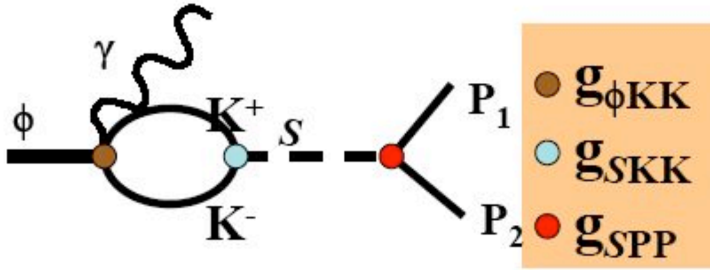

$$\left(\begin{array}{l} \text{CMD-2 (1999)}: 1.90 \pm 0.16 \\ \text{SND (2000)}: 2.14 \pm 0.19 \\ \text{BES (2005)}: 2.05 \pm 0.11 \\ (J/\psi \to \phi\pi^+\pi^-, \phi K^+K^-) \end{array}\right)$$




( SND (2000) : $1.33 \pm 0.92$)

# Models

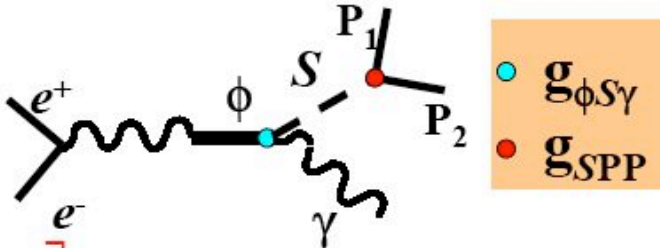
- **Kaon loop**




$$M_{KL} = \frac{g_{SK\bar{K}}\, g_{SPP}\, g(m^2) e^{i\delta_m(\theta)}}{D_S(m^2)}$$

(Achasov - Ivanchenko Nucl.Phys.B315(1989)465,
Achasov - Gubin Phys.Rev.D63(2001)094007,
Achasov - Kiselev Phys.Rev.D68(2003)014006 )

- **"No Structure"**




$$M_{NS} = \frac{e}{4F_\phi} \frac{sM_\phi^2}{D_\phi(s)} \left[ \frac{g_{SPP} g_{\varphi S\gamma}}{D_S(m^2)} + \frac{c_0}{m_\varphi^2} + c_1 \frac{m^2 - m_S^2}{m_\varphi^4} \right]$$

(G.Isidori et al., JHEP0605(2006)049)

# Dalitz fit: KL model

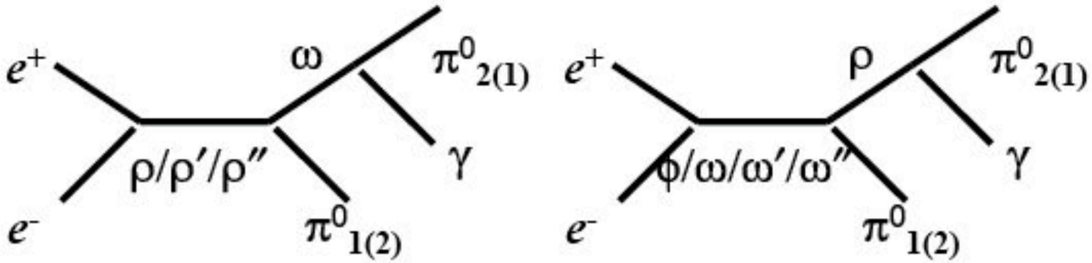
- **Kaon Loop with σ(600) contribution**

$$\mathbf{M_{KL} = g(m^2)e^{i\delta_m} \sum_{S,S'=f_0,\sigma} g_{SK\bar{K}}\ G^{-1}_{SS'}\ g_{S'\pi\pi}}$$

**($\delta_m$ takes into account for both pp and KK scattering)**

**σ(600) and pp and KK scattering parameters fixed to the values of Achasov-Kiselev PRD73(2006)054029**

- **Interfering background (VDM parametrization)**




- **Free parameters: $M_{f0}$, $g_{fKK}$, $g_{fpp}$ + 7 VDM parameters**

# Dalitz fit: NS model

- "No structure" :

$$M_{NS} = (s - m^2)\left[\frac{g_{f_0\pi\pi} g_{\phi f_0\gamma}}{D_{f_0}(m^2)} + \frac{c_0 e^{ib_0 \frac{v_\pi(m)}{m_\phi}}}{m_\phi^2} + c_1 e^{ib_1 \frac{v_\pi(m)}{m_\phi}} \frac{m^2 - m_{f_0}^2}{m_\phi^4}\right]$$

- Scalar term: B-W with energy dependent width:

$$\Gamma_{f_0}(m) = g_{f_0\pi\pi}^2 \frac{v_\pi(m)}{8\pi m^2} + g_{f_0K\bar{K}}^2 \frac{v_{K^\pm}(m) + v_{K^0}(m)}{8\pi m^2}$$

$$v_P(m) = \sqrt{m^2/4 - m_P^2}$$

- Background: same VDM parametrization as Kaon loop
- Free parameters: $M_{f0}$, $g_{fKK}$, $g_{fpp}$, $g_{\phi f\gamma}$, $c_0$, $c_1$, $b_1$ + VDM parameters ($b_0$ can be expressed in terms of other parameters)

# Structures

$\mathbf{q\bar{q}}$ **case:**

$$f_0 = -\frac{1}{\sqrt{2}}(u\bar{u}+d\bar{d})\sin\varphi_S + s\bar{s}\cos\varphi_S$$

$$\sigma = \frac{1}{\sqrt{2}}(u\bar{u}+d\bar{d})\cos\varphi_S + s\bar{s}\sin\varphi_S$$

$$\frac{g_{f_0\pi\pi}}{g_{f_0KK}} = \frac{-2\sin\varphi_S}{\sqrt{2}\cos\varphi_S - \sin\varphi_S}$$

$\mathbf{qq\bar{q}\bar{q}}$ **case:**

$$f_0 = -\frac{1}{\sqrt{2}}(u\bar{u}+d\bar{d})s\bar{s}\sin\varphi_S + u\bar{u}d\bar{d}\cos\varphi_S$$

$$\sigma = \frac{1}{\sqrt{2}}(u\bar{u}+d\bar{d})s\bar{s}\cos\varphi_S + u\bar{u}d\bar{d}\sin\varphi_S$$

$$\frac{g_{f_0\pi\pi}}{g_{f_0KK}} = \sqrt{2}\cot\varphi_S$$

**$a_0$ patterns:**

$$\frac{1}{\sqrt{2}}(u\bar{u}-d\bar{d})s\bar{s}:$$

$$\frac{g_{a_0\eta\pi}}{g_{a_0KK}} = 2\cos\varphi_P \quad = 1.54$$

$$\frac{1}{\sqrt{2}}(u\bar{u}-d\bar{d}):$$

$$\frac{g_{a_0\eta\pi}}{g_{a_0KK}} = \sqrt{2}\sin\varphi_P = 0.90$$

assuming $\varphi_P = 39.7^\circ$