

# Eightfold Way From Dynamics in Strongly Coupled Lattice QCD

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This set of slides includes some overlays. It is better visualized, with Acrobat Reader, CTRL+L (to get full screen text). Use Page up and down keys.

## OUR LONG STANDING PROGRAM:

**GOAL:** Try to FILL In The GAP  
Between QCD and NUCLEAR PHYSICS

Starting from FIRST PRINCIPLES (Quarks, Gluons and QCD dynamics): Prove from the Dynamics that Hadrons and Their BOUND STATES Are Part of the ENERGY-MOMENTUM (EM) SPECTRUM

Possibly, UNDERSTAND Better, From Theory, The Nature of BINDING POTENTIALS

PART OF IT: DONE Already.  $2 + 1$  and  $3 + 1$  Dimensional Models,  $2 \times 2$ ,  $4 \times 4$  Spin Matrices, 1 and 2 Flavors.

FRAMEWORK: Imaginary-time LATTICE Models Within STRONG COUPLING and Functional Integral Formulation.

Namely, in the richest model we analyzed up to now: 2 Flavors,  $3 + 1$  Dimensions,  $4 \times 4$  Dirac Spin Matrices. Proved Existence of Baryons and Mesons and Analyzed  $I = 0, 3$  Sectors of 2-Baryon Bound States, including the  $I = 0, J = 1$   $p - n$  State: DEUTERIUM.

Bound State Results: Also INCLUDE The MESON PARTICLES in Some Cases.

BS OBTAINED in a LADDER Approximation, Using Lattice BETHE-SALPETER Equation.

TREATMENT IS TUNED TO CONTROL CONTRIBUTIONS BEYOND LADDER APPROX.

IN ALL THIS, an IMPORTANT TOOL is the HYPERPLANE DECOUPLING Method.

A LOT STILL TO BE DONE!

$I = 2, 3$ , BS With MESONS,  $1/r$  CORRECTION to  $e^{-m_\pi r}$  IN YUKAWA, etc...

TODAY: Continuing Our Program, We Will Apply Our ANALYTICAL METHODS to ANALYZE the 3+1 Dimensional, 3 Color, 3 Flavor Case, Strongly Coupled Lattice QCD.

OBTAIN The BARYON and MESON SPECTRUM

OBTAIN THE GELL'MANN-NE'EMAN EIGHTFOLD WAY FROM DYNAMICS

# THE MODEL

## PARTITION FCT & EXPECTATIONS

$$Z = \int e^{-S(\psi, \bar{\psi}, g)} d\psi d\bar{\psi} d\mu(g),$$

$$\langle F(\bar{\psi}, \psi, g) \rangle = \frac{1}{Z} \int F(\bar{\psi}, \psi, g) e^{-S(\psi, \bar{\psi}, g)} d\psi d\bar{\psi} d\mu(g).$$

$\hat{\psi}_{a\alpha f}(u)$  Grassmann quark variable at site  $u$   
 ( $\hat{\phantom{x}}$  = bar/no-bar),  $d\hat{\psi}$  associated 'Berezin' measures  
 $g \in \text{SU}(3)$  on oriented lattice bonds  
 $d\mu(g)$  is product measure of  $\text{SU}(3)$  Haar measures

## MODEL ACTION (Wilson's Action)

$$S(\psi, \bar{\psi}, g) = \frac{\kappa}{2} \sum' \bar{\psi}_{a\alpha f}(u) \Gamma_{\alpha\beta}^{\epsilon e^\mu}(g_{u, u+\epsilon e^\mu})_{ab} \psi_{b\beta f}(u + \epsilon e^\mu) \\ + \sum \bar{\psi}_{a\alpha f}(u) \psi_{a\alpha f}(u) \\ - \beta \sum_{\mathbf{p}} \chi(g_{\mathbf{p}}),$$

$\sum$  is over  $u = (u^0, \vec{u})$ ,  $u^0 \in \mathbb{Z} + \frac{1}{2}$ ,  $\vec{u} \in \mathbb{Z}^d$ ,  $d = 3$ ,  $a, b = 1, 2, 3$ ,  
 $\alpha, \beta = \pm$  and  $f = 1, 2, 3$  OR  $f = u, d, s$ , and  $\sum'$  ALSO sums  
 over  $\epsilon = \pm 1$ ,  $\mu = 0, 1, \dots, d$ .

$\Gamma^{\pm e^\mu} = 1 \pm \gamma^\mu$ ,  $\gamma^\mu$  = Pauli or Dirac spin matrices

$\sum_{\mathbf{p}}$  is over **Plaquettes**.  $\chi$  is  $\Re(\text{Character})$ .

$M \equiv M(m, \kappa) = (m + 2\kappa) \mathbb{I}_{\text{spin}}$  Is Set To  $\mathbb{I}_{\text{spin}} \equiv 1$  by suitably  
 choosing  $m > 0$ .

## GAUGE INVARIANCE:

For  $x \in \mathbb{Z}_0^{d+1} = (\mathbb{Z} + \frac{1}{2}) \times \mathbb{Z}^d$  and  $h(x) \in \text{SU}(3)$ ,  
 $\psi(x) \mapsto h(x)\psi(x), \quad \bar{\psi}(x) \mapsto \bar{\psi}(x)[h(x)]^{-1},$   
 $g_{x+e^\mu, x} \mapsto h(x + e^\mu) g_{x+e^\mu, x} [h(x)]^{-1}.$

FLAVOR or **ISOSPIN SYMMETRY**: GLOBAL  $\text{SU}(3)_f$

NO CONTINUOUS Rotation Symmetry. ONLY Discrete  $\pi/2$  rotations.

## FOR SPIN

At  $\kappa = 0$ , RECOVER  $\text{SU}(2) \oplus \text{SU}(2)$  SPIN STRUCTURE,  $J^2$  and  $J_z$  CONSERVATION, Separately for UPPER and LOWER Spin Components

WHEN  $\kappa \neq 0$ , We Have a **PARTIAL RESTORATION** of Continuous Symmetry **FOR ZERO SPATIAL MOMENTUM STATES**. MEANING THAT the Discrete Group is ENOUGH to INHERIT STRUCTURE From the Continuous.

DEFINE Generators Using the Log and NOT Taking  $\lim_{\theta \searrow 0} [\exp(i\theta/2) - 1]$ . May find Difficulties for HIGH SPIN VALUES (fix the Log branch!)

THIS IS WHAT WE USE TO TALK ABOUT SPIN!  
Define Spin Operators  $J^2$  and  $J_z$ . Lowering  $J_-$  and  
Raising  $J_+$  Spin Operators.

Isospin  $(I, I_3)$ , Spin  $(J, J_z)$ , Parity  $\mathcal{P}$ , Time Reversal  $\mathcal{T}$ , Charge Conjugation  $\mathcal{C}$ , Coordinate Reflection and Rotation,... OPERATORS Can Be Raised  
From Correlation Function Level To Hilbert Space  
Operators!

We Also Have  $\mathcal{F}_s = -i\mathcal{TCT}$ , A SPIN FLIP Symmetry ( $J_z \leftrightarrow -J_z$ ) in Lower / Upper components

T is a **NEW** TIME REFLECTION SYMMETRY

**This is NOT the FIRST TIME this Problem is Treated in the Literature!**

In the 80's some papers (Smit, Hoeck, ...) were devoted to analyze this problem.

**OUR CLAIM: We Do It Correctly!**

**BESIDES: Many Results Emerge In Our Treatment That Were Not Obtained Previously:**

*Masses as convergent expansions in hopping parameter, possibility of reliable perturbation in gauge coupling  $\beta$ , gauge invariance for free, confinement in Hilbert space up to an energy threshold, no group structure and right particle multiplicities, dispersions and some of their properties,...*

**AND The One-Particle Spectrum, As It Is Done HERE, IS A NECESSARY STEP To Go Up In Spectrum and Obtain Two-Particle Spectrum.**

## DOMAIN: STRONG COUPLING REGIME

Small HOPPING Parameter  $\kappa$ ,  $0 < \kappa \ll 1$

Large GLUEBALL MASS  $\beta$ ,  $0 < \beta \ll \kappa$

FAR from SCALING LIMIT But Can Manage It

CONFINEMENT Shows Up In This Way.

## DESCRIPTION of MAIN RESULTS

**Main Tool: Hyperplane Decoupling Expansion  
(Hopping Term)**

## BARYONS

1. **EXISTENCE OF 56 (ANTI-)BARYONS:** (Masses  $\approx -3 \ln \kappa$ ). Manifested by Isolated Dispersion Curves in the EM Spectrum  $w(\vec{p}) \equiv w(p^1, p^2, p^3)$ ,  $p^{i=1,\dots,3} \in (-\pi, \pi]$ . (Isolated up to near the Meson-Baryon Threshold  $\approx -5 \ln \kappa$ ).
2. **EIGHTFOLD WAY:** Consider the Usual **ISOSPIN SU(2) & HYPERCHARGE U(1)** Subgroups of  $SU(3)_f$ .

A) 2 Sets of EIGHT  $J = 1/2$  particles. That is, **Two OCTETS**, ONE for  $J_z = 1/2$  and Another for  $J_z = -1/2$ .

By FLAVOR AND SPIN FLIP SYMMETRY **ALL** the Particles in the OCTETS HAVE SAME MASS!

B) 4 Sets of TEN  $J = 3/2$  Particles. That is, **Four DECUPLETS**, Two for  $J_z = \pm 1/2$  and Two Other for  $J_z = \pm 3/2$ .

By FLAVOR and SPIN FLIP SYMMETRIES, and SPIN LOWERING for zero-momentum states, the Decuplet Masses are INDEPENDENT of  $J_z$ . Therefore, **ALL** Decuplet Baryons have the Same Mass.

NO NEED for Particles in the OCTETS and in the DECUPLETS To Have the SAME MASS!

3. **MASS SPLITTING:** MASS SPLITTING Between Baryon OCTETS and DECUPLETS of  $\mathcal{O}(\kappa^6)$ .

#### 4. DISPERSION RELATIONS:

$$w_c(\kappa, \vec{p}) = -3 \ln \kappa - 3\kappa^3/4 + \kappa^3 \sum_{j=1}^3 (1 - \cos p^j)/4 \\ + r_c(\kappa, \vec{p}) \quad , \quad c = o, d$$

$r_c(\kappa, \vec{p})$  is of  $\mathcal{O}(\kappa^6)$ .  $r_o(\kappa, \vec{p})$  is jointly analytic in  $\kappa$  and in each  $p^j$ , for small  $|\Im p^j|$ .

OBSERVATION: ALL MEMBERS of The Octets Have the SAME Value for the Quadratic Casimir  $C_2 = 3$ . For the Decuplet Members,  $C_2 = 6$

DISPERSIONS ARE **ALL EQUAL** for OCTET Members And **EQUAL** For The DECUPLET Members With The Same  $|J_z|$ .

### MESONS

1'. **EXISTENCE OF 36 MESONS:** (Masses  $\approx -2 \ln \kappa$ ). Manifested by Isolated Dispersion Curves in the EM Spectrum  $w(\vec{p}) \equiv w(p^1, p^2, p^3)$ ,  $p^{i=1, \dots, 3} \in (-\pi, \pi]$ . (Isolated up to near the Meson-Meson Threshold  $\approx -4 \ln \kappa$ ).

2'. **MESON EIGHTFOLD WAY:** Consider ISOSPIN, HYPERCHARGE and  $C_2$

The 36 Mesons Can Be Grouped Into FOUR ISOSPIN NONETS: The Pseudo-Scalar Mesons and The Vector Mesons.

There are: 9 Pseudo-Scalar Mesons with  $J = 0$  and 27 Vector Mesons with  $J = 1$ .

Each NONET: Decomposed Into a Singlet ( $C_2 = 0$ ) and Octet ( $C_2 = 3$ ) Isospin.

Up To and Including  $\mathcal{O}(\kappa^4)$ : they have the SAME MASS

$$M(\kappa) = -2 \ln \kappa - 3\kappa^2/2 + \kappa^4 r(\kappa),$$

with  $r(\kappa)$  analytic and  $r(0) \neq 0$ .

For zero momentum states, the masses are independent of  $J_z$ . Therefore, As For The PSEUDO-SCALAR MESONS, **ALL** Vector Mesons Have the SAME MASS.

3'. **MESON MASS SPLITTING:** Between Vector and Pseudo-Scalar Mesons

$$[r_p(\kappa) - r_v(\kappa)]\kappa^4 = 2\kappa^4 + \mathcal{O}(\kappa^6)$$

#### 4'. MESON DISPERSION RELATIONS:

$$w_c(\kappa, \vec{p}) = -2 \ln \kappa - 3\kappa^2/2 + (1/4)\kappa^2 \sum_{j=1}^3 2(1 - \cos p^j) \\ + \kappa^4 r_c(\kappa, \vec{p}) \quad , \quad c = p, v$$

$|r_c(\kappa, \vec{p})| \leq \text{const}$  and  $r_p(\kappa, \vec{p})$  is jointly analytic in  $\kappa$  and  $p^j$  for  $|\kappa|, |\Im p^j|$  small.

The various  $w_v(\kappa, \vec{p})$  may depend on  $|J_z|$ .

5. BARYON SPECTRAL RESULTS: In ODD SUB-SPACE of Quantum Mechanical Hilbert Space  $\mathcal{H}$  (ODD # of fermions)

MESON SPECTRAL RESULTS: In EVEN SUB-SPACE of Quantum Mechanical Hilbert Space  $\mathcal{H}$

This is Due To SUBTRACTION METHODS For Two-Baryon and Two-Meson Correlations.

COMBINING BOTH SPECTRAL RESULTS: CONFINEMENT IS PROVEN UP To NEAR The TWO-MESON THRESHOLD.

5. NEW Symmetry: TIME REFLECTION T.

Time **Reflection** Transformation  $T$  is **Nonlocal** and *Linear*.

Time **Reversion** Transformation  $\mathcal{T}$  is **Nonlocal** and *Antilinear*.

SUPPOSING THE CONTINUUM LIMIT EXISTS  
 $T$  SURVIVES This Limit.

Consequences of Time Reflection (If Any?!).

What happens in other models?

6. **SYMMETRIES**: Implemented on  $\mathcal{H}$  by Unitary (Anti-unitary, for  $\mathcal{T}$  and  $\mathcal{F}_s$ ) Operators.

**IMPORTANT**: In Obtaining The EXISTENCE Of Particles, **NO GROUP STRUCTURE IS NEEDED!**

To Prove the Existence of Particles: Use INDIVIDUAL ISOSPIN and SPIN BASIS. NO GUESS-WORK On the FORM Of The Fields.

**GAUGE INVARIANCE of HADRON STATES**: EMERGES FOR FREE, from Hyperplane decoupling expansion and Intermediate Gauge Field Integration.

ONLY To Make Contact With 8fdWay: MAKE APPEAL TO  $SU(3)_f$  GROUP When Re-Expressing Baryon Fields in PARTICLE BASIS.

An ORTHOGONAL TRANSFORMATION RELATES THE TWO BASIS.

IT is in the PARTICLE BASIS that the 2-Point Fcts get the Closest as POSSIBLE To DIAGONAL FORM!

FOR MESONS: NEED  $\mathcal{G}$ -parity  $\mathcal{G}_p$ .

$\mathcal{G}_p$  is a Composition of  $\mathcal{C}$  and Discrete  $SU(3)_f$  Symmetry of Permutations of Flavor Indices

## TASTE OF THE METHOD:

### Hilbert Space $\mathcal{H}$

With Our Parameters and SPIN Matrices, there is a QM **Hilbert Space  $\mathcal{H}$**  of Physical States

FREE FERMION: Dispersions Increase in Each  $p^i$

**Concentrate Only** On **BARYONS** For Simplicity:

$$\mathcal{H}_{odd} \equiv \mathcal{H}_o .$$

### **In Strong Coupling: By Polymer Expansions**

- ◇ Thermodynamic Limit of correlations (CF)  
EXISTS
- ◇ Truncated CF have exponential tree decay
- ◇ CF are Lattice Translational invariant and  
Extend to Analytic Fcts of  $\kappa$  and  $\beta$

## UNIT TRANSLATION Operators (Linear) in $\mathcal{H}$

$\check{T}_\mu, \mu = 0, 1, \dots, d$ , are commuting

$\check{T}_0$  is self-adjoint, with  $-1 \leq \check{T}_0 \leq 1$

$\check{T}_{j=1,\dots,d}$  are unitary;  $\check{T}_j = e^{iP^j}$

$\vec{P} = (P^1, \dots, P^d)$  is the Self-Adjoint MOMENTUM

Spectral points  $\vec{p} \in \mathbf{T}^d \equiv (-\pi, \pi]^d$

$\check{T}_0^2 \geq 0$ , defines the ENERGY OP.  $H \geq 0$  by  
 $\check{T}_0^2 = e^{-2H}$

A point in EM SPECTRUM with  $\vec{p} = \vec{0}$  is a MASS

### FEYNMAN-KAC (F-K) FORMULA

$G(\psi, \bar{\psi}, g), F(\psi, \bar{\psi}, g)$ , supported on  $u^0 = 1/2$ .

For  $x^0 > 0$ ,

$$(G, \check{T}_0^{x^0} \check{T}_1^{x^1} \check{T}_2^{x^2} \check{T}_3^{x^3} F)_{\mathcal{H}} = \langle [T_0^{x^0} T_1^{x^1} T_2^{x^2} T_3^{x^3} F] \Theta G \rangle,$$

$$\check{T}_0^{x^0} \check{T}_1^{x^1} \check{T}_2^{x^2} \check{T}_3^{x^3} = \int_{-1}^1 \int_{\mathbf{T}^d} [\lambda^0]^{x^0} e^{i\vec{\lambda}^0 \cdot \vec{x}} d_{\lambda^0} d_{\vec{\lambda}} \mathcal{E}(\lambda^0, \vec{\lambda})$$

$\mathcal{E}(\lambda^0, \vec{\lambda})$  is Product of Spectral Families for  $H, P^i$ ,  
 $i = 1, 2, 3$

## PARTICLE DETECTION

**HYPERPLANE EXPANSION:**  $\kappa \rightarrow \kappa_p$  in Hopping Term Between  $x^0 = p$  and  $x^0 = p + 1$ .

$$G_{LM}(x, y) = \langle L(x)M(y) \rangle$$

*Arbitrary* functions  $L$  and  $M$  with odd # of quarks.

Temporally Separated.  $\kappa \rightarrow \kappa_p$ ,  $x^0 \leq p < p + 1 \leq y^0$ .

$\kappa_p = 0$  **DISCONNECTS** the  $x^0$  and  $y^0$  hyperplanes since any link crossing the  $p$ -hyperplane is forbidden.

$\frac{d}{d\kappa_p}|_{\kappa_p=0}$  **FORCES CONNECTION.**

$G_{LM}(x, y)$  is jointly analytic in *all*  $\kappa_p$ 's.

**IMBALANCE** in # Quarks or by Gauge Integration,  $\frac{d^{r=0,1,2}}{d\kappa_p^r}|_{\kappa_p=0}$  Give **ZERO**.

### THIRD $\kappa_p$ Derivative

$$G_{LM}^{(3)}(x, y) = - \sum_{\vec{\gamma}, \vec{g}} \sum_{\vec{w}} G_{L\bar{\psi}_{\vec{\gamma}\vec{g}}}^{(0)}(x, (p, \vec{w})) \\ \times G_{\psi_{\vec{\gamma}\vec{g}}^3 M}^{(0)}((p+1, \vec{w}), y) + (\textit{Anti}B),$$

Notation:

$$\psi_{\vec{\gamma}\vec{f}}^3(x) = \frac{1}{6} \epsilon_{a_1 a_2 a_3} \psi_{a_1 \alpha_1 f_1}(x) \psi_{a_2 \alpha_2 f_2}(x) \psi_{a_3 \alpha_3 f_3}(x)$$

For CLOSURE (same type of correlations):

$$L = \psi_{\vec{\alpha}\vec{f}}^3, \quad M = \bar{\psi}_{\vec{\beta}\vec{h}}^3$$

$(\textit{Anti}B)$  vanishes for this choice.

This is How BARYON Fields Emerge NATURALLY  
As Gauge Invariant Fields! NO  $SU(3)_f$  was used!

REMARK: Important PRODUCT STRUCTURE!

## BARYON FIELDS:

With All Fields LOWER,  $\alpha_i = 3, 4$ , and at the same point (Tilde = presence/absence of bars):

$$\tilde{B}_{\vec{\alpha}\vec{f}} \equiv \frac{1}{n_{\vec{\alpha}\vec{f}}} \epsilon_{abc} \tilde{\psi}_{a\alpha_1 f_1} \tilde{\psi}_{b\alpha_2 f_2} \tilde{\psi}_{c\alpha_3 f_3},$$

$$\langle B_{\vec{\alpha}\vec{f}} \bar{B}_{\vec{\beta}\vec{h}} \rangle^{(0)} = -\frac{1}{(n_{\vec{\alpha}\vec{f}})^2} \text{per}(\delta_{\vec{\alpha}\vec{\beta}} \delta_{\vec{f}\vec{g}}),$$

**NORMALIZATION:** Such that the **2-BARYON FCT**

$$G_{\ell\ell'}(u, v) = \langle B_{\ell}(u) \bar{B}_{\ell'} \rangle \chi_{u^0 \leq v^0} - \langle \bar{B}_{\ell}(u) B_{\ell'} \rangle^* \chi_{u^0 > v^0},$$

is  $-1$  at  $\kappa = 0$ . Orthonormal at  $\kappa = 0$ .

**PRODUCT STRUCTURE:** ( $u^0 \neq v^0$ )

$$\begin{aligned} G_{\ell_1 \ell_2}^{(3)} &= -\sum_{\ell_3, \vec{w}} [ \\ &\quad G_{\ell_1 \ell_3}^{(0)}(u, (p, \vec{w})) G_{\ell_3 \ell_2}^{(0)}((p+1, \vec{w}), v) \chi_{u^0 < v^0} \\ &\quad + G_{\ell_1 \ell_3}^{(0)}(u, (p+1, \vec{w})) G_{\ell_3 \ell_2}^{(0)}((p, \vec{w}), v) \chi_{u^0 > v^0} ] \\ &\equiv -G^{(0)} \circ G^{(0)}. \end{aligned}$$

**SPECTRAL REPRESENTATION** for  $G$ : For  $u^0 \neq v^0$ , and  $\bar{B}_\ell \equiv \bar{B}_\ell(1/2, \vec{0})$ ,  $x \equiv u - v$ , since  $G_{\ell_1 \ell_2}(u, v) = G_{\ell_1 \ell_2}(u - v)$

$$G_{\ell_1 \ell_2}(x) = - \int_{-1}^1 \int_{\mathbb{T}^3} (\lambda^0)^{|x^0|-1} e^{-i\vec{\lambda} \cdot \vec{x}} \\ \times d_\lambda(\bar{B}_{\ell_1}, \mathcal{E}(\lambda^0, \vec{\lambda}) \bar{B}_{\ell_2})_{\mathcal{H}};$$

Here  $x \in \mathbb{Z}^4$ ,  $x^0 \neq 0$ .  $G$  is an even function of  $\vec{x}$ .

**FOURIER TRANSFORM:**

$$\tilde{G}_{\ell_1 \ell_2}(p) = \sum_{x \in \mathbb{Z}^4} G_{\ell_1 \ell_2}(x) e^{-ip \cdot x}$$

After Separating the Equal-Time Contribution, has the **Spectral Representation**

$$\tilde{G}_{\ell_1 \ell_2}(p) = \tilde{G}_{\ell_1 \ell_2}(\vec{p}) - (2\pi)^3 \int_{-1}^1 f(p^0, \lambda^0) d_{\lambda^0} \alpha_{\vec{p}, \ell_1 \ell_2}(\lambda^0),$$

where

$$d_{\lambda^0} \alpha_{\vec{p}, \ell_1 \ell_2}(\lambda^0) = \int_{\mathbb{T}^3} \delta(\vec{p} - \vec{\lambda}) d_{\lambda^0} d_{\vec{\lambda}}(\bar{B}_{\ell_1}, \mathcal{E}(\lambda^0, \vec{\lambda}) \bar{B}_{\ell_2})_{\mathcal{H}},$$

with  $f(x, y) \equiv (e^{ix} - y)^{-1} + (e^{-ix} - y)^{-1}$ , and we set  $\tilde{G}(\vec{p}) = \sum_{\vec{x}} e^{-i\vec{p} \cdot \vec{x}} G(x^0 = 0, \vec{x})$ .

**USE  $\tilde{G}_{\ell_1 \ell_2}(p)$  to DETECT PARTICLES.**

Particles correspond to **Singularities of  $\tilde{G}_{\ell_1 \ell_2}(p)$** , for **Imaginary  $p^0$** , and are Points in the E-M Spectrum.

$\mathcal{T}$ ,  $\mathcal{P}$  & Spectral Rep.:  $G$  is Self-Adjoint.

Theorem GLOBAL BOUND for  $\tilde{G}_{\ell_1 \ell_2}(p)$ :

$$|G_{\ell_1 \ell_2}(u, v)| < \leq \mathcal{O}(1) |\kappa|^{3|u-v|},$$

$\mathcal{O}(1) > 0$  uniform in  $\kappa$  and  $\ell$ 's.

Follows a Spectral MASS GAP of at least

$$-(3 - \epsilon) \ln \kappa, \quad 0 < \epsilon \ll 1$$

To GO HIGHER in Spectrum: Use Meromorphic extension of  $\tilde{G}(p)$ . For Fixed  $\vec{p}$  and  $\kappa$

$$\tilde{\Gamma}^{-1}(p) = \{\text{cof}[\tilde{\Gamma}(p)]\}^t / \det \tilde{\Gamma}(p),$$

$\tilde{\Gamma}(p)\tilde{G}(p) = 1$ ,  $\Gamma$  is Convolution Inverse of  $G$ .

Thus, the SINGULARITIES of  $\tilde{G}(p)$  Are in Zeroes of  $\det \tilde{\Gamma}(p)$ .

That  $\tilde{\Gamma}^{-1}(p)$  provides an extension of  $\tilde{G}(p)$  follows from the FASTER TIME FALLOFF of  $\Gamma$ .

PRODUCT STRUCTURE Is INSTRUMENTAL In Showing The Faster Falloff of  $\Gamma$  !

Define  $\Gamma$  by NEUMANN SERIES:

$$\begin{aligned}\Gamma &= (1 + G_d^{-1} G_n)^{-1} G_d^{-1} \\ &= \sum_{n=0}^{\infty} (-1)^n [G_d^{-1} G_n]^n G_d^{-1},\end{aligned}$$

$G = G_d + G_n$ ,  $G_d$  diagonal.

$G_d$  and  $G_d^{-1}$  bounded since  $G_d^{(0)} = -1$ .

Neumann Series CONVERGES: Recall from THM  $|G_n|$  is  $\mathcal{O}(\kappa^3)$ . Using This, we have

Theorem GLOBAL BOUND for  $\tilde{\Gamma}_{\ell_1 \ell_2}(p)$ :

$$|\Gamma_{\ell_1 \ell_2}(u, v)| \leq \mathcal{O}(1) |\kappa|^{3|\vec{u}-\vec{v}|} |\kappa|^{5(|u^0-v^0|-1)}, |u^0-v^0| \neq 0,$$

$\mathcal{O}(1) > 0$  uniform in  $\kappa$  and  $\ell$ 's

**Proof of Both Theorems:** Applications of the Hyperplane Decoupling Expansion. (Product Structure Enters in Essential and Instrumental Way for  $\Gamma$ ).

ANALYTICITY of  $\tilde{\Gamma}(p)$ : Strip  $|\Im p^0| \leq -(5-\epsilon) \ln \kappa$ .

DISPERSION CURVES:

$$\det \tilde{\Gamma}(p^0 = iw(\vec{p}), \vec{p}) = 0,$$

For FIXED  $\vec{p}$ , the Curves Are ISOLATED from the Rest of the Spectrum.

## NUMBER of SOLUTIONS: INTUITIVE ARGUMENT

Retain Only  $\mathcal{O}(\kappa^3)$  terms.

$$\tilde{G}_{\ell_1 \ell_2}(p) = [-1 - 2\kappa^3 \cos p^0 - \frac{\kappa^3}{4} \sum_{j=1,2,3} \cos p^j] \delta_{\ell_1 \ell_2} + \mathcal{O}(\kappa^6)$$

$$\tilde{\Gamma}_{\ell_1 \ell_2}(p) = [-1 + 2\kappa^3 \cos p^0 + \frac{\kappa^3}{4} \sum_{j=1,2,3} \cos p^j] \delta_{\ell_1 \ell_2} + \mathcal{O}(\kappa^6)$$

$\Gamma$  is Multiple of Identity Under This Approximation.

Dropping the  $\kappa^{\geq 4}$  terms,  $\det \tilde{\Gamma}(p)$  **FACTORIZES INTO 56 FACTORS**.

Let 
$$p_\ell^2 \equiv 2 \sum_{i=1}^3 (1 - \cos p^i).$$

EACH FACTOR Gives (Identical) DISP. CURVE

$$w(\vec{p}) \equiv w(\vec{p}, \kappa) = [-3 \ln \kappa - \frac{3\kappa^3}{4} + \frac{\kappa^3}{8} p_\ell^2] + \mathcal{O}(\kappa^6)$$

PARTICLE MASS:

$$M \equiv w(\vec{0}, \kappa) = [-3 \ln \kappa - \frac{3\kappa^3}{4}] + \mathcal{O}(\kappa^6)$$

OBS.: Solution in  $\Im m p^0$  runs out to Infinity as  $\kappa \searrow 0$ .

## SOLUTION Without Approximation: In Contrast to ABOVE INDIVIDUAL SPIN and ISOSPIN BASIS

### PARTICLE BASIS

In this Basis,  $G$  is more diagonal By Using:  $SU(3)_f$  and Other Symmetries as Charge Conjugation, Parity, Time Reflection. ANALYSIS Becomes SIMPLER.

Related to INDIVIDUAL Basis By ORTHOGONAL TRANSFORMATION. Orthonormal at  $\kappa = 0$ .

FASTER DECAY of  $\Gamma$  is MAINTAINED in PARTICLE Basis. (Isolated Dispersion Curves).

**PARTICLE Basis:** Elements Labelled by EIGHT-FOLD WAY Quantum Numbers and SPIN.

Total Isospin  $I$ ,  $z$ -component  $I_z$ , Hypercharge  $Y$ , Value of Quadratic Casimir  $c_2$  of  $SU(3)_f$ .  $c_3$  not needed! LABELS of Total Spin  $J$  and  $J_z$ .

**Orthogonality Relations** Ensure 2pf is DIAGONAL In ALL Quantum Numbers BUT Spin, for all  $\kappa > 0$ .

DECOMPOSITION of  $G$ : TWO *Identical*  $8 \times 8$  blocks ( $J = 1/2$ ) + FOUR *Identical*  $10 \times 10$  ( $J = 3/2$ ).

DECOMPOSITION of  $\Gamma$ : SAME Block Struct.

ADDITIONAL SYMMETRIES: Parity  $\mathcal{P}$ , Charge Conjugation  $\mathcal{C}$ , Discrete Spatial Rotations of  $\pi/2$  About  $e^3$  and Time Reversal  $\mathcal{T}$ .

$\tilde{\Gamma}(\vec{p})$  Is Simplified:  $\tilde{\Gamma}(\vec{p} = \vec{0})$  Is DIAGONAL.

$\tilde{\Gamma}(\vec{p})$  Still NOT DIAGONAL for  $\vec{p} \neq 0$ .

Here: For  $\vec{p} = \vec{0}$ ,  $\det \tilde{\Gamma}$  Factorizes. USE **AUXILIARY FCT Method, Rouché's Thm** To Get MASS Spectrum.

MASSES Given By Convergent Expansions in  $\kappa$ .

## NEW SYMMETRY: TIME REFLECTION T.

$\mathcal{PCT}$ , Further Diagonalizes  $\tilde{\Gamma}(\vec{p})$ .  $\mathcal{PCT}$  is nonlocal!

OCTET GETS DIAGONAL. DECUPLET GETS AS CLOSEST AS POSSIBLE TO DIAGONAL.

### PARENTHESIS ON SYMMETRIES

- Time Reflection  $T$ :  $\psi_\alpha(x) \rightarrow A_{\alpha\beta}\psi_\beta(-x^0, \vec{x})$ ,  $\bar{\psi}_\alpha(x) \rightarrow \bar{\psi}_\beta(-x^0, \vec{x})B_{\beta\alpha}$  where  $A = B = A^{-1} = \begin{pmatrix} 0 & -iI_2 \\ iI_2 & 0 \end{pmatrix}$ ,  $f(g_{xy}) \rightarrow f(g_{\bar{x}\bar{y}})$ , with  $\bar{z} = (-z^0, \vec{z})$ ;
- Time Reversal  $\mathcal{T}$ :  $\psi_\alpha(x) \rightarrow \bar{\psi}_\beta(x^t)A_{\beta\alpha}$ ,  $x^t \equiv (-x^0, \vec{x})$ ,  $\bar{\psi}_\alpha(x) \rightarrow B_{\alpha\beta}\psi_\beta(x^t)$ ,  $A = B = B^{-1} = \gamma^0$ ,  $f(g_{xy}) \rightarrow [f(g_{x^t y^t})]^*$ .

Composed Operation  $\mathcal{F}_s \equiv -i\mathcal{TCT}$  Gives Spin Flip (LOCAL) is a Symmetry. Leaves Invariant EACH Term in Action.  $\tilde{\psi}_1 \rightarrow \tilde{\psi}_2$ ,  $\tilde{\psi}_2 \rightarrow -\tilde{\psi}_1$ ,  $(12 \rightarrow 34)$ ,  $g_{xy} \rightarrow g_{xy}$

### END OF PARENTHESIS ON SYMMETRIES

**HYPERPLANE EXPANSION:** Fields Which Create Baryons are (with all fields at the same point). For the anti-baryons, *all* the fields must be unbarred.

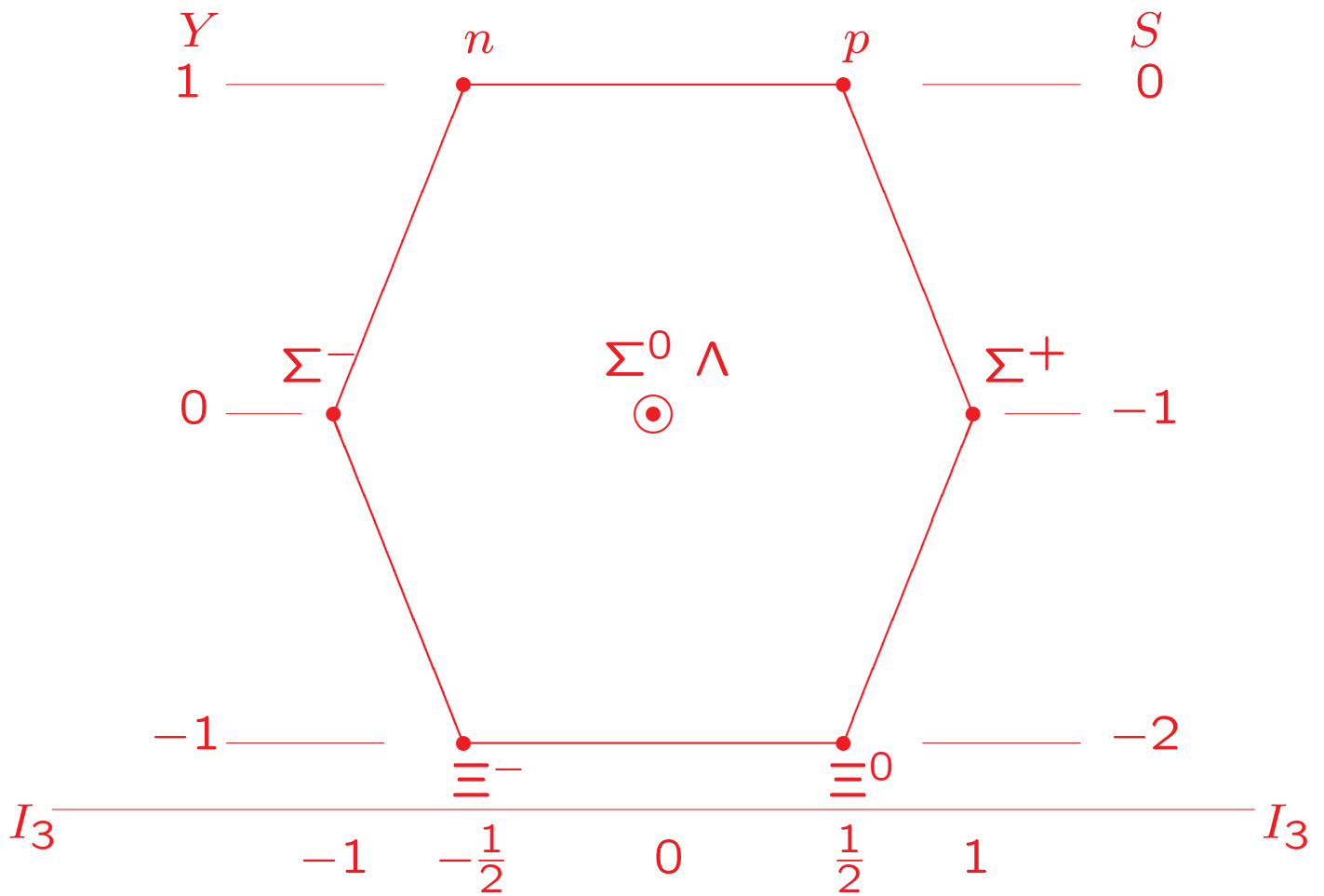
## OCTET

$$\left\{ \begin{array}{l} p_{\pm} = \frac{\epsilon_{abc}}{3\sqrt{2}} (\bar{\psi}_{a+u}\bar{\psi}_{b-d} - \bar{\psi}_{a+d}\bar{\psi}_{b-u})\bar{\psi}_{c\pm u}, \\ n_{\pm} = \frac{\epsilon_{abc}}{3\sqrt{2}} (\bar{\psi}_{a+u}\bar{\psi}_{b-d} - \bar{\psi}_{a+d}\bar{\psi}_{b-u})\bar{\psi}_{c\pm d}, \\ \Xi_{\pm}^0 = \frac{\epsilon_{abc}}{3\sqrt{2}} (\bar{\psi}_{a+u}\bar{\psi}_{b-s} - \bar{\psi}_{a+s}\bar{\psi}_{b-u})\bar{\psi}_{c\pm s}, \\ \Xi_{\pm}^{-} = \frac{\epsilon_{abc}}{3\sqrt{2}} (\bar{\psi}_{a+d}\bar{\psi}_{b-s} - \bar{\psi}_{a+s}\bar{\psi}_{b-d})\bar{\psi}_{c\pm s}, \\ \Sigma_{\pm}^{+} = \frac{\epsilon_{abc}}{3\sqrt{2}} (\bar{\psi}_{a+u}\bar{\psi}_{b-s} - \bar{\psi}_{a+s}\bar{\psi}_{b-u})\bar{\psi}_{c\pm u}, \\ \Sigma_{\pm}^0 = \frac{\epsilon_{abc}}{6} (2\bar{\psi}_{a\pm u}\bar{\psi}_{b\pm d}\bar{\psi}_{c\mp s} - \bar{\psi}_{a-u}\bar{\psi}_{b+d}\bar{\psi}_{c\pm s} - \\ \qquad \qquad \qquad \bar{\psi}_{a+u}\bar{\psi}_{b-d}\bar{\psi}_{c\pm s}), \\ \Sigma_{\pm}^{-} = \frac{\epsilon_{abc}}{3\sqrt{2}} (\bar{\psi}_{a+d}\bar{\psi}_{b-s} - \bar{\psi}_{a+s}\bar{\psi}_{b-d})\bar{\psi}_{c\pm d}, \\ \Lambda_{\pm} = \frac{\epsilon_{abc}}{2\sqrt{3}} (\bar{\psi}_{a+u}\bar{\psi}_{b-d} - \bar{\psi}_{a+d}\bar{\psi}_{b-u})\bar{\psi}_{c\pm s}, \end{array} \right.$$

Subindices  $\pm$  denote  $J_z = \pm\frac{1}{2}$ .

$n$ ,  $p$ ,  $\Xi^{-}$  and  $\Xi^0$  have total isospin  $I = 1/2$ ;  $\Sigma^{+}$ ,  $\Sigma^0$  and  $\Sigma^{-}$  have  $I = 1$  and  $\Lambda$  has  $I = 0$ .

$c_2 = 3$  for Each Member of the Octet.

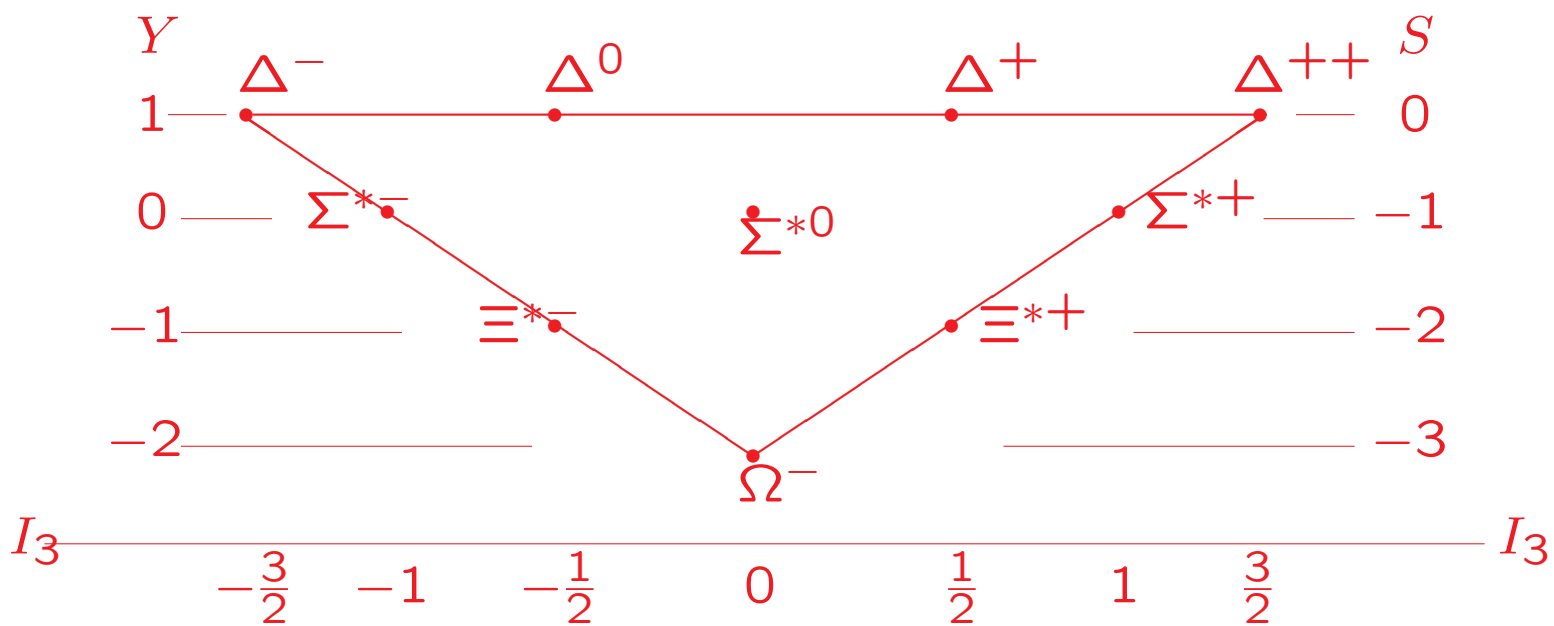


The particle total Hypercharge  $Y$ , Strangeness  $S$ , Total Isospin  $I$  and  $z$ -component of Total Isospin  $I_3$  are indicated.

# DECUPLET

$$\left\{ \begin{aligned}
 \Delta_{\frac{\pm 1}{2}}^+ &= \frac{\epsilon_{abc}}{6} (\bar{\psi}_{a\pm u} \bar{\psi}_{b\pm u} \bar{\psi}_{c\mp d} + 2\bar{\psi}_{a\pm u} \bar{\psi}_{b\mp u} \bar{\psi}_{c\pm d}), \\
 \Delta_{\frac{\pm 3}{2}}^+ &= \frac{\epsilon_{abc}}{2\sqrt{3}} \bar{\psi}_{a\pm u} \bar{\psi}_{b\pm u} \bar{\psi}_{c\pm d}, \\
 \Delta_{\frac{\pm 1}{2}}^0 &= \frac{\epsilon_{abc}}{6} (2\bar{\psi}_{a\pm u} \bar{\psi}_{b\pm d} \bar{\psi}_{c\mp d} + \bar{\psi}_{a\mp u} \bar{\psi}_{b\pm d} \bar{\psi}_{c\pm d}), \\
 \Delta_{\frac{\pm 3}{2}}^0 &= \frac{\epsilon_{abc}}{2\sqrt{3}} \bar{\psi}_{a\pm u} \bar{\psi}_{b\pm d} \bar{\psi}_{c\pm d}, \\
 \Delta_{\frac{\pm 1}{2}}^- &= \frac{\epsilon_{abc}}{2\sqrt{3}} \bar{\psi}_{a\pm d} \bar{\psi}_{b\pm d} \bar{\psi}_{c\mp d}, \\
 \Delta_{\frac{\pm 3}{2}}^- &= \frac{\epsilon_{abc}}{6} \bar{\psi}_{a\pm d} \bar{\psi}_{b\pm d} \bar{\psi}_{c\pm d}, \\
 \Delta_{\frac{\pm 1}{2}}^{++} &= \frac{\epsilon_{abc}}{2\sqrt{3}} \bar{\psi}_{a\pm u} \bar{\psi}_{b\pm u} \bar{\psi}_{c\mp u}, \\
 \Delta_{\frac{\pm 3}{2}}^{++} &= \frac{\epsilon_{abc}}{6} \bar{\psi}_{a\pm u} \bar{\psi}_{b\pm u} \bar{\psi}_{c\pm u}, \\
 \Sigma_{\frac{\pm 3}{2}}^{*+} &= \frac{\epsilon_{abc}}{2\sqrt{3}} \bar{\psi}_{a\pm u} \bar{\psi}_{b\pm u} \bar{\psi}_{c\pm s}, \\
 \Sigma_{\frac{\pm 1}{2}}^{*+} &= \frac{\epsilon_{abc}}{6} (\bar{\psi}_{a\pm u} \bar{\psi}_{b\pm u} \bar{\psi}_{c\mp s} + 2\bar{\psi}_{a\pm u} \bar{\psi}_{b\mp u} \bar{\psi}_{c\pm s}), \\
 \Sigma_{\frac{\pm 3}{2}}^{*0} &= \frac{\epsilon_{abc}}{6} \bar{\psi}_{a\pm u} \bar{\psi}_{b\pm d} \bar{\psi}_{c\pm s}, \\
 \Sigma_{\frac{\pm 1}{2}}^{*0} &= \frac{\epsilon_{abc}}{3\sqrt{2}} (\bar{\psi}_{a\pm u} \bar{\psi}_{b\pm d} \bar{\psi}_{c\mp s} + \bar{\psi}_{a\pm u} \bar{\psi}_{b\mp d} \bar{\psi}_{c\pm s} + \bar{\psi}_{a\mp u} \bar{\psi}_{b\pm d} \bar{\psi}_{c\pm s}), \\
 \Sigma_{\frac{\pm 3}{2}}^{*-} &= \frac{\epsilon_{abc}}{2\sqrt{3}} \bar{\psi}_{a\pm d} \bar{\psi}_{b\pm d} \bar{\psi}_{c\pm s}, \\
 \Sigma_{\frac{\pm 1}{2}}^{*-} &= \frac{\epsilon_{abc}}{6} (\bar{\psi}_{a\pm d} \bar{\psi}_{b\pm d} \bar{\psi}_{c\mp s} + 2\bar{\psi}_{a\pm d} \bar{\psi}_{b\mp d} \bar{\psi}_{c\pm s}), \\
 \Xi_{\frac{\pm 3}{2}}^{*0} &= \frac{\epsilon_{abc}}{2\sqrt{3}} \bar{\psi}_{a\pm u} \bar{\psi}_{b\pm s} \bar{\psi}_{c\pm s}, \\
 \Xi_{\frac{\pm 1}{2}}^{*0} &= \frac{\epsilon_{abc}}{6} (\bar{\psi}_{a\mp u} \bar{\psi}_{b\pm s} + 2\bar{\psi}_{a\pm u} \bar{\psi}_{b\mp s}) \bar{\psi}_{c\pm s}, \\
 \Xi_{\frac{\pm 3}{2}}^{*-} &= \frac{\epsilon_{abc}}{2\sqrt{3}} \bar{\psi}_{a\pm d} \bar{\psi}_{b\pm s} \bar{\psi}_{c\pm s}, \\
 \Xi_{\frac{\pm 1}{2}}^{*-} &= \frac{\epsilon_{abc}}{6} (\bar{\psi}_{a\mp d} \bar{\psi}_{b\pm s} + 2\bar{\psi}_{a\pm d} \bar{\psi}_{b\mp s}) \bar{\psi}_{c\pm s}, \\
 \Omega_{\frac{\pm 3}{2}}^- &= \frac{\epsilon_{abc}}{6} \bar{\psi}_{a\pm s} \bar{\psi}_{b\pm s} \bar{\psi}_{c\pm s}, \\
 \Omega_{\frac{\pm 1}{2}}^- &= \frac{\epsilon_{abc}}{2\sqrt{3}} \bar{\psi}_{a\pm s} \bar{\psi}_{b\pm s} \bar{\psi}_{c\mp s},
 \end{aligned} \right.$$

$c_2 = 6$  for Each Member of the Decuplet.



$e$ —Charges verify Gell'Mann-Nishijima relation

$$Q = I_3 + \frac{Y}{2}$$

For DECUPLET, In  $\tilde{\Gamma}$  ( $\vec{p} \neq \vec{0}$ ): Four  $10 \times 10 \rightarrow$   
 Ten  $4 \times 4$  Blocks With Fixed  $J_z$ . Multiplicity Two  
 Eigenvalues ( $a, b \in \mathbb{R}$ )

$$\begin{pmatrix} a & 0 & c & d \\ 0 & a & \bar{d} & -\bar{c} \\ \bar{c} & d & b & 0 \\ \bar{d} & -c & 0 & b \end{pmatrix},$$

1, 2, 3, 4 label  $J_z = 3/2, -3/2, 1/2, -1/2$ , respectively.

We Cast the Problem of Determining Dispersion  
 Curves and Masses into Framework of the **ANA-  
 LYTIC IMPLICIT FCT THM**

To obtain  $M$  up to and including order  $\kappa^6$ , we must  
 compute the values of the  $\kappa^{3r+6}$  contributions to  
 $\Gamma(x^0 = r\epsilon e^0, \vec{x})$  for  $r = 0, |\vec{x}| \leq 2$ ;  $r = 1, |\vec{x}| \leq 2$ ;  
 $r = 2, |\vec{x}| \leq 1$ ;  $r = 3, 4, \vec{x} = \vec{0}$ .

Need  $G$  for the above points and also the  $\kappa^{3r+3}$   
 contributions to  $G(x)$  for  $x = r\epsilon e^0 + \epsilon' e^j$ ,  $r = 3, 4$ ,  
 as They are Needed in Neumann Series.

SHORT DISTANCE behavior of  $G(x)$  and  $\Gamma(x)$ :  
 ( $c$ 's,  $c'$ 's,  $d$ 's and  $d'$ 's are computable  $\kappa$  and spin independent constants.)

Decuplet 2pf. In this case, **FOR**  $G_{r_1 r_2}(x)$ , we have:

- $[-1 + c_8 \kappa^8 + \mathcal{O}(\kappa^9)] \delta_{r_1 r_2}$  ,  $x = 0$  ;
- $(-\kappa^3 + c_9 \kappa^9) \delta_{r_1 r_2} + \mathcal{O}(\kappa^{11})$  ,  $x = \epsilon e^0$  ;
- $-\frac{1}{8} \kappa^3 \delta_{r_1 r_2} + \mathcal{O}(\kappa^9)$  ,  $x = \epsilon e^j$  ;
- $[(-\kappa^6 + c_{12} \kappa^{12}) \delta_{r_1 r_2} + \mathcal{O}(\kappa^{13})] \delta_{\mu 0} + [-\frac{1}{8} \kappa^6 \delta_{r_1 r_2} + \mathcal{O}(\kappa^{10})] \delta_{\mu j}$  ,  
 $x = 2\epsilon e^\mu$  ;
- $(\frac{1}{16} \delta_{r_1 \frac{3}{2}} - \frac{1}{16} \delta_{r_1 \frac{1}{2}}) \kappa^6 \delta_{r_1 r_2} + \mathcal{O}(\kappa^{10})$  ,  $x = \epsilon e^1 + \epsilon' e^2$  ;
- $(-\frac{1}{32} \delta_{r_1 \frac{3}{2}} + \frac{1}{32} \delta_{r_1 \frac{1}{2}}) \kappa^6 \delta_{r_1 r_2} + \mathcal{O}(\kappa^{10})$  ,  $x = \epsilon e^1 + \epsilon' e^3, \epsilon e^2 + \epsilon' e^3$  ;
- $-\frac{1}{4} \kappa^6 \delta_{r_1 r_2} + \mathcal{O}(\kappa^{10})$  ,  $x = \epsilon e^0 + \epsilon' e^j$  ;
- $\frac{17}{64} \kappa^9 \delta_{r_1 r_2} + \mathcal{O}(\kappa^{10})$  ,  $x = \epsilon e^0 + 2\epsilon' e^j$  ;
- $-\frac{3}{8} \kappa^9 \delta_{r_1 r_2} + \mathcal{O}(\kappa^{13})$  ,  $x = 2\epsilon e^0 + \epsilon' e^j$  ;
- $(\frac{3}{32} \delta_{s \frac{3}{2}} - \frac{5}{32} \delta_{s \frac{1}{2}}) \kappa^9 \delta_{r_1 r_2} + \mathcal{O}(\kappa^{10})$  ,  $x = \epsilon e^0 + \epsilon' e^1 + \epsilon'' e^2$  ;
- $(-\frac{3}{32} \delta_{s \frac{3}{2}} + \frac{1}{32} \delta_{s \frac{1}{2}}) \kappa^9 \delta_{r_1 r_2} + \mathcal{O}(\kappa^{10})$  ,  $x = \epsilon e^0 + \epsilon' e^i + \epsilon'' e^3, i = 1, 2$  ;
- $(-\kappa^9 + c_{15} \kappa^{15}) \delta_{r_1 r_2} + \mathcal{O}(\kappa^{16})$  ,  $x = 3\epsilon e^0$  ;
- $d_{12} \kappa^{12} \delta_{r_1 r_2} + \mathcal{O}(\kappa^{13})$  ,  $x = 3\epsilon e^0 + \epsilon' e^j$  ;
- $(-\kappa^{12} + c_{18} \kappa^{18}) \delta_{r_1 r_2} + \mathcal{O}(\kappa^{19})$  ,  $x = 4\epsilon e^0$  ;
- $d_{15} \kappa^{15} \delta_{r_1 r_2} + \mathcal{O}(\kappa^{16})$  ,  $x = 4\epsilon e^0 + \epsilon' e^j$  ;

## FOR $\Gamma_{r_1 r_2}(x)$

- $[-1 - \frac{67}{32}\kappa^6 + \mathcal{O}(\kappa^8)]\delta_{r_1 r_2}$  ,  $x = 0$  ;
- $(\kappa^3 + c'_9 \kappa^9)\delta_{r_1 r_2} + \mathcal{O}(\kappa^{10})$  ,  $x = \epsilon e^0$  ;
- $\frac{1}{8}\kappa^3 \delta_{r_1 r_2} + \mathcal{O}(\kappa^9)$  ,  $x = \epsilon e^j$  ;
- $[c'_{12}\kappa^{12}\delta_{r_1 r_2} + \mathcal{O}(\kappa^{13})] \delta_{\mu 0} + [\frac{7}{64}\kappa^6 \delta_{r_1 r_2} + \mathcal{O}(\kappa^{10})] \delta_{\mu j}$  ,  $x = 2\epsilon e^\mu$  ;
- $(-\frac{3}{32}\delta_{r_1 \frac{3}{2}} + \frac{1}{32}\delta_{r_1 \frac{1}{2}})\kappa^6 \delta_{r_1 r_2} + \mathcal{O}(\kappa^{10})$  ,  $x = \epsilon e^1 + \epsilon' e^2$  ;
- $(0 \delta_{r_1 \frac{3}{2}} - \frac{1}{16}\delta_{r_1 \frac{1}{2}})\kappa^6 \delta_{r_1 r_2} + \mathcal{O}(\kappa^{10})$  ,  $x = \epsilon e^1 + \epsilon' e^3, \epsilon e^2 + \epsilon' e^3$  ;
- $\mathcal{O}(\kappa^{10})$  ,  $x = \epsilon e^0 + \epsilon' e^j$  ;
- $\mathcal{O}(\kappa^{10})$  ,  $x = \epsilon e^0 + 2\epsilon' e^j$  ;
- $\mathcal{O}(\kappa^{13})$  ,  $x = 2\epsilon e^0 + \epsilon' e^j$  ;
- $\mathcal{O}(\kappa^{10})$  ,  $x = \epsilon e^0 + \epsilon' e^i + \epsilon'' e^{j>i}$  ;
- $c'_{15}\kappa^{15}\delta_{r_1 r_2} + \mathcal{O}(\kappa^{16})$  ,  $x = 3\epsilon e^0$  ;
- $c'_{18}\kappa^{18}\delta_{r_1 r_2} + \mathcal{O}(\kappa^{19})$  ,  $x = 4\epsilon e^0$  ;

**Octet 2pf:** SAME FOR  $x = r\epsilon e^\mu$ ,  $r = 1, 2, 3, 4$ ,  $\epsilon e^0 + \epsilon' e^j$ ,  $\epsilon e^0 + 2\epsilon' e^j$ ,  $2\epsilon e^0 + \epsilon' e^j$ ,  $3\epsilon e^0 + \epsilon' e^j$ ,  $4\epsilon e^0 + \epsilon' e^j$ .

For  $x = \epsilon e^i + \epsilon' e^j$ ,  $x = \epsilon e^0 + \epsilon' e^i + \epsilon'' e^j$ , for  $ij = 12, 13, 23$ , the results are different (Former is **RESPONSIBLE FOR MASS SPLITTING!**)

$$G_{r_1 r_2} = -\frac{1}{16}\kappa^6 \delta_{r_1 r_2} + \mathcal{O}(\kappa^7), \quad \Gamma_{r_1 r_2} = \frac{1}{32}\kappa^6 \delta_{r_1 r_2} + \mathcal{O}(\kappa^7),$$

$$G_{r_1 r_2} = -\frac{5}{32}\kappa^9 \delta_{r_1 r_2} + \mathcal{O}(\kappa^{10}), \quad \Gamma_{r_1 r_2} = \mathcal{O}(\kappa^{10}).$$

Continuing: FOR OCTET, Fix  $\kappa$  and  $\vec{p}$ . Apply a **ROUCHÉ's theorem** Argument. (Recall  $\Gamma = \text{const } I$ ).

Find OCTET Dispersion Curve

$$w_o(\vec{p}) \equiv w(\vec{p}, \kappa) = \left[ -3 \ln \kappa - 3\kappa^3/4 + p_\ell^2 \kappa^3/8 \right] + r_o(\vec{p}),$$

$r_o(\vec{p})$  is of  $\mathcal{O}(\kappa^6)$  and is jointly analytic in each  $p^i$  and  $\kappa$ .

FOR DECUPLET: In Terms of  $4 \times 4$ : Two-By-Two Dispersion Curves  $w_d(\vec{p})$  Are Determined By ( $\vec{p}$  Dependence Omitted in  $\tilde{\Gamma}_{ij}$ )

$$\frac{1}{2} [\tilde{\Gamma}_{11} + \tilde{\Gamma}_{33}] \pm \sqrt{\frac{1}{4} [\tilde{\Gamma}_{11} - \tilde{\Gamma}_{33}]^2 + |\tilde{\Gamma}_{13}|^2 + |\tilde{\Gamma}_{14}|^2} = 0.$$

Due to SQUARE ROOT, and Analyticity Difficulties, the Auxiliary Function Method is NOT DIRECTLY Applicable.

Rouché's argument can be applied and shows the existence of Exactly Four solutions.

Taking the union of these solutions for all  $\vec{p}$  It Does NOT tell us How to Decompose Them Into Curves.

TO CONSTRUCT DISPERSION CURVES: FIXED-POINT Method Is Sketched in Our Previous Reference.

That each Dispersion Curve is Isolated In ALL  $\mathcal{H}_0$   
Up To Near the Meson-Baryon Threshold Follows from a SUBTRACTION METHOD. (Omitted Here!)

SHORT DISTANCE BHV: CORRECTS OUR PREVIOUS: TWO-FLAVOR RESULTS. BS Are OK!

BOSONIC EIGHTFOLD WAY: AVAILABLE SOON.

MESONIC EIGHTFOLD WAY: On the Way, With A.F. Neto.

## RELATED PUBLICATIONS:

- 1)** *Understanding Baryons From First Principles*, Phys. Rev. D67, 017501 (2003);
- 2)** *On Baryon-Baryon Bound States in a  $2+1$  Lattice QCD Model*, Phys. Rev. D68, 037501 (2003);
- 3)** *Existence of Baryons, Baryon Spectrum and Mass Splitting in Strong Coupling Lattice QCD*, Commun. Math. Phys. 245, 383 (2004);
- 4)** *Existence of Mesons and Mass Splitting in Strong Coupling Lattice QCD*, J. Math. Phys. 45, 628 (2004);
- 5)** *Meson-Meson Bound States in a  $(2+1)$ -dimensional Strongly Coupled Lattice QCD Model*, Phys. Rev. D69, 097501 (2004);
- 6)** *A Baryon-Baryon Bound State in a  $2+1$  Lattice QCD Model With Two Flavors and Strong Coupling*, Phys. Rev. D71, 017503 (2005);
- 7)** *A Meson-Meson Bound State in a  $2+1$  Lattice QCD Model With Two Flavors & Strong Coupling*, Phys. Rev. D72, 034057 (2005) , pp. 1-18;
- 8)** *Baryon-Baryon Bound States From First Principles in  $3+1$  Lattice QCD With 2 Flavors & Strong Coupling*, Phys. Lett. B243, 109 (2006).
- 9)** *Baryon-Baryon Bound States in Strongly Coupled Lattice QCD*, Phys. Rev. D75, 074503 (2007), pp. 1-29.
- 10)** *Towards A More Complete Understanding of Nuclear Forces*, In Festschrift for Roberto Salmeron, R. Aldrovandi et al. eds., AIAFEX, RJ, 2003.
- 11)** *Hadron-Hadron Bound States*, AIP Conference Proceedings, J.E.Ribeiro ed., QCHS-7, Açores, Portugal, 2006.

ALL PAPERS AVAILABLE AT THE SITE: [www.icmc.usp.br/~veiga](http://www.icmc.usp.br/~veiga)

- 12)** by A.F. Neto: *Meson-Baryon Bound States in a  $2+1$ -dimensional Strongly Coupled Lattice QCD Model*, Phys. Rev. D70, 037502 (2004);
- 13)** by A.F. Neto: *A Meson-Baryon Bound State in a  $2+1$  Lattice QCD Model With Two Flavors and Strong Coupling*, Phys. Scripta (2007).