Meson-Baryon Scattering and Resonances with Strangeness –1

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1. Introduction. Interest.
2. UCHPT
3. S-Wave, S=-1 Meson-Baryon Scattering
4. Scattering
5. Spectroscopy
6. Conclusions
1. INTRODUCTION. INTEREST.

\[ \mathcal{K}N \] SCATTERING, TEN TWO BODY COUPLED CHANNELS:

\[ \pi^0 \Lambda \pi^0 \Sigma^0 \pi^- \Sigma^+ \pi^+ \Sigma^- K^- p \bar{K}^0 p \eta \Lambda \pi^0 \Sigma^0 K^0 \Xi^0 K^- \Xi^+ \]

\[ 8 \times 8 = 1 + 8_s + 8_a + 10 + 10 + 27 \]

The representations 1, 8_s, 8_a and 27 (exotic) give rise to resonances.

• Potential Models, Quark Models, (Chiral) Bag Models, etc
• CHPT+Unitarization (UCHPT)

Kaiser, Siegel, Weise NPA594,325(’95)
Oset, Ramos NPA635,99(’98)
Meissner, JAO PLB500,263(’01)
Lutz, Kolomeitsev NPA700,193(’02);
Garcia-Recio, Lutz, Nieves, PLB582,49 (’04);
Borasoy, Nissler, Weise PRL94,213401 (05), EPJA25,79(’05)
Borasoy, Meissner, Nissler, PRC74,055201(’06), etc
Renewed interest with the precise measurement by DEAR Coll. of strong shift and width of kaonic hydrogen 1s energy level

\[ K^- p \rightarrow \left\{ \begin{array}{c} \pi^0 \Lambda, \pi^\mp \Sigma^\pm \text{[strong]} \\ \Sigma \pi \gamma, \Sigma \pi e^+ e^-, \Sigma \gamma, \ldots < 1\% \end{array} \right. \]

Unstable

DEAR:
\[ \Delta E = 193 \pm 37 \text{(stat.)} \pm 6 \text{(syst.)} \text{ eV} \]
\[ \Gamma = 249 \pm 111 \text{(stat.)} \pm 39 \text{(syst.)} \text{ eV} \]

KEK:
\[ \Delta E = 323 \pm 63 \pm 11 \text{ eV} \]
\[ \Gamma = 407 \pm 208 \pm 100 \text{ eV}. \]

G. Beer et al., PRL94,212302(05)

Unstable Meissner,Raha,Rusetsky EPJ C35,349(04);
Borasoy,Nissler,Weise PRL94,213401(05),
EPJA25,79(05) pointed out a possible inconsistency between DEAR and previous scattering data. SU(3) chiral dynamics results agree with KEK but disagrees with the factor 2 more precise DEAR measurement.
\[ E_{1s} = E_{1s}^{em} + \epsilon_{1s}, \quad \epsilon_{1s} \text{ is complex} \]

Deser Formula \[ \epsilon_{1s} = -2\alpha^3 \mu_C^2 T_{K^-p} \]

Precise knowledge \[ \epsilon_{1s} \leftrightarrow T_{K^-p} \text{ at threshold} \]

Meissner, Raha, Rusetsky EPJ C35, 349 (’04) include isospin breaking correction on the Deser formula up to an including \( O(\alpha^4, \alpha^3(m_u-m_d)) \) \( \sim 9\% \)

Cusp Effect: \( \sim 50\% \) \( O(d^{1/2}) \)

Coulomb Effects: \( \sim 10 - 15\% \)

\[ \Delta E_{1s} = \frac{\alpha^3 \mu_C^3}{2\pi M_{K^+}} T_{K^-p} \left\{ 1 - \frac{\alpha \mu_C^2 s_1(\alpha)}{4\pi M_{K^+}} T_{K^-p} \right\} \]

Vacuum Polarization: \( \sim 1\% \)

\( d \sim a \sim m_u-m_d \)

DEAR/SIDDHARTA Coll. Aims to finally measure it up to eV level, a few percent (nowadays the precision is 20\%).

http://www.lnf.infn.it/esperimenti/dear/DEAR_RPR.pdf
1. A systematic scheme able to be applied when the interactions between the hadrons are not perturbative (even at low energies) but still using Chiral Lagrangians and Chiral Perturbation Theory (though to be valid only for low energies).
   - Meson-meson processes, both scattering, production and decays, involving $I=0,1,1/2$ S-waves, $J^{PC}=0^{++}$ (vacuum quantum numbers)
     - $I=0$ $σ(500)$ - really low energies
     - Not low energies. More resonances come up: $I=0$ $f_0(980)$, $f_0(1370)$, $f_0(1500)$, $f_0(1710)$, $f_0(1790)$; $I=1$ $a_0(980)$, $a_0(1450)$; $I=1/2$ $κ(700)$, $K^*_0(1430)$
     - Related by SU(3) symmetry.
   - Processes involving $S=–1$ (strangeness) S-waves meson-baryon interactions $J^P=1/2^-$. $I=0$ $Λ(1405)$‘s, $Λ(1670)$, $Λ(1800)$; $I=1$ possible $Σ(1430)$, $Σ(1620)$, $Σ(1750)$
     - One also finds other resonances in $S=–2$, 0, +1, and even with $I=2$...
   - Processes involving scattering or production of, particularly, the lowest Nucleon-Nucleon partial waves like the $^1S_0$, $^3S_1$ or P-waves. Deuteron, Nuclear matter, Nuclei.

2. Then one can handle with:
   - Strongly interacting coupled channels.
   - Large unitarity loops.
   - Resonances.

2. UNITARY CHPT (UCHPT).
In all these examples the unitarity cut (sum over the unitarity bubbles) is enhanced.

UCHPT makes an expansion of an ``Interacting Kernel'' from the appropriate EFT and then the unitarity cut is fulfilled to all orders (non-perturbatively)
• Above threshold and on the real axis (physical region), a partial wave amplitude must fulfill because of unitarity:

\[
\text{Im } T_{ij} = \sum_k T_{ik} \rho_k T_{kj}^* \quad \rightarrow \quad \text{Im } T_{ij}^{-1} = -\rho_i \delta_{ij}
\]

Unitarity Cut

\[ W = \sqrt{s} \]

We perform a dispersion relation for the inverse of the partial wave (the discontinuity when crossing the unitarity cut is known)

\[
T_{ij}^{-1} = R_{ij}^{-1} + \delta_{ij} \left( g(s_0)_i - \frac{s - s_0}{\pi} \int_{s_{\text{th},i}}^{\infty} \frac{\rho(s')_i}{(s' - s - i0^+)(s' - s_0)} ds' \right)
\]

The rest

\[ g(s)_i : \text{ Single unitarity bubble} \]
\[ g(s) = \frac{1}{4\pi^2} \left( a_{SL} + \sigma(s) \log \frac{\sigma(s) - 1}{\sigma(s) + 1} \right) \]

\[ T = \left[ R^{-1} + g(s) \right]^{-1} = [I + R \cdot g]^{-1} \cdot R \]

1. \( T \) obeys a CHPT/alike expansion
\[ g(s) = \frac{1}{4\pi^2} \left( a_{SL} + \sigma(s) \log \frac{\sigma(s) - 1}{\sigma(s) + 1} \right) \]

\[ T = \left[ R^{-1} + g(s) \right]^{-1} = \left[ I + R \cdot g \right]^{-1} \cdot R \quad \sigma(s) = \frac{2q}{\sqrt{s}} \]

1. \( T \) obeys a CHPT/alike expansion \( T = T_1 + T_2 + T_4 + \ldots \)
2. \( R \) is fixed by matching algebraically with the CHPT/alike expressions of \( T \), \( R = R_1 + R_2 + R_3 + \ldots \)

In doing that, one makes use of the CHPT/alike counting for \( g(s) \)

The counting of \( R(s) \) is consequence of the known ones of \( g(s) \) and \( T(s) \)
\[ g(s) = \frac{1}{4\pi^2} \left( a_{\text{SL}} + \sigma(s) \log \frac{\sigma(s) - 1}{\sigma(s) + 1} \right) \]

\[ T = \left[ R^{-1} + g(s) \right]^{-1} = [I + R \cdot g]^{-1} \cdot R \quad \sigma(s) = \frac{2g}{\sqrt{s}} \]

1. \( T \) obeys a CHPT/alike expansion \( T = T_1 + T_2 + T_4 + \ldots \)

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The counting of \( R(s) \) is consequence of the known ones of \( g(s) \) and \( T(s) \)

3. The CHPT/alike expansion is done to \( R(s) \). Crossed channel dynamics is included perturbatively.
3. S-WAVE, $S=-1$ MESON-BARYON SCATTERING

\[ T = \left[ R^{-1} + g(s) \right]^{-1} = \left[ I + R \cdot g(s) \right]^{-1} \cdot R(s) \]

$g(s)$ is $O(p)$

$R = R_1 = T_1$ \hspace{1cm} LEADING ORDER, $O(p)$

$R = R_1 + R_2 = T_1 + T_2$ \hspace{1cm} NLO, $O(p^2)$

$R_2 = T_2$, for $O(p^3)$ and higher $R_n \neq T_n$

Seagull \hspace{1cm} Direct \hspace{1cm} Crossed

$O(p)$ \hspace{1cm} $O(p^2)$
\(\mathcal{O}(p)\) and \(\mathcal{O}(p^2)\) Chiral Lagrangians

\[
\mathcal{L}_1 = \langle i \bar{B} \gamma^\mu [D_\mu, B] \rangle - m_0 \langle \bar{B} B \rangle \\
+ \frac{D}{2} \langle \bar{B} \gamma^\mu \gamma_5 \{u_\mu, B\} \rangle + \frac{F}{2} \langle \bar{B} \gamma^\mu \gamma_5 [u_\mu, B] \rangle,
\]

\(D = 0.8, F = 0.46, m_0 = \text{proton mass in SU(3) chiral limit}\)

\[
\mathcal{L}_2 = b_0 \langle \bar{B} B \rangle \langle \chi_+ \rangle + b_D \langle \bar{B} \{\chi_+, B\} \rangle + b_F \langle \bar{B} [\chi_+, B] \rangle \\
+ b_1 \langle \bar{B} [u_\mu, [u^\mu, B]] \rangle + b_2 \langle \bar{B} \{u_\mu, \{u^\mu, B\}\} \rangle \\
+ b_3 \langle \bar{B} \{u_\mu, [u^\mu, B]\} \rangle + b_4 \langle \bar{B} B \rangle \langle u_\mu u^\mu \rangle + \cdots.
\]

\[
U = e^{i \Phi/f}, U = u^2, u = e^{i \Phi/2f}, u_\mu = u^\dagger (\partial_\mu U) u^\dagger \\
\chi_+ = u^\dagger \chi u^\dagger + u \chi^\dagger u, \chi = \begin{pmatrix} m_n^2 & 0 & 0 \\ 0 & m_n^2 & 0 \\ 0 & 0 & 2m_K^2 - m_n^2 \end{pmatrix} \\
\Phi = \begin{pmatrix} \pi^0 + \frac{\eta}{\sqrt{6}} & \pi^+ & K^+ \\ \frac{\pi^0}{\sqrt{2}} + \frac{\eta}{\sqrt{6}} & 0 & K^0 \\ K^- & \bar{K}^0 & -\frac{2}{\sqrt{6}} \eta \end{pmatrix} \\
B = \begin{pmatrix} \frac{\Sigma^0}{\sqrt{2}} + \frac{\Lambda}{\sqrt{6}} & \Sigma^+ & p \\ \Sigma^- & -\frac{\Sigma^0}{\sqrt{2}} + \frac{\Lambda}{\sqrt{6}} & n \\ \Xi^- & \Xi^0 & -\frac{2}{\sqrt{6}} \Lambda \end{pmatrix}
\]
From these Lagrangians one calculates the $O(p)$, $R_1$, and $O(p^2)$, $R_2$, CHPT meson-baryon scattering amplitudes. $R = R_1 + R_2$.

**EXPERIMENTAL DATA**

S= – 1 meson-baryon sector is plenty of data

Good ground test for SU(3) chiral dynamics, where very strong SU(3) breaking effects due to the explicit presence of mesons/baryons with strangeness– Explicit breaking of chiral symmetry

Important isospin breaking effects due to cusps at thresholds, we work with the physical basis.
1) CROSS SECTIONS:

\[ K^- p \to K^- p , \bar{K}^0 n , \pi^+ \Sigma^- , \pi^- \Sigma^+ , \pi^0 \Sigma^0 , \pi^0 \Lambda \]

In the fit we include data from threshold up to \( p_{lab} = 0.2 \) GeV.

2) Precisely Measured Ratios

\[
\gamma = \frac{\sigma(K^- p \to \pi^+ \Sigma^-)}{\sigma(K^- p \to \pi^- \Sigma^+)} = 2.36 \pm 0.04 ,
\]

\[
R_c = \frac{\sigma(K^- p \to \text{charged particles})}{\sigma(K^- p \to \text{all})} = 0.664 \pm 0.011 ,
\]

\[
R_n = \frac{\sigma(K^- p \to \pi^0 \Lambda)}{\sigma(K^- p \to \text{all neutral states})} = 0.189 \pm 0.015 ,
\]
3) \( \pi \Sigma \) EVENT DISTRIBUTION AROUND THE \( \Lambda(1405) \) RESONANCE

4) DEAR and KEK STRONG SHIFT AND WIDTH OF KAONIC HYDROGEN

5) WE ALSO CONSTRAINT OUR FITS CALCULATING AT O(p^2) IN PURE BARYON CHPT SEVERAL PION-NUCLEON OBSERVABLES, WHERE CHPT EXPANSION IS RELIABLE:

\[
\begin{align*}
\sigma_{\pi N} &= -2m_\pi^2(2b_0 + b_F + b_D) , \\
a_{0+}^+ &= \frac{m_\pi^2}{2\pi f^2} \left(-2b_1 + b_2 + b_3 - \frac{g_A^2}{8m}\right), \\
m_0 &= m_p + 4m_K^2(b_0 + b_D - b_F) + 2m_\pi^2(b_0 + 2b_F).
\end{align*}
\]

\( \sigma_{\pi N} = 20, 30, 40 \text{ MeV} \) (45\( \pm \)8 from Gasser, Leutwyler, Sainio PLB253,252 (’91), higher order corrections \( \pm 10 \) MeV Gasser, AP254,192(’97))

\( m_0 = 0.7 \) or 0.8 GeV

\( a_{0+}^+ = (-1\pm1) m_\pi \ 10^{-2} \) Exp. \(-0.25\pm0.49\) Schroder et al., PLB469,25(’99) and expected higher order corrections \(+m_\pi \ 10^{-2}\) from unitarity Bernard et al. PLB309,421(’93).
6) $\sigma(K^-p \rightarrow \eta\Lambda)$ cross-section
On top of the $\Lambda(1670)$ resonance.

7) $\sigma(K^-p \rightarrow \Sigma^0\pi^0\pi^0)$
total cross-section and event distribution.

6) and 7) measured by the Crystall-Barrell Collaboration, 2001 and 2004, respectively. Precise experimental data.

8) $\Lambda\pi$ P- and S-wave phase shift difference at the $\Xi^-$ mass $\delta_P - \delta_S = (4.6 \pm 1.4)^\circ$.
HyperCP Collaboration (2004)
For the production of the process $K^- p \rightarrow \pi^0 \pi^0 \Sigma^0$ we take as the production vertex the mechanism:

It is largely enhanced due to the almost on-shell character of the intermediate proton

4. RESULTS

Two classes of fits A,B with 1s kaonic hydrogen $\Delta E$ and $\Gamma$:
A: Around DEAR (The fits are numerically more stable)
B: Away from DEAR.
Reproduction of the data by the new A-type fits (agree with DEAR) of JAO, EPJA28,63(’06)
Three b’s are fixed in terms of the others from the $O(p^2)$ constraints

<table>
<thead>
<tr>
<th>$\sigma_{\pi N}$</th>
<th>20$^*$</th>
<th>30$^*$</th>
<th>40$^*$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\gamma$</td>
<td>2.36</td>
<td>2.36</td>
<td>2.37</td>
</tr>
<tr>
<td>$R_c$</td>
<td>0.629</td>
<td>0.628</td>
<td>0.628</td>
</tr>
<tr>
<td>$R_n$</td>
<td>0.168</td>
<td>0.171</td>
<td>0.173</td>
</tr>
<tr>
<td>$\Delta E$ (eV)</td>
<td>194</td>
<td>192</td>
<td>192</td>
</tr>
<tr>
<td>$\Gamma$ (eV)</td>
<td>324</td>
<td>302</td>
<td>270</td>
</tr>
<tr>
<td>$\Delta E_D$ (eV)</td>
<td>204</td>
<td>204</td>
<td>207</td>
</tr>
<tr>
<td>$\Gamma_D$ (eV)</td>
<td>361</td>
<td>338</td>
<td>305</td>
</tr>
<tr>
<td>$a_{K-p}$ (fm)</td>
<td>$-0.49 + i 0.44$</td>
<td>$-0.49 + i 0.41$</td>
<td>$-0.50 + i 0.37$</td>
</tr>
<tr>
<td>$a_0$ (fm)</td>
<td>$-1.07 + i 0.53$</td>
<td>$-1.04 + i 0.50$</td>
<td>$-1.02 + i 0.45$</td>
</tr>
<tr>
<td>$a_1$ (fm)</td>
<td>$0.44 + i 0.15$</td>
<td>$0.40 + i 0.15$</td>
<td>$0.33 + i 0.14$</td>
</tr>
<tr>
<td>$\delta_{\pi\Lambda}(^\circ)$</td>
<td>3.4</td>
<td>4.5</td>
<td>5.7</td>
</tr>
<tr>
<td>$m_0$ (GeV)</td>
<td>1.2</td>
<td>1.1</td>
<td>1.0</td>
</tr>
<tr>
<td>$a_{0+}^\dagger (10^{-2} \cdot M_\pi^{-1})$</td>
<td>$-2.0$</td>
<td>$-2.2$</td>
<td>$-2.2$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Units</th>
<th>$\sigma_{\pi N}$ MeV</th>
<th>20$^*$</th>
<th>30$^*$</th>
<th>40$^*$</th>
</tr>
</thead>
<tbody>
<tr>
<td>MeV$^{-1}$</td>
<td>$f$</td>
<td>75.2</td>
<td>71.8</td>
<td>67.8</td>
</tr>
<tr>
<td>GeV$^{-1}$</td>
<td>$b_0$</td>
<td>$-0.615$</td>
<td>$-0.750$</td>
<td>$-0.884$</td>
</tr>
<tr>
<td>GeV$^{-1}$</td>
<td>$b_D$</td>
<td>$+0.818$</td>
<td>$+0.848$</td>
<td>$+0.873$</td>
</tr>
<tr>
<td>GeV$^{-1}$</td>
<td>$b_F$</td>
<td>$-0.114$</td>
<td>$-0.130$</td>
<td>$-0.138$</td>
</tr>
<tr>
<td>GeV$^{-1}$</td>
<td>$b_1$</td>
<td>$+0.660$</td>
<td>$+0.670$</td>
<td>$+0.676$</td>
</tr>
<tr>
<td>GeV$^{-1}$</td>
<td>$b_2$</td>
<td>$+1.144$</td>
<td>$+1.169$</td>
<td>$+1.189$</td>
</tr>
<tr>
<td>GeV$^{-1}$</td>
<td>$b_3$</td>
<td>$-0.297$</td>
<td>$-0.316$</td>
<td>$-0.315$</td>
</tr>
<tr>
<td>GeV$^{-1}$</td>
<td>$b_4$</td>
<td>$-1.048$</td>
<td>$-1.181$</td>
<td>$-1.307$</td>
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<tr>
<td>GeV$^{-1}$</td>
<td>$a_1$</td>
<td>$-1.786$</td>
<td>$-1.591$</td>
<td>$-1.413$</td>
</tr>
<tr>
<td>GeV$^{-1}$</td>
<td>$a_2$</td>
<td>$-0.519$</td>
<td>$-0.454$</td>
<td>$-0.386$</td>
</tr>
<tr>
<td>GeV$^{-1}$</td>
<td>$a_5$</td>
<td>$-1.185$</td>
<td>$-1.170$</td>
<td>$-1.156$</td>
</tr>
<tr>
<td>GeV$^{-1}$</td>
<td>$a_7$</td>
<td>$-5.251$</td>
<td>$-5.209$</td>
<td>$-5.123$</td>
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<tr>
<td>GeV$^{-1}$</td>
<td>$a_8$</td>
<td>$-1.316$</td>
<td>$-1.310$</td>
<td>$-1.308$</td>
</tr>
<tr>
<td>GeV$^{-1}$</td>
<td>$a_9$</td>
<td>$-1.186$</td>
<td>$-1.132$</td>
<td>$-1.050$</td>
</tr>
</tbody>
</table>

**Experiment**

| $\gamma$ | $2.36 \pm 0.04$ |
| $R_c$ | $0.664 \pm 0.011$ |
| $R_n$ | $0.189 \pm 0.015$ |
| $\Delta E$ | $193 \pm 38$ |
| $\Gamma$ | $249 \pm 118$ |
| $\delta_{\pi\Lambda}$ | $4.6 \pm 2$ |

These fits agree with the present experimental data, both on scattering and kaonic hydrogen.
Reproduction of the data by the new B-type fits (do not agree with DEAR) of JAO, EPJA28,63(’06)
Three b’s are fixed in terms of the others from the $O(p^2)$ constraints.

<table>
<thead>
<tr>
<th>$\sigma_{\pi N}$</th>
<th>20°</th>
<th>30°</th>
<th>40°</th>
<th>$O(p)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\gamma$</td>
<td>2.34</td>
<td>2.35</td>
<td>2.34</td>
<td>2.32</td>
</tr>
<tr>
<td>$R_c$</td>
<td>0.643</td>
<td>0.643</td>
<td>0.644</td>
<td>0.637</td>
</tr>
<tr>
<td>$R_n$</td>
<td>0.160</td>
<td>0.163</td>
<td>0.176</td>
<td>0.193</td>
</tr>
<tr>
<td>$\Delta E$ (eV)</td>
<td>436</td>
<td>409</td>
<td>450</td>
<td>348</td>
</tr>
<tr>
<td>$\Gamma$ (eV)</td>
<td>614</td>
<td>681</td>
<td>591</td>
<td>611</td>
</tr>
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<td>$\Delta E_D$ (eV)</td>
<td>418</td>
<td>385</td>
<td>436</td>
<td>325</td>
</tr>
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<td>$I_D$ (eV)</td>
<td>848</td>
<td>880</td>
<td>844</td>
<td>775</td>
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<td>$a_0$ (fm)</td>
<td>$-1.75 + i 1.15$</td>
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<td>$-1.79 + i 1.10$</td>
<td>$-1.50 + i 1.00$</td>
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<td>$0.32 + i 0.46$</td>
</tr>
<tr>
<td>$\delta_{\pi \Lambda} (\text{°})$</td>
<td>-1.4</td>
<td>1.7</td>
<td>-1.2</td>
<td>-1.4</td>
</tr>
<tr>
<td>$m_0$ (GeV)</td>
<td>0.8</td>
<td>0.6</td>
<td>0.7</td>
<td>...</td>
</tr>
<tr>
<td>$a_{0+}^- (10^{-2} \cdot M_\pi^{-1})$</td>
<td>-0.5</td>
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<td>+0.3</td>
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<tr>
<td>$\delta_{\pi \Lambda}$</td>
<td>4.6 ± 2</td>
</tr>
</tbody>
</table>

These fits disagree with DEAR but agree with KEK.

The scattering length $a_{K-p}$ is much larger than in the A-type fits.

Numerically it is simpler to obtain A-type fits.

Three b’s are fixed in terms of the others from the $O(p^2)$ constraints.
**K⁻p Scattering Length:**

- Martin, NPB179,33(’81): \( a_{K^-p} = -0.67 + i0.64 \) fm
- Kaiser, Siegel, Weise, NPA594,325(’95): \( a_{K^-p} = -0.97 + i1.1 \) fm
- Oset, Ramos, NPA635,99(’98): \( a_{K^-p} = -0.99 + i0.97 \) fm
- Meissner, JAO PLB500,263(’01): \( a_{K^-p} = -0.75 + i1.2 \) fm
- Borasoy, Nissler, Weise, PRL94,213401(’05), EPJA25,79(’05):

\[
a_{K^-p} = -0.51 + i 0.82 \text{ fm. They cannot reproduce the elastic } K^-p \rightarrow K^-p \text{ cross section together with the DEAR measurement (compromise).}
\]

- Previous work, J. Prades, M. Verbeni and JAO, Phys. Rev. Lett. 95,172502(2005)
  
  Fit: \( A^+_4: a_{K^-p} = -0.50 + i0.42 \) fm. \( B^+_4: a_{K^-p} = -1.01 + i0.80 \) fm

<table>
<thead>
<tr>
<th>( \sigma_{\pi N} )</th>
<th>20*</th>
<th>30*</th>
<th>40*</th>
</tr>
</thead>
<tbody>
<tr>
<td>( a_{K^-p} ) (fm)</td>
<td>-0.49 + i 0.44</td>
<td>-0.49 + i 0.41</td>
<td>-0.50 + i 0.37</td>
</tr>
<tr>
<td>( a_0 ) (fm)</td>
<td>-1.07 + i 0.53</td>
<td>-1.04 + i 0.50</td>
<td>-1.02 + i 0.45</td>
</tr>
<tr>
<td>( a_1 ) (fm)</td>
<td>0.44 + i 0.15</td>
<td>0.40 + i 0.15</td>
<td>0.33 + i 0.14</td>
</tr>
</tbody>
</table>

**New A-type:**

<table>
<thead>
<tr>
<th>( a_{K^-p} ) (fm)</th>
<th>20*</th>
<th>30*</th>
<th>40*</th>
</tr>
</thead>
<tbody>
<tr>
<td>( a_0 ) (fm)</td>
<td>-1.01 + i 1.03</td>
<td>-0.93 + i 1.07</td>
<td>-1.06 + i 1.02</td>
</tr>
<tr>
<td>( a_1 ) (fm)</td>
<td>-1.75 + i 1.15</td>
<td>-1.65 + i 1.30</td>
<td>-1.79 + i 1.10</td>
</tr>
</tbody>
</table>

**New B-type:**
5. SPECTROSCOPY

\[ T_{ij} = \lim_{s \to s_R} \frac{\gamma_i \gamma_j}{s - s_R} \]

*Residues*

*Pole Position* \((M_R - i\Gamma_R/2)^2\)

Physical Riemann Sheet

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th>GeV</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\pi\Lambda)</td>
<td>(\pi\Sigma)</td>
<td>(KN)</td>
<td>(\eta\Lambda)</td>
<td>(\eta\Sigma)</td>
<td>(K\Xi)</td>
</tr>
<tr>
<td>1RS</td>
<td>2RS</td>
<td>3RS</td>
<td>4RS</td>
<td>5RS</td>
<td>6RS</td>
</tr>
</tbody>
</table>

*Different Riemann Sheets:*
### Fit I: New $\Lambda$-type fit with $\sigma_{\pi N}=40$ MeV

#### $I=0$ Poles (MeV)

| Re(Pole) | $-\text{Im}(\text{Pole})$ | Sheet | $|\gamma_{\pi\Sigma}|_0$ | $|\gamma_{\pi\Sigma}|_1$ | $|\gamma_{\pi\Sigma}|_2$ | $|\gamma_{\bar{K}N}|_0$ | $|\gamma_{\bar{K}N}|_1$ | $|\gamma_{\eta\Lambda}|$ | $|\gamma_{\eta\Sigma}|$ | $|\gamma_{K\Xi}|_0$ | $|\gamma_{K\Xi}|_1$ |
|----------|-----------------|-------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|
| $\Lambda(1310)$ | 1301 | 0.03 | 13 | 1.12 | 0.02 | 0.01 | 5.83 | 0.05 | 0.41 | 0.04 | 2.11 | 0.03 |
| \hspace{1cm} | 1309 | 0.02 | 13 | 3.66 | 0.02 | 0.02 | 4.46 | 0.04 | 0.21 | 0.04 | 3.05 | 0.03 |
| $\Lambda(1405)$ | 1414 | 0.14 | 23 | 4.24 | 0.13 | 0.01 | 4.87 | 0.39 | 0.85 | 0.20 | 9.35 | 0.11 |
| \hspace{1cm} | 1388 | 0.02 | 17 | 3.81 | 0.02 | 0.02 | 1.33 | 0.04 | 0.42 | 0.04 | 9.55 | 0.04 |
| $\Lambda(1670)$ | 1676 | 0.01 | 10 | 1.28 | 0.03 | 0.00 | 1.67 | 0.01 | 2.19 | 0.07 | 5.29 | 0.07 |
| \hspace{1cm} | 1673 | 0.01 | 18 | 1.26 | 0.02 | 0.00 | 1.82 | 0.01 | 2.13 | 0.06 | 5.32 | 0.06 |
| $\Lambda(1800)$ | 1825 | 0.02 | 49 | 2.29 | 0.02 | 0.00 | 2.10 | 0.02 | 0.89 | 0.03 | 7.43 | 0.09 |

**PDG:**
- $\Lambda(1310)$: $M=1660-1680$, $\Gamma=25-50$
- $\Lambda(1405)$: $M=1720-1850$, $\Gamma=65-400$
- $\Lambda(1670)$: $M=1660-1680$, $\Gamma=25-50$
- $\Lambda(1800)$: $M=1720-1850$, $\Gamma=65-400$
Assymetry in the width, before the $\eta\Lambda$ threshold $\Gamma=20$ MeV and above $\Gamma=36$ MeV
### $I=1$ Poles (MeV)

| Re(Pole) | -Im(Pole) | Sheet | $|\gamma_{\pi \Lambda}|$ | $|\gamma_{\pi \Sigma}|$ | $|\gamma_{\pi \Sigma}|$ | $|\gamma_{\bar{K}N}|$ | $|\gamma_{K\bar{N}}|$ | $|\gamma_{\eta A}|$ | $|\gamma_{\eta \Sigma}|$ | $|\gamma_{K\Xi}|$ | $|\gamma_{K\Xi}|$ |
|----------|-----------|-------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|
| 1425     | 6.5       | 2RS   | 1.35           | 0.24           | 1.66           | 0.01           | 0.35           | 3.92           | 0.05           | 4.23           | 0.49           | 2.98           |
| 1468     | 13        | 2RS   | 2.80           | 0.16           | 5.96           | 0.02           | 0.23           | 8.74           | 0.04           | 10.66          | 0.19           | 2.48           |
| 1433     | 3.7       | 3RS   | 0.65           | 0.08           | 0.80           | 0.00           | 0.12           | 1.58           | 0.02           | 5.82           | 0.20           | 2.14           |
| 1720     | 18        | 4RS   | 1.82           | 0.02           | 1.21           | 0.00           | 0.02           | 0.95           | 0.02           | 6.78           | 0.05           | 5.31           |
| 1769     | 96        | 6RS   | 2.65           | 0.00           | 0.61           | 0.00           | 0.00           | 2.48           | 0.00           | 3.32           | 0.01           | 4.22           |
| 1340     | 143       | 3-4RS | 1.33           | 0.14           | 5.50           | 0.02           | 0.02           | 1.58           | 0.00           | 3.28           | 0.03           | 1.20           |
| 1395     | 311       | 3-4RS | 2.08           | 0.01           | 1.49           | 0.01           | 0.00           | 1.24           | 0.00           | 7.63           | 0.01           | 3.97           |

**Σ(1750)**

PDG: $M=1730-1800$

$\Gamma: 50 - 160$
On the physical axis between 1.4 and 1.5 GeV

\[ |t(\eta\Sigma \rightarrow \eta\Sigma)|^2 \]

20 MeV

\[ |t(\pi\Lambda \rightarrow \pi\Lambda)|^2 \]
I=1 is much smaller than I=0. This is why these narrow peaks in I=1 are not seen in $\pi \Sigma$ event distributions (up to now). One needs and I=1 ‘filter.'
For the open channels $\pi\Lambda$, $\pi\Sigma$, $\bar{K}N$

it is a distorted bump

For the closed channels $\eta\Sigma$ and $K\Xi$ it is a clear resonance shape
The amplitudes show a broad bump after the $\bar{K}N$ threshold and before that of the $\eta\Sigma$.

Multipole interference effect

$\Sigma(1620)$ Bump

Physical Axis

$I=1 \ |\pi\Sigma|^2$

$|\pi\Lambda|^2$
I=2 Pole (MeV) at 1722-i 181 MeV   Exotic state

\[ I=2 \mid t(\pi \Sigma \rightarrow \pi \Sigma) \mid^2 \]

The only resonance in I=2

Non uniform shape.
I=2 is of a size not negligibly small compared with other spin channels.
• Fit I: New A-type with $\sigma_{\pi N} = 40$ MeV
  I=0: $\Lambda(1305), \Lambda(1405), \Lambda(1670), \Lambda(1800)$
  I=1: $\Sigma(1430), \Sigma(1620), \Sigma(1750)$

• Fit II: New B-type with $\sigma_{\pi N} = 40$ MeV
  I=0: $\Lambda(1305), \Lambda(1405), \Lambda(1670), \Lambda(1800)$
  I=1: $\Sigma(1430), \Sigma(1620), \Sigma(1750)$

Only in $\bar{K}N$

$8 \otimes 8 = 1 \oplus 8_s \oplus 8_a \oplus 10 \oplus \bar{10} \oplus 27$

Fit I: Has attractive SU(3) kernels for 1, 8$_s$, 8$_a$, 27
It can accomadate 4 I=0 and 3 I=1 resonances.

Fit II: Has attractive SU(3) kernels for 1, 8$_s$, 8$_a$ and $\bar{10}$
It can accomodate 3 I=0 and 3 I=1 resonances.
UCHPT study of meson-baryon dynamics with strangeness $-1$ in S-wave up to NNLO or $O(p^2)$

Simultaneous reproduction of scattering and kaonic hydrogen data. Including the recent and precise data.

The A-type fits also generate the resonances: $\Lambda(1305)$, $\Lambda(1405)$, $\Lambda(1607)$, $\Lambda(1800)$ for $I=0$ and $\Sigma(1430)$, $\Sigma(1620)$, $\Sigma(1750)$ for $I=1$.

All the ones quoted in the PDG up to 1.8 GeV for $1/2^-$ and strangeness $-1$.

The B-type fits do not reproduce DEAR but agree with KEK and scattering data.

The B-type fits are not able to generate a comparable set of resonances. The $\Lambda(1800)$ and $\Sigma(1750)$ are missing.

4. CONCLUSIONS
• The fits A are then preferred over the B ones, based on the present experimental information from scattering, spectroscopy and kaonic hydrogen data.

• $a_{K^-p} = -0.50 + i0.40$ fm (A) preferred over $a_{K^-p} = -1.0 + i1.0$ fm (B)
The only place where Wigner bound is somewhat violated but there cannot be applied because the phase is not differentiable – inelastic cusp

Reply to Borasoı̈, Nissler, Weise Comment PRL96, 199201 ('06)
Figure 1: First panel: 3p-ionic hydrogen strong energy shift and width. In the rest, the solid lines correspond to the fit $A_4^+$ and the dashed ones to $B_4^+$. For further details see the text.
Typically one takes: 

\[ \frac{dN_{\pi\Sigma}}{dE} = C |T^{I=0}_{\pi\Sigma \rightarrow \pi\Sigma}|^2 p_{\pi\Sigma} \]  

As if the process were elastic

E.g: Dalitz, Deloff, JPG 17,289 (’91); Müller, Holinde, Speth NPA513,557 (’90), Kaiser, Siegel, Weise NPB594,325 (’95); Oset, Ramos NPA635, 99 (’89)

But the \( \vec{K}N \) threshold is only 100 MeV above the \( \pi\Sigma \) one, comparable with the widths of the present resonances in this region and with the width of the shown invariant mass distribution. This prescription is ambiguous, why not?

\[ \frac{dN_{\pi\Sigma}}{dE} = C |T^{I=0}_{\vec{K}N \rightarrow \pi\Sigma}|^2 p_{\pi\Sigma} \]

We follow the Production Process scheme previously shown: already employed for this case in Meissner, JAO PLB500,263(’01)

\[ F = (I + R \cdot g)^{-1} \cdot \xi \text{ , } \xi^T = (0, r_1, r_1, r_1, r_2, r_2, 0, 0, 0, 0) \]

\[ \frac{r_2}{r_1} = -0.28 \text{ \quad I=0 Source} \]

\( r_2 = 0 \) (previous approach)
\( \delta_P - \delta_S \) \( \Lambda \pi \) PHASE SHIFTS DIFFERENCE

AT THE \( \Xi^- \) MASS, RECENT MEASUREMENTS FROM THE DECAY PARAMETERS \( \Xi^- \rightarrow \Lambda \pi^- \):

(4.6 \( \pm \) 1.4 \( \pm \) 1.2)\(^0\) Huang et al. (HyperCP Coll.) PRL93,011802 ('04)

(3.2 \( \pm \) 5.3 \( \pm \) 0.7)\(^0\) Chakravorty et al. (E756 Coll.) PRL91,031601 ('03)

**Fit** \( A^+_4 \) **PREDICTS**: 2.5\(^0\) COMPATIBLE WITH DATA

For **Fit** \( B^+_4 \): 0.2\(^0\)
Our calculation at NLO supports a pronounced two pole structure in the $L(1405)$ region as obtained in the LO studies of Meissner, JAO PLB500, 263 (’01) (other later works) Jido, Oset, Ramos, Meissner, JAO, NPA725, 181 (’03) at odds with the claims of Borasoy, Nissler, Weise, PRL94, 213401 (’05)

**THERE ARE MORE RESONANCES...**

We also confirm the scheme Jido, Oset, Ramos, Meissner, JAO, NPA725, 181 (’03)
Poles from the SU(3) representations: 1, 8s, 8a (similar pattern as shown before):

\[ I=0 \]

\[(1321-i 43.5) \text{ MeV} \quad \begin{cases} \lambda \leftarrow 140 \\ 5 \end{cases} \]
\[(1402-i39.6) \text{ MeV} \]
\[(1756-i150) \text{ MeV} \quad L(1670) \quad ?? \]
(position very sensitive to particular values of a’s, TO THE INPUT)

\[ I=1 \]

\[(1487-i46) \text{ MeV} \quad S(1480) \]
\[(1694-i 149) \text{ MeV} \quad S(1620) \]

EXOTIC 27 REPRESENTATION:

\[ I=0 \]

\[(1773-i219) \text{ MeV} \quad L(1800) \]

\[ I=1 \]

\[(1822-i217) \text{ MeV} \quad S(1750) \]

\[ I=2 \]

\[(1862-i238) \text{ MeV} \quad \text{This is the } p^+ S^+ \text{ channel and it can be observed since there are no additional resonances} !! \]
\[ |T_{\pi\Sigma}^{I=2}|^2 \]

Resonance signal

Destructive interference with background

GeV

\[ \pi \pi \ I=0 \]

\[ f_0(980) \]
Narrow resonance just on top the $\bar{K}N$ threshold:
$I=1 \ (1431-i1.3) \ MeV$
\( I=1 \quad |T_{\pi\Sigma}|^2 \)

\( I=1 \quad |T_{\bar{K}N}|^2 \)

\( I=1 \quad |T_{\pi\Lambda}|^2 \)

\( I=1 \quad |T_{\bar{K}N}|^2 \)

WITHOUT NARROW \( I = 1 \) Pole
Narrow resonance just on top the $\bar{K}N$ threshold: 
$I=1 \ (1431-i1.3) \ MeV$

\[ \text{ISOSPIN } K^-N \text{ SCATTERING LENGTHS}, \text{fm:} \]
\[ A_4^+ \]
\[ a_0 = -1.23 + i0.45 \quad a_0 = -1.63 + i0.81 \]
\[ a_1 = 0.98 + i0.35 \quad a_1 = -0.01 + i0.54 \]

IN THE ISOSPIN LIMIT
INFLUENCE OF THE I=1 RESONANCES IN $pS$ EVENT DISTRIBUTION

$gp \blacklozenge K^+ L(1405) \blacklozenge K^+ p^+ S^-, p^- S^+$

J.K. Ahn, NP A721 (’03) 715c

$K^- p \rightarrow \Sigma^\pm \pi^\mp \pi^- \pi^+$

Hemingway, NPB253, 742(’85)

LINE:
Nacher, Oset, Toki, Ramos PL B455 (’99) 55

Magas, Ramos, Oset PRL95, 052301(’05)

K$^-p$ $\blacklozenge$ $\square^0 \square^0 \blacklozenge$ $0$ & $p$ $\blacklozenge$ $K^0$ $\square$ $\blacklozenge$
MORE WORK IS NEEDED:

1. MATCH THEORETICAL PRECISION WITH DEAR/SIDDHARTA measurement of width and shift of kaonic hydrogen at the eV level
   a) Going to order $d^{3/2}$ (or better $d^2$) in the correction of the Deser formula.
   b) Order $p^3$ (one loop) in the calculation of strong amplitude.

2. CLARIFY ISSUES ON SPECTROSCOPY:
   a) I=1 Narrow Resonance (1430-i 1.3)MeV, Disentangle experimentally the I=1 broad one (1487-i 46) MeV.
   b) Exotic Resonances (I=2)
Table 3: Pole positions and couplings to $I = 0$ physical states from the model of Ref. [3]

| $z_R$ ($I = 0$) | $g_i$   | $|g_i|$ | $g_{|i|}$ | $|g_{|i|}|$ |
|-----------------|---------|---------|-----------|-----------|
| $\pi \Sigma$    | -1.76 - 0.62i| 1.87    | -0.56 - 1.02i| 1.16     |
| $\bar{K}N$      | 0.86 + 0.70i | 1.11    | -1.74 + 0.63i| 1.85     |
| $\eta \Lambda$  | 0.19 + 0.33i | 0.38    | -1.20 + 0.23i| 1.23     |
| $K \Xi$         | -0.52 - 0.19i| 0.55    | -0.20 - 0.30i| 0.36     |

(a) is more than twice wider than b) (Quite Different Shape)

(b) Couples stronger to $\bar{K}N$ than to $\pi \Sigma$

contrarily to a)

It depends to which resonance the production mechanism couples stronger that the shape will move from one to the other resonance.

![Graph showing pole positions and couplings with labels for a) and b) and resonance moves from one to another.](image)
Simple parametrization of our own results with BW like expressions

$$\pi \Sigma \rightarrow \bar{K}N$$

$$\pi \Sigma \rightarrow \pi \Sigma$$
\[ \varepsilon_{12} = -2\alpha^3 \mu_r^2 T_{Kp}^{th} (1 + X) \]

Isospin violating corrections
Meißner, Raha, Rusetsky

DEAR

\[ X = 0 \rightarrow T_{Kp}^{th} = (-0.33 \pm 0.10) + i(0.28 \pm 0.10) \text{ fm} \]

M. Iwasaki et al. PRL 78 (1997) 3067

\[ \varepsilon_{12} - i\frac{\Gamma}{2} = (323\pm 63 \pm 11) - i(200\pm 100 \pm 50) \text{ eV} \]

Scattering experiment B. R. Martin NP B94 (1975) 413

\[ T_{Kp}^{th} = (-0.67 \pm 0.10) + i(0.64 \pm 0.10) \text{ fm} \]

Oset, Ramos: -0.85+i 1.24 // Kaiser et al.: -0.97+i 1.1 //
Meißner, J. A. O.: -0.51+i 0.9 LO (relativistic) UCHPT

Rather controversial (not very precise) situation:
Experimentally: more precision is needed in kaonic atoms experiments (hopefully DEAR)

Theoretically: 1) Higher orders are necessary to be considered and one must check the convergence of the UCHPT expansion to calculate \( T_{Kp}^{th} \)

2) To compute \( X \)
Physical picture of Kaonic Hydrogen:

A kaonic hydrogen atom is a quasistable bound state of a kaon ($K^-$) and a proton ($p$), in which the interaction is predominantly electromagnetic with strong interactions that can be treated as perturbations giving rise to small corrections.

- $(K^- p)_{1s} \rightarrow \{ \pi^0 \Lambda, \Sigma^\pm \pi^0, \Sigma^0 \pi^\pm, \gamma Y, Y = \Lambda, \Sigma^0 \}$ [strong]

- Very small momenta.

$$\langle p^2 \rangle^{1/2} = \alpha \mu_c \approx 2 \text{ MeV} \ll \mu_c, \quad \mu_c = \frac{m_p M_{K^-}}{m_p + M_{K^-}}$$

- $R = (\alpha \mu_c)^{-1} \approx 100 \text{ fm} \gg R_{\text{Strong}}$

- $E_{1s} = \frac{1}{2} \mu_c \alpha^2 + ... \approx 8 \text{ KeV} \ll \mu_c$

- $\Gamma_{1s} \approx 250 \text{ eV} \ll E_{1s}$

- Mass($K^0 n$) > Mass($K^- p$) $\rightarrow$ Cusp Effect

The observable characteristics of hadronic atoms obtained from the study of the spectrum and decays of kaonic hydrogen:

- (Small) shifts of energy levels $\Delta E_{nl}$ from purely Coulomb values and total decay width $\Gamma_{nl}$
The set of Feynman diagrams contributing to the energy shift of the kaonic hydrogen up-to-and-including $\mathcal{O}(\alpha^4, \alpha^3(m_d - m_u))$. Solid, dashed, double, dotted, wiggly and spring lines correspond to the proton, $K^-$, neutron, $K^0$, Coulomb and transverse photons, respectively. The electrons run in the closed loops shown in diagrams (d) and (i). The diagrams (f) and (i) contain Coulomb ladders — the contributions with 0, 1, 2, ... Coulomb photons exchanged.
Modified Deser Formula:

Our modified formula, up to and including $O(\alpha^4, \alpha^3 (m_d - m_u))$, here large nonanalytic corrections due to cusp effect are explicitly included, is best suited for the analysis of experimental data:

$$\Delta E_n^3 - \frac{i}{2} \Gamma_n = -\frac{\alpha^3 \mu_c^3}{2\pi M_{K^+} + n^3} \left( T_{KNN}^{(0)} + \delta T_{KNN} \right) \left\{ 1 - \frac{\alpha \mu_c^2 s_n(\alpha)}{4\pi M_{K^+}} T_{KNN}^{(0)} + \delta_{\text{vac}} \right\}$$

Coulomb Corrections

Corrections to the Deser Formula (Rough estimate):

- Cusp Effect $\sim 50\%$ at $O(\sqrt{\delta M})$
- Coulomb Effects $\sim (10$ to $15)\%$
- Vacuum Polarization $\sim 1\%$
- CHPT $\sim (-0.5 \pm 0.4)\%$ at $O(p^2)$ (or $O(\delta M)$)
Production Processes

The re-scattering is due to the strong „final“ state interactions from some „weak“ production mechanism.

We first consider the case with only the right hand cut for the strong interacting amplitude, \( R^{-1} \) is then a sum of poles (CDD) and a constant. It can be easily shown then:

\[
F = [I + R \cdot g]^{-1} \cdot \xi
\]
Finally, $\xi$ is also expanded pertubatively (in the same way as $R$) by the matching process with CHPT/alike expressions for $F$, order by order, $\xi = \xi_1 + \xi_2 + \xi_3 ...$ 

The crossed dynamics, as well for the production mechanism, are then included pertubatively.
E.Oset, J.A.O. NP A620(1997)438 (E NPA652(’99)407) applied it to meson-meson interactions in S-wave $s$, $f_0(980)$, $a_0(980)$ resonances.

However, the approach was fully ON-SHELL, and algebraic since it was demonstrated that the off-shell part of the potential (LO CHPT) when iterated in the LS equation only renormalizes the potential itself.
• There are good and well established reasons why the unitarity corrections are so enhanced in the previous examples giving rise to non-perturbative physics

• New scales or numerical enhancements can appear that makes definitively smaller the overall scale $\Lambda_{\text{CHPT}}$, e.g:
  
  – Scalar Sector (S-waves) of meson-meson interactions with $I=0,1,1/2$ the unitarity loops are enhanced by numerical factors.

$$\frac{s - 4m^2_\pi}{6f^2} \rightarrow \frac{s - m^2_\pi}{f^2}$$

  Enhancement by a factor $6^L$

  - Presence of large masses compared with the typical momenta, e.g: Kaon masses in driving the appearance of the $\Lambda(1405)$ close to threshold in $\bar{K}N$. This also occurs similarly in Nucleon-Nucleon scattering with the nucleon mass.
3. S-WAVE, S=-1 MESON-BARYON SCATTERING

J. A. Oller, EPJA (2006)

HBCHPT calculation at one loop level fails miserably for the $\bar{K}N$ scattering lengths (opposite signs) N. Kaiser, EPJ64,045204(’01)

\[
T = [R^{-1} + g(s)]^{-1} = [I + R \cdot g(s)]^{-1} \cdot R(s)
\]

$R = R_1 = T_1$ \hspace{1cm} LEADING ORDER, $\mathcal{O}(p)$

$R = R_1 + R_2 = T_1 + T_2$, NLO, $\mathcal{O}(p^2)$

for $\mathcal{O}(p^3)$ and higher $R_n \neq T_n$
Comparison of New and Old Data:

Predictions of the ground-state strong shift $\Delta E_1^o$ and width $\Gamma_1$. Filled circles correspond to using the original Deser formula, empty circles to using $T_{KN}^{(0)}$ instead of $\frac{1}{3}(a_0 + a_1)$ in this formula and filled boxes to our final formula with $\delta T_{KN} = \delta^{\text{vac}}_{\pi} = 0$.


$$a_0 = -1.31 + 1.24i \quad ; \quad a_1 = 0.26 + 0.66i$$


$$a_0 = -1.70 + 0.68i \quad ; \quad a_1 = 0.37 + 0.60i$$

**DEAR**

$$X = 0 \rightarrow T_{KP}^{\text{th}} = (-0.33 \pm 0.10) + i(0.28 \pm 0.10) \text{ fm}$$

M.Iwasaki et al. PRL78(1997)3067

$$X = 0 \rightarrow T_{KP}^{\text{th}} = (-0.78 \pm 0.15) + i(0.50 \pm 0.50)$$
4. This allows as well to use the Chiral Lagrangians for higher energies. (BONUS)

5. Since one can also use the chiral Lagrangians for higher energies it is possible to establish a connection with perturbative QCD, 
\[ \alpha_S (4 \text{ GeV}^2) / \pi \approx 0.1 \] (OPE). E.g. providing phenomenological spectral functions for QCD Sum Rules (going definitively beyond the sometimes insufficient hadronic scheme of narrow resonance+resonance dominance).  

\textbf{Jamin, Pich, JAO}  
\textit{V}^v_{\text{bs}}: \text{JHEP 0402, 047 ('04)}  
\textbf{m}_{u,d,s}: \text{E\Pi\Theta X24, 237 (02); hep-ph/0605095}

6. The same scheme can be applied to productions mechanisms. Some examples: →

- **Photoproduction:** \( \gamma \gamma \rightarrow \pi^0 \pi^0, \pi^+ \pi^-, K^+ K^-, K^0 \bar{K}^0, \pi^0 \eta \); \( D \rightarrow 3\pi, K 2\pi, ... \)  
  \( \gamma p \rightarrow K^+ \Lambda(1405); (\gamma, \pi\pi) ; \gamma d \rightarrow d \); \( \gamma \text{NN} \rightarrow \text{NN} \); \( \gamma d \rightarrow \gamma d \); ...

- **Decays:** \( \phi \rightarrow \gamma \pi^0 \pi^0, \pi^0 \eta, K^0 \bar{K}^0 \); \( J/\Psi \rightarrow \phi (\omega) \pi\pi, KK \);  
  \( f_0(980) \rightarrow \gamma\gamma \); branching ratios ...

\textbf{JAO PRD 71, 054030 (’05) on D ! 3p, K 2p}  
\textbf{and D_s ! 3p , and references therein}
Let us keep track of the kaon mass, $M_K \approx 500$ MeV
We follow similar arguments to those of S. Weinberg in NPB363,3 (’91)
respect to NN scattering (nucleon mass).

Unitarity Diagram

\[
\frac{1}{k^0 - E(q)} \frac{1}{2E(q)} \approx \frac{2M_K}{k^2 - q^2} \frac{1}{2M_K}
\]

Let us take now the crossed diagram $k \rightarrow -k$

\[
\frac{1}{k^0 + E(q)} \frac{1}{2E(q)} \approx \frac{1}{4M_K^2}
\]

Unitarity enhancement for low three-momenta:

\[
\frac{4M_K^2}{k^2 - q^2}
\]

\[
\frac{2M_K}{q}
\]
Chiral Perturbation Theory

Weinberg, Physica A96,32 (79); Gasser, Leutwyler, Ann.Phys. (NY) 158,142 (84)

QCD Lagrangian

Hilbert Space

Physical States

$u, d, s$ massless quarks

SU(3)$_L$ $\otimes$ SU(3)$_R$

Spontaneous Chiral Symmetry Breaking

SU(3)$_V$

Goldstone Theorem

Octet of massless pseudoscalars

$\pi, K, \eta$

Energy gap

$m_q \neq 0$. Explicit breaking of Chiral Symmetry

Non-zero masses

$m_p^2 \propto m_q$

Perturbative expansion in powers of the external four-momenta of the pseudo-Goldstone bosons over $\Lambda_{\text{CHPT}}$

$L = L_2 + L_4 + \ldots$

$\frac{L_4}{L_2} = O\left(\frac{p^2}{\Lambda_{\text{CHPT}}^2}\right)$

$\Lambda_{\text{CHPT}} \approx 1$ GeV $\approx M_\rho$

$\approx 4\pi f_\pi \approx 1$ GeV
Enhancement of the unitarity cut that makes definitively smaller the overall scale $\Lambda_{\text{CHPT}}$ in meson-baryon scattering with strangeness:

Presence of large masses compared with the typical low three-momenta (Baryon+Kaon masses) drive the appearance of the $\Lambda(1405)$ close to threshold in $\bar{K}N$ scattering.

This also occurs similarly in Nucleon-Nucleon scattering with the nucleon mass
Reproduction of the data by the fits of Prades, Verbeni, JAO PRL95(’05), plus an O(p) fit.

<table>
<thead>
<tr>
<th>Units</th>
<th>(A_t^+)</th>
<th>(B_t^+)</th>
<th>(O(p))</th>
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<td>MeV</td>
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<tr>
<td>GeV(^{-1})</td>
<td>(a_9)</td>
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<td>-0.886</td>
</tr>
</tbody>
</table>

**Three b’s are fixed in terms of the others from the \(O(p^2)\) constraints**

\[\sigma_{\pi N} = 40 \text{ MeV}\]

\[m_0 = 0.8 \text{ GeV}\]

\[a_{0+} = (-1 \pm 1)m_\pi^{-1}10^{-2}\]

\[a_2 = a_3 = a_4, \quad a_5 = a_6, \quad a_9 = a_{10}\]
Fit A reproduces simultaneously scattering data plus DEAR measurement

It was the first chiral fit to accomplish this

However, it fails to reproduce the Crystall Ball data.
Reproduction of the data by the fits of Prades, Verbeni, JAO PRL95('05), plus an O(p) fit.

Other results for which a precise knowledge of $\bar{K}N$ scattering is important:

- Nature of $\Lambda(1405)$, problems in lattice QCD and quarks models. Dynamically generated resonance.

- Two poles making up the $\Lambda(1405)$
  Meissner, JAO PLB500, 263 (’01); Jido, Oset, Ramos, Meissner, J.A.O, NPA 725(03)181
  Magas, Oset, Ramos PRL 95, 052301(’05); S. Prakhov et al. (Crystall Ball Coll.), PRC 70, 034605(’04);

- Discover of tri-baryons $S^0(3115), S^1(3140)$. IF SO, it is established that $\bar{K}$-nucleus potential is definitely strong. Suzuki et al., PLB597, 263(’04)
  CONTROVERSY ON THE CORRECT INTERPRETATION OF EXPERIMENT: Oset, Toki, PRC74, 015207(06). The situation is still contentious.

- Strangeness content of the proton and large pion-nucleon sigma terms,
  $\langle p|\bar{s}s|p\rangle$ strange proton-scalar form factor related by unitarity with $\bar{K}N$ amplitudes.
Historically, the first approach to apply a Chiral expansion to an interacting KERNEL was:


The Chiral expansion was applied to the set of two nucleon irreducible diagrams, THE POTENCIAL, which was then iterated through a Lippmann-Schwinger equation.

The solution to the LS equation is NUMERICAL.

Further regularization is needed when solving the LS equation (cut-off dependence) so that the new divergences are not reabsorbed by the counterterms introduced in $V$. These drawbacks are solved when using UCHPT. The solution is algebraic and there is no cut-off dependence.

Let us keep track of the kaon mass, $M_K \approx 500$ MeV.

We follow similar arguments to those of S. Weinberg in NPB363,3 (ʼ91) respect to NN scattering (nucleon mass).

Unitarity Diagram

$$\int \frac{dq^0}{(k^0 - q^0 + i\varepsilon)(q^0 + E(q) - i\varepsilon)(q^0 - E(q) + i\varepsilon)}$$

Unitarity enhancement for low three-momenta:

Around one order of magnitude in the region of the \(\Lambda(1405)\) region, $|q| \approx 100$ MeV.