Meson-Baryon Scattering and Resonances with Strangeness –1

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1. **Introduction. Interest.**
2. **UCHPT**
3. **S-Wave, S=-1 Meson-Baryon Scattering**
4. **Scattering**
5. **Spectroscopy**
6. **Conclusions**

1. INTRODUCTION. INTEREST.

$\bar{K}N$ SCATTERING, TEN TWO BODY COUPLED CHANNELS:

$$\pi^0\Lambda \;\; \pi^0\Sigma^0 \;\; \pi^-\Sigma^+ \;\; \pi^+\Sigma^- \;\; K^-p \;\; \bar{K}^0p \;\; \eta\Lambda \;\; \pi^0\Sigma^0 \;\; K^0\Xi^0 \;\; K^-\Xi^+$$

$$8 \times 8 = 1 + 8_s + 8_a + 10 + \overline{10} + 27$$

The representations 1, 8_s, 8_a and 27(exotic) give rise to resonances.

- Potential Models, Quark Models, (Chiral) Bag Models, etc
- CHPT+Unitarization (UCHPT)

Kaiser,Siegel,Weise NPA594,325('95)

Oset, Ramos NPA635,99('98)

Meissner,JAO PLB500,263('01)

Lutz,Kolomeitsev NPA700,193('02) ;

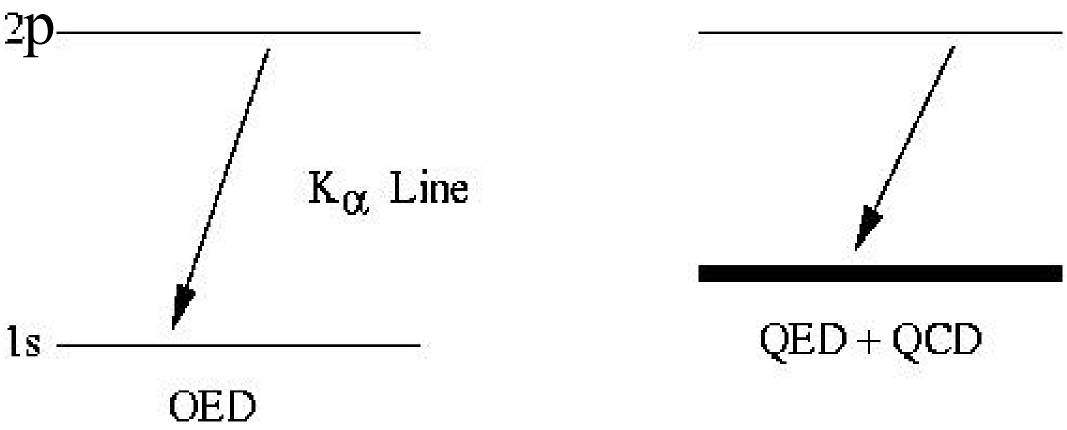
Garcia-Recio, Lutz, Nieves, PLB582,49 ('04);

Borasoy, Nissler, Weise PRL94,213401 (05), EPJA25,79('05)

Borasoy, Meissner, Nissler, PRC74,055201('06), etc

Renewed interest with the precise measurement by DEAR Coll. of strong shift and width of kaonic hydrogen 1s energy level

G. Beer et al., PRL94,212302('05)



Upwards shift

Repulsive $K^-p \to K^-p$

$$K^-p \to \begin{cases} \pi^0\Lambda\ ,\ \pi^\mp\Sigma^\pm\ \text{[strong]} \\ \Sigma\pi\gamma\ ,\ \Sigma\pi e^+e^-\ ,\ \Sigma\gamma\ ,\ldots < 1\% \end{cases}$$

Unstable

DEAR:

$$\Delta E = 193 \pm 37(stat.) \pm 6(syst.)\ \text{eV}$$
$$\Gamma = 249 \pm 111(stat.) \pm 39(syst.)\ \text{eV}$$

KEK:

$$\Delta E = 323 \pm 63 \pm 11\ \text{eV}$$
$$\Gamma = 407 \pm 208 \pm 100\ \text{eV}.$$

Meissner,Raha,Rusetsky EPJ C35,349('04); Borasoy,Nissler,Weise PRL94,213401(05), EPJA25,79(05) pointed out a possible inconsistency between DEAR and previous scattering data. SU(3) chiral dynamics results agree with KEK but disagrees with the factor 2 more precise DEAR measurement

$$E_{1s} = E_{1s}^{em} + \epsilon_{1s}$$, ϵ_{1s} is complex

Deser Formula $\epsilon_{1s} = -2\alpha^3\mu_C^2 T_{K^-p}$

Precise knowledge Precise determination

$\epsilon_{1s} \Longleftrightarrow T_{K^-p}$ at threshold

Meissner,Raha,Rusetsky EPJ C35,349('04) include isospin breaking correction on the Deser formula up to an including $O(\alpha^4, \alpha^3(m_u - m_d)) \sim 9\%$

Cusp Effect: ~ 50% $O(d^{1/2})$

Coulomb Effects: ~ 10 – 15 %

Vacuum Polarization: ~ 1%

$$\Delta E_{1s} - \frac{i}{2}\Gamma_{1s} = -\frac{\alpha^3\mu_C^3}{2\pi M_{K^+}} T_{K^-p}\left\{1 - \frac{\alpha\mu_C^2 s_1(\alpha)}{4\pi M_{K^+}} T_{K^-p}\right\}$$

$d \sim a \sim m_u - m_d$

DEAR/SIDDHARTA Coll. Aims to finally measure it up to eV level, a few percent (nowadays the precission is 20%).

http://www.lnf.infn.it/esperimenti/dear/DEAR_RPR.pdf

2. UNITARY CHPT (UCHPT).

1. A systematic scheme able to be applied when the interactions between the hadrons are not perturbative (even at low energies) but still using Chiral Lagrangians and Chiral Perturbation Theory (though to be valid only for low energies).
    - Meson-meson processes, both scattering, production and decays, involving I=0,1,1/2 S-waves, $J^{PC}=0^{++}$ (vaccum quantum numbers)

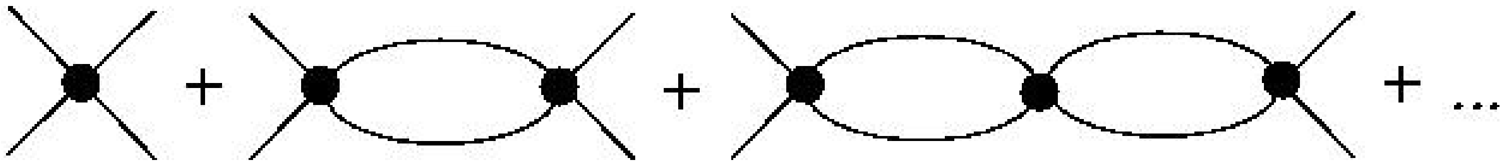
      I=0 $\sigma(500)$ - really low energies

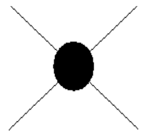
      Not low energies. More resonances come up: I=0 $f_0(980)$, $f_0(1370)$, $f_0(1500)$, $f_0(1710)$, $f_0(1790)$; I=1 $a_0(980)$, $a_0(1450)$; I=1/2 $\kappa(700)$, $K^*_0(1430)$

      Related by SU(3) symmetry.
    - Processes involving S=–1 (strangeness) S-waves meson-baryon interactions $J^P=1/2^-$. I=0 $\Lambda(1405)$'s , $\Lambda(1670)$, $\Lambda(1800)$; I=1 possible $\Sigma(1430)$, $\Sigma(1620)$, $\Sigma(1750)$

      One also finds other resonances in S=−2, 0, +1, and even with I=2...
    - Processes involving scattering or production of, particularly, the lowest Nucleon-Nucleon partial waves like the ${}^1S_0$, ${}^3S_1$ or P-waves. Deuteron, Nuclear matter, Nuclei.
2. Then one can handle with:
    - Strongly interacting coupled channels.
    - Large unitarity loops.
    - Resonances.

In all these examples the unitarity cut (sum over the unitarity bubbles) is enhanced.



UCHPT makes an expansion of an ``Interacting Kernel´´ 

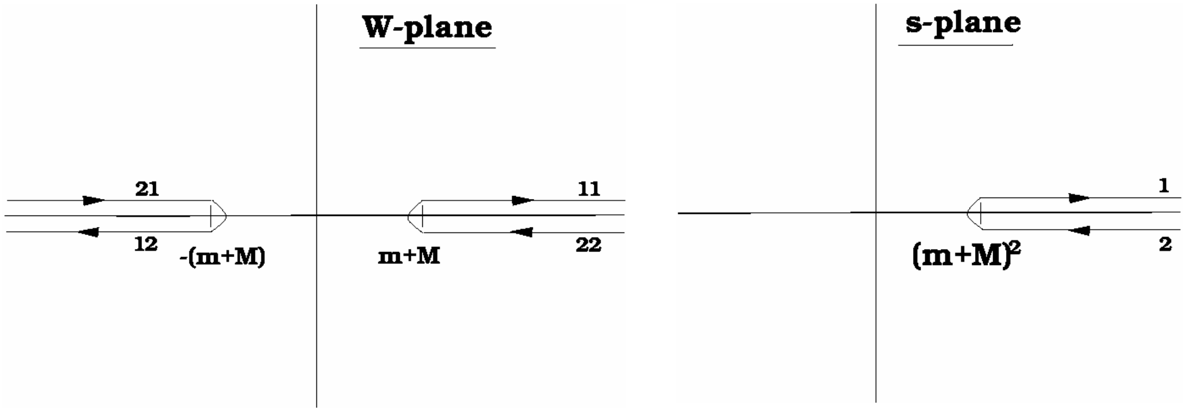
from the appropriate EFT and then the unitarity cut is fulfilled to all orders (non-perturbatively)

General Expression for a Partial Wave Amplitude

- Above threshold and on the real axis (physical region), a partial wave amplitude must fulfill because of unitarity:

$$\operatorname{Im} T_{ij} = \sum_k T_{ik}\rho_k T^*_{kj} \longrightarrow \operatorname{Im} T^{-1}_{ij} = -\rho_i\,\delta_{ij}$$ Unitarity Cut

$$W = \sqrt{s}$$

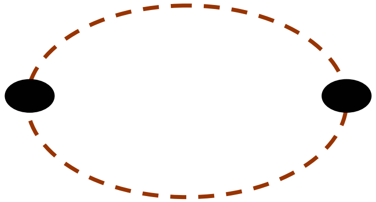


We perform a dispersion relation for the inverse of the partial wave (the discontinuity when crossing the unitarity cut is known)

$$T^{-1}_{ij} = R^{-1}_{ij} + \delta_{ij}\underbrace{\left(g(s_0)_i - \frac{s-s_0}{\pi}\int_{s_{th;i}}^{\infty}\frac{\rho(s')_i}{(s'-s-i0^+)(s'-s_0)}ds'\right)}$$

The rest

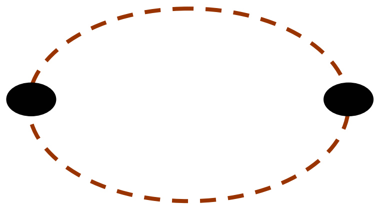
$g(s)_i$: Single unitarity bubble

g(s)= 

$$g(s) = \frac{1}{4\pi^2}\left(a_{SL} + \sigma(s)\log\frac{\sigma(s)-1}{\sigma(s)+1}\right)$$

$$T = \left[R^{-1} + g(s)\right]^{-1} = [I + R\cdot g]^{-1}\cdot R \qquad \sigma(s) = \frac{2q}{\sqrt{s}}$$

1. T obeys a CHPT/alike expansion

g(s)= 

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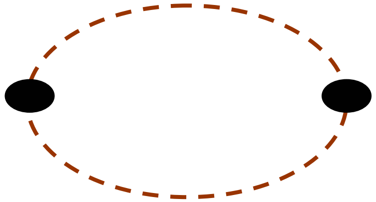
1. T obeys a CHPT/alike expansion $T = T_1 + T_2 + T_4 + \ldots$
2. R is fixed by matching algebraically with the CHPT/alike

expressions of T, **R= R_1+R_2 R_3+...**

In doing that, one makes use of the CHPT/alike counting for g(s)

The countingof R(s) is consequence of the known ones of g(s) and T(s) g(s)= 

$$g(s) = \frac{1}{4\pi^2}\left(a_{SL} + \sigma(s)\log\frac{\sigma(s)-1}{\sigma(s)+1}\right)$$

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CHPT/alike+Resonances
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The countingof R(s) is consequence of the known ones of g(s) and T(s)

3. The CHPT/alike expansion is done to R(s). Crossed channel dynamics is included perturbatively.

3. S-WAVE, S=-1 MESON-BARYON SCATTERING

J.Prades, M. Verbeni, JAO PRL95,172502(05), PRL96,199202(06)(Reply)

J.A. Oller, EPJA 28,63(2006)

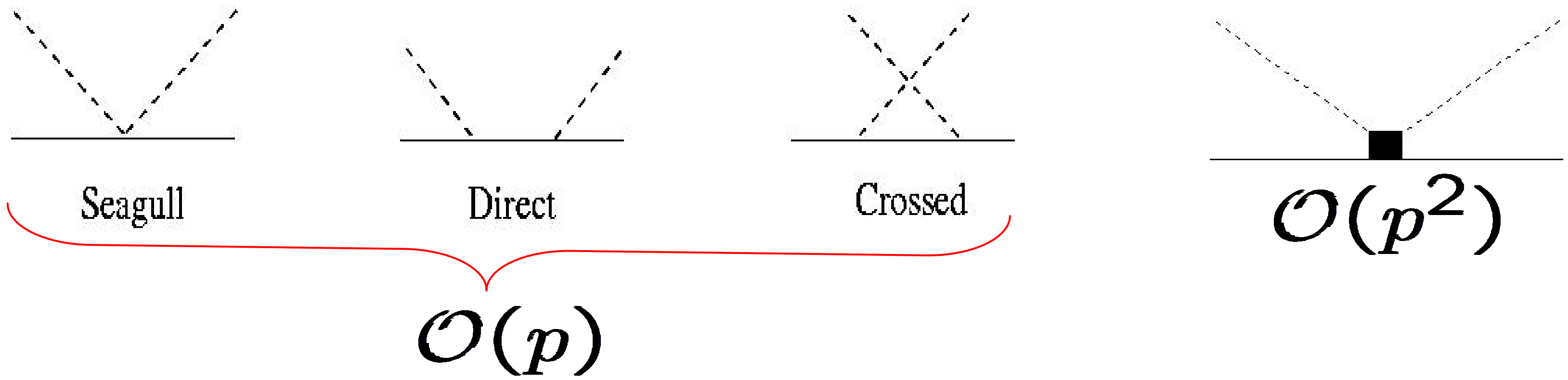
$$T = \left[R^{-1} + g(s)\right]^{-1} = [I + R \cdot g(s)]^{-1} \cdot R(s)$$

$g(s)$ is $\mathcal{O}(p)$

$R = R_1 = T_1$ LEADING ORDER, $\mathcal{O}(p)$

$R = R_1 + R_2 = T_1 + T_2$, NLO, $\mathcal{O}(p^2)$

$R_2 = T_2$, for $O(p^3)$ and higher $R_n \neq T_n$



$\mathcal{O}(p)$ and $\mathcal{O}(p^2)$ Chiral Lagrangians

$$\begin{aligned}\mathcal{L}_1 &= \langle i\bar{B}\gamma^\mu[D_\mu, B]\rangle - m_0\langle\bar{B}B\rangle \\ &+ \frac{D}{2}\langle\bar{B}\gamma^\mu\gamma_5\{u_\mu, B\}\rangle + \frac{F}{2}\langle\bar{B}\gamma^\mu\gamma_5[u_\mu, B]\rangle\ ,\end{aligned}$$

$D = 0.8, F = 0.46, m_0 =$proton mass in SU(3) chiral limit

$$\begin{aligned}\mathcal{L}_2 &= b_0\langle\bar{B}B\rangle\langle\chi_+\rangle + b_D\langle\bar{B}\{\chi_+, B\}\rangle + b_F\langle\bar{B}[\chi_+, B]\rangle \\ &+ b_1\langle\bar{B}[u_\mu, [u^\mu, B]]\rangle + b_2\langle\bar{B}\{u_\mu, \{u^\mu, B\}\}\rangle \\ &+ b_3\langle\bar{B}\{u_\mu, [u^\mu, B]\}\rangle + b_4\langle\bar{B}B\rangle\langle u_\mu u^\mu\rangle + \cdots\ .\end{aligned}$$

$$U = e^{i\Phi/f}\ ,\ U = u^2\ ,\ u = e^{i\Phi/2f}\ ,\ u_\mu = iu^\dagger(\partial_\mu U)u^\dagger$$

$$\chi_+ = u^\dagger\chi u^\dagger + u\chi^\dagger u\ ,\ \chi = \begin{pmatrix} m_\pi^2 & 0 & 0 \\ 0 & m_\pi^2 & 0 \\ 0 & 0 & 2m_K^2 - m_\pi^2 \end{pmatrix}$$

$$\Phi = \begin{pmatrix} \frac{\pi^0}{\sqrt{2}} + \frac{\eta}{\sqrt{6}} & \pi^+ & K^+ \\ \pi^- & -\frac{\pi^0}{\sqrt{2}} + \frac{\eta}{\sqrt{6}} & K^0 \\ K^- & \bar{K}^0 & -\frac{2}{\sqrt{6}}\eta \end{pmatrix}\quad B = \begin{pmatrix} \frac{\Sigma^0}{\sqrt{2}} + \frac{\Lambda}{\sqrt{6}} & \Sigma^+ & p \\ \Sigma^- & -\frac{\Sigma^0}{\sqrt{2}} + \frac{\Lambda}{\sqrt{6}} & n \\ \Xi^- & \Xi^0 & -\frac{2}{\sqrt{6}}\Lambda \end{pmatrix}$$

From these Lagrangians one calculates the $\mathcal{O}(p)$, R_1, and $\mathcal{O}(p^2)$, R_2, CHPT meson-baryon scattering amplitudes. $R = R_1 + R_2$.

EXPERIMENTAL DATA

S= – 1 meson-baryon sector is plenty of data

Good ground test for SU(3) chiral dynamics, where very strong SU(3) breaking effects due to the explicit presence of mesons/baryons with strangeness– Explicit breaking of chiral symmetry

Important isospin breaking effects due to cusps at thresholds, we work with the physical basis.

I) DATA INCLUDED IN THE ANALYSIS

Prades,Verbeni,JAO PRL95,172502(05)

1) CROSS SECTIONS:

$$K^- p \to K^- p \, , \bar{K}^0 n \, , \pi^+\Sigma^- \, , \pi^-\Sigma^+ \, , \pi^0\Sigma^0 \, , \pi^0\Lambda$$

In the fit we include data from threshold up to $p_{lab} = 0.2$ GeV.

2) Precisely Measured Ratios

$$\gamma = \frac{\sigma(K^- p \to \pi^+\Sigma^-)}{\sigma(K^- p \to \pi^-\Sigma^+)} = 2.36 \pm 0.04 \, ,$$

$$R_c = \frac{\sigma(K^- p \to \text{charged particles})}{\sigma(K^- p \to \text{all})} = 0.664 \pm 0.011 \, ,$$

$$R_n = \frac{\sigma(K^- p \to \pi^0\Lambda)}{\sigma(K^- p \to \text{all neutral states})} = 0.189 \pm 0.015,$$

3) $\pi\Sigma$ EVENT DISTRIBUTION AROUND THE $\Lambda(1405)$ RESONANCE

4) DEAR and KEK STRONG SHIFT AND WIDTH OF KAONIC HYDROGEN

5) WE ALSO CONSTRAINT OUR FITS CALCULATING AT O(p²) IN PURE BARYON CHPT SEVERAL PION-NUCLEON OBSERVABLES, WHERE CHPT EXPANSION IS RELIABLE:

$$\begin{aligned}
\sigma_{\pi N} &= -2m_\pi^2(2b_0+b_F+b_D)\ , \\
a_{0+}^{+} &= \frac{m_\pi^2}{2\pi f^2}\left(-2b_1+b_2+b_3-\frac{g_A^2}{8m}\right) \\
m_0 &= m_p + 4m_K^2(b_0+b_D-b_F)+2m_\pi^2(b_0+2b_F)\ .
\end{aligned}$$

b_i from the fits

b_D, b_f and b_3 in terms of b_0, b_1 and b_2.

$\sigma_{\pi N}$=20, 30, 40 MeV (45±8 from **Gasser,Leutwyler,Sainio PLB253,252 ('91),** higher order corrections ±10 MeV **Gasser, AP254,192('97))**

m_0=0.7 or 0.8 GeV

a_{0+}^{+}=(−1±1) m_π 10^{-2} Exp. –0.25±0.49 **Schroder et al.,PLB469,25('99)** and expected higher order corrections +m_π 10^{-2} from unitarity **Bernard et al. PLB309,421('93).**

100 points

I) RECENT FURTHER DATA INCLUDED IN THE EXTENDED ANALYSIS ***JAO EPJA28,63(2006)***

6) $\sigma(K^- p \to \eta\Lambda)$ cross-section

On top of the $\Lambda(1670)$ resonance.

7) $\sigma(K^- p \to \Sigma^0\pi^0\pi^0)$

total cross-section and event distribution.

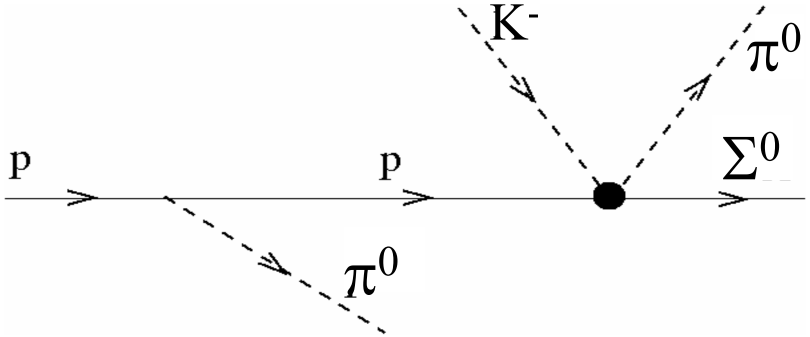
6) and 7) measured by the Crystall-Barrell Collaboration , 2001 and 2004, respectively. Precise experimental data.

8) $\Lambda\pi$ P- and S-wave phase shift difference at the Ξ^- mass $\delta_P - \delta_S = (4.6 \pm 1.4)^\circ$.

HyperCP Collaboration (2004)

155 points

For the production of the process $K^- p \to \pi^0 \pi^0 \Sigma^0$ we take as the production vertex the mechanism:



Magas, Oset, Ramos PRL95,052301('05).

It is largely enhanced due to the almost on-shell character of the intermediate proton

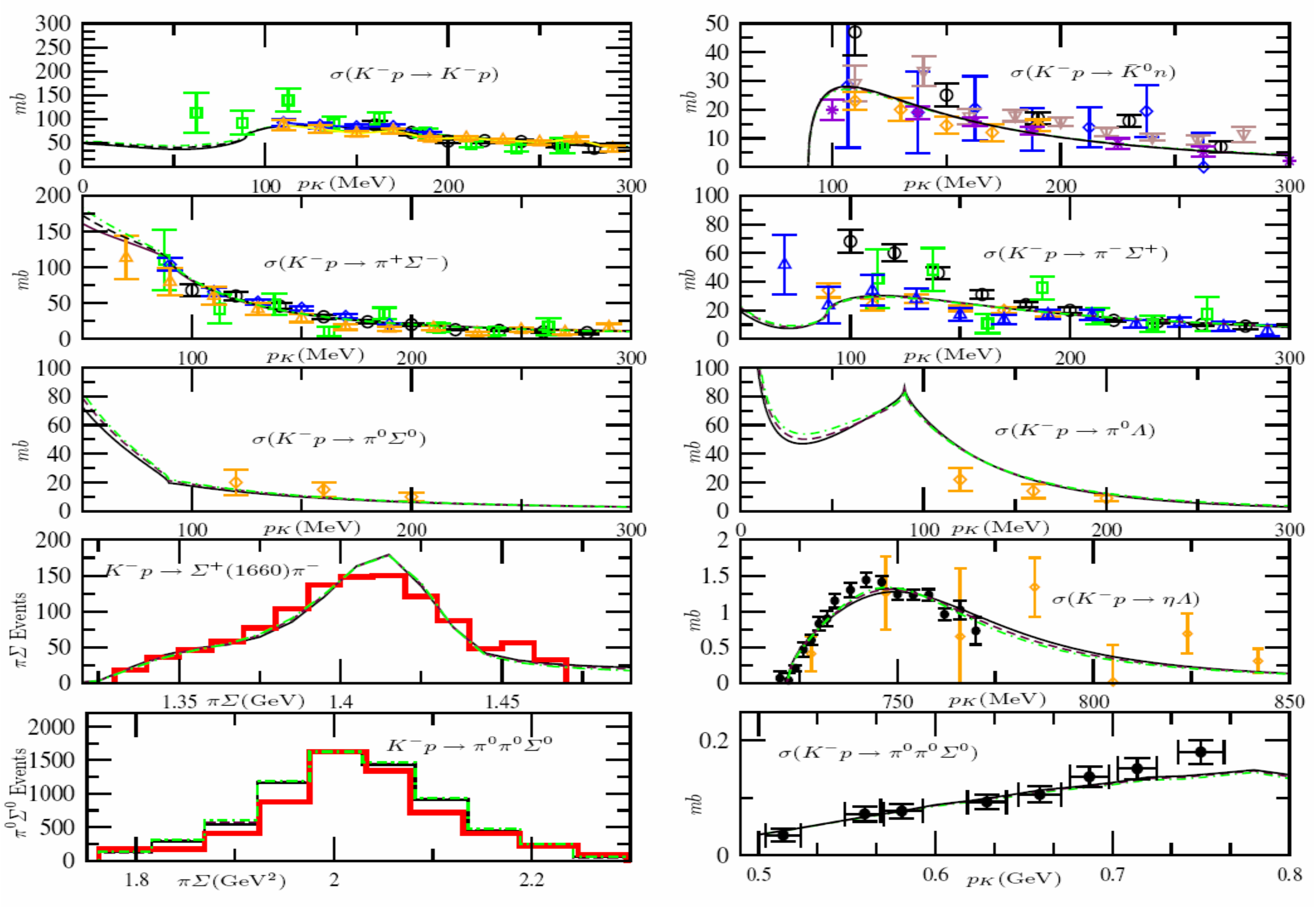
4. RESULTS

Two classes of fits A,B with $1s$ kaonic hydrogen ΔE and Γ :

A: Around DEAR (The fits are numerically more stable)

B: Away from DEAR.

Reproduction of the data by the new A-type fits (agree with DEAR) of **JAO, EPJA28,63('06)**



$\sigma_{\pi N}$	20^*	30^*	40^*
γ	2.36	2.36	2.37
R_c	0.629	0.628	0.628
R_n	0.168	0.171	0.173
ΔE (eV)	194	192	192
Γ (eV)	324	302	270
ΔE_D (eV)	204	204	207
Γ_D (eV)	361	338	305
a_{K^-p} (fm)	$-0.49 + i\,0.44$	$-0.49 + i\,0.41$	$-0.50 + i\,0.37$
a_0 (fm)	$-1.07 + i\,0.53$	$-1.04 + i\,0.50$	$-1.02 + i\,0.45$
a_1 (fm)	$0.44 + i\,0.15$	$0.40 + i\,0.15$	$0.33 + i\,0.14$
$\delta_{\pi\Lambda}(\Xi)$ ($^\circ$)	3.4	4.5	5.7
m_0 (GeV)	1.2	1.1	1.0
a^+_{0+} ($10^{-2} \cdot M_\pi^{-1}$)	-2.0	-2.2	-2.2

Experiment

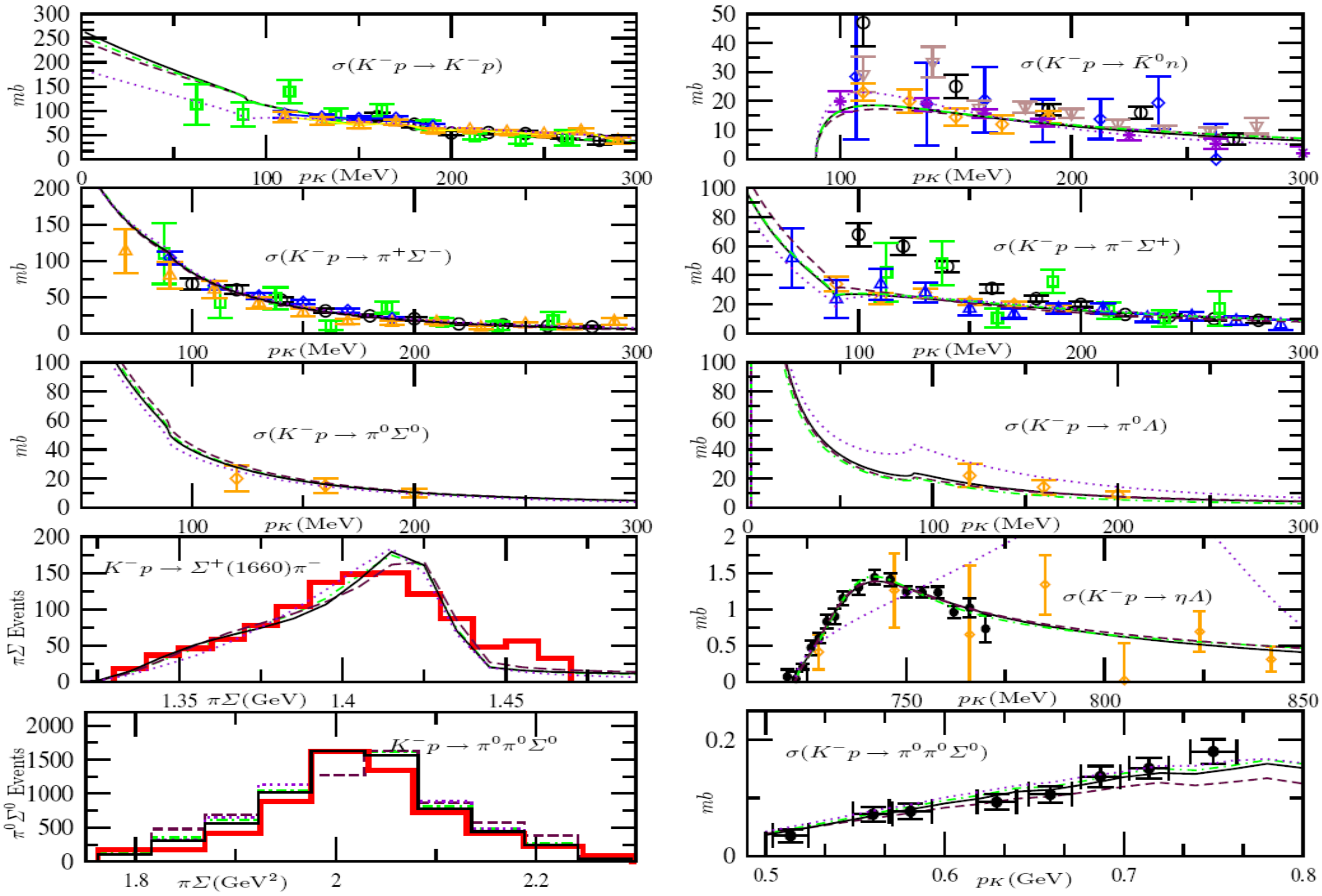
γ	2.36 ± 0.04
R_c	0.664 ± 0.011
R_n	0.189 ± 0.015
Δ_E	193 ± 38
Γ	249 ± 118
$\delta_{\pi\Lambda}$	4.6 ± 2

Units	$\sigma_{\pi N}$ MeV	20^*	30^*	40^*
MeV	f	75.2	71.8	67.8
GeV^{-1}	b_0	-0.615	-0.750	-0.884
GeV^{-1}	b_D	$+0.818$	$+0.848$	$+0.873$
GeV^{-1}	b_F	-0.114	-0.130	-0.138
GeV^{-1}	b_1	$+0.660$	$+0.670$	$+0.676$
GeV^{-1}	b_2	$+1.144$	$+1.169$	$+1.189$
GeV^{-1}	b_3	-0.297	-0.316	-0.315
GeV^{-1}	b_4	-1.048	-1.181	-1.307
	a_1	-1.786	-1.591	-1.413
	a_2	-0.519	-0.454	-0.386
	a_5	-1.185	-1.170	-1.156
	a_7	-5.251	-5.209	-5.123
	a_8	-1.316	-1.310	-1.308
	a_9	-1.186	-1.132	-1.050

These fits agree with the present experimental dada, both on scattering and kaonic hydrogen.

Three b's are fixed in terms of the others from the $O(p^2)$ constraints

Reproduction of the data by the new B-type fits (do not agree with DEAR) of **JAO, EPJA28,63('06)**



$\sigma_{\pi N}$	20^*	30^*	40^*	$\mathcal{O}(p)$
γ	2.34	2.35	2.34	2.32
R_c	0.643	0.643	0.644	0.637
R_n	0.160	0.163	0.176	0.193
ΔE (eV)	436	409	450	348
Γ (eV)	614	681	591	611
ΔE_D (eV)	418	385	436	325
Γ_D (eV)	848	880	844	775
a_{K^-p} (fm)	$-1.01 + i\,1.03$	$-0.93 + i\,1.07$	$-1.06 + i\,1.02$	$-0.79 + i\,0.94$
a_0 (fm)	$-1.75 + i\,1.15$	$-1.65 + i\,1.30$	$-1.79 + i\,1.10$	$-1.50 + i\,1.00$
a_1 (fm)	$-0.13 + i\,0.39$	$-0.14 + i\,0.36$	$-0.12 + i\,0.46$	$0.32 + i\,0.46$
$\delta_{\pi\Lambda}(\Xi)$ (°)	-1.4	1.7	-1.2	-1.4
m_0 (GeV)	0.8	0.6	0.7	...
a^+_{0+} $(10^{-2} \cdot M_\pi^{-1})$	-0.5	-1.4	$+0.3$	...

Experiment

γ	2.36 ± 0.04
R_c	0.664 ± 0.011
R_n	0.189 ± 0.015
Δ_E	193 ± 38
Γ	249 ± 118
$\delta_{\pi\Lambda}$	4.6 ± 2

These fits disagree with DEAR but agree with KEK

The scattering length a_{K^-p} is much larger than in the A-type fits.

Units	$\sigma_{\pi N}$ MeV	20^*	30^*	40^*	$\mathcal{O}(p)$
MeV	f	95.8	113.2	100.0	93.9
GeV^{-1}	b_0	-0.201	-0.159	-0.487	0^*
GeV^{-1}	b_D	-0.005	-0.297	0.127	0^*
GeV^{-1}	b_F	-0.133	-0.157	-0.188	0^*
GeV^{-1}	b_1	$+0.122$	$+0.016$	$+0.135$	0^*
GeV^{-1}	b_2	-0.080	-0.151	-0.037	0^*
GeV^{-1}	b_3	-0.533	-0.281	-0.494	0^*
GeV^{-1}	b_4	$+0.028$	-0.291	-0.173	0^*
	a_1	$+4.037$	$+4.188$	$+2.930$	-2.958
	a_2	-2.063	-3.129	-2.400	-1.479
	a_5	-1.131	-1.214	-1.225	-1.330
	a_7	-3.488	-3.000	-2.795	-1.805
	a_8	-0.347	$+0.642$	$+2.906$	-0.655
	a_9	-1.767	-2.109	-1.913	-1.918

Numerically it is simpler to obtain A-type fits

Three b's are fixed in terms of the others from the $O(p^2)$ constraints

K^-p Scattering Length:

- Martin, NPB179,33('81): $a_{K^-p} = -0.67 + i0.64$ fm
- Kaiser,Siegel,Weise, NPA594,325('95): $a_{K^-p} = -0.97 + i1.1$ fm
- Oset,Ramos, NPA635,99('98): $a_{K^-p} = -0.99 + i0.97$ fm
- Meissner,JAO PLB500,263('01): $a_{K^-p} = -0.75 + i1.2$ fm
- Borasoy,Nissler,Weise,PRL94,213401('05), EPJA25,79('05):

  $a_{K^-p} = -0.51 + i\,0.82$ fm. They cannot reproduce the elastic $K^-p \to K^-p$ cross section together with the DEAR measuremen (compromise).

- Previous work, J. Prades, M. Verbeni and JAO, Phys. Rev. Lett. 95,172502(2005)

Fit: A_4^+: $a_{K^-p} = -0.50 + i\,0.42$ fm. B_4^+: $a_{K^-p} = -1.01 + i\,0.80$ fm

New A-type:

$\sigma_{\pi N}$	20^*	30^*	40^*
a_{K^-p} (fm)	$-0.49 + i\,0.44$	$-0.49 + i\,0.41$	$-0.50 + i\,0.37$
a_0 (fm)	$-1.07 + i\,0.53$	$-1.04 + i\,0.50$	$-1.02 + i\,0.45$
a_1 (fm)	$0.44 + i\,0.15$	$0.40 + i\,0.15$	$0.33 + i\,0.14$

New B-type:

a_{K^-p} (fm)	$-1.01 + i\,1.03$	$-0.93 + i\,1.07$	$-1.06 + i\,1.02$
a_0 (fm)	$-1.75 + i\,1.15$	$-1.65 + i\,1.30$	$-1.79 + i\,1.10$
a_1 (fm)	$-0.13 + i\,0.39$	$-0.14 + i\,0.36$	$-0.12 + i\,0.46$

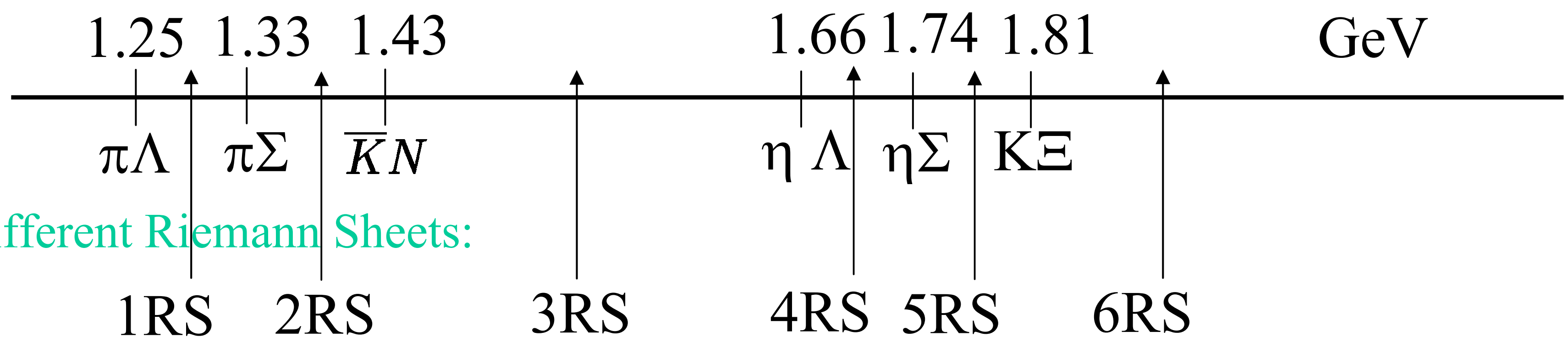
5. SPECTROSCOPY

$$T_{ij} = \lim_{s \to s_R} - \frac{\boxed{\gamma_i \gamma_j}}{s - \boxed{s_R}}$$

Residues

Pole Position ' $(M_R - i\Gamma_R/2)^2$

Physical Riemann Shet



Fit I: New A-type fit with $\sigma_{\pi N}$=40 MeV

I=0 Poles (MeV)

Λ(1310)

Λ(1405)

PDG: $M = 1406.5 \pm 4.0$

$\Gamma = 50 \pm 2$

Λ(1670)

PDG:M=1660-1680

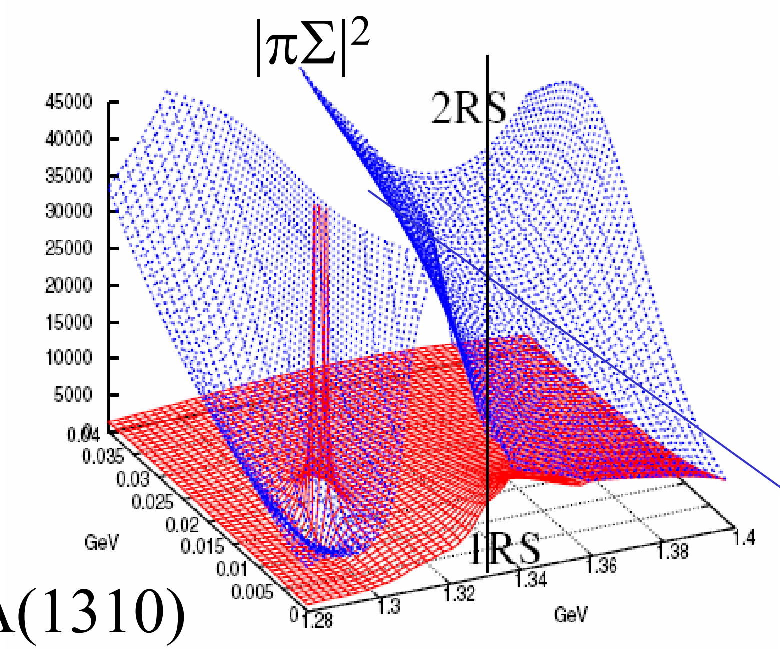
Γ=25-50

Λ(1800)

PDG:M=1720-1850

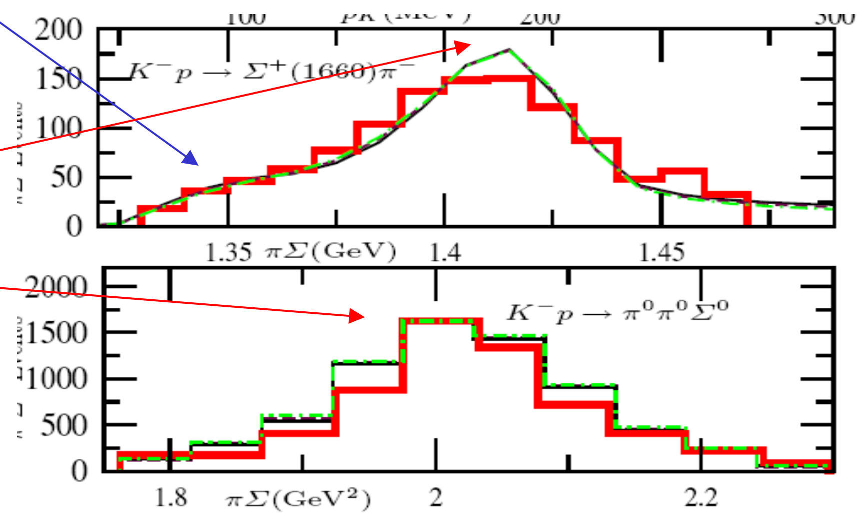
Γ=65 - 400

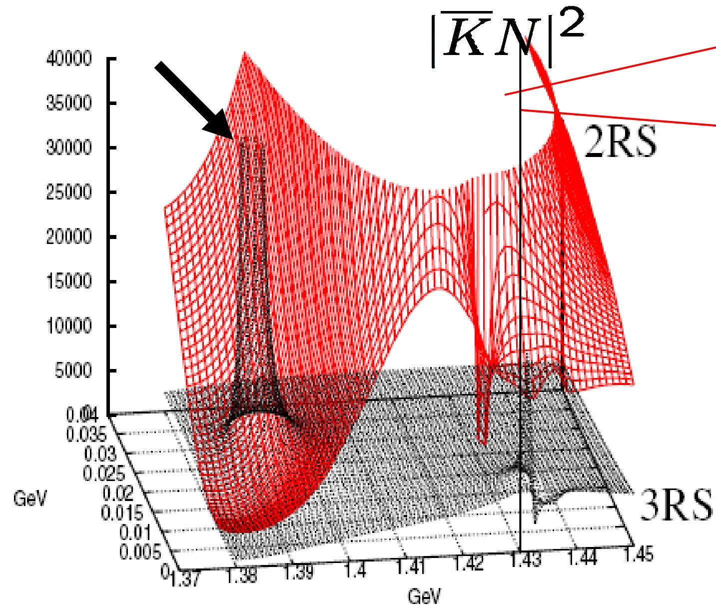
Re(Pole) $\lvert\gamma_{\pi\Lambda}\rvert$	−Im(Pole) $\lvert\gamma_{\pi\Sigma}\rvert_0$	Sheet $\lvert\gamma_{\pi\Sigma}\rvert_1$	$\lvert\gamma_{\pi\Sigma}\rvert_2$	$\lvert\gamma_{\bar{K}N}\rvert_0$	$\lvert\gamma_{\bar{K}N}\rvert_1$	$\lvert\gamma_{\eta\Lambda}\rvert$	$\lvert\gamma_{\eta\Sigma}\rvert$	$\lvert\gamma_{K\Xi}\rvert_0$	$\lvert\gamma_{K\Xi}\rvert_1$
1301	13	1RS							
0.03	1.12	0.02	0.01	5.83	0.05	0.41	0.04	2.11	0.03
1309	13	2RS							
0.02	3.66	0.02	0.02	4.46	0.04	0.21	0.04	3.05	0.03
1414	23	2RS							
0.14	4.24	0.13	0.01	4.87	0.39	0.85	0.20	9.35	0.11
1388	17	3RS							
0.02	3.81	0.02	0.02	1.33	0.04	0.42	0.04	9.55	0.04
1676	10	3RS							
0.01	1.28	0.03	0.00	1.67	0.01	2.19	0.07	5.29	0.07
1673	18	4RS							
0.01	1.26	0.02	0.00	1.82	0.01	2.13	0.06	5.32	0.06
1825	49	5RS							
0.02	2.29	0.02	0.00	2.10	0.02	0.89	0.03	7.43	0.09



Λ(1310)

$\lvert\gamma_{\pi\Lambda}\rvert$	$\lvert\gamma_{\pi\Sigma}\rvert_0$	$\lvert\gamma_{\pi\Sigma}\rvert_1$	$\lvert\gamma_{\pi\Sigma}\rvert_2$	$\lvert\gamma_{\bar{K}N}\rvert_0$	$\lvert\gamma_{\bar{K}N}\rvert_1$	$\lvert\gamma_{\eta\Lambda}\rvert$	$\lvert\gamma_{\eta\Sigma}\rvert$	$\lvert\gamma_{K\Xi}\rvert_0$
1301	13	1RS						
0.03	1.12	0.02	0.01	5.83	0.05	0.41	0.04	2.11
1309	13	2RS						
0.02	3.66	0.02	0.02	4.46	0.04	0.21	0.04	3.05





Λ(1405)

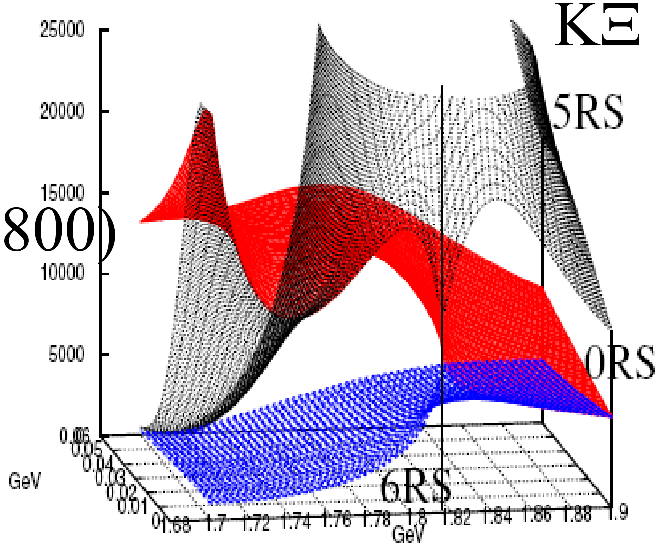
$\lvert\gamma_{\pi\Lambda}\rvert$	$\lvert\gamma_{\pi\Sigma}\rvert_0$	$\lvert\gamma_{\pi\Sigma}\rvert_1$	$\lvert\gamma_{\pi\Sigma}\rvert_2$	$\lvert\gamma_{\bar{K}N}\rvert_0$	$\lvert\gamma_{\bar{K}N}\rvert_1$	$\lvert\gamma_{\eta\Lambda}\rvert$	$\lvert\gamma_{\eta\Sigma}\rvert$	$\lvert\gamma_{K\Xi}\rvert_0$
1414	23	2RS						
0.14	4.24	0.13	0.01	4.87	0.39	0.85	0.20	9.35
1388	17	3RS						
0.02	3.81	0.02	0.02	1.33	0.04	0.42	0.04	9.55

Λ(1670)

$\lvert\gamma_{\pi\Lambda}\rvert$	$\lvert\gamma_{\pi\Sigma}\rvert_0$	$\lvert\gamma_{\pi\Sigma}\rvert_1$	$\lvert\gamma_{\pi\Sigma}\rvert_2$	$\lvert\gamma_{\bar{K}N}\rvert_0$	$\lvert\gamma_{\bar{K}N}\rvert_1$	$\lvert\gamma_{\eta\Lambda}\rvert$	$\lvert\gamma_{\eta\Sigma}\rvert$	$\lvert\gamma_{K\Xi}\rvert_0$
1676	10	3RS						
0.01	1.28	0.03	0.00	1.67	0.01	2.19	0.07	5.29
1673	18	4RS						
0.01	1.26	0.02	0.00	1.82	0.01	2.13	0.06	5.32

Assymetry in the width, before the ηΛ threshold Γ=20 MeV and above Γ=36 MeV

Λ(1800)



$\lvert\gamma_{\pi\Lambda}\rvert$	$\lvert\gamma_{\pi\Sigma}\rvert_0$	$\lvert\gamma_{\pi\Sigma}\rvert_1$	$\lvert\gamma_{\pi\Sigma}\rvert_2$	$\lvert\gamma_{\bar{K}N}\rvert_0$	$\lvert\gamma_{\bar{K}N}\rvert_1$	$\lvert\gamma_{\eta\Lambda}\rvert$	$\lvert\gamma_{\eta\Sigma}\rvert$	$\lvert\gamma_{K\Xi}\rvert_0$
1825	49	5RS						
0.02	2.29	0.02	0.00	2.10	0.02	0.89	0.03	7.43

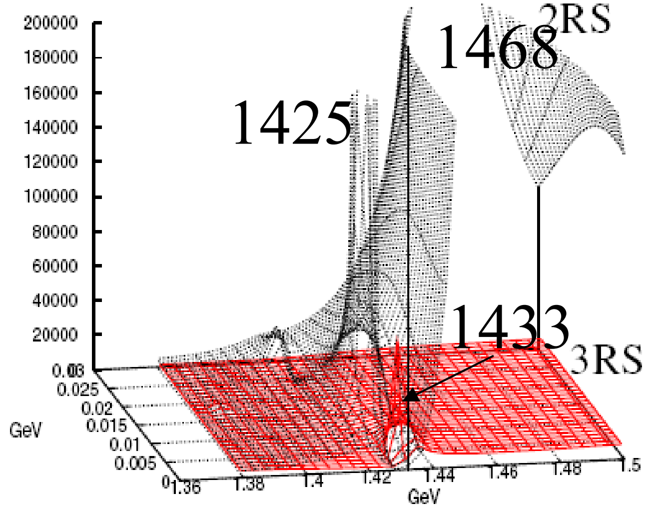
I=1 Poles (MeV)

Σ(1750)

PDG:M=1730-1800

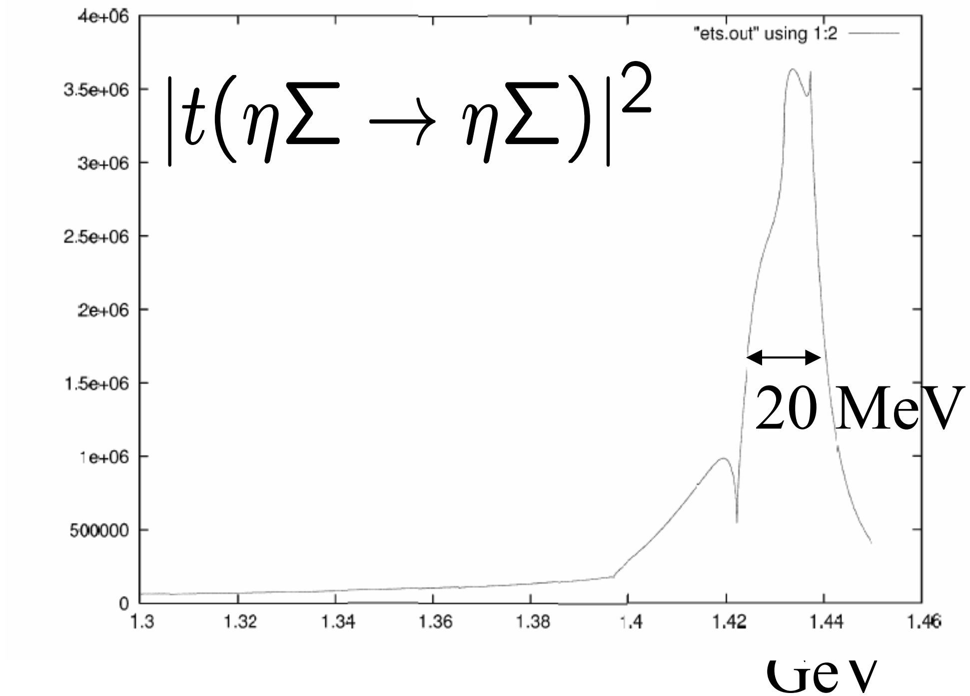
Γ: 50 - 160

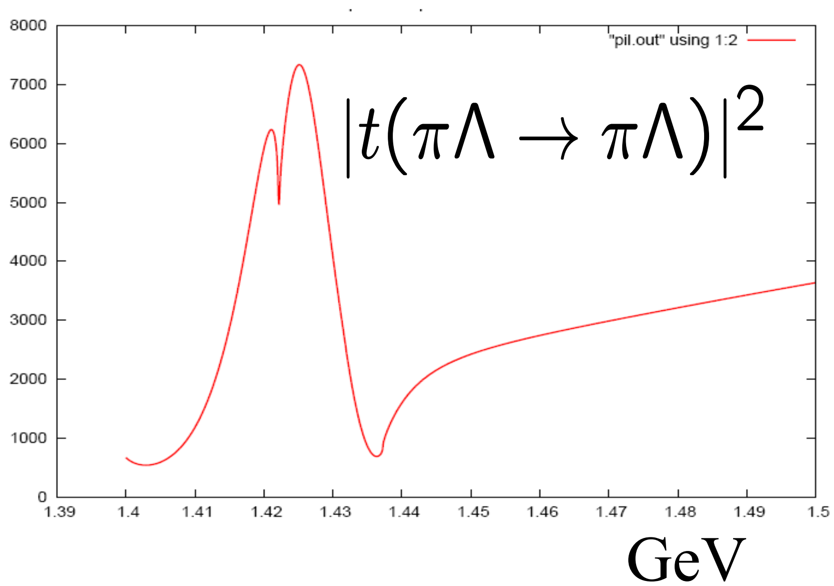
Re(Pole)	−Im(Pole)	Sheet							
$\lvert\gamma_{\pi\Lambda}\rvert$	$\lvert\gamma_{\pi\Sigma}\rvert_0$	$\lvert\gamma_{\pi\Sigma}\rvert_1$	$\lvert\gamma_{\pi\Sigma}\rvert_2$	$\lvert\gamma_{\bar{K}N}\rvert_0$	$\lvert\gamma_{\bar{K}N}\rvert_1$	$\lvert\gamma_{\eta\Lambda}\rvert$	$\lvert\gamma_{\eta\Sigma}\rvert$	$\lvert\gamma_{K\Xi}\rvert_0$	$\lvert\gamma_{K\Xi}\rvert_1$
1425	6.5	2RS							
1.35	0.24	1.66	0.01	0.35	3.92	0.05	4.23	0.49	2.98
1468	13	2RS							
2.80	0.16	5.96	0.02	0.23	8.74	0.04	10.66	0.19	2.48
1433	3.7	3RS							
0.65	0.08	0.80	0.00	0.12	1.58	0.02	5.82	0.20	2.14
1720	18	4RS							
1.82	0.02	1.21	0.00	0.02	0.95	0.02	6.78	0.05	5.31
1769	96	6RS							
2.65	0.00	0.61	0.00	0.00	2.48	0.00	3.32	0.01	4.22
1340	143	3-4RS							
1.33	0.14	5.50	0.02	0.02	1.58	0.00	3.28	0.03	1.20
1395	311	3-4RS							
2.08	0.01	1.49	0.01	0.00	1.24	0.00	7.63	0.01	3.97



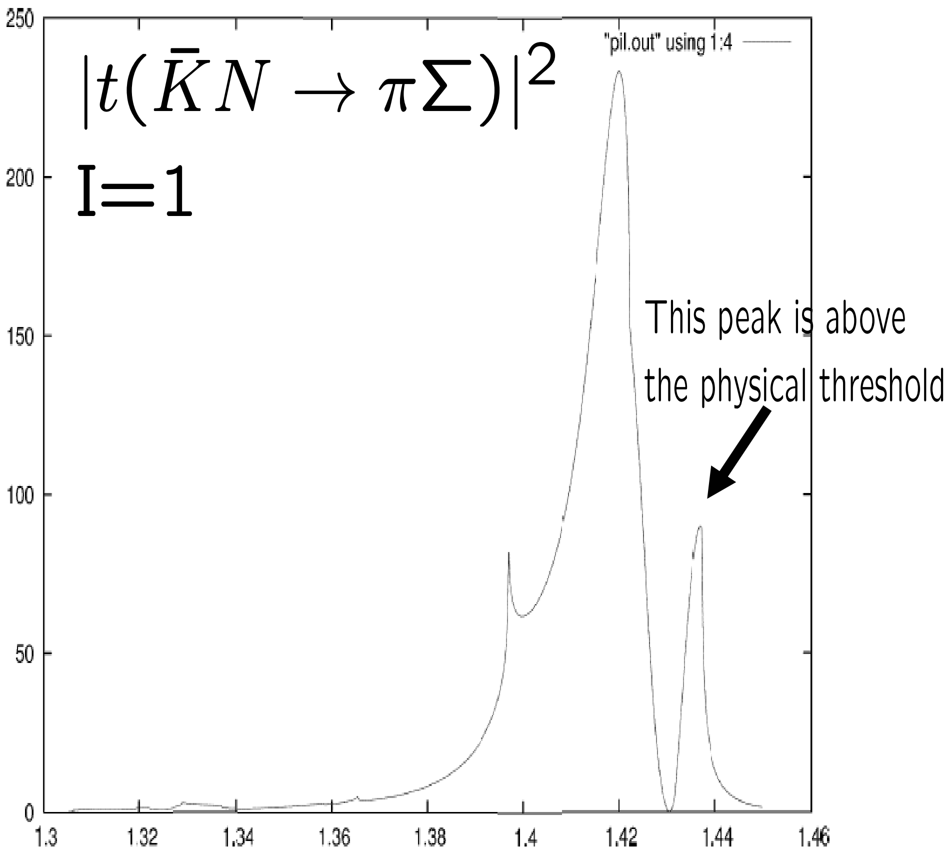
$\lvert\gamma_{\pi\Lambda}\rvert$	$\lvert\gamma_{\pi\Sigma}\rvert_0$	$\lvert\gamma_{\pi\Sigma}\rvert_1$	$\lvert\gamma_{\pi\Sigma}\rvert_2$	$\lvert\gamma_{\bar{K}N}\rvert_0$	$\lvert\gamma_{\bar{K}N}\rvert_1$	$\lvert\gamma_{\eta\Lambda}\rvert$	$\lvert\gamma_{\eta\Sigma}\rvert$	$\lvert\gamma_{K\Xi}\rvert_0$	$\lvert\gamma_{K\Xi}\rvert_1$
1425	6.5	2RS							
1.35	0.24	1.66	0.01	0.35	3.92	0.05	4.23	0.49	2.98
1468	13	2RS							
2.80	0.16	5.96	0.02	0.23	8.74	0.04	10.66	0.19	2.48
1433	3.7	3RS							
0.65	0.08	0.80	0.00	0.12	1.58	0.02	5.82	0.20	2.14

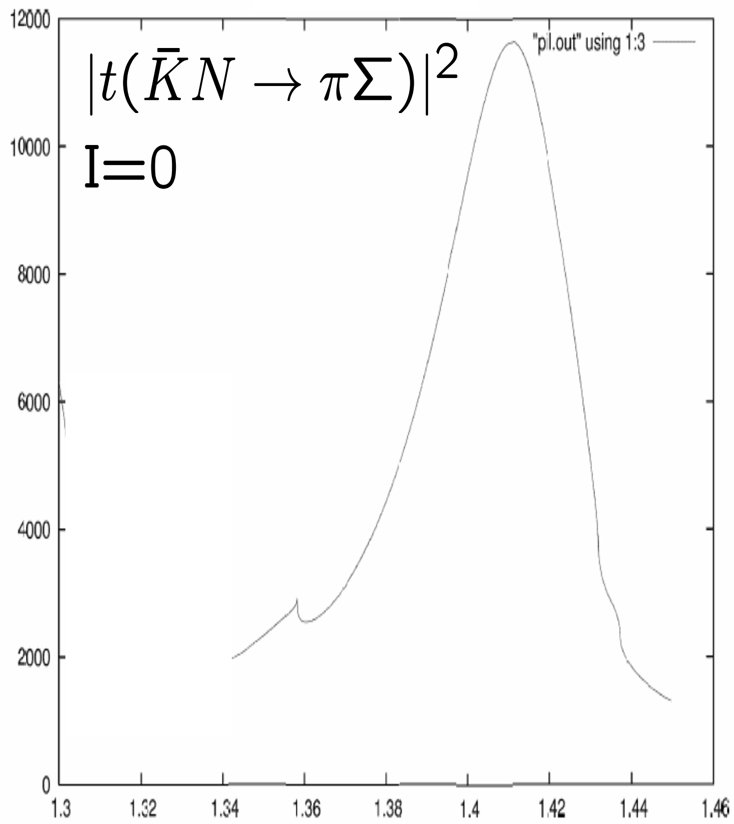
On the physical axis between 1.4 and 1.5 GeV



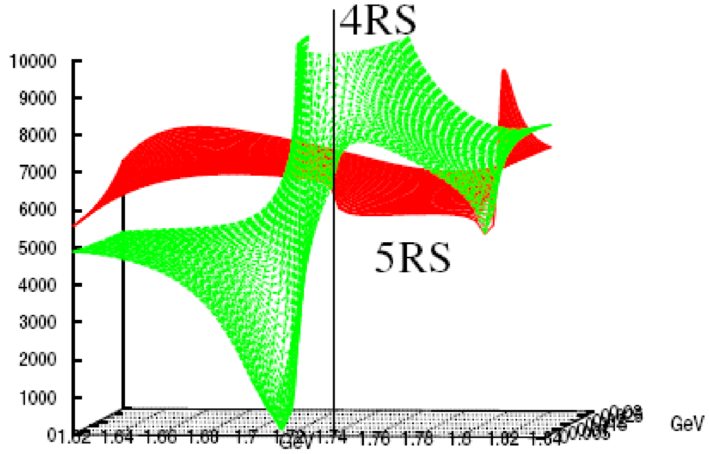


I=1 is much smaller than I=0. This is why these narrow peaks in I=1 are not seen in $\pi\Sigma$ event distributions (up to now).
One needs and I=1 ‘filter.





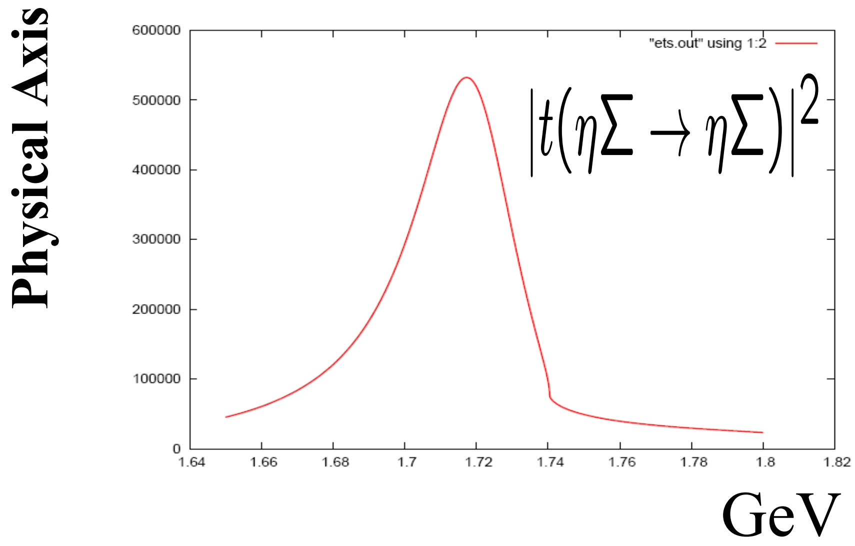
Σ(1750)



$\lvert\gamma_{\pi\Lambda}\rvert$	$\lvert\gamma_{\pi\Sigma}\rvert_0$	$\lvert\gamma_{\pi\Sigma}\rvert_1$	$\lvert\gamma_{\pi\Sigma}\rvert_2$	$\lvert\gamma_{\bar{K}N}\rvert_0$	$\lvert\gamma_{\bar{K}N}\rvert_1$	$\lvert\gamma_{\eta\Lambda}\rvert$	$\lvert\gamma_{\eta\Sigma}\rvert$	$\lvert\gamma_{K\Xi}\rvert_0$	$\lvert\gamma_{K\Xi}\rvert_1$
1720	18	4RS							
1.82	0.02	1.21	0.00	0.02	0.95	0.02	6.78	0.05	5.31

For the open channels πΛ, πΣ , $\overline{K}N$ it is a distorted bump

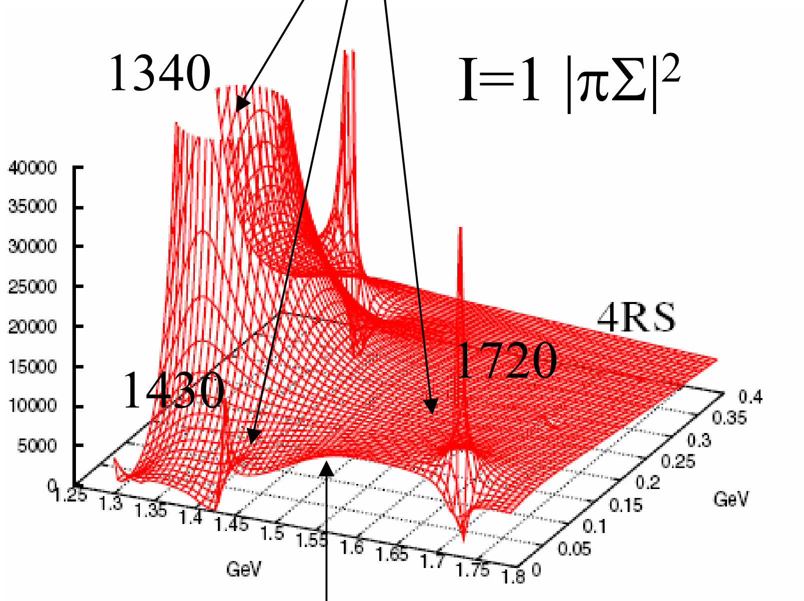
For the closed channels ηΣ and KΞ it is a clear resonance shape



$\Sigma(1620)$

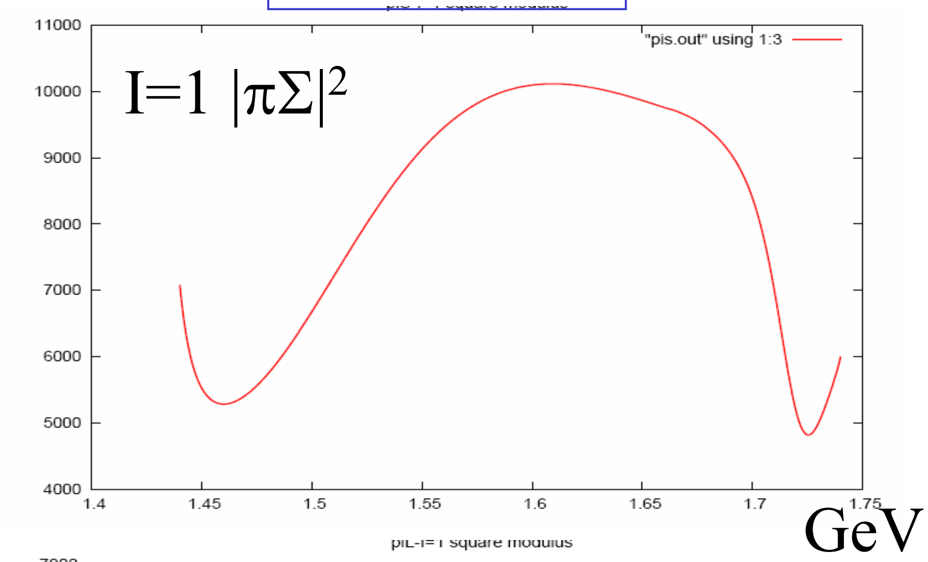
The amplitudes show a broad bumpt after the $\overline{K}N$ threshold and before that of the $\eta\Sigma$

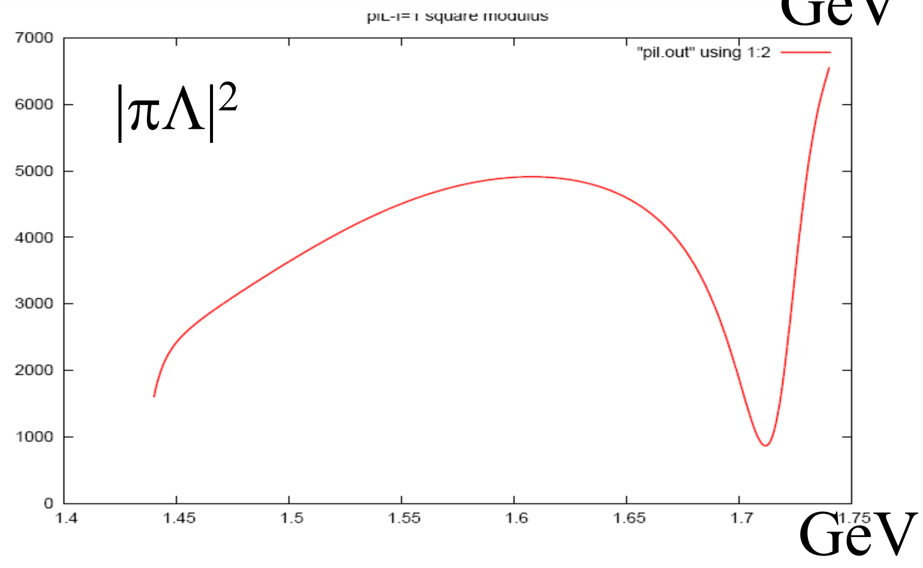
Multipole interference effect



Σ(1620) Bump

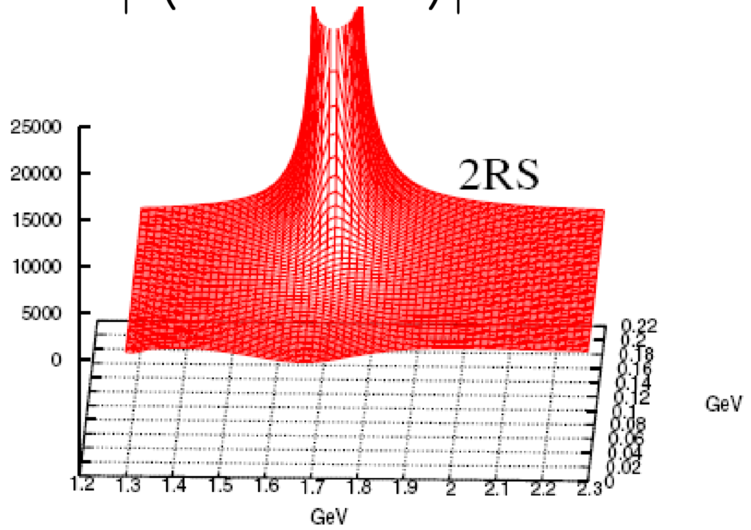
Physical Axis





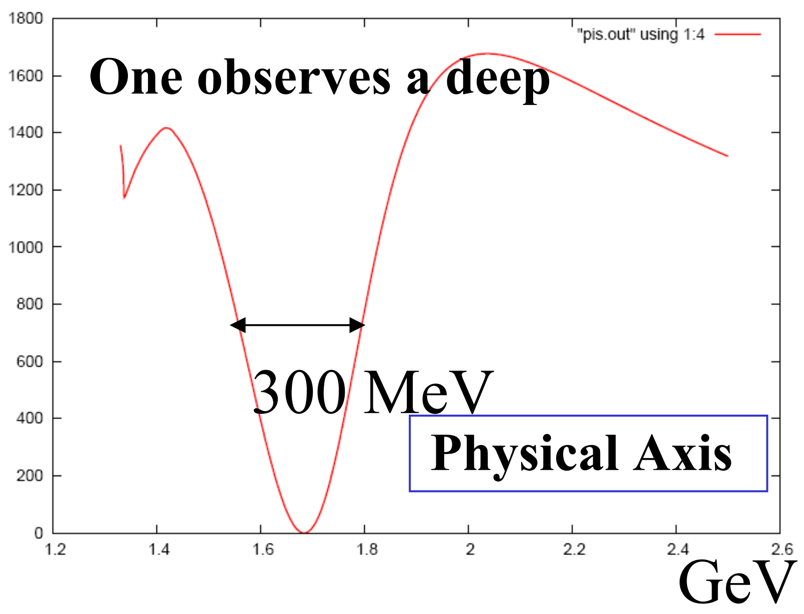
I=2 Pole (MeV) at 1722-i 181 MeV   Exotic state

$$I=2\ \left|t(\pi\Sigma\rightarrow\pi\Sigma)\right|^2$$



The only resonance in I=2

Non uniform shape.
I=2 is of a size not negligibly small compared with other spin channels.



$$I=2\ \left|t(\pi\Sigma\rightarrow\pi\Sigma)\right|^2$$

• Fit I: New A-type with $\sigma_{\pi N} = 40$ MeV

I=0: $\Lambda(1305)$, $\Lambda(1405)$, $\Lambda(1670)$, $\Lambda(1800)$

I=1: $\Sigma(1430)$, $\Sigma(1620)$, $\Sigma(1750)$

• Fit II: New B-type with $\sigma_{\pi N} = 40$ MeV

I=0: $\Lambda(1305)$, $\Lambda(1405)$, $\Lambda(1670)$, $\cancel{\Lambda(1800)}$

I=1: $\Sigma(1430)$, $\cancel{\Sigma(1620)}$, $\cancel{\Sigma(1750)}$

Only in $\bar{K}N$

$8 \otimes 8 = 1 \oplus 8_s \oplus 8_a \oplus 10 \oplus \bar{10} \oplus 27$

Fit I: Has attractive SU(3) kernels for 1, 8_s, 8_a, 27
It can accomadate 4 I=0 and 3 I=1 resonances.

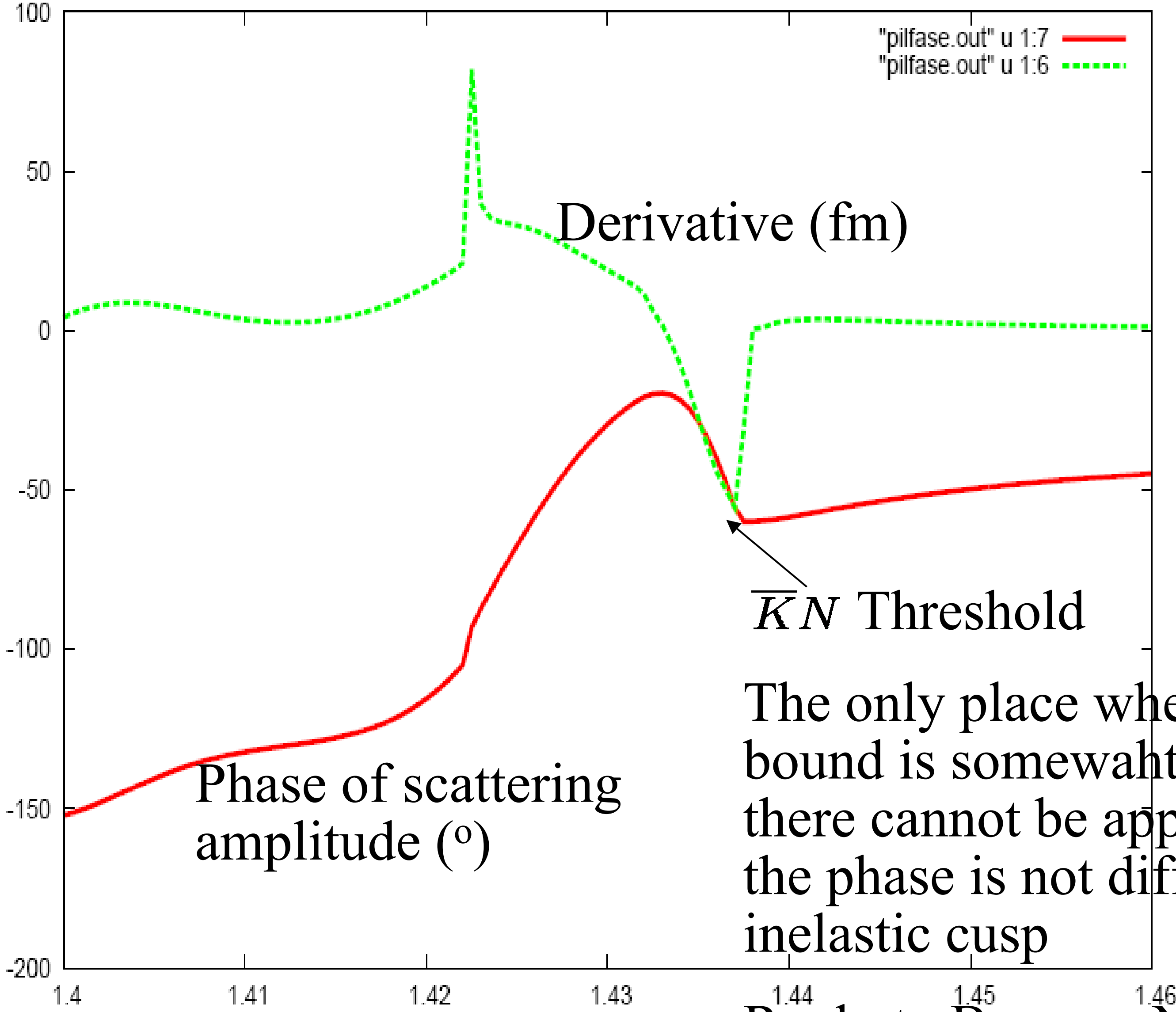
Fit II: Has attractive SU(3) kernels for 1, 8_s, 8_a and $\bar{10}$
It can accomodate 3 I=0 and 3 I=1 resonances.

4. CONCLUSIONS

- UCHPT study of meson-baryon dynamics with strangeness -1 in S-wave up to NNLO or $\mathcal{O}(p^2)$
- Simultaneous reproduction of scattering and kaonic hydrogen data. Including the recent and precise data.
- The A-type fits also generate the resonances: $\Lambda(1305)$, $\Lambda(1405)$, $\Lambda(1607)$, $\Lambda(1800)$ for I=0 and $\Sigma(1430)$, $\Sigma(1620)$, $\Sigma(1750)$ for I=1.
  All the ones quoted in the PDG up to 1.8 GeV for $1/2^-$ and strangeness -1.
- The B-type fits do not reproduce DEAR but agree with KEK and scattering data.
- The B-type fits are not able to generate a comparable set of resonances. The $\Lambda(1800)$ and $\Sigma(1750)$ are missing.

- The fits A are then preferred over the B ones, based on the present experimental information from scattering, spectroscopy and kaonic hydrogen data.

- $a_{K^-p} = -0.50 + i\,0.40$ fm (A) preferred over $a_{K^-p} = -1.0 + i1.0$ fm (B)



The only place where Wigner bound is somewaht violated but there cannot be applied because the phase is not differetiable – inelastic cusp

Reply to Borasoy,Nissler,Weise Comment PRL96,199201('06)

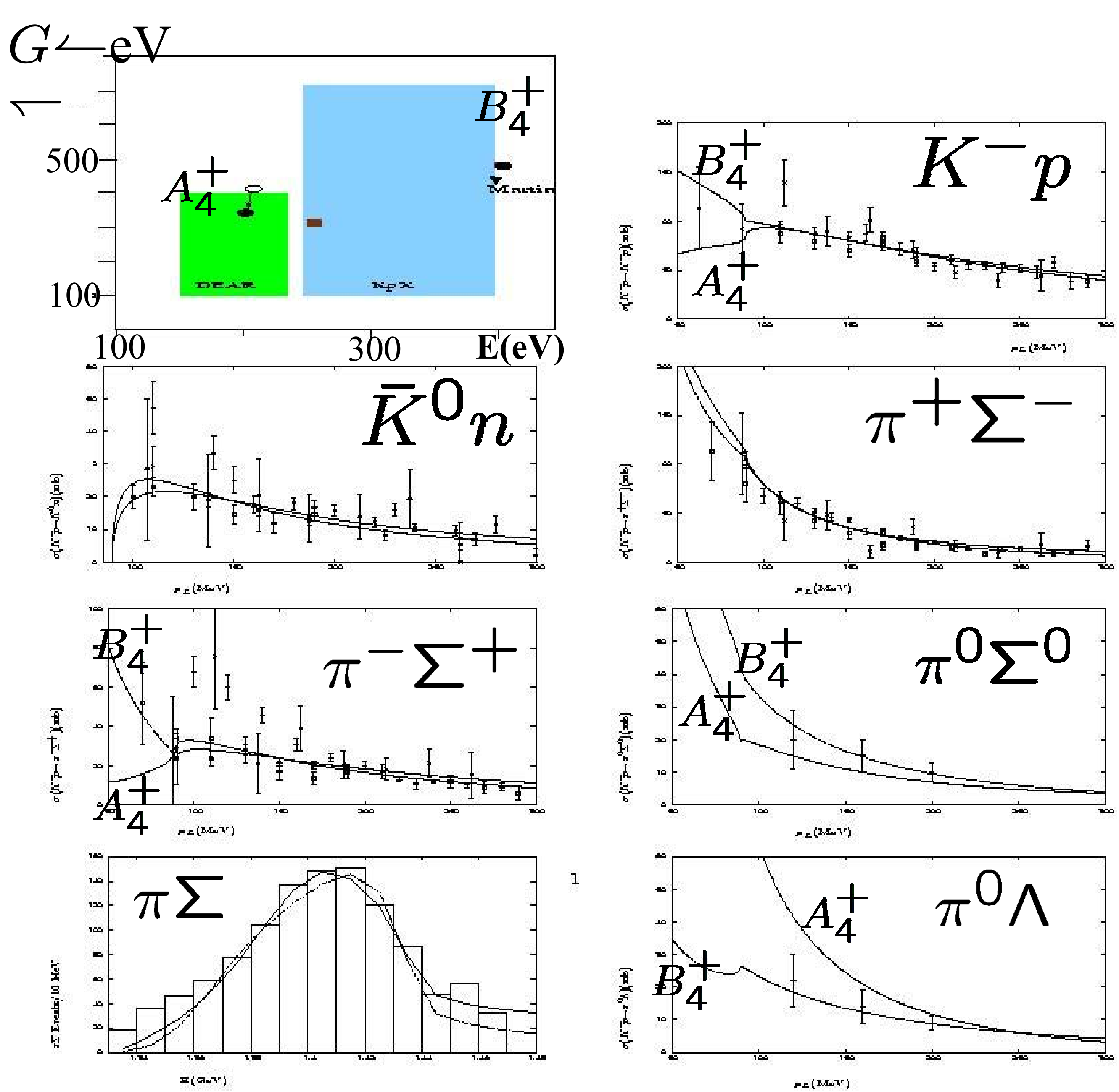


Figure 1: First panel: 1s kaonic hydrogen strong energy shift and width. In the rest, the solid lines correspond to the fit A_4^+ and the dashed ones to B_4^+. For further details see the text.

$A_4^+(B_4^+)$	Experiment
$\gamma = 2.36(2.36)$	$\gamma = 2.36 \pm 0.04$
$R_c = 0.628(0.655)$	$R_c = 0.664 \pm 0.011$
$R_n = 0.172(0.195)$	$R_n = 0.189 \pm 0.015$

$$\sigma_{\pi N} = 40 \text{ MeV}$$

$$m_0 = 0.8 \text{ GeV}$$

$$a_{0+}^+ = (-1 \pm 1) m_\pi^{-1} 10^{-2}$$

$\pi\Sigma$ I=0 Mass Distribution

Hemingway, NPB253,742('85)

$K^- p \to \Sigma^\pm \pi^\mp \pi^- \pi^+$ from them $\Sigma^\pm \pi^\mp$ event distributions are obtained. The I=0 corresponds to the average of both.

Typically one takes: $$\frac{dN_{\pi\Sigma}}{dE} = C|T^{I=0}_{\pi\Sigma\to\pi\Sigma}|^2 p_{\pi\Sigma}$$ As if the process were elastic

E.g: Dalitz, Deloff, JPG 17,289 (´91); Müller,Holinde,Speth NPA513,557(´90), Kaiser, Siegel, Weise NPB594,325 (´95); Oset, Ramos NPA635, 99 (´89)

But the $\bar{K}N$ threshold is only 100 MeV above the $\pi\Sigma$ one, comparable with the widths of the present resonances in this region and with the width of the shown invariant mass distribution. This prescription is ambiguous, why not?

$$\frac{dN_{\pi\Sigma}}{dE} = C|T^{I=0}_{\bar{K}N\to\pi\Sigma}|^2 p_{\pi\Sigma}$$

We follow the Production Process scheme previously shown: already employed for this case in **Meissner, JAO PLB500,263('01)**

$$F = (I + R\cdot g)^{-1}\cdot\xi\ ,\quad \xi^T = (0, r_1, r_1, r_1, r_2, r_2, 0, 0, 0, 0)$$

$$\frac{r_2}{r_1} = -0.28$$

I=0 Source

$r_2=0$ (previous approach)

$\delta_P - \delta_S$ $\Lambda\pi$ PHASE SHIFTS DIFFERENCE

AT THE Ξ^- MASS, RECENT MEASUREMENTS

FROM THE DECAY PARAMETERS $\Xi^- \to \Lambda\pi^-$:

$(4.6 \pm 1.4 \pm 1.2)^{\mathrm{o}}$ Huang et al. (HyperCP Coll.) PRL93,011802 ('04)

$(3.2 \pm 5.3 \pm 0.7)^{\mathrm{o}}$ Chakravorty et al. (E756 Coll.) PRL91,031601 ('03)

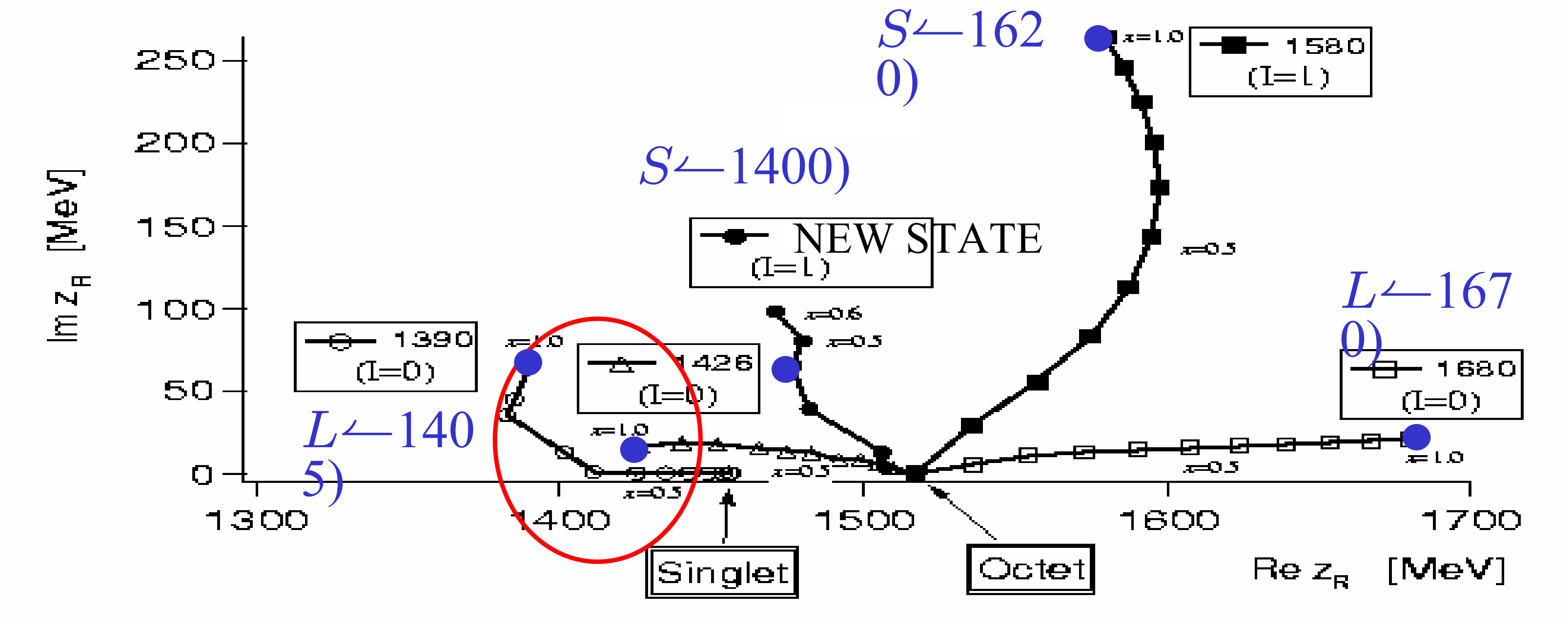
Fit A_4^+ PREDICTS: 2.5^{o} COMPATIBLE WITH DATA

For Fit B_4^+: 0.2^{o}

Our calculation at NLO supports a pronounced two pole structure in the *L*(1405) region as obtained in the LO studies of
Meissner,JAO PLB500,263('01) (other later works)
Jido,Oset,Ramos,Meissner,JAO, NPA725,181('03)
at odds with the claims of Borasoy,Nissler,Weise,PRL94,213401('05)

THERE ARE MORE RESONANCES...

We also confirm the scheme Jido,Oset,Ramos,Meissner,JAO, NPA725,181('03)



Poles from the SU(3) representations: 1, 8s, 8a (similar pattern as shown before):

I=0

(1321-i 43.5) MeV

(1402-i39.6) MeV

L←140 5)

(1756-i150) MeV *L*(1670) ??

(position very sensitive to particular values of a's, TO THE INPUT)

I=1

(1487-i46) MeV *S*(1480)

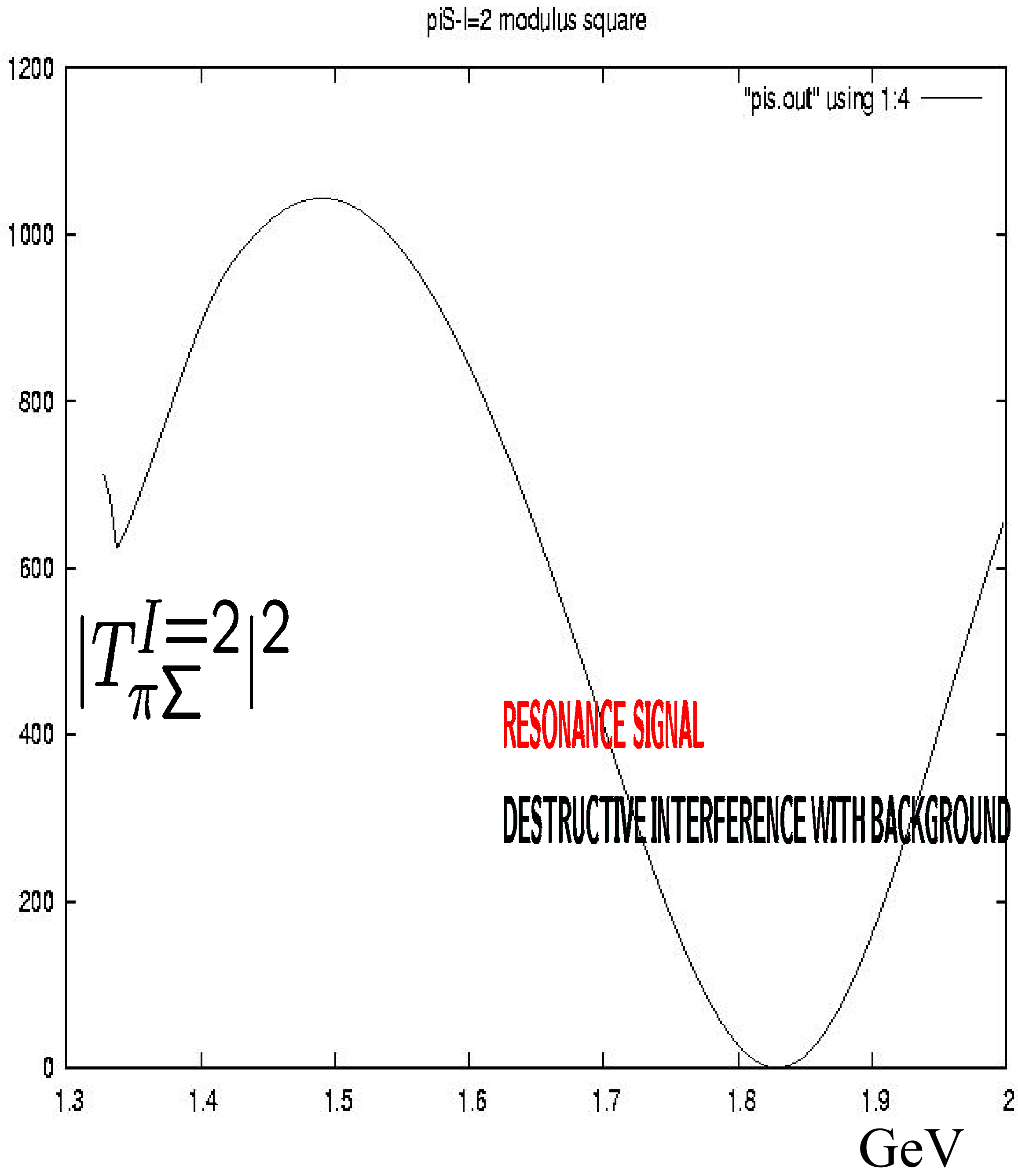
(1694-i 149) MeV *S*(1620)

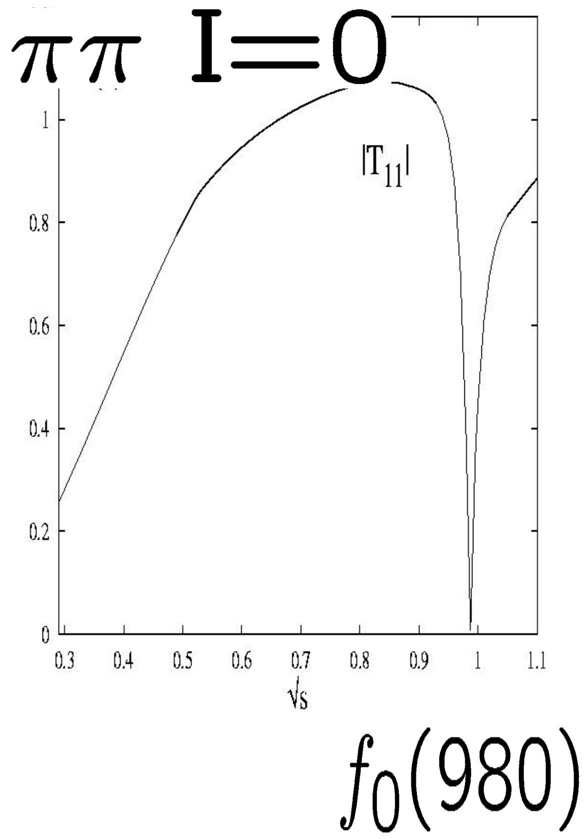
EXOTIC 27 REPRESENTATION:

I=0 (1773-i219) MeV *L*(1800)

I=1 (1822-i217) MeV *S*(1750)

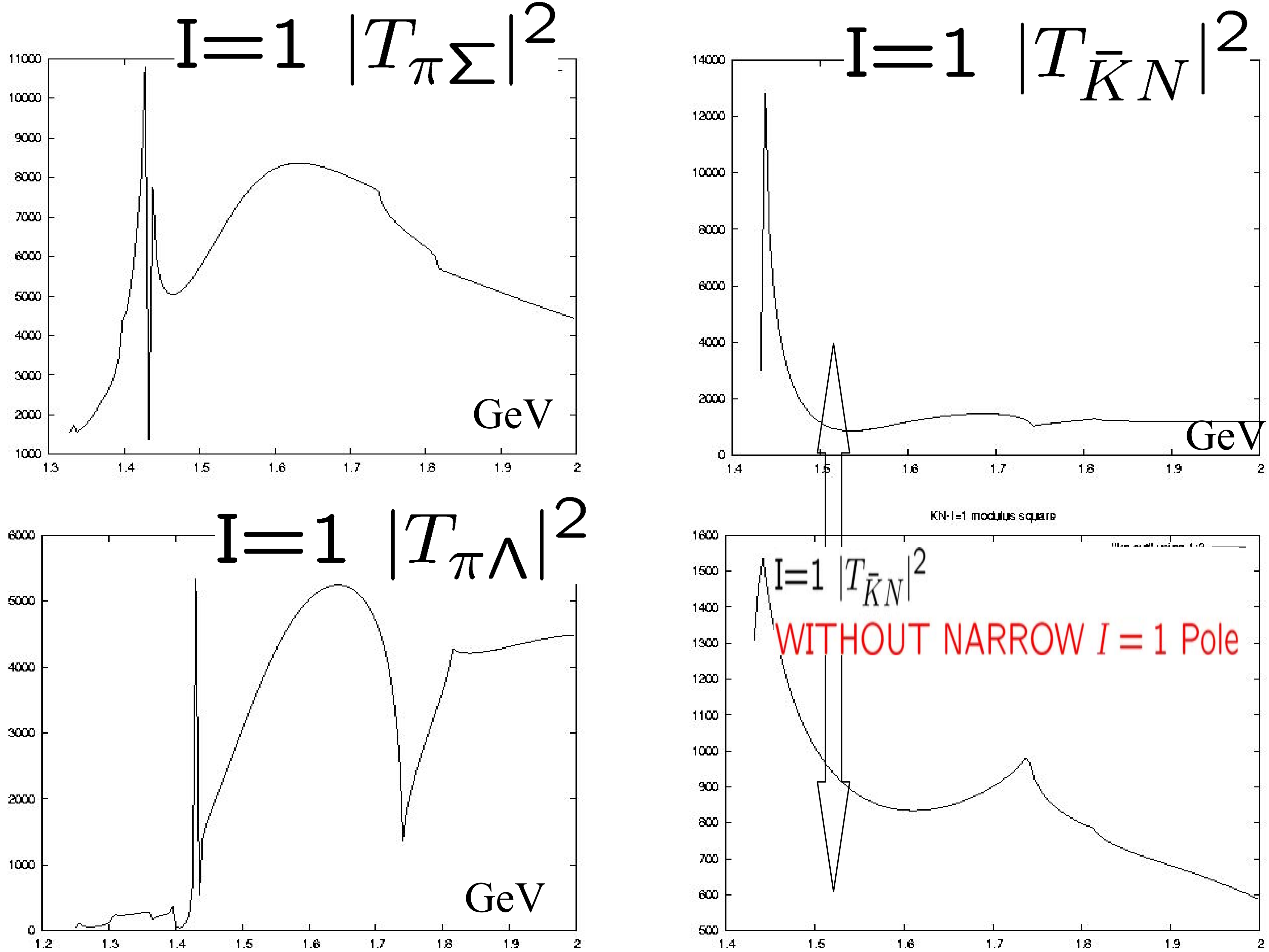
I=2 (1862-i238) MeV This is the p^+ S^+ channel and it can be observed since there are no additional resonances !!


piS-I=2 modulus square
"pis.out" using 1:4
$|T^{I=2}_{\pi\Sigma}|^2$
RESONANCE SIGNAL
DESTRUCTIVE INTERFERENCE WITH BACKGROUND
GeV
1200
1000
800
600
400
200
0
1.3
1.4
1.5
1.6
1.7
1.8
1.9
2


$\pi\pi$ I=0
$|T_{11}|$
$\sqrt{s}$
$f_0(980)$
1
0.8
0.6
0.4
0.2
0
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
1.1

Narrow resonance just on top the $\bar{K}N$ threshold:

I=1 (1431-i1.3) MeV



I=1 $|T_{\pi\Sigma}|^2$
GeV
I=1 $|T_{\bar{K}N}|^2$
GeV
I=1 $|T_{\pi\Lambda}|^2$
GeV
KN-I=1 modulus square
I=1 $|T_{\bar{K}N}|^2$
WITHOUT NARROW $I=1$ Pole

Narrow resonance just on top the $\bar{K}N$ threshold:

I=1 (1431-i1.3) MeV

ISOSPIN $K^{-}N$ SCATTERING LENGTHS,fm:

$$A_4^{+} \qquad\qquad\qquad B_4^{+}$$

$$a_0 = -1.23 + i0.45 \quad a_0 = -1.63 + i0.81$$

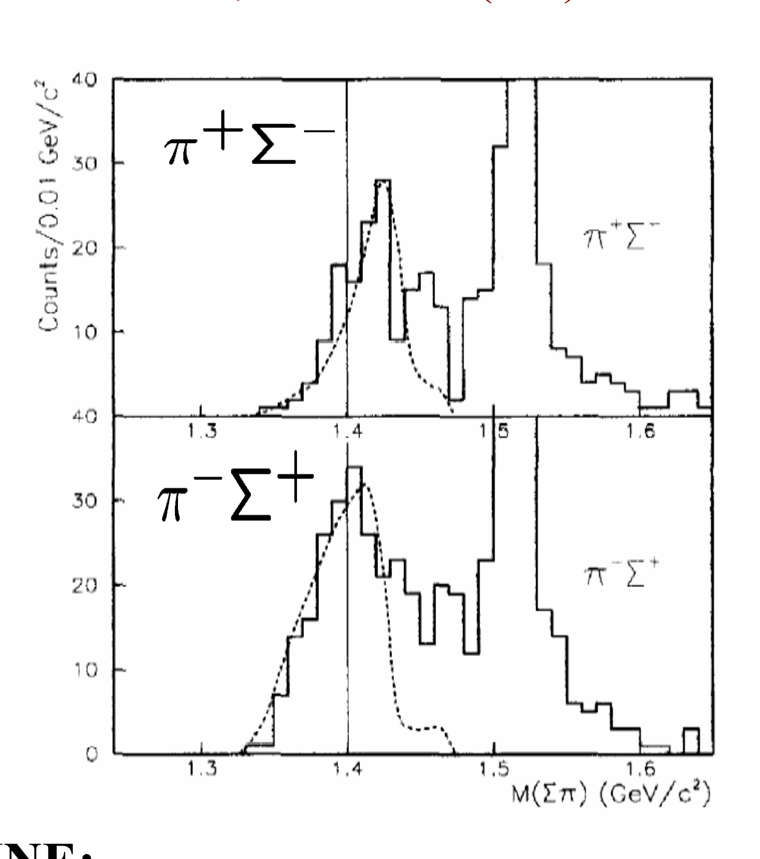
$$a_1 = 0.98 + i0.35 >> a_1 = -0.01 + i0.54$$

IN THE ISOSPIN LIMIT

INFLUENCE OF THE I=1 RESONANCES IN pS EVENT DISTRIBUTION

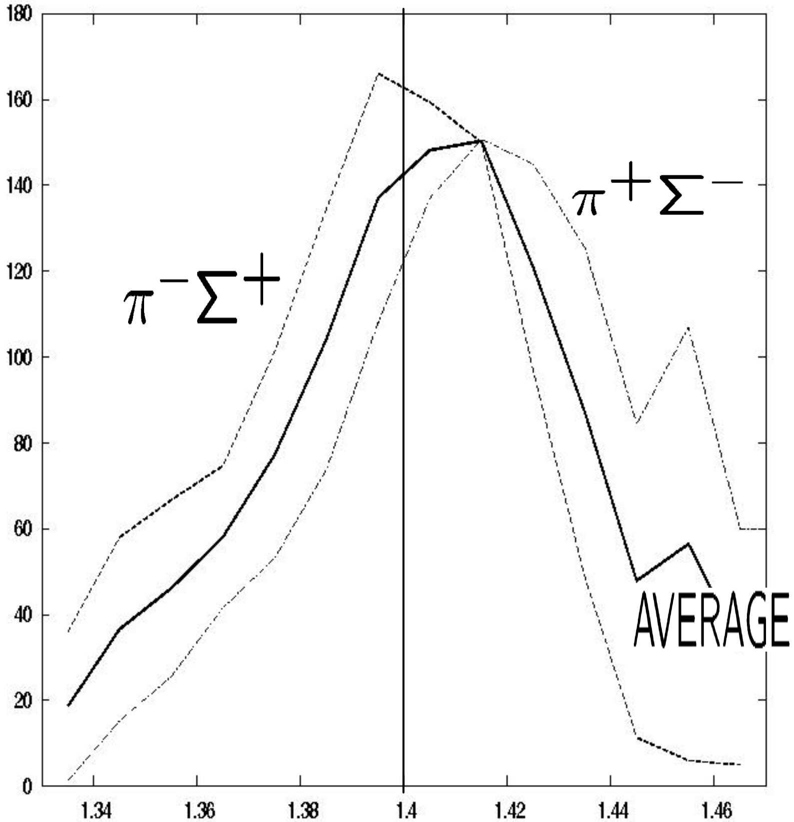
gp ✹ K^+ L(1405) ✹ K^+ p^+S^-, p^- S^+

J.K. Ahn, NP A721 ('03) 715c



$$K^- p \longrightarrow \Sigma^{\pm}\pi^{\mp}\pi^-\pi^+$$

Hemingway,NPB253,742('85)



LINE:

Nacher,Oset,Toki,Ramos PL B455 ('99)55

Magas,Ramos,Oset PRL95,052301(´05)

K^-p ✹□^{0}□^{0}♦ ^{0} & □ $^-$p ✹K^0 □ ♦

MORE WORK IS NEEDED:

1. MATCH THEORETICAL PRECISION WITH DEAR/SIDDHARTA measurement of width and shift of kaonic hydrogen at the eV level
   a) Going to order $d^{3/2}$ (or better d^2) in the correction of the Deser formula.
   b) Order p^3 (one loop) in the calculation of strong amplitude.

2. CLARIFY ISSUES ON SPECTROSCOPY:
   a) I=1 Narrow Resonance (1430-i 1.3)MeV, Disentangle experimentally the I=1 broad one (1487-i 46) MeV.
   b) Exotic Resonances (I=2)

Table 3: Pole positions and couplings to $I = 0$ physical states from the model of Ref. [3]

z_R $(I=0)$	$1379+27i$		$1434+11i$		$1692+14i$	
	g_i	$\lvert g_i \rvert$	g_i	$\lvert g_i \rvert$	g_i	$\lvert g_i \rvert$
$\pi\Sigma$	$-1.76-0.62i$	1.87	$-0.56-1.02i$	1.16	$-0.08-0.32i$	0.33
$\bar{K}N$	$0.86+0.70i$	1.11	$-1.74+0.63i$	1.85	$0.32+0.41i$	0.52
$\eta\Lambda$	$0.19+0.33i$	0.38	$-1.20+0.23i$	1.23	$-0.83-0.19i$	0.85
$K\Xi$	$-0.52-0.19i$	0.55	$-0.20-0.30i$	0.36	$3.87+0.05i$	3.87

a) b)

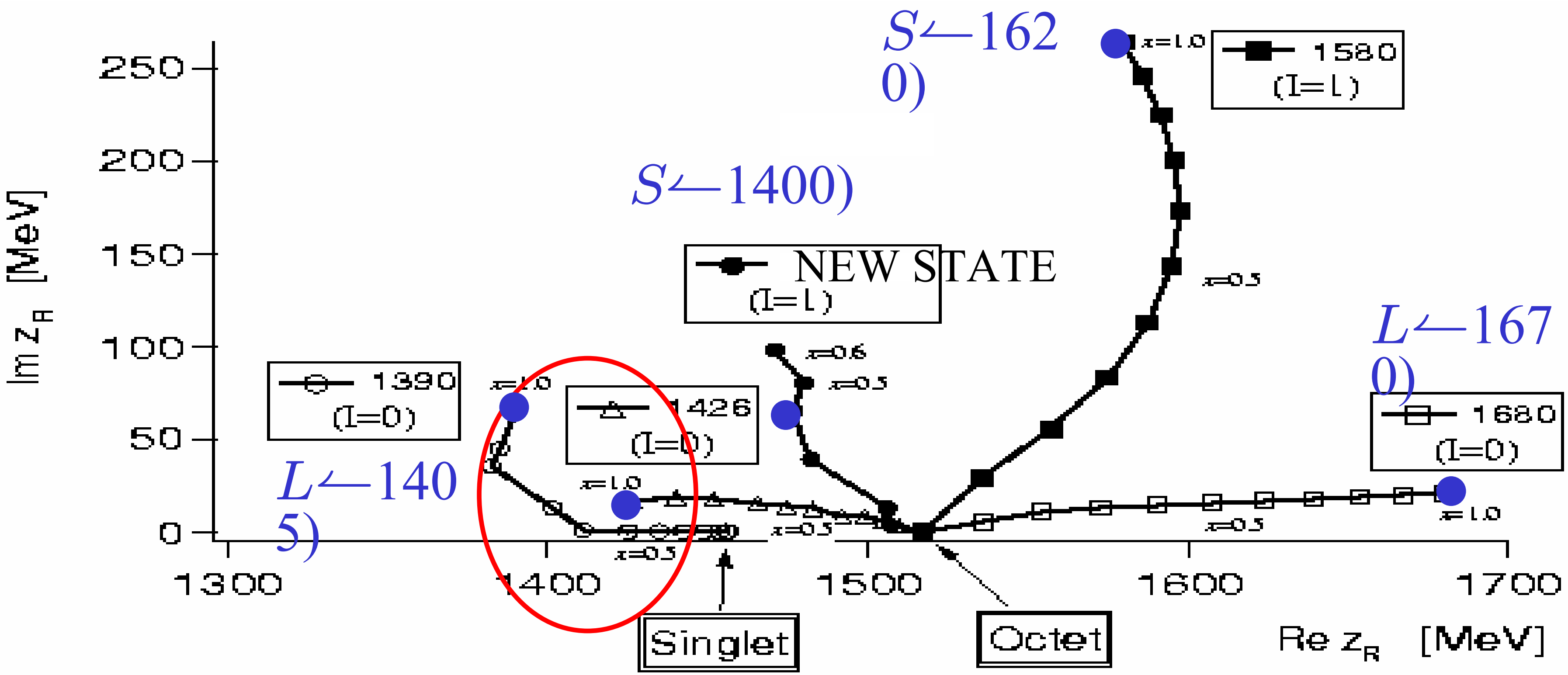
L↼1405)

L↼1670)

a) is more than twice wider than b)
(Quite Different Shape)

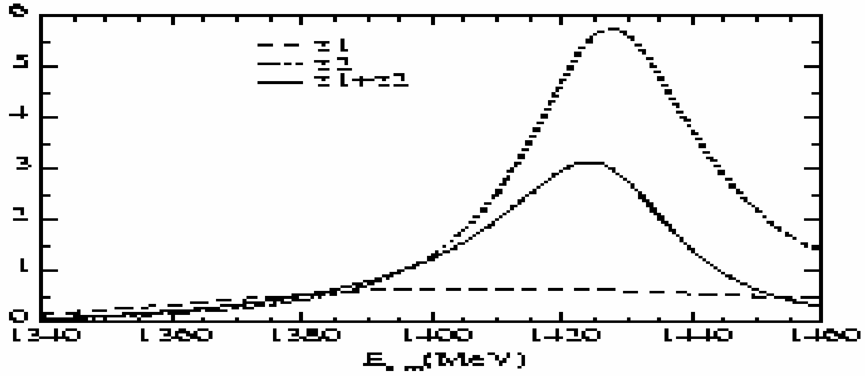
b) Couples stronger to $\bar{K}N$ than to $\pi\Sigma$ contrarily to a)

It depends to which resonance the production mechanism couples stronger that the shape will move from one to the other resonance



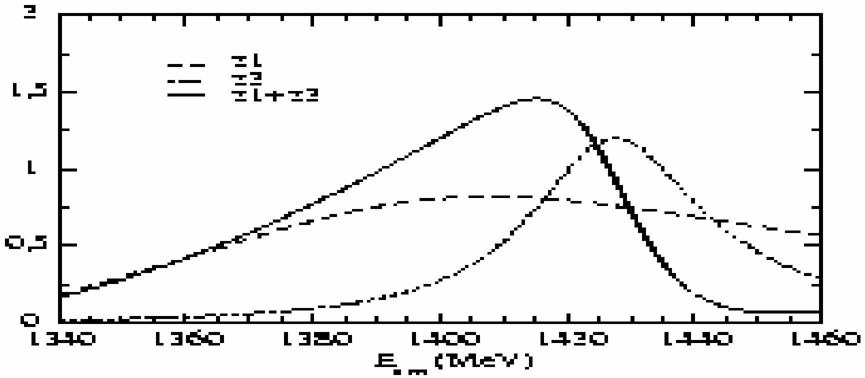
Simple parametrization of our own results with BW like expressions

$$g^{R_1}_{\bar{K}N}\frac{1}{W - M_{R_1} + i\Gamma_{R_1}/2}g^{R_1}_{\pi\Sigma} + g^{R_2}_{\bar{K}N}\frac{1}{W - M_{R_2} + i\Gamma_{R_2}/2}g^{R_2}_{\pi\Sigma} \,,$$



$\pi\Sigma \to \bar{K}N$

$$g^{R_1}_{\pi\Sigma}\frac{1}{W - M_{R_1} + i\Gamma_{R_1}/2}g^{R_1}_{\pi\Sigma} + g^{R_2}_{\pi\Sigma}\frac{1}{W - M_{R_2} + i\Gamma_{R_2}/2}g^{R_2}_{\pi\Sigma} \,.$$



$\pi\Sigma \to \pi\Sigma$

$$\epsilon_{12} = -2\alpha^3\mu_r^2 T^{th}_{\bar{K}p}(1+X)$$

Isospin violating corrections
Meißner,Raha,Rusetsky

DEAR $X=0 \rightarrow T^{th}_{\bar{K}p} = (-0.33\pm 0.10) + i(0.28\pm 0.10)\ \text{fm}$

M.Iwasaki et al. PRL78(1997)3067 $\epsilon_{12} - i\frac{\Gamma}{2} = (323\pm 63\pm 11) - i(200\pm 100\pm 50)\ \text{eV}$

$$X=0 \rightarrow T^{th}_{\bar{K}p} = (-0.78\pm 0.15) + i(0.50\pm 0.30)\ \text{fm}$$

Scattering experiment B.R. Martin NP B94 (1975)413

$$T^{th}_{\bar{K}p} = (-0.67\pm 0.10) + i(0.64\pm 0.10)\ \text{fm}$$

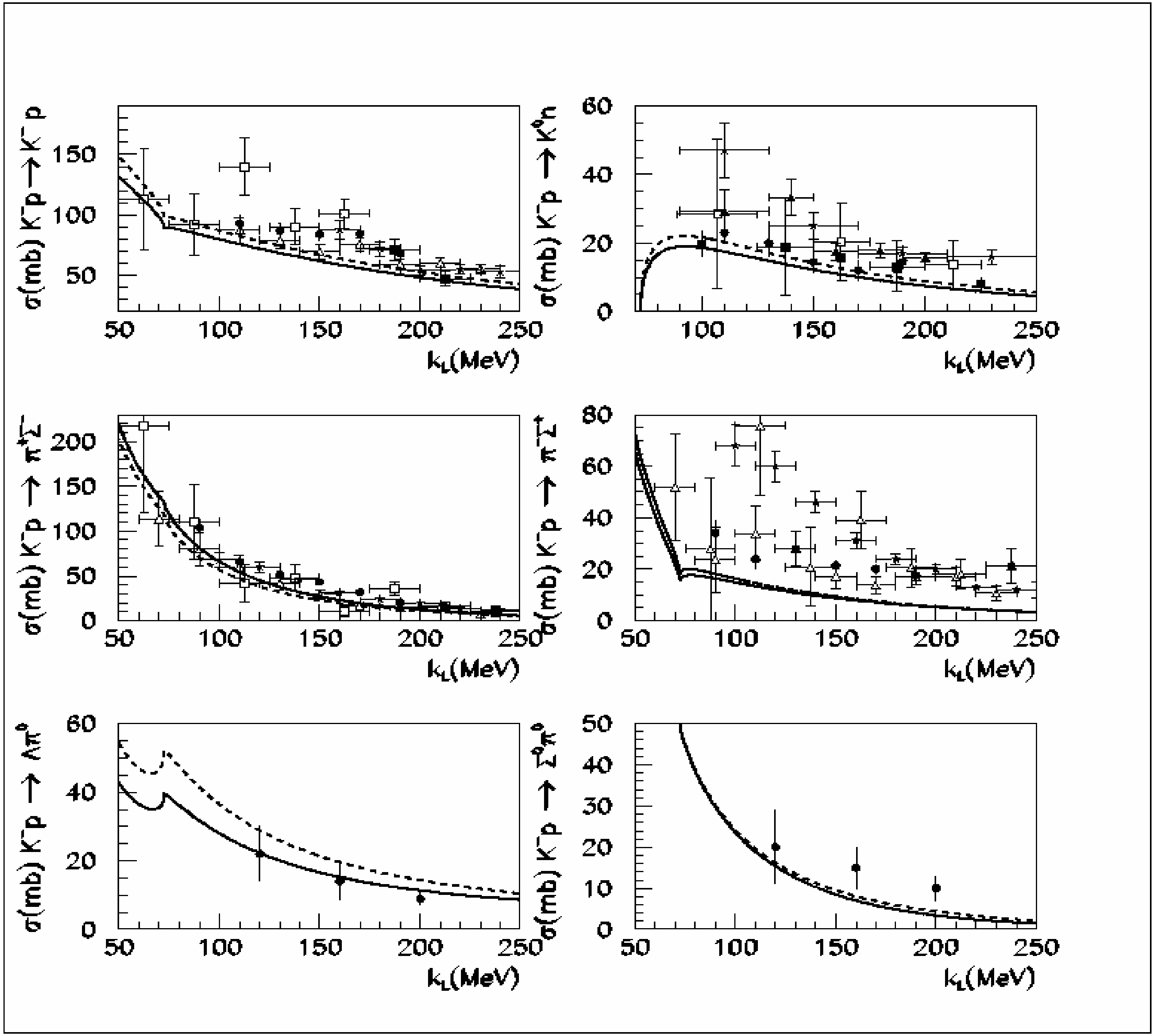
Oset,Ramos: -0.85+i 1.24 // Kaiser et al.:-0.97+i1.1 //
Meißner, J.A.O.: -0.51+i0.9 LO (relativistic) UCHPT

Rather controversial (not very precise) situation:

Experimentally: more precision is needed in kaonic atoms experiments (hopefully DEAR)

Theoretically: 1) Higher orders are necessary to be considered and one must check the convergence of the UCHPT expansion to calculate $T^{th}_{\bar{K}p}$

2) To compute X


σ(mb) K⁻p → K⁻p
σ(mb) K⁻p → K̄⁰n
σ(mb) K⁻p → π⁺Σ⁻
σ(mb) K⁻p → π⁻Σ⁺
σ(mb) K⁻p → Λπ⁰
σ(mb) K⁻p → Σ⁰π⁰
k_L(MeV)

3 Physical picture of Kaonic Hydrogen:

A kaonic hydrogen atom is a quasistable bound state of a kaon (K^-) and a proton(p), in which the interaction is predominantly *electromagnetic* with strong interactions that can be treated as perturbations giving rise to small corrections.

- $(K^- p)_{1s} \longrightarrow \begin{cases} \pi^0 \Lambda, \quad \Sigma^\pm \pi^\mp \quad [strong] \\ \gamma Y, \quad Y = \Lambda, \Sigma^0 \quad [electromagnetic] \leq 1\% \end{cases}$
- Very small momenta.

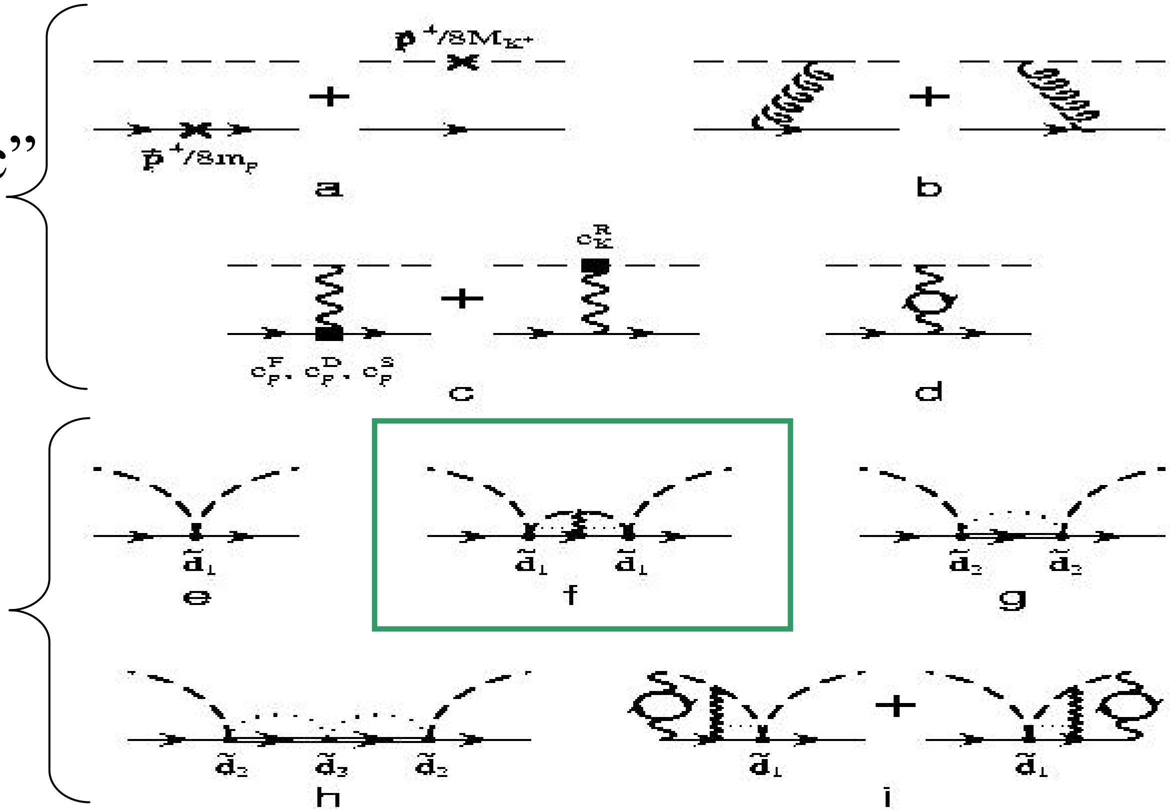
  $\langle p^2 \rangle^{1/2} = \alpha \mu_c \approx 2 \text{ MeV} << \mu_c \,, \qquad \mu_c = \frac{m_p M_{K^-}}{m_p + M_{K^-}}$
- $R = (\alpha \mu_c)^{-1} \approx 100 \text{ fm} >> R_{Strong}$
- $E_{1s} = \frac{1}{2} \mu_c \alpha^2 + \ldots \approx 8 \text{ KeV} << \mu_c$
- $\Gamma_{1s} \approx 250 \text{ eV} << E_{1s}$
- $\text{Mass}(\bar{K}^0 n) > \text{Mass}(K^- p) \longrightarrow$ Cusp Effect

The observable charecteristics of hadronic atoms obtained from the study of the spectrum and decays of kaonic hydrogen:

- (Small) shifts of energy levels $\triangle E_{nl}$ from purely Coulomb values and total decay width Γ_{nl}

romagnetic”

ig” Part



- The set of Feynman diagrams contributing to the energy shift of the kaonic hydrogen up-to-and-including $\mathcal{O}\left(\alpha^4, \alpha^3(m_d - m_u)\right)$. Solid, dashed, double, dotted, wiggly and spring lines correspond to the proton, K^-, neutron, $\bar{K}^0$, Coulomb and transverse photons, respectively. The electrons run in the closed loops shown in diagrams (d) and (i). The diagrams (f) and (i) contain Coulomb ladders – the contributions with $0, 1, 2, \cdots$ Coulomb photons exchanged.

Modified Deser Formula :

ur modified formula, upto-and-including $\mathcal{O}(\alpha^4, \alpha^3(m_d - m_u))$, here large nonanalytic corrections due to cusp effect are explicitely cluded, is best suited for the analysis of experimental data:

$$\Delta E_n^s - \frac{i}{2}\Gamma_n = -\frac{\alpha^3\mu_c^3}{2\pi M_{K^+} n^3}\left(\mathcal{T}_{KN}^{(0)} + \delta\mathcal{T}_{KN}\right)$$

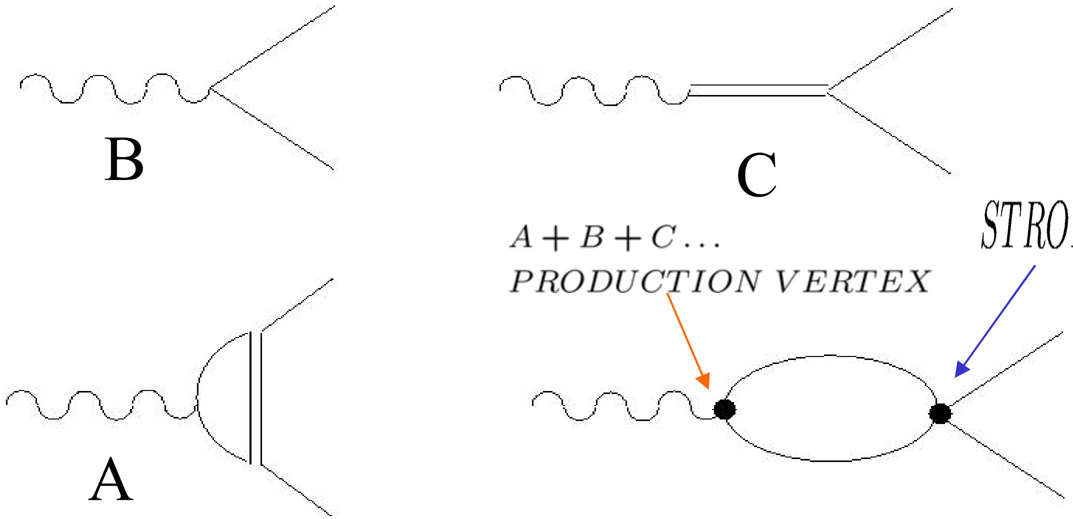
$$\left\{ 1 - \underbrace{\frac{\alpha\mu_c^2 s_n(\alpha)}{4\pi M_{K^+}}\mathcal{T}_{KN}^{(0)}}_{\text{Coulomb Corrections}} + \delta_n^{\text{vac}} \right\}$$

Corrections to the Deser Formula (Rough estimate):

- **Cusp Effect** $\sim 50\%$ at $\mathcal{O}(\sqrt{\delta\mathcal{M}})$
- **Coulomb Effects** $\sim (10 \text{ to } 15)\%$
- **Vacuum Polarization** $\sim 1\%$
- **CHPT** $\sim (-0.5 \pm 0.4)\%$ at $\mathcal{O}(p^2)$(or $\mathcal{O}(\delta\mathcal{M})$)

Production Processes

The re-scattering is due to the strong „final“ state interactions from some „weak“ production mechanism.



$$\mathrm{Im} F_i = \sum_k F_k \, \rho_k \, T_{ki}^*$$

We first consider the case with only the right hand cut for the strong interacting amplitude, R^{-1} is then a sum of poles (CDD) and a constant. It can be easily shown then:

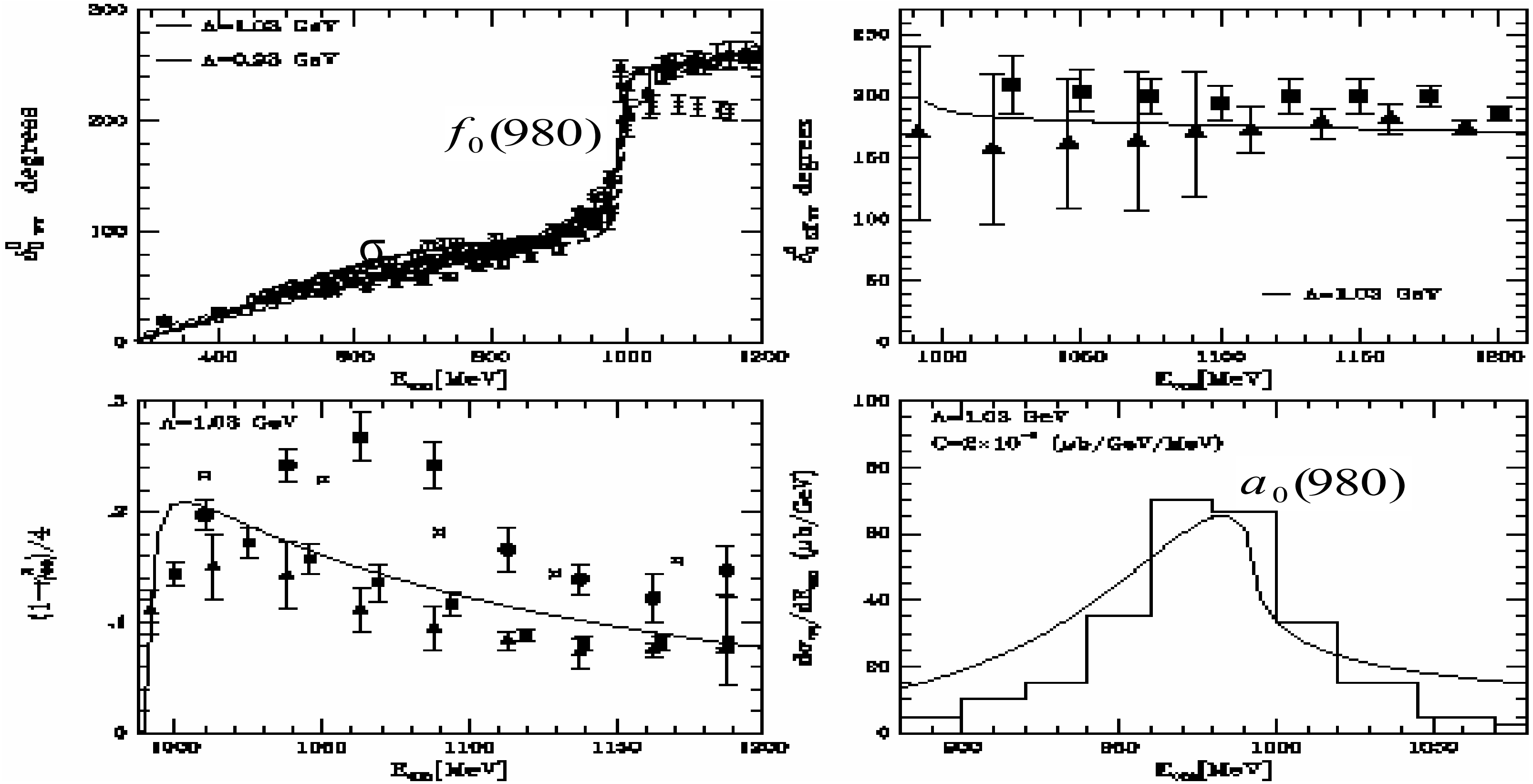
$$F = [I + R \cdot g]^{-1} \cdot \xi$$

Finally, ξ is also expanded pertubatively (in the same way as R) by the matching process with CHPT/alike expressions for F, order by order, $\xi = \xi_1 + \xi_2 + \xi_3 \ldots$

The crossed dynamics, as well for the production mechanism, are then included pertubatively.

E.Oset, J.A.O. NP A620(1997)438 (E NPA652('99)407) applied it to meson-meson interactions in S-wave s, $f_0(980)$, $a_0(980)$ resonances

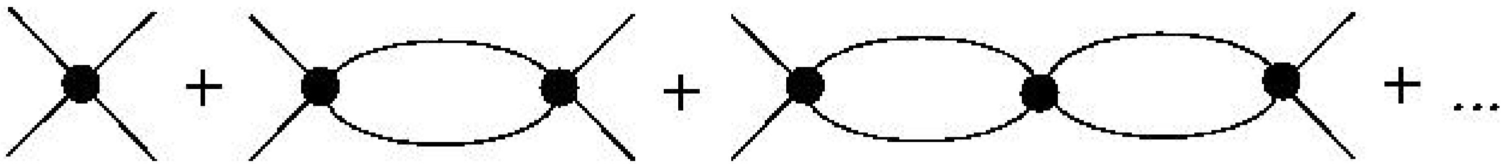
± However the approach was fully ON-SHELL, and algebraic since it was demonstrated that the off-shell part of the potential (LO CHPT) when iterated in the LS equation only renormalizes the potential itself.



- **There are good and well established reasons why the unitarity corrections are so enhanced in the previous examples giving rise to non-perturbative physics**
- New scales or numerical enhancements can appear that makes definitively smaller the overall scale Λ_{CHPT}, e.g:
  - Scalar Sector (S-waves) of meson-meson interactions with I=0,1,1/2 the unitarity loops are enhanced by numerical factors.

P-WAVE S-WAVE

$$\frac{s-4m_\pi^2}{6f^2} \longrightarrow \frac{s-m_\pi^2}{f^2}$$ Enhancement by a factor 6^L

*s*meson 

  - Presence of large masses compared with the typical momenta, e.g: Kaon masses in driving the appearance of the $\Lambda(1405)$ close to tresholed in $\bar{K}N$.

  This also occurs similarly in Nucleon-Nucleon scattering with the nucleon mass.

3. S-WAVE, S=-1 MESON-BARYON SCATTERING

J.Prades, M. Verbeni, JAO PRLXXX, PRL (2006)(Reply)

J.A. Oller, EPJA (2006)

HBCHPT calculation at one loop level fails miserably for the $\bar{K}N$ scattering lengths (opposite signs) N. Kaiser, EPJ64,045204(´01)

$$T = \left[R^{-1} + g(s)\right]^{-1} = [I + R \cdot g(s)]^{-1} \cdot R(s)$$

$R = R_1 = T_1$ LEADING ORDER, $\mathcal{O}(p)$

$R = R_1 + R_2 = T_1 + T_2$, NLO, $\mathcal{O}(p^2)$

for $O(p^3)$ and higher $R_n \neq T_n$

8 Comparison of New and Old Data:

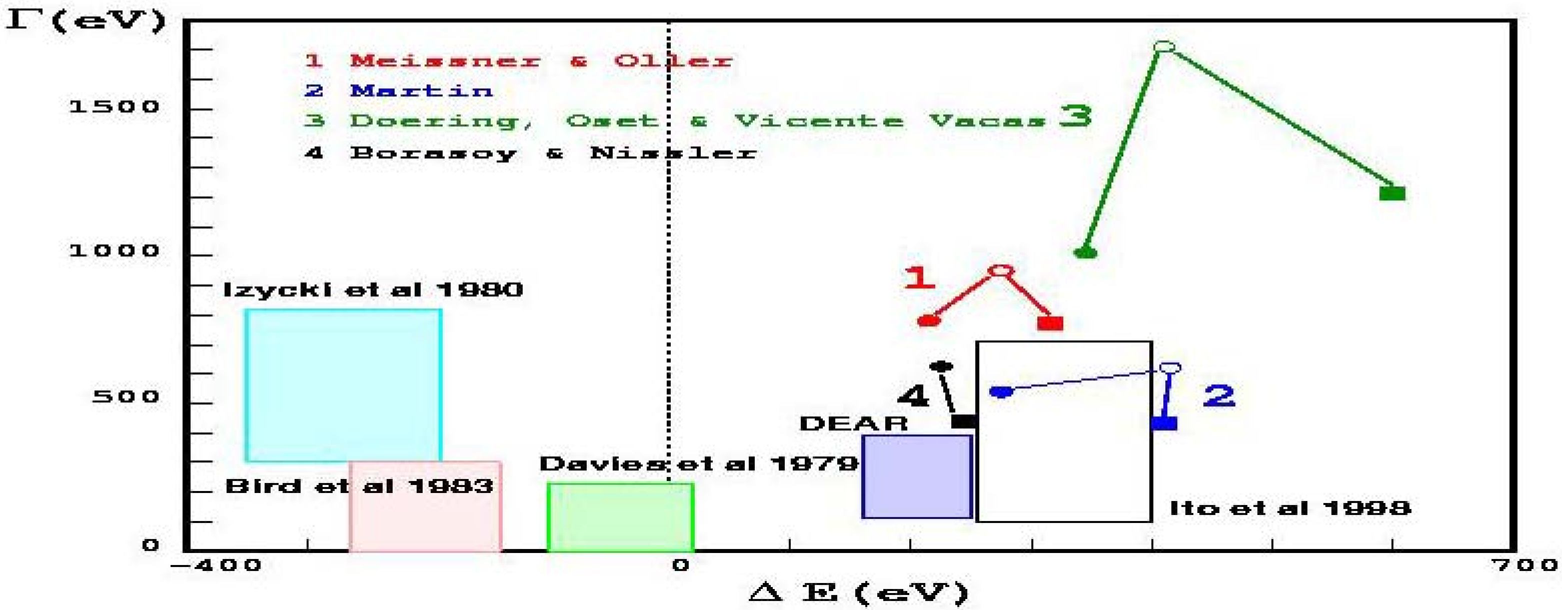
Predictions of the ground-state strong shift $\triangle E_1^s$ and width Γ_1. Filled circles correspond to using the original Deser formula, empty circles to using $T^{(0)}_{\bar{K}N}$ instead of $\frac{1}{2}(a_0 + a_1)$ in this formula and filled boxes to our final formula with $\delta T_{\bar{K}N} = \delta_n^{\rm vac} = 0$.

J.A. Oller and U.-G. Meißner, Phys. Lett. B **500** *(2001) 263.* [arXiv:hep-ph/0011146].

$$a_0 = -1.31 + 1.24i \quad ; \quad a_1 = 0.26 + 0.66i$$

A.D. Martin, Nucl. Phys. B **179** *(1981) 33.*

$$a_0 = -1.70 + 0.68i \quad ; \quad a_1 = 0.37 + 0.60i$$



DEAR $\quad X = 0 \rightarrow T^{th}_{\bar{K}p} = (-0.33 \pm 0.10) + i(0.28 \pm 0.10)$ fm

M.Iwasaki et al. PRL78(1997)3067 $\quad X = 0 \rightarrow T^{th}_{\bar{K}p} = (-0.78 \pm 0.15) + i(0.50 \pm 0.$

4. This allows as well to use the Chiral Lagrangians for higher energies. (BONUS)

5. Since one can also use the chiral Lagrangians for higher energies it is possible to establish a connection with perturbative QCD, α_S (4 GeV^2)/$\pi\approx$0.1. (OPE). E.g. providing phenomenological spectral functions for QCD Sum Rules (going definitively beyond the sometimes insufficient hadronic scheme of narrow resonance+resonance dominance).

Jamin,Pich, JAO

$V_{\upsilon\sigma}$:JHEP 0402,047('04)

$m_{u,d,s}$: EΠϑ X24, 237 (□02)**; hep-ph/0605095**

6. The same scheme can be applied to productions mechanisms. Some examples: $\rightarrow$

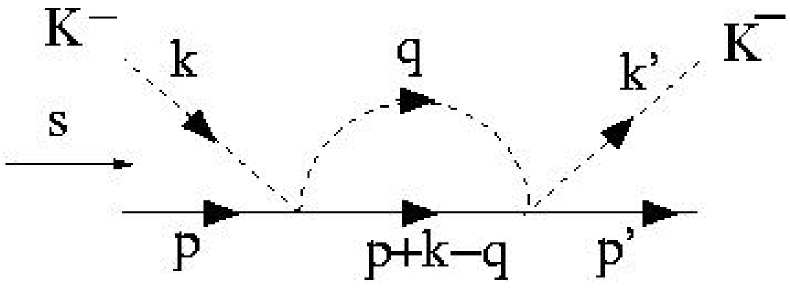
- Photoproduction: $\gamma\gamma \rightarrow \pi^0\pi^0$, $\pi^+\pi^-$, K^+K^-, K^0K^0, $\pi^0\eta$; D $\rightarrow$ 3π, K 2π, ...
  γ p $\rightarrow$ K^+ Λ(1405) ; **(γ, $\pi\pi$)** ; γ d $\rightarrow$ d ; γ NN $\rightarrow$ NN ; γ d $\rightarrow$ γ d ; ...
- Decays: $\phi \rightarrow \gamma\, \pi^0\pi^0$, $\pi^0\eta$, K^0K^0 ; J/$\Psi \rightarrow \phi$ (ω) $\pi\pi$, KK ;
  f_0(980) $\rightarrow \gamma\gamma$; branching ratios ...

JAO PRD 71,054030 ('05) on D ! 3*p*, K 2*p*

and D_s ! 3*p* , and references therein

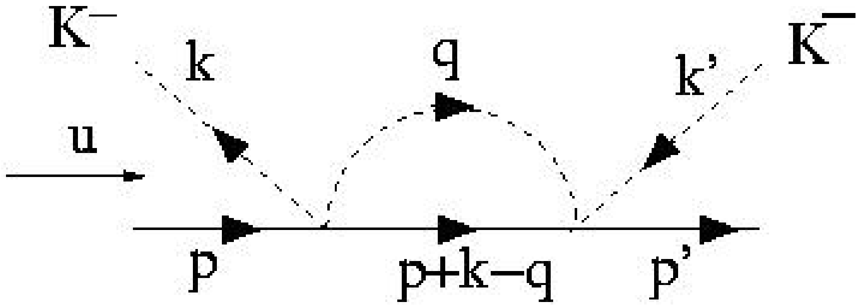
Let us keep track of the kaon mass, $M_K \approx 500$ MeV

We follow similar arguments to those of S. Weinberg in NPB363,3 (´91) respect to NN scattering (nucleon mass).



Unitarity Diagram

$$\frac{1}{k^0 - \mathrm{E}(q)} \frac{1}{2\mathrm{E}(q)} \cong \frac{2M_K}{k^2 - q^2} \frac{1}{2M_K}$$



Let us take now the crossed diagram

$$k \to -k$$

$$\frac{1}{k^0 + \mathrm{E}(q)} \frac{1}{2\mathrm{E}(q)} \cong \frac{1}{4M_K^2}$$

Unitarity&Crossed loop diagram: $\frac{4M_K^2}{k^2 - q^2}$

Unitarity enhancement for low three-momenta: $\frac{2M_K}{q}$

Chiral Perturbation Theory

Weinberg, Physica A96,32 (79); Gasser, Leutwyler, Ann.Phys. (NY) 158,142 (84)

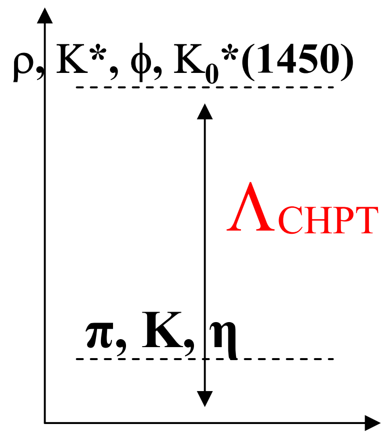
QCD Lagrangian | **Hilbert Space**
Physical States

u, d, s massless quarks **Spontaneous Chiral Symmetry Breaking**

$SU(3)_L \otimes SU(3)_R \longrightarrow SU(3)_V$

Goldstone Theorem

Octet of massles pseudoscalars
π, K, η
Energy gap



$m_q \neq 0$. Explicit breaking of Chiral Symmetry | **Non-zero masses $m_P^2 \propto m_q$**

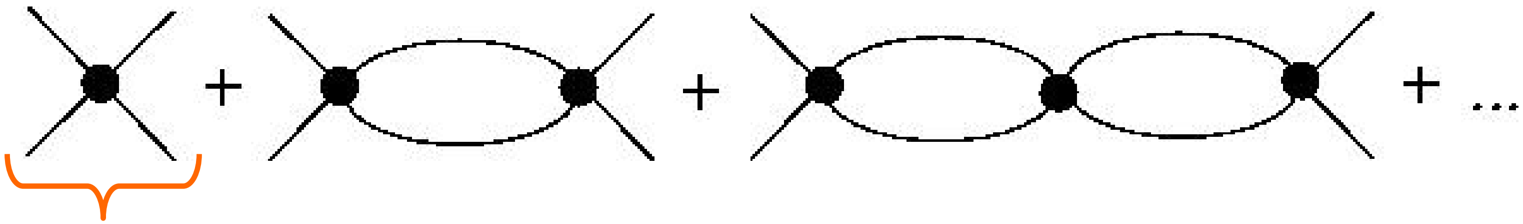
Perturbative expansion in powers of the external four-momenta of the pseudo-Goldstone bosons over Λ^2_{CHPT}

$$L = L_2 + L_4 + \ldots$$

$$\frac{L_4}{L_2} = O\left(\frac{p^2}{\Lambda^2_{CHPT}}\right)$$

$$\Lambda_{CHPT} \approx 1\ \text{GeV} \approx M_\rho \approx 4\pi f_\pi \approx 1\ \text{GeV}$$

- **Enhancement of the unitarity cut that makes definitively simaller the overall scale Λ_{CHPT} in meson-baryon scattering with strangeness:**

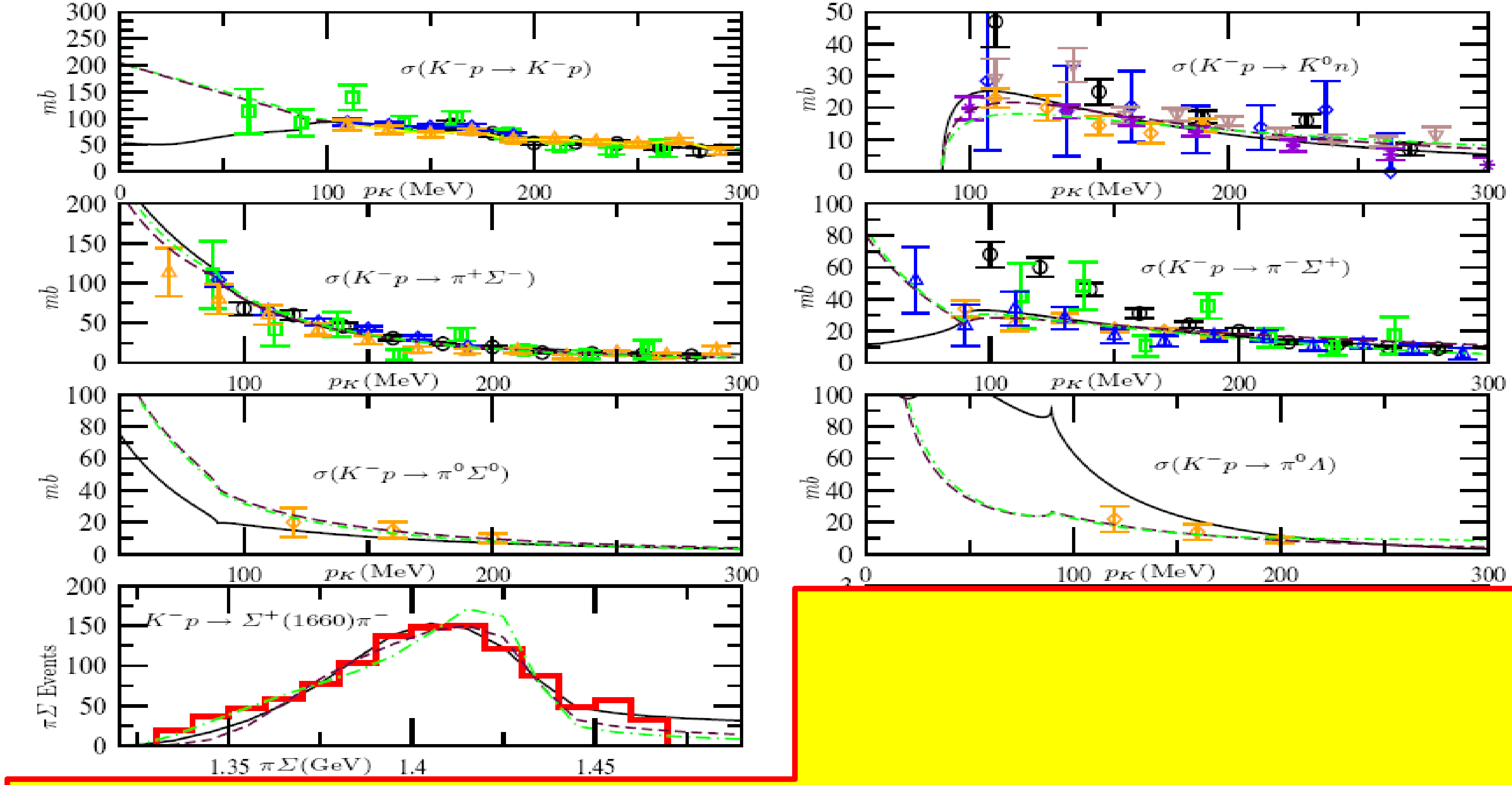


Arbitrary Meson-Baryon Vertex

–Presence of large masses compared with the typical low three-momenta (Baryon+Kaon masses) drive the appearance of the $\Lambda(1405)$ close to threshold in $\bar{K}N$ scattering.

This also occurs similarly in Nucleon-Nucleon scattering with the nucleon mass

Reproduction of the data by the fits of Prades,Verbeni,JAO PRL95('05), plus an O(p) fit.



Solid: Fit A. **Dashed**: Fit B. **Dash-Dotted**: O(p) Fit.

	A_4^+	B_4^+	$\mathcal{O}(p)$
γ	2.36	2.36	2.35
R_c	0.628	0.655	0.667
R_n	0.172	0.195	0.205
ΔE (eV)	201	403	390
Γ (eV)	338	477	525
ΔE_D (eV)	209	416	394
Γ_D (eV)	346	662	716
a_{K^-p} (fm)	$-0.51 + i\,0.42$	$-1.01 + i\,0.80$	$-0.96 + i\,0.87$
a_0 (fm)	$-1.23 + i\,0.45$	$-1.63 + i\,0.81$	$-1.55 + i\,0.87$
a_1 (fm)	$0.98 + i\,0.35$	$-0.01 + i\,0.54$	$-0.03 + i\,0.65$
$\delta_{\pi\Lambda}(\Xi)$ (°)	2.5	0.2	−1.9
m_0 (GeV)	0.8*	0.8*	…
a_{0+}^+ ($10^{-2} \cdot M_\pi^{-1}$)	−1.2	−1.7	…
$\sigma_{\pi N}$ (MeV)	40*	40*	…

Experiment

γ	2.36 ± 0.04
R_c	0.664 ± 0.011
R_n	0.189 ± 0.015
Δ_E	193 ± 38
Γ	249 ± 118
$\delta_{\pi\Lambda}$	4.6 ± 2

Units		A_4^+	B_4^+	$\mathcal{O}(p)$
MeV	f	79.8	89.2	88.0
GeV^{-1}	b_0	−0.855	−0.318	0*
GeV^{-1}	b_D	+0.715	−0.101	0*
GeV^{-1}	b_F	−0.036	−0.314	0*
GeV^{-1}	b_1	+0.605	−0.193	0*
GeV^{-1}	b_2	+1.075	−0.275	0*
GeV^{-1}	b_3	−0.189	−0.153	0*
GeV^{-1}	b_4	−1.249	−0.277	0*
	a_1	−1.155	−1.570	−0.472
	a_2	−0.383	−2.062	−1.572
	a_5	−1.304	−2.605	−1.266
	a_7	−1.519	−1.568	−1.853
	a_8	−1.212	−2.064	−1.210
	a_9	−0.145	−0.886	+3.337

Three b's are fixed in terms of the others from the O(p^2) constraints

$$\sigma_{\pi N} = 40 \text{ MeV}$$
$$m_0 = 0.8 \text{ GeV}$$
$$a_{0+}^+ = (-1 \pm 1) m_\pi^{-1} 10^{-2}$$

$$a_2 = a_3 = a_4 \ , \ a_5 = a_6 \ , \ a_9 = a_{10}$$

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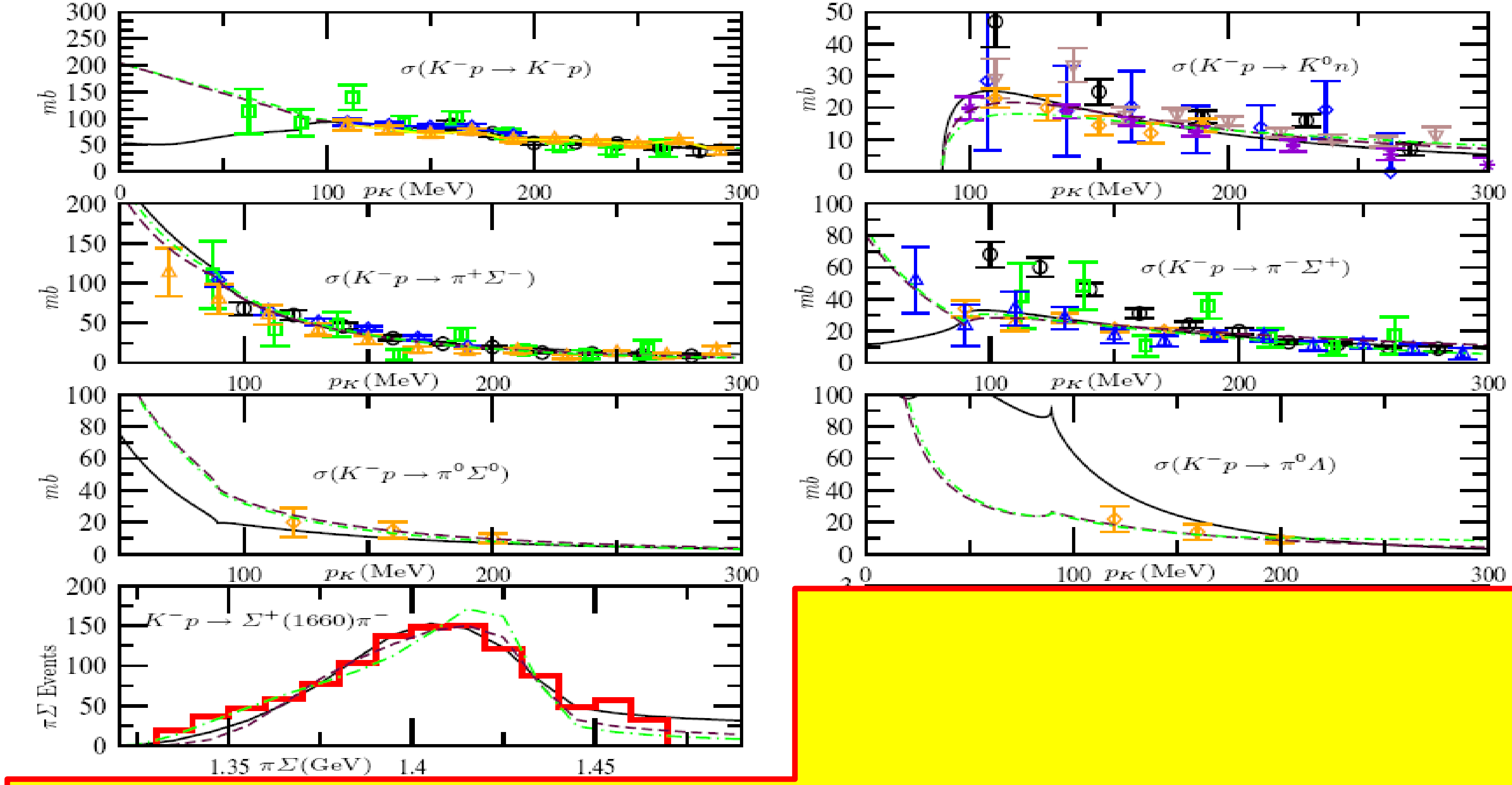
	a_8	-1.212	-2.064	-1.210
	a_9	-0.145	-0.886	$+3.337$

Fit A reproduces simultaneously scattering data plus DEAR measurement

It was the first chiral fit to accomplish this

However, it fails to reproduce the Crystall Ball data.

Reproduction of the data by the fits of Prades,Verbeni,JAO PRL95('05), plus an O(p) fit.



Solid: Fit A. **Dashed**: Fit B. **Dash-Dotted**: O(p) Fit.

Other results for which a precise knowledge of $\bar{K}N$ scattering is important:

- Nature of $\Lambda(1405)$, problems in lattice QCD and quarks models. Dynamically generated resonance.

- Two poles making up the $\Lambda(1405)$

Meissner,JAO PLB500,263('01) ; Jido, Oset, Ramos, Meissner, J.A.O, NPA725(03)181

Magas, Oset, Ramos PRL95,052301('05); S. Prakhov et al. (Crystall Ball Coll.), PRC70,034605('04));

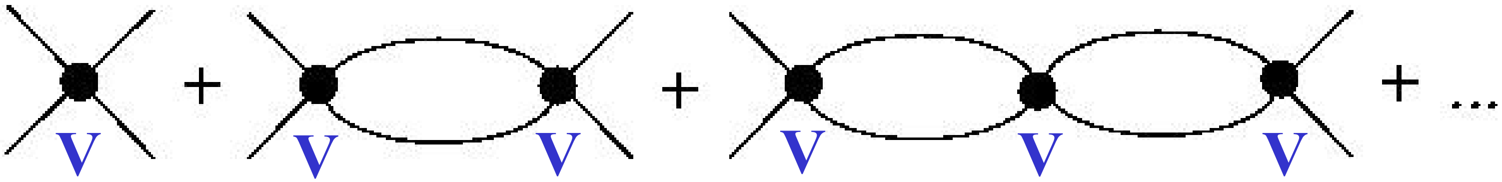
- Descover of tri-baryons $S^0(3115)$, $S^1(3140)$. IF SO, it is established that $\bar{K}$-nucleus potential is definitely strong. Suzuki et al., PLB597,263('04) **CONTROVERSY ON THE CORRECT INTERPRETATION OF EXPERIMENT: Oset, Toki, PRC74,015207(06). The situation is still contentious.**

- Strangeness content of the proton and large pion-nucleon sigma terms, $\langle p|\bar{s}s|p\rangle$ strange proton-scalar form factor related by unitarity with $\bar{K}N$ amplitiudes.

Historically, the first approach to apply a Chiral expansion to an interacting KERNEL was:

S. Weinberg, PL B251(1990)288, NP B363(1991)3, PL B295 (1992)114 FOR THE NUCLEON-NUCLEON INTERACTIONS.

The Chiral expansion was applied to the set of two nucleon irreducible diagrams, THE POTENCIAL, which was then iterated through a Lippmann-Schwinger equation.



The solution to the LS equation is NUMERICAL

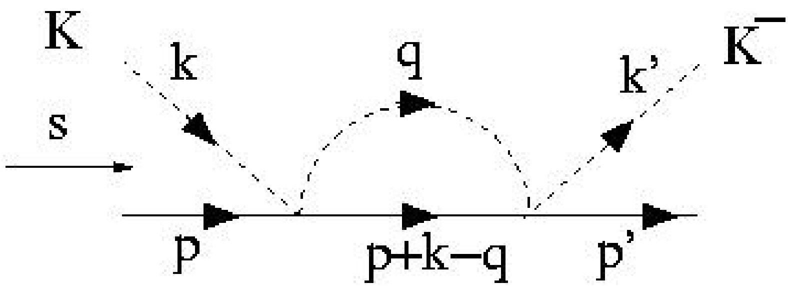
Further regularization is needed when solving the LS equation (cut-off dependence) so that the new divergences are not reabsorbed by the counterterms introduced in V. These drawbacks are solved when using UCHPT. The solution is algebraic and there is no cut-off dependence.

N. Kaiser, P.B. Siegel and W.Weise NP A594(1995)325 proceeded similarly as the Weinberg's scheme in the S-wave strangeness= –1 meson-baryon sector

Unitarity Enhancement. Large Kaon Masses.

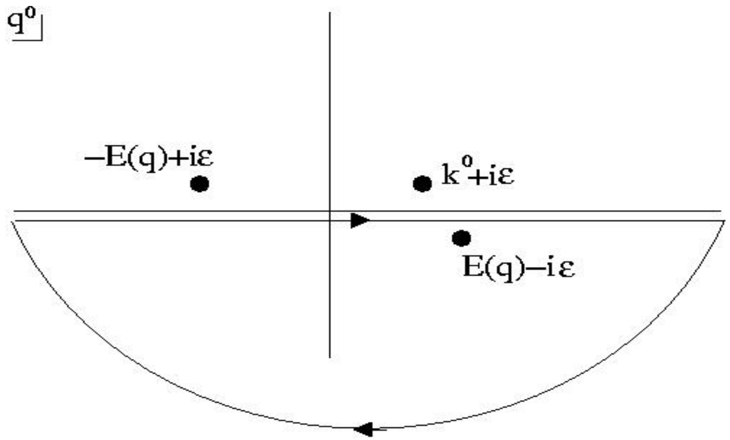
Let us keep track of the kaon mass, $M_K \approx 500$ MeV

We follow similar arguments to those of S. Weinberg in NPB363,3 (´91) respect to NN scattering (nucleon mass).



Unitarity Diagram

$$\int \frac{dq^0}{(k^0 - q^0 + i\varepsilon)(q^0 + \mathrm{E}(q) - i\varepsilon)(q^0 - \mathrm{E}(q) + i\varepsilon)}$$



$$\frac{1}{k^0 - \mathrm{E}(q)} \frac{1}{2\mathrm{E}(q)} \cong \frac{2M_K}{k^2 - q^2} \frac{1}{2M_K}$$

Unitarity enhancement for low three-momenta: $\frac{2M_K}{q}$

Around one order of magnitude in the region of the Λ(1405) region,
|q| ' 100 MeV