

HEAVY HADRONS IN THE RELATIVISTIC QUARK MODEL

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INTRODUCTION

Quark-diquark picture of heavy hadrons

[Anselmino et al. (1993), Ebert, Reinhardt, Volkov (1994), Jaffe (2005), Wilczek (2004)]

* Heavy baryons (qqQ): heavy-quark–light-diquark picture $[qq]Q$

Three-body calculation \longrightarrow two-step two-body calculation

* Heavy tetraquarks ($qQ\bar{q}\bar{Q}$ and $QQ'\bar{q}\bar{q}'$): heavy-diquark–heavy-antidiquark picture $(qQ)(\bar{q}\bar{Q})$ and $(QQ')(\bar{q}\bar{q}')$

Four-body calculation \longrightarrow two-step two-body calculation

Different ways to consider diquark:

- completely phenomenological object
- bound qq system

Difference in dynamics of heavy and light quarks:

- slow relative motion of heavy quarks Q
- fast motion of light quarks q (as in heavy-light mesons, $v/c \sim 0.7 - 0.8$) \rightarrow light quark should be treated fully relativistically

Diquark is in antitriplet colour state

Diquark is a composite system with spin $S = 0, 1$:

- diquark is nonlocal object: Its interaction with gluons is smeared by the form factor expressed through the overlap integral of diquark wave functions
- diquark excitations can contribute to the excitation spectrum of hadron masses

Pauli principle for ground state diquarks:

- (qq') diquark can have $S = 0, 1$ [scalar $[q, q']$ ("good"), axial vector $\{q, q'\}$ ("bad")]
- (qq) diquarks can have only $S = 1$ (axial vector $\{q, q\}$)

Spectator (di)quark influences dynamics of the quarks forming the diquark. We assume that such influence is considerably smaller than the diquark correlation and can be neglected. If it could not be neglected than properties of the diquark will be different in baryons, tetraquarks, pentaquarks etc.

RELATIVISTIC QUARK MODEL

Quasipotential equation of Schrödinger type:

$$\left(\frac{b^2(M)}{2\mu_R} - \frac{\mathbf{p}^2}{2\mu_R} \right) \Psi_M(\mathbf{p}) = \int \frac{d^3q}{(2\pi)^3} V(\mathbf{p}, \mathbf{q}; M) \Psi_M(\mathbf{q})$$

\mathbf{p} - relative momentum of quarks

M - bound state mass ($M = E_1 + E_2$)

μ_R - relativistic reduced mass:

$$\mu_R = \frac{E_1 E_2}{E_1 + E_2} = \frac{M^4 - (m_1^2 - m_2^2)^2}{4M^3}$$

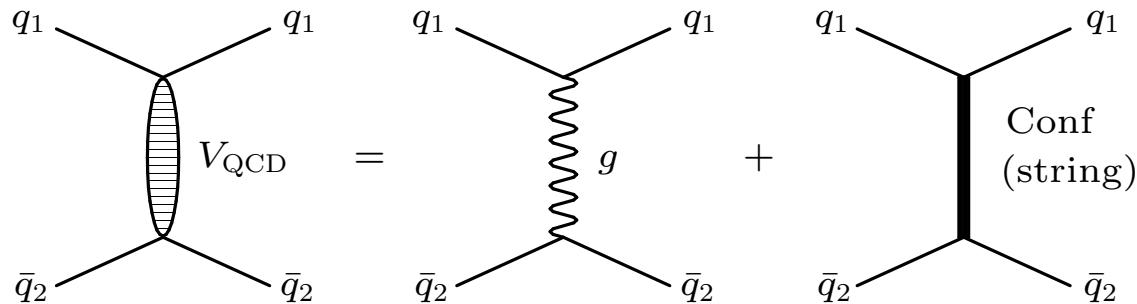
$b(M)$ - on-mass-shell relative momentum in cms:

$$b^2(M) = \frac{[M^2 - (m_1 + m_2)^2][M^2 - (m_1 - m_2)^2]}{4M^2}$$

$E_{1,2}$ - center of mass energies:

$$E_1 = \frac{M^2 - m_2^2 + m_1^2}{2M}, \quad E_2 = \frac{M^2 - m_1^2 + m_2^2}{2M}$$

- Parameters of the model fixed from meson sector
- $q\bar{q}$ quasipotential



$$V(\mathbf{p}, \mathbf{q}; M) = \bar{u}_1(p)\bar{u}_2(-p) \left\{ \frac{4}{3}\alpha_S D_{\mu\nu}(\mathbf{k})\gamma_1^\mu\gamma_2^\nu + V_{\text{conf}}^V(\mathbf{k})\Gamma_1^\mu\Gamma_{2;\mu} + V_{\text{conf}}^S(\mathbf{k}) \right\} u_1(q)u_2(-q)$$

$$\mathbf{k} = \mathbf{p} - \mathbf{q}$$

$D_{\mu\nu}(\mathbf{k})$ - (perturbative) gluon propagator

$\Gamma_\mu(\mathbf{k})$ - effective long-range vertex with Pauli term:

$$\Gamma_\mu(\mathbf{k}) = \gamma_\mu + \frac{i\kappa}{2m}\sigma_{\mu\nu}k^\nu,$$

κ - anomalous chromomagnetic moment of quark,

$$u^\lambda(p) = \sqrt{\frac{\epsilon(p) + m}{2\epsilon(p)}} \begin{pmatrix} 1 \\ \sigma \mathbf{p} \\ \hline \epsilon(p) + m \end{pmatrix} \chi^\lambda,$$

with $\epsilon(p) = \sqrt{\mathbf{p}^2 + m^2}$.

- Lorentz structure of $V_{\text{conf}} = V_{\text{conf}}^V + V_{\text{conf}}^S$

In nonrelativistic limit

$$\left. \begin{array}{rcl} V_{\text{conf}}^V & = & (1 - \varepsilon)(Ar + B) \\ V_{\text{conf}}^S & = & \varepsilon(Ar + B) \end{array} \right\} \text{ Sum : } (Ar + B)$$

ε - mixing parameter

Parameters A , B , κ , ε and quark masses fixed from analysis of meson masses and radiative decays:

$\varepsilon = -1$ from heavy quarkonium radiative decays ($J/\psi \rightarrow \eta_c + \gamma$) and HQET

$\kappa = -1$ from fine splitting of heavy quarkonium 3P_J states and HQET

$(1 + \kappa) = 0 \implies$ vanishing long-range chromomagnetic interaction !

Freezing of α_s for light quarks (Simonov, Badalyan)

$$\alpha_s(\mu) = \frac{4\pi}{\beta_0 \ln \frac{\mu^2 + M_0^2}{\Lambda^2}}, \quad \beta_0 = 11 - \frac{2}{3}n_f, \quad \mu = \frac{2m_1 m_2}{m_1 + m_2},$$

$$M_0 = 2.24\sqrt{A} = 0.95 \text{ GeV}$$

Quasipotential parameters:

$$A = 0.18 \text{ GeV}^2, \quad B = -0.30 \text{ GeV},$$

$$\Lambda = 0.413 \text{ GeV} \text{ (from } M_\rho \text{)}$$

Quark masses:

$$m_b = 4.88 \text{ GeV} \quad m_s = 0.50 \text{ GeV}$$

$$m_c = 1.55 \text{ GeV} \quad m_{u,d} = 0.33 \text{ GeV}$$

- Heavy baryons in quark-diquark picture

(qq)-interaction:

$$V_{qq} = \frac{1}{2} V_{q\bar{q}}$$

$$V(\mathbf{p}, \mathbf{q}; M) = \bar{u}_1(p)\bar{u}_2(-p)\mathcal{V}(\mathbf{p}, \mathbf{q}; M)u_1(q)u_2(-q),$$

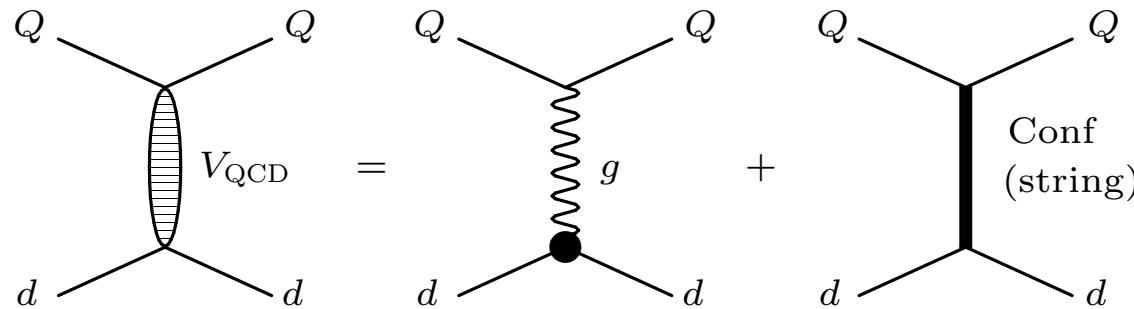
where

$$\mathcal{V}(\mathbf{p}, \mathbf{q}; M) = \frac{2}{3}\alpha_S D_{\mu\nu}(\mathbf{k})\gamma_1^\mu\gamma_2^\nu + \frac{1}{2}V_{\text{conf}}^V(\mathbf{k})\Gamma_1^\mu\Gamma_{2;\mu} + \frac{1}{2}V_{\text{conf}}^S(\mathbf{k})$$

(dQ)-interaction:

$$d = (qq')$$

$$V(\mathbf{p}, \mathbf{q}; M) = \frac{\langle d(P)|J_\mu|d(Q)\rangle}{2\sqrt{E_d(p)E_d(q)}}\bar{u}_Q(p)\frac{4}{3}\alpha_S D_{\mu\nu}(\mathbf{k})\gamma^\nu u_Q(q) \\ + \psi_d^*(P)\bar{u}_Q(p)J_{d;\mu}\Gamma_Q^\mu V_{\text{conf}}^V(\mathbf{k})u_Q(q)\psi_d(Q) + \psi_d^*(P)\bar{u}_Q(p)V_{\text{conf}}^S(\mathbf{k})u_Q(q)\psi_d(Q)$$



$J_{d,\mu}$ – effective long-range vector vertex of diquark:

$$J_{d;\mu} = \begin{cases} \frac{(P+Q)_\mu}{2\sqrt{E_d(p)E_d(q)}} & \text{for scalar diquark} \\ \frac{(P+Q)_\mu}{2\sqrt{E_d(p)E_d(q)}} + \frac{i\mu_d}{2M_d}\Sigma_\mu^\nu k_\nu & \text{for axial vector diquark } (\mu_d = 0) \end{cases}$$

μ_d - total chromomagnetic moment of axial vector diquark

diquark spin matrix: $(\Sigma_{\rho\sigma})_\mu^\nu = -i(g_{\mu\rho}\delta_\sigma^\nu - g_{\mu\sigma}\delta_\rho^\nu)$

\mathbf{S}_d - axial vector diquark spin: $(S_{d;k})_{il} = -i\varepsilon_{kil}$

$\psi_d(P)$ – diquark wave function:

$$\psi_d(p) = \begin{cases} 1 & \text{for scalar diquark} \\ \varepsilon_d(p) & \text{for axial vector diquark} \end{cases}$$

$\varepsilon_d(p)$ – polarization vector of axial vector diquark

$\langle d(P) | J_\mu | d(Q) \rangle$ – vertex of diquark-gluon interaction:

$$\langle d(P) | J_\mu(0) | d(Q) \rangle = \int \frac{d^3 p d^3 q}{(2\pi)^6} \bar{\Psi}_P^d(\mathbf{p}) \Gamma_\mu(\mathbf{p}, \mathbf{q}) \Psi_Q^d(\mathbf{q}) \Rightarrow F(k^2)$$

Γ_μ – two-particle vertex function of the diquark-gluon interaction:

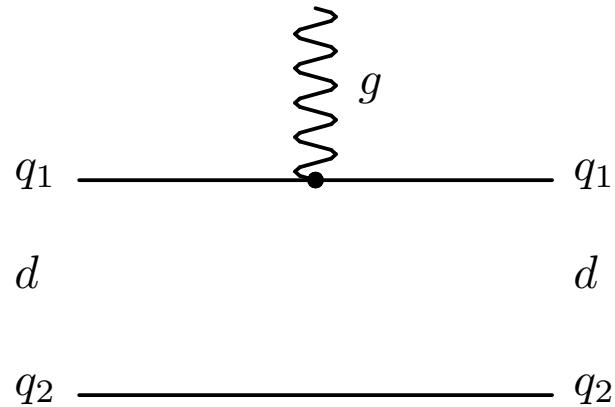


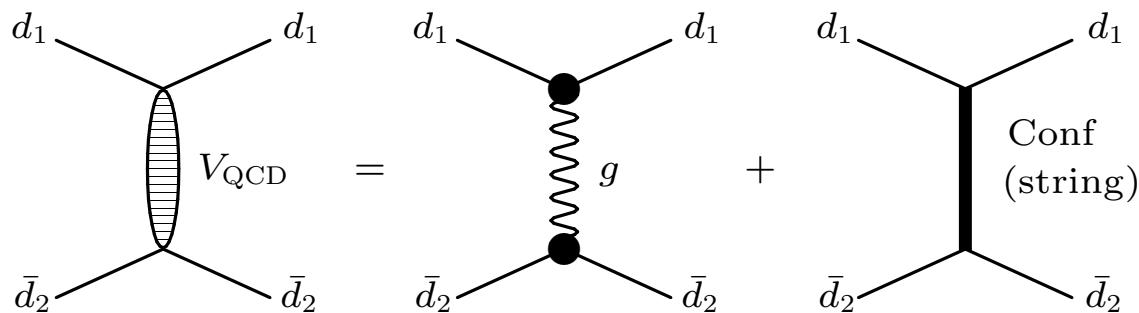
Figure 1: The vertex function Γ of the diquark-gluon interaction in the impulse approximation. The gluon interaction only with one quark is shown.

- Heavy tetraquarks in diquark-antidiquark picture

$(d_1 \bar{d}_2)$ -interaction:

$$d = (Qq)$$

$$\begin{aligned} V(\mathbf{p}, \mathbf{q}; M) &= \frac{\langle d_1(P) | J_\mu | d_1(Q) \rangle}{2\sqrt{E_{d_1} E_{\bar{d}_1}}} \frac{4}{3} \alpha_S D^{\mu\nu}(\mathbf{k}) \frac{\langle d_2(P') | J_\nu | d_2(Q') \rangle}{2\sqrt{E_{d_2} E_{\bar{d}_2}}} \\ &+ \psi_{d_1}^*(P) \psi_{d_2}^*(P') \left[J_{d_1;\mu} J_{d_2}^\mu V_{\text{conf}}^V(\mathbf{k}) + V_{\text{conf}}^S(\mathbf{k}) \right] \psi_{d_1}(Q) \psi_{d_2}(Q'), \end{aligned}$$



DIQUARKS

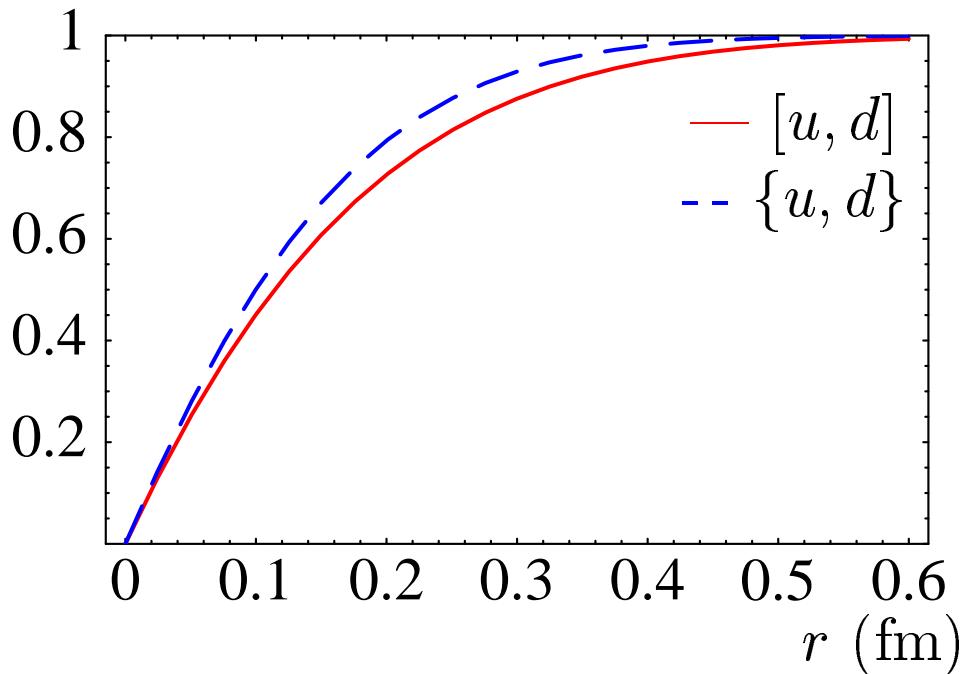
Table 1: Masses of light ground state diquarks (in MeV). S and A denotes scalar and axial vector diquarks antisymmetric $[q, q']$ and symmetric $\{q, q'\}$ in flavour, respectively.

| Quark content | Diquark type | Mass | | | | |
|------------------|-----------------|------------|---------------------|----------------------|--------------|------------------------|
| | | our RQM | Ebert et al. NJL | Burden et al. BSE | Maris BSE | Hess et al. Lattice |
| $[u, d]$ | S | 710 | 705 | 737 | 820 | 694(22) |
| $\{u, d\}$ | A | 909 | 875 | 949 | 1020 | 806(50) |
| $[u, s]$ | S | 948 | 895 | 882 | 1100 | |
| $\{u, s\}$ | A | 1069 | 1050 | 1050 | 1300 | |
| $\{s, s\}$ | A | 1203 | 1215 | 1130 | 1440 | |

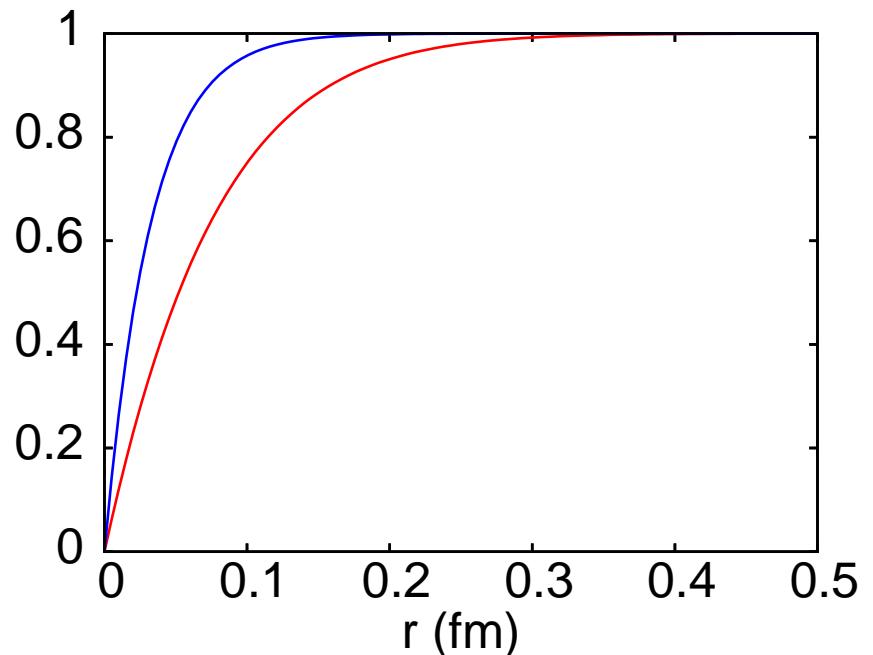
Table 2: Masses of heavy-light and doubly heavy diquarks (MeV).

| Quark content | Diquark type | Mass | |
|------------------|-----------------|---------|---------|
| | | $Q = c$ | $Q = b$ |
| $[Q, q]$ | S | 1973 | 5359 |
| $\{Q, q\}$ | A | 2036 | 5381 |
| $[Q, s]$ | S | 2091 | 5462 |
| $\{Q, s\}$ | A | 2158 | 5482 |
| $[Q, c]$ | S | | 6519 |
| $\{Q, c\}$ | A | 3226 | 6526 |
| $\{Q, b\}$ | A | 6526 | 9778 |

The form factors $F(r)$ for the scalar $[u, d]$ (solid line) and axial vector $\{u, d\}$ (dashed line) diquarks:



The form factors $F(r)$ for $\{c, q\}$ (red line) and $\{b, q\}$ (blue line) axial vector diquarks.



MASSES OF HEAVY BARYONS

- ★ p/m_Q expansion for heavy quark
- ★ relativistic treatment of light diquark $d = (qq)$

- leading order in p/m_Q

for scalar diquark

$$V^{(0)}(r) = \hat{V}_{\text{Coul}}(r) + V_{\text{conf}}(r),$$

for axial vector diquark

$$V^{(0)}(r) = \hat{V}_{\text{Coul}}(r) + V_{\text{conf}}(r) + \frac{1}{E_d(E_d + M_d)} \frac{1}{r} \left[\frac{M_d}{E_d} \hat{V}'_{\text{Coul}}(r) - V'_{\text{conf}}(r) + \mu_d \frac{E_d + M_d}{2M_d} V'^V_{\text{conf}}(r) \right] \mathbf{LS}_d,$$

$$\hat{V}_{\text{Coul}}(r) = -\frac{4}{3} \alpha_s \frac{F(r)}{r}, \quad V_{\text{conf}}(r) = Ar + B,$$

where $\hat{V}_{\text{Coul}}(r)$ is the smeared Coulomb potential (which accounts for diquark structure).

- $\delta V(r)$ corrections up to second order in p/m_Q (spin-independent + \mathbf{LS}_Q , $\mathbf{S}_d \mathbf{S}_Q$, T terms)

Mass formula

$$\frac{b^2(M)}{2\mu_R} = \frac{\langle \mathbf{p}^2 \rangle}{2\mu_R} + \langle V^{(0)}(r) \rangle + \langle \delta V(r) \rangle.$$

Table 3: Masses of the ground state heavy baryons (in MeV).

| Baryon | $I(J^P)$ | Theory | | | | | Experiment |
|--------------|------------------------------|---------------|-------------------|---------------------|--------|--------------|------------|
| | | our (2005) | Capstick Isgur | Roncaglia et al. | Savage | Jenkins | |
| Λ_c | $0(\frac{1}{2}^+)$ | 2297 | 2265 | 2285 | | | 2290 |
| Σ_c | $1(\frac{1}{2}^+)$ | 2439 | 2440 | 2453 | | | 2452 |
| Σ_c^* | $1(\frac{3}{2}^+)$ | 2518 | 2495 | 2520 | 2518 | | 2538 |
| Ξ_c | $\frac{1}{2}(\frac{1}{2}^+)$ | 2481 | | 2468 | | | 2473 |
| Ξ'_c | $\frac{1}{2}(\frac{1}{2}^+)$ | 2578 | | 2580 | 2579 | 2580.8(2.1) | 2599 |
| Ξ_c^* | $\frac{1}{2}(\frac{3}{2}^+)$ | 2654 | | 2650 | | | 2680 |
| Ω_c | $0(\frac{1}{2}^+)$ | 2698 | | 2710 | | | 2678 |
| Ω_c^* | $0(\frac{3}{2}^+)$ | 2768 | | 2770 | 2768 | 2760.5(4.9) | 2752 |
| Λ_b | $0(\frac{1}{2}^+)$ | 5622 | 5585 | 5620 | | | 5672 |
| Σ_b | $1(\frac{1}{2}^+)$ | 5805 | 5795 | 5820 | | 5824.2(9.0) | 5847 |
| Σ_b^* | $1(\frac{3}{2}^+)$ | 5834 | 5805 | 5850 | | 5840.0(8.8) | 5871 |
| Ξ_b | $\frac{1}{2}(\frac{1}{2}^+)$ | 5812 | | 5810 | | 5805.7(8.1) | 5788 |
| Ξ'_b | $\frac{1}{2}(\frac{1}{2}^+)$ | 5937 | | 5950 | | 5950.9(8.5) | 5936 |
| Ξ_b^* | $\frac{1}{2}(\frac{3}{2}^+)$ | 5963 | | 5980 | | 5966.1(8.3) | 5959 |
| Ω_b | $0(\frac{1}{2}^+)$ | 6065 | | 6060 | | 6068.7(11.1) | 6040 |
| Ω_b^* | $0(\frac{3}{2}^+)$ | 6088 | | 6090 | | 6083.2(11.0) | 6060 |

* error estimates of lattice calculations — ~ 50 MeV for charmed, ~ 100 MeV for bottom baryons

† BaBar 2006; ‡ CDF 2006; * D0 2007

Table 4: Masses of the excited Λ_Q ($Q = c, b$) heavy baryons (in MeV) (scalar diquark)

| $I(J^P)$ | Qd state | $Q = c$ | | | $Q = b$ | | | | |
|--------------------|------------|-----------------|------------------|-----|-----------------|------------------|-----|------------------|-----|
| | | $M(\text{our})$ | M^{exp} | PDG | $M(\text{our})$ | M^{exp} | PDG | M^{exp} | CDF |
| $0(\frac{1}{2}^+)$ | $1S$ | 2297 | 2286.46(14) | | 5622 | 5624(9) | | 5619.7(2.4) | |
| $0(\frac{1}{2}^-)$ | $1P$ | 2598 | 2595.4(6) | | 5930 | | | | |
| $0(\frac{3}{2}^-)$ | $1P$ | 2628 | 2628.1(6) | | 5947 | | | | |
| $0(\frac{1}{2}^+)$ | $2S$ | 2772 | 2766.6(2.4)? | | 6086 | | | | |
| $0(\frac{3}{2}^+)$ | $1D$ | 2874 | | | 6189 | | | | |
| $0(\frac{5}{2}^+)$ | $1D$ | 2883 | 2882.5(2.2) | | 6197 | | | | |
| $0(\frac{1}{2}^-)$ | $2P$ | 3017 | | | 6328 | | | | |
| $0(\frac{3}{2}^-)$ | $2P$ | 3034 | | | 6337 | | | | |

 Table 5: Masses of the excited Σ_Q ($Q = c, b$) heavy baryons (in MeV) (axial vector diquark)

| $I(J^P)$ | Qd state | $Q = c$ | | | $Q = b$ | | | | | | |
|--------------------|------------|-----------------|-----------------------|-----|------------------|-------|------------------------|-------|-----------------|------------------|-----|
| | | $M(\text{our})$ | M^{exp} | PDG | M^{exp} | BaBar | M^{exp} | Belle | $M(\text{our})$ | M^{exp} | CDF |
| $1(\frac{1}{2}^+)$ | $1S$ | 2439 | 2453.76(18) | | | | | | 5805 | 5807.5(3.6) | |
| $1(\frac{3}{2}^+)$ | $1S$ | 2518 | 2518.0(5) | | | | | | 5834 | 5829.0(3.3) | |
| $1(\frac{1}{2}^-)$ | $1P$ | 2805 | | | | | | | 6122 | | |
| $1(\frac{1}{2}^-)$ | $1P$ | 2795 | | | | | | | 6108 | | |
| $1(\frac{3}{2}^-)$ | $1P$ | 2799 | 2802($\frac{4}{7}$) | | | | | | 6106 | | |
| $1(\frac{3}{2}^-)$ | $1P$ | 2761 | 2766.6(2.4)? | | | | | | 6076 | | |
| $1(\frac{5}{2}^-)$ | $1P$ | 2790 | | | | | | | 6083 | | |
| $1(\frac{1}{2}^+)$ | $2S$ | 2864 | | | | | | | 6202 | | |
| $1(\frac{3}{2}^+)$ | $2S$ | 2912 | | | 2939.8(2.3)? | | 2938($\frac{3}{5}$)? | | 6222 | | |

Table 6: Masses of the excited Ξ_Q ($Q = c, b$) heavy baryons with scalar diquark (in MeV).

| $I(J^P)$ | Qd state | $Q = c$ | | | $Q = b$ | | | |
|------------------------------|------------|-----------------|------------------|-----|-----------------|------------------|----|------------------|
| | | $M(\text{our})$ | M^{exp} | PDG | $M(\text{our})$ | M^{exp} | D0 | M^{exp} |
| $\frac{1}{2}(\frac{1}{2}^+)$ | $1S$ | 2481 | 2471.0(4) | | 5812 | 5774(26) | | 5793(4) |
| $\frac{1}{2}(\frac{1}{2}^-)$ | $1P$ | 2801 | 2791.9(3.3) | | 6119 | | | |
| $\frac{1}{2}(\frac{3}{2}^-)$ | $1P$ | 2820 | 2818.2(2.1) | | 6130 | | | |
| $\frac{1}{2}(\frac{1}{2}^+)$ | $2S$ | 2923 | | | 6264 | | | |

Table 7: Masses of the excited Ξ_Q ($Q = c, b$) heavy baryons with axial vector diquark (in MeV).

| $I(J^P)$ | Qd state | $Q = c$ | | | $Q = b$ | | | | |
|------------------------------|------------|-----------------|------------------|-----|------------------|-------|------------------|-------|---------------------------------------|
| | | $M(\text{our})$ | M^{exp} | PDG | M^{exp} | Belle | M^{exp} | BaBar | $\frac{M(\text{our})}{M(\text{our})}$ |
| $\frac{1}{2}(\frac{1}{2}^+)$ | $1S$ | 2578 | 2578.0(2.9) | | | | | | 5937 |
| $\frac{1}{2}(\frac{3}{2}^+)$ | $1S$ | 2654 | 2646.1(1.2) | | | | | | 5963 |
| $\frac{1}{2}(\frac{1}{2}^-)$ | $1P$ | 2934 | | | | | | | 6249 |
| $\frac{1}{2}(\frac{1}{2}^-)$ | $1P$ | 2928 | | | | | | | 6238 |
| $\frac{1}{2}(\frac{3}{2}^-)$ | $1P$ | 2931 | | | | | | | 6237 |
| $\frac{1}{2}(\frac{3}{2}^-)$ | $1P$ | 2900 | | | | | | | 6212 |
| $\frac{1}{2}(\frac{5}{2}^-)$ | $1P$ | 2921 | | | | | | | 6218 |
| $\frac{1}{2}(\frac{1}{2}^+)$ | $2S$ | 2984 | | | 2978.5(4.1) | | 2967.1(2.9) | | 6327 |
| $\frac{1}{2}(\frac{3}{2}^+)$ | $2S$ | 3035 | | | | | | | 6341 |
| $\frac{1}{2}(\frac{1}{2}^+)$ | $1D$ | 3132 | | | | | | | 6420 |
| $\frac{1}{2}(\frac{3}{2}^+)$ | $1D$ | 3127 | | | | | | | 6410 |
| $\frac{1}{2}(\frac{3}{2}^+)$ | $1D$ | 3131 | | | | | | | 6412 |
| $\frac{1}{2}(\frac{5}{2}^+)$ | $1D$ | 3123 | | | | | | | 6403 |
| $\frac{1}{2}(\frac{5}{2}^+)$ | $1D$ | 3087 | | | 3082.8(3.3) | | 3076.4(1.0) | | 6377 |

HEAVY TETRAQUARKS

Motivation:

- ★ Tetraquarks with hidden charm and bottom ($Qq)(\bar{Q}\bar{q})$: New charmonium states ($X(3872)$, $Y(3940)$, $Y(4260)$) cannot be simply interpreted as $c\bar{c}$ bound states.
- ★ Tetraquarks with open charm and bottom ($QQ'(\bar{q}\bar{q}')$: Explicitly exotic states with heavy flavour number equal to 2
⇒ their observation would be a direct proof of existence of multiquark states

The diquark-antidiquark model of heavy tetraquarks predicts existence of the $SU(3)$ nonet of states with hidden charm or bottom ($Q = c, b$):

- four tetraquarks ($[Qq][\bar{Q}\bar{q}]$, $q = u, d$) with neither open nor hidden strangeness, which have electric charges 0 or ± 1 and isospin 0 or 1
- four tetraquarks ($[Qs][\bar{Q}\bar{q}]$ and $[Qq][\bar{Q}\bar{s}]$, $q = u, d$) with open strangeness ($S = \pm 1$), which have electric charges 0 or ± 1 and isospin $\frac{1}{2}$
- one tetraquark ($[Qs][\bar{Q}\bar{s}]$) with hidden strangeness and zero electric charge.

In our model we neglect the mass difference of u and d quarks and electromagnetic interactions – thus corresponding tetraquarks will be degenerate in mass. More detailed analysis predicts that such mass differences can be of few MeV.

The (non)observation of such states will be a crucial test of the tetraquark model.

Table 8: Masses of hidden charm tetraquark states (in MeV).

| State J^{PC} | Diquark content | Tetraquark mass | | |
|-------------------|------------------------------------|--------------------|--------------------|-------------------------------------|
| | | $cq\bar{c}\bar{q}$ | $cs\bar{c}\bar{s}$ | $cs\bar{c}\bar{q}/cq\bar{c}\bar{s}$ |
| $1S$ | | | | |
| 0^{++} | $S\bar{S}$ | 3812 | 4051 | 3922 |
| $1^{+\pm}$ | $(S\bar{A} \pm \bar{S}A)/\sqrt{2}$ | 3871 | 4113 | 3982 |
| 0^{++} | $A\bar{A}$ | 3852 | 4110 | 3967 |
| 1^{+-} | $A\bar{A}$ | 3890 | 4143 | 4004 |
| 2^{++} | $A\bar{A}$ | 3968 | 4209 | 4080 |
| $1P$ | | | | |
| 1^{--} | $S\bar{S}$ | 4244 | 4466 | 4350 |

Table 9: Thresholds for open charm decays and nearby hidden-charm thresholds.

| Channel | Threshold (MeV) | Channel | Threshold (MeV) | Channel | Threshold (MeV) |
|----------------------|-----------------|----------------------|-----------------|--------------------|-----------------|
| $D^0\bar{D}^0$ | 3729.4 | $D_s^+D_s^-$ | 3936.2 | $D^0D_s^\pm$ | 3832.9 |
| D^+D^- | 3738.8 | $\eta'J/\psi$ | 4054.7 | $D^\pm D_s^\mp$ | 3837.7 |
| $D^0\bar{D}^{*0}$ | 3871.3 | $D_s^\pm D_s^{*\mp}$ | 4080.0 | $D^{*0}D_s^\pm$ | 3975.0 |
| $\rho J/\psi$ | 3872.7 | $\phi J/\psi$ | 4116.4 | $D^0D_s^{*\pm}$ | 3976.7 |
| $D^\pm D^{*\mp}$ | 3879.5 | $D_s^{*+}D_s^{*-}$ | 4223.8 | $K^{*\pm}J/\psi$ | 3988.6 |
| $\omega J/\psi$ | 3879.6 | | | $K^{*0}J/\psi$ | 3993.0 |
| $D^{*0}\bar{D}^{*0}$ | 4013.6 | | | $D^{*0}D_s^{*\pm}$ | 4118.8 |

Table 10: Masses of hidden bottom tetraquark states (in MeV).

| State J^{PC} | Diquark content | Tetraquark mass | | |
|-------------------|------------------------------------|--------------------|--------------------|-------------------------------------|
| | | $bq\bar{b}\bar{q}$ | $bs\bar{b}\bar{s}$ | $bs\bar{b}\bar{q}/bq\bar{b}\bar{s}$ |
| $1S$ | | | | |
| 0^{++} | $S\bar{S}$ | 10471 | 10662 | 10572 |
| $1^{+\pm}$ | $(S\bar{A} \pm \bar{S}A)/\sqrt{2}$ | 10492 | 10682 | 10593 |
| 0^{++} | $A\bar{A}$ | 10473 | 10671 | 10584 |
| 1^{+-} | $A\bar{A}$ | 10494 | 10686 | 10599 |
| 2^{++} | $A\bar{A}$ | 10534 | 10716 | 10628 |
| $1P$ | | | | |
| 1^{--} | $S\bar{S}$ | 10807 | 11002 | 10907 |

Table 11: Thresholds for open bottom decays.

| Channel | Threshold (MeV) | Channel | Threshold (MeV) | Channel | Threshold (MeV) |
|----------------|-----------------|----------------------|-----------------|------------|-----------------|
| $B\bar{B}$ | 10558 | $B_s^+ B_s^-$ | 10739 | BB_s | 10649 |
| $B\bar{B}^*$ | 10604 | $B_s^\pm B_s^{*\mp}$ | 10786 | B^*B_s | 10695 |
| $B^*\bar{B}^*$ | 10650 | $B_s^{*+} B_s^{*-}$ | 10833 | $B^*B_s^*$ | 10742 |

Table 12: Comparison of theoretical predictions for the masses of charm diquark-antidiquark states $cq\bar{c}\bar{q}$ (in MeV) and possible experimental candidates.

| State J^{PC} | Theory | | | Experiment | |
|-------------------|--------|-------------------|--------------------------------------|------------|------------------------|
| | EFG | Maiani et al. | Maiani et al. ($cs\bar{c}\bar{s}$) | state | mass |
| $1S$ | | | | | |
| 0^{++} | 3812 | 3723 | | | |
| 1^{++} | 3871 | 3872 [†] | | $X(3872)$ | 3871.9 ± 0.5 |
| 1^{+-} | 3871 | 3754 | | | |
| 0^{++} | 3852 | 3832 | | | |
| 1^{+-} | 3890 | 3882 | | | |
| 2^{++} | 3968 | 3952 | | $Y(3943)$ | $3943 \pm 11 \pm 13$ |
| $1P$ | | | | | |
| 1^{--} | 4244 | | 4330 ± 70 | $Y(4260)$ | $4259 \pm 8_{-6}^{+2}$ |

[†] input

Table 13: Masses M of heavy-diquark (QQ')–light-antidiquark ($\bar{q}\bar{q}$) states. T is the lowest threshold for decays into two heavy-light ($Q\bar{q}$) mesons and $\Delta = M - T$. All values are given in MeV.

| System | State $I(J^P)$ | $Q = Q' = c$ | | | $Q = Q' = b$ | | | $Q = c, Q' = b$ | | |
|-------------------------|--------------------|--------------|------|----------|--------------|-------|----------|-----------------|------|----------|
| | | M | T | Δ | M | T | Δ | M | T | Δ |
| $(QQ')(\bar{u}\bar{d})$ | | | | | | | | | | |
| | $0(0^+)$ | | | | | | | 7239 | 7144 | 95 |
| | $0(1^+)$ | 3935 | 3871 | 64 | 10502 | 10604 | -102 | 7246 | 7190 | 56 |
| | $1(1^+)$ | | | | | | | 7403 | 7190 | 213 |
| | $1(0^+)$ | 4056 | 3729 | 327 | 10648 | 10558 | 90 | 7383 | 7144 | 239 |
| | $1(1^+)$ | 4079 | 3871 | 208 | 10657 | 10604 | 53 | 7396 | 7190 | 206 |
| | $1(2^+)$ | 4118 | 4014 | 104 | 10673 | 10650 | 23 | 7422 | 7332 | 90 |
| $(QQ')(\bar{u}\bar{s})$ | | | | | | | | | | |
| | $\frac{1}{2}(0^+)$ | | | | | | | 7444 | 7232 | 212 |
| | $\frac{1}{2}(1^+)$ | 4143 | 3975 | 168 | 10706 | 10693 | 13 | 7451 | 7277 | 174 |
| | $\frac{1}{2}(1^+)$ | | | | | | | 7555 | 7277 | 278 |
| | $\frac{1}{2}(0^+)$ | 4221 | 3833 | 388 | 10802 | 10649 | 153 | 7540 | 7232 | 308 |
| | $\frac{1}{2}(1^+)$ | 4239 | 3975 | 264 | 10809 | 10693 | 116 | 7552 | 7277 | 275 |
| | $\frac{1}{2}(2^+)$ | 4271 | 4119 | 152 | 10823 | 10742 | 81 | 7572 | 7420 | 152 |
| $(QQ')(\bar{s}\bar{s})$ | | | | | | | | | | |
| | $0(1^+)$ | | | | | | | 7684 | 7381 | 303 |
| | $0(0^+)$ | 4359 | 3936 | 423 | 10932 | 10739 | 193 | 7673 | 7336 | 337 |
| | $0(1^+)$ | 4375 | 4080 | 295 | 10939 | 10786 | 153 | 7683 | 7381 | 302 |
| | $0(2^+)$ | 4402 | 4224 | 178 | 10950 | 10833 | 117 | 7701 | 7525 | 176 |

SUMMARY

- Dynamical approach based on the relativistic quark model was used, where diquark, baryon and tetraquark masses were obtained by numerical solution of the quasipotential equation with the corresponding relativistic potentials.
- Light quarks were treated fully relativistically
- v/c or $1/m_Q$ expansions were used only for heavy quarks
- The diquark structure was taken into account with the help of diquark-gluon form factor in terms of diquark wave functions.
- The results were obtained with values of parameters taken from previous considerations of meson properties.
 - Heavy baryons qqQ were studied in heavy-quark–light-diquark picture.
 - Predictions for the masses of the Ω_c^* , Σ_b , Σ_b^* and Ξ_b were recently confirmed experimentally.
 - Calculated masses of ground state and excited heavy baryons are in good agreement with available experimental data.
 - All presently available data on masses of heavy baryons can be accommodated in the picture treating a heavy baryon as the bound state of light diquark and heavy quark, experiencing orbital and radial excitations.

- Masses of heavy tetraquarks with hidden and open charm and bottom were calculated in the diquark-antidiquark picture.
- $X(3872)$ can be the 1^{++} neutral charm tetraquark state. If it is really a tetraquark, one more neutral and two charged tetraquark states must exist with close masses.
- $Y(4260)$ can be the 1^{--} P -wave ($[cq]_{S=0}[\bar{c}\bar{q}]_{S=0}$) tetraquark state. Then its dominant decay mode should be $Y(4260) \rightarrow D\bar{D}$.
- The ground states of bottom tetraquarks are predicted to have masses below the open bottom threshold and thus should be narrow.
- All the $(cc)(\bar{q}\bar{q}')$ tetraquarks are predicted to be above the decay threshold into the open charm mesons.
- Only the $I(J^P) = 0(1^+)$ state of $(bb)(\bar{u}\bar{d})$ was found to lie below the BB^* threshold.

The (non)observation of the additional states will be an important test of the tetraquark model.