Baryon Regge Trajectories in the Light of the N_c Expansion

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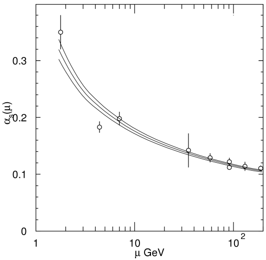
Outline

Introduction

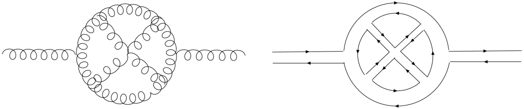
- Impossible to solve QCD exactly
- No perturbative expansion of QCD at low energies with respect to g



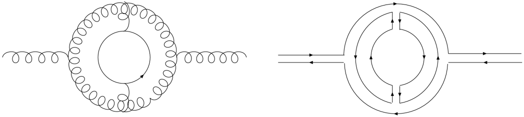
- 't Hooft suggested to generalize QCD to N_c color (1974)
- $1/N_c$ should be the expansion parameter of QCD
- Witten power counting rules (1979)

Large N_c power counting rules for Feynman diagrams

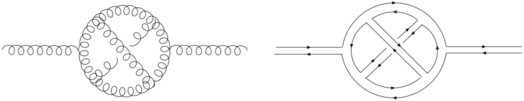
- Planar gluon insertions do not change the order of the diagram



- Internal quark loops are suppressed by factors of $1/N_c$



- Each non-planar gluon line is suppressed by a factor of $1/N_c^2$



The leading Feynman diagrams are planar and contain a minimum number of quark loops

The total wave function of baryons Ψ

$$\Psi = \psi_{lm}\chi\phi C$$

where ψ_{lm}, χ, ϕ and C are the space, spin, flavor and color part.

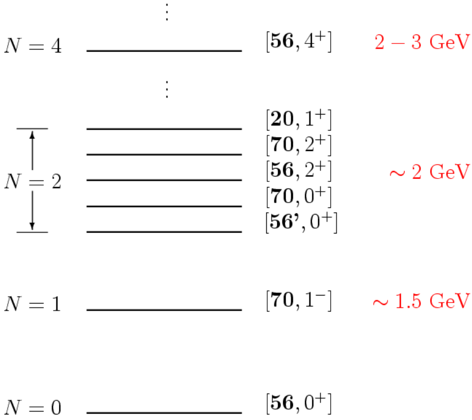
Baryons in large N_c QCD

$$\varepsilon_{i_1 i_2 i_3 \cdots i_{N_c}} q^{i_1} q^{i_2} q^{i_3} \cdots q^{i_{N_c}}$$

bound state of N_c quarks completely antisymmetric in color because baryons are colorless

Baryon mass grows with N_c

Baryon spectrum (SU(6) notation: $[\mathbf{X}, l^P]$)



SU($2N_f$) is a good symmetry in $N_c \to \infty$ limit

$$\mathrm{SU}(2N_f) \supset \mathrm{SU_S}(2) \times \mathrm{SU}_f(N_f)$$

Assume a SU(6) $\times$ O(3) symmetry at leading order.

SU($2N_f$) generators

$$\begin{array}{rcll} S^i & = & q^\dagger (S^i \otimes \mathbb{1}) q & (3,1) \\ T^a & = & q^\dagger (\mathbb{1} \otimes T^a) q & (1,\mathrm{adj}) \\ G^{ia} & = & q^\dagger (S^i \otimes T^a) q & (3,\mathrm{adj}) \end{array}$$

SO(3) generators

$$\ell^i = q^\dagger \ell^i q \quad (3)$$

$1/N_c$ Expansion of a QCD 1-body operator

(a)

(b)

$1/N_c$ expansion of a static QCD 1-body operator (baryon mass, axial vector current, magnetic moment) transforming according to a given SU(2)×SU(N_f) representation

$$\mathcal{O}^{1-\text{body}}_{\text{QCD}} = \sum_n \frac{1}{N_c^{n-1}} c_n O^n, \quad O^n = O_\ell \cdot O_{SF}$$

where O_ℓ and O_{SF} are expressed in terms of products of SO(3) and SU($2N_f$) generators

Baryon Regge Trajectories [1]

- Analyze the evolution of the dynamical coefficients c_i vs. ℓ

$$M = \sum c_i O_i$$

- Operators considered: spin-flavor singlet, hyperfine, strangeness and hyperfine $SU(3)$ breaking for symmetric multiplets only
- One-body contributions in the hyperfine operator removed

$$\frac{1}{N_c} \sum_{i \neq j} \vec{s}_i \cdot \vec{s}_j$$

- Large N_c Hartree picture for all excited multiplets: core composed of $N_c - 1$ quarks in the ground state + excited quark
- Hyperfine interactions between core quarks only

[1] J. L. Goity and N. Matagne, arXiv:0705.3055 [hep-ph], to be published in Phys. Lett. **B**.

Mixings between multiplets **neglected**

Mass formulas used to the analysis

- **For the ground state baryons**

$$\hat{M}_{\mathrm{GS}} = N_c\, c_1 \mathbb{1} + \frac{1}{N_c} c_{\mathrm{HF}} \left(\hat{S}^2 - \frac{3}{4} N_c \right) - c_{\mathcal{S}}\, \hat{\mathcal{S}} + \frac{1}{N_c}\, c_4 \left(\hat{I}^2 - \hat{S}^2 - \frac{1}{4} \hat{\mathcal{S}}^2 \right)$$

- **For excited baryons**

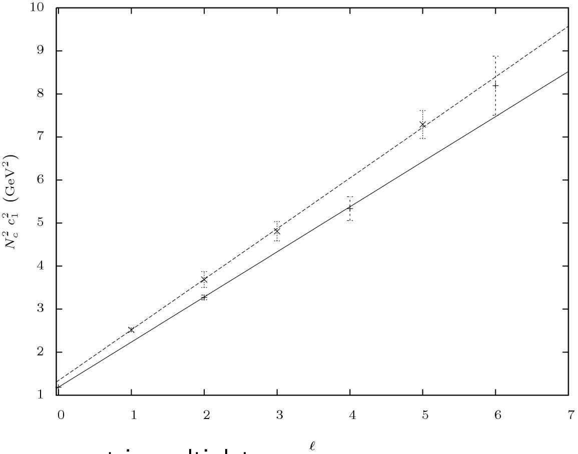
$$\begin{aligned} \hat{M}' &= N_c\, c_1 \mathbb{1} + \frac{c_{\mathrm{HF}}}{N_c} \left(\hat{S}^{c\;2} - \frac{3}{4} (N_c - 1) \mathbb{1} \right) - c_{\mathcal{S}}\, \hat{\mathcal{S}} \\ &+ \frac{4\, c_4}{3\, N_c} \left(\sqrt{3}\, \hat{S}^c \cdot \hat{G}_8^c - \frac{1}{2}\, \hat{S}^{c\;2} - \frac{1}{8}\, N_c\, \hat{\mathcal{S}}^c \right) \end{aligned}$$

Symmetric multiplets

Multiplet	Baryon	Name, status	Exp. (MeV)	Theo (MeV)	c_1 (MeV)	c_{HF} (MeV)	c_S (MeV)	c_4 (MeV)	χ^2_{dof}
$[56, 0^+]$	$N_{1/2}$	N(939)****	939 ± 1	939 ± 2	362 ± 1	295 ± 3	208 ± 3	90 ± 5	9.1
	$\Lambda_{1/2}$	Λ(1116)****	1116 ± 1	1117 ± 1					
	${}^8\Sigma_{1/2}$	Σ(1193)****	1192 ± 4	1177 ± 4					
	${}^8\Xi_{1/2}$	Ξ(1318)****	1318 ± 3	1325 ± 4					
	$\Delta_{3/2}$	Δ(1232)****	1232 ± 1	1233 ± 2					
	${}^{10}\Sigma_{3/2}$	Σ(1385)****	1383 ± 3	1381 ± 1					
	${}^{10}\Xi_{3/2}$	Ξ(1530)****	1532 ± 1	1529 ± 2					
	$\Omega_{3/2}$	Ω(1672)****	1672 ± 2	1677 ± 2					
$[56, 2^+]$	$N_{3/2}$	N(1720)****	1700 ± 50	1682 ± 18	603 ± 5	767 ± 66	233 ± 46	416 ± 124	1.9
	$\Lambda_{3/2}$	Λ(1890)****	1880 ± 30	1822 ± 11					
	$N_{5/2}$	N(1680)****	1683 ± 8	1682 ± 17					
	$\Lambda_{5/2}$	Λ(1820)****	1820 ± 5	1822 ± 11					
	${}^8\Sigma_{5/2}$	Σ(1915)****	1918 ± 18	1915 ± 38					
	$\Delta_{1/2}$	Δ(1910)****	1895 ± 25	1938 ± 18					
	$\Delta_{3/2}$	Δ(1920)***	1935 ± 35	1938 ± 18					
	$\Delta_{5/2}$	Δ(1905)****	1895 ± 25	1938 ± 18					
	$\Delta_{7/2}$	Δ(1950)****	1950 ± 10	1938 ± 18					
	${}^{10}\Sigma_{7/2}$	Σ(2030)****	2033 ± 8	2032 ± 18					
$[56, 4^+]$	$N_{9/2}$	N(2220)****	2245 ± 65	2245 ± 92	770 ± 20	398 ± 372	110 ± 94		0.13
	$\Lambda_{9/2}$	Λ(2350)***	2355 ± 15	2355 ± 21					
	$\Delta_{7/2}$	Δ(2390)*	2387 ± 88	2378 ± 84					
	$\Delta_{9/2}$	Δ(2300)*	2318 ± 132	2378 ± 84					
	$\Delta_{11/2}$	Δ(2420)*	2400 ± 100	2378 ± 84					
$[56, 6^+]$	$N_{13/2}$	N(2700)**	2806 ± 207	2806 ± 207	954 ± 40	342 ± 720			
	$\Delta_{15/2}$	Δ(2950)**	2920 ± 122	2920 ± 122					

Mixed symmetric multiplets

Multiplet	Baryon	Name, status	Exp. (MeV)	Theo (MeV)	c_1 (MeV)	$c_{\rm HF}$ (MeV)	c_S (MeV)	$\chi^2_{\rm dof}$
$[\mathbf{70},1^-]$	$\mathrm{N}_{1/2}$	N(1535)****	1538 ± 18	1513 ± 14	529 ± 5	443 ± 19	148 ± 13	61
	${}^8\Lambda_{1/2}$	Λ(1670)****	1670 ± 10	1662 ± 6				
	$\mathrm{N}_{3/2}$	N(1520)****	1523 ± 8	1513 ± 14				
	${}^8\Lambda_{3/2}$	Λ(1690)****	1690 ± 5	1662 ± 6				
	${}^8\Sigma_{3/2}$	Σ(1670)****	1675 ± 10	1662 ± 6				
	${}^8\Xi_{3/2}$	Ξ(1820)***	1823 ± 5	1810 ± 15				
	$\mathrm{N}'_{1/2}$	N(1650)****	1660 ± 20	1661 ± 17				
	${}^8\Lambda'_{1/2}$	Λ(1800)***	1785 ± 65	1809 ± 12				
	${}^8\Sigma'_{1/2}$	Σ(1750)***	1765 ± 35	1809 ± 12				
	$\mathrm{N}'_{3/2}$	N(1700)***	1700 ± 50	1661 ± 17				
	$\mathrm{N}'_{5/2}$	N(1675)****	1678 ± 8	1661 ± 17				
	${}^8\Lambda'_{5/2}$	Λ(1830)****	1820 ± 10	1809 ± 12				
	${}^8\Sigma'_{5/2}$	Σ(1775)****	1775 ± 5	1809 ± 12				
	$\Delta_{1/2}$	Δ(1620)****	1645 ± 30	1661 ± 17				
	$\Delta_{3/2}$	Δ(1700)****	1720 ± 50	1661 ± 17				
	${}^1\Lambda_{1/2}$	Λ(1405)****	1407 ± 4	1514 ± 4				
	${}^1\Lambda_{3/2}$	Λ(1520)****	1520 ± 1	1514 ± 4				
$[\mathbf{70},2^+]$	$\mathrm{N}'_{1/2}$	N(2100)*	1926 ± 26	1987 ± 50	640 ± 16	400 (input)	120 ± 86	0.03
	$\mathrm{N}'_{5/2}$	N(2000)**	1981 ± 200	1987 ± 50				
	$\Lambda'_{5/2}$	Λ(2110)***	2112 ± 40	2108 ± 71				
	$\mathrm{N}'_{7/2}$	N(1990)**	2016 ± 104	1987 ± 50				
	$\Lambda'_{7/2}$	Λ(2020)*	2094 ± 78	2108 ± 71				
	$\Delta_{5/2}$	Δ(2000)**	1976 ± 237	1987 ± 50				
$[\mathbf{70},3^-]$	$\mathrm{N}_{5/2}$	N(2200)**	2057 ± 180	2153 ± 67	731 ± 17	249 ± 315	30 ± 159	0.15
	$\mathrm{N}_{7/2}$	N(2190)****	2160 ± 49	2153 ± 67				
	$\mathrm{N}'_{9/2}$	N(2250)****	2239 ± 76	2236 ± 81				
	$\Delta_{7/2}$	Δ(2200)*	2232 ± 87	2236 ± 81				
	${}^1\Lambda_{7/2}$	Λ(2100)****	2100 ± 20	2100 ± 28				
$[\mathbf{70},5^-]$	$\mathrm{N}_{11/2}$	N(2600)***	2638 ± 97		900 ± 20 (Est)			



- Solid line: symmetric multiplets

$$(3\ c_1([\mathbf{56}, \ell]))^2 = (1.179 \pm 0.003) + (1.05 \pm 0.01)\ \ell$$

- dashed line: mixed symmetric multiplets

$$(3\ c_1([\mathbf{70}, \ell]))^2 = (1.34 \pm 0.02) + (1.18 \pm 0.02)\ \ell$$

Conclusions

- $1/N_c$ expansion is successful for baryon spectroscopy
- N $=$ 0,...,6 bands have been studied
- Classification of resonances into octets, decuplets and singlets (Gell-Mann–Ne'eman classification for excited states)
- Regge trajectories for symmetric and mixed symmetric multiplets