

# Spacelike and Timelike Nucleon Form Factors within Light-Front dynamics

J. P. B. C. de Melo (CCET, Univ. de Cruzeiro do Sul, São Paulo)

Tobias Frederico (ITA-CT, São Paulo)

Emanuele Pace ( Tor Vergata University & INFN )

Silvia Pisano ( Rome University & INFN - Tor Vergata )

G. S. (INFN - Roma)

**Electromagnetic Hadron Form Factors and Higher Fock Components** → J. P. B. C. de Melo, T. Frederico, E. Pace, S. Pisano, G. Salme', Nucl. Phys. A **782** (2007) 69c

**Timelike and spacelike hadron form factors, Fock state components and light-front dynamics** J. P. B. C. de Melo, T. Frederico, E. Pace, S. Pisano, G. Salme', Nucl. Phys. A **790** (2007) 606c

**Pion ff in the space-like and time-like regions** → J. P. B. C. de Melo, T. Frederico, E. Pace, G. Salme', Phys. Lett. B **581** (2004) 75

**Space-like and time-like pion electromagnetic form factor and Fock state components within the Light-Front dynamics** → J. P. B. C. de Melo, T. Frederico, E. Pace, G. Salme', Phys. Rev. D **73**, 074013 (2006)

## Outline

- Motivations
- A covariant approach for the Hadron EM Current: i) the Mandelstam Formula - ii) phenomenological Bethe - Salpeter amplitudes for Hadrons - iii) quark-photon vertex
- Pion EM Form Factor in the space- and time-like regions: tuning our model - Microscopic Vector Meson Dominance
- Nucleon EM Form Factors in the space- and time-like regions: quark-photon vertex  $\equiv$  Bare + VMD
- Conclusion & Perspectives: Nucleon GPD's ?

The investigation of hadron EM form factors in the space- and time-like regions, within the light-front dynamics,

- opens a unique possibility to study the hadronic state, in both the valence and the nonvalence sector (Brodsky, Pauli & Pinsky, Phys. Rep. **301** (1998) 299 )

$$\begin{aligned} |meson\rangle &= |q\bar{q}\rangle + |q\bar{q}q\bar{q}\rangle + |q\bar{q}g\rangle + \dots \\ |baryon\rangle &= \underbrace{|qqq\rangle}_{\text{valence}} + \underbrace{|qqq q\bar{q}\rangle + |qqq g\rangle}_{\text{nonvalence}} + \dots \end{aligned}$$

★ A meaningful Fock expansion within LF framework, due to the properties of the LF vacuum.

★★ Zero modes  $\rightarrow \chi_{\text{SB}}$  for fermions

- yields the possibility to address the vast phenomenology of hadronic resonances (Vector Meson propagation...) in the time-like region, and then to impose strong constraints on dynamical models pointing to a microscopical description of hadrons
- allows one to obtain insights into the two-body currents associated to the  $q\bar{q}$  pair production, (very important in reference frames where  $q^+ \neq 0$ ).

## The Mandelstam Formula for the EM current

Our guidance  $\Rightarrow$  the Mandelstam formula, that yields a covariant expression for the em current of Hadrons, (to be considered as an interacting systems).

For a global investigation of SL and TL regions we need to change frame, from the  $q^+ = 0$  frame (a standard choice within LF) to a  $q^+ \neq 0$  frame (F.M. Lev, Pace and G.S. NPA 641 (1998) 229).

In the SL region

$$j^\mu = -ie \int \frac{d^4 k}{(2\pi)^4} \text{Tr} [S_Q(k - P_h) \bar{\Lambda}_h(k - P'_h, P'_h) S(k - q) \times \Gamma^\mu(k, q) S(k) \Lambda_h(k, P_h)]$$

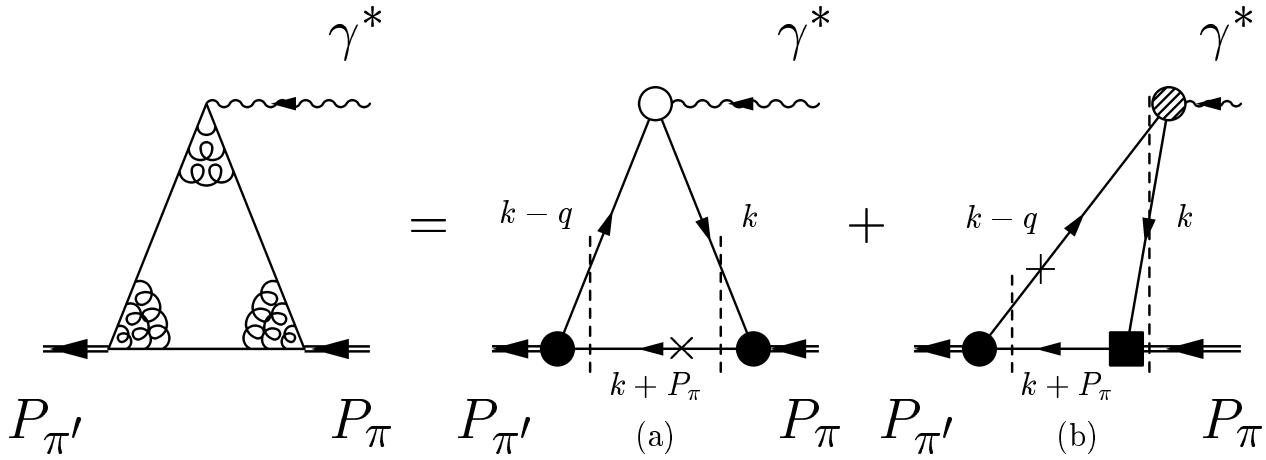
- $S(p) = \frac{1}{\not{p} - m + i\epsilon}$ , with  $m$  the mass of the constituent quark struck by the virtual photon
- $S_Q(p)$ : propagator(s) of the spectator constituent quark(s) for a meson (baryon)
- $\Lambda_h(k, P_h)$  is the hadron vertex function;  $P_h^\mu$  and  $P_{h'}^\mu$  are the hadron momenta. Very important: it contains a Dirac structure, i.e. a proper combination of Dirac matrices.
- $\Gamma^\mu(k, q)$  is the quark-photon vertex ( $q^\mu$  the virtual photon momentum)

$$\text{SL} \rightarrow \text{TL} \quad P_h^\mu \rightarrow -P_{\bar{h}}^\mu$$

# Projecting out the Mandelstam Formula on the Light Front

...through a  $k^-$  integration, (only the poles of the Dirac propagators taken into account), in a reference frame where  $q^+ > 0$ ,  $\mathbf{q}_\perp = 0$

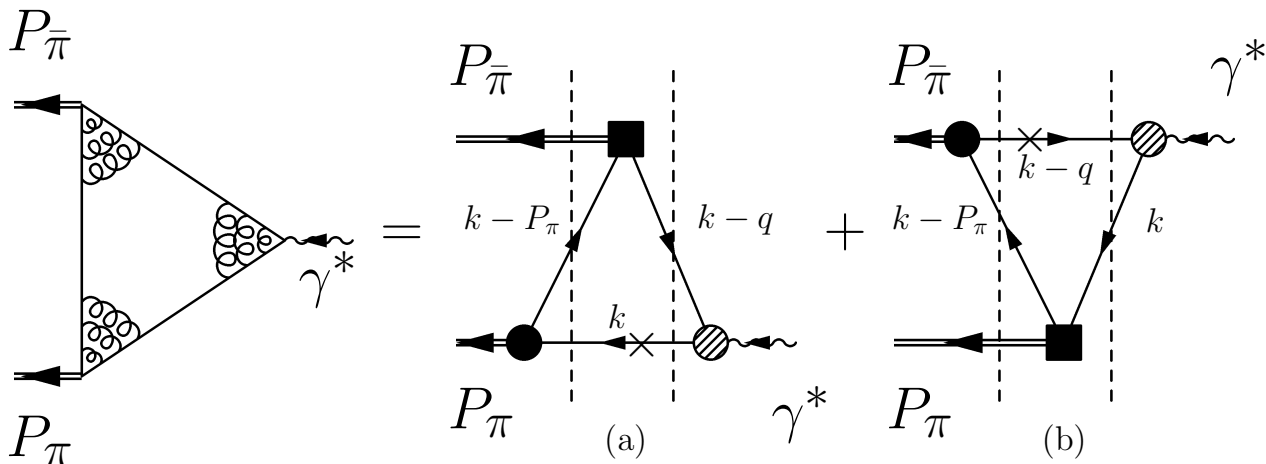
## Pion Space-like region



$$0 < k^+ + P_{\pi}^+ < P_{\pi}^+$$

$$0 < k^+ < q^+$$

## Time-like region



$$(\text{val.}) 0 < k^+ < P_{\pi}^+$$

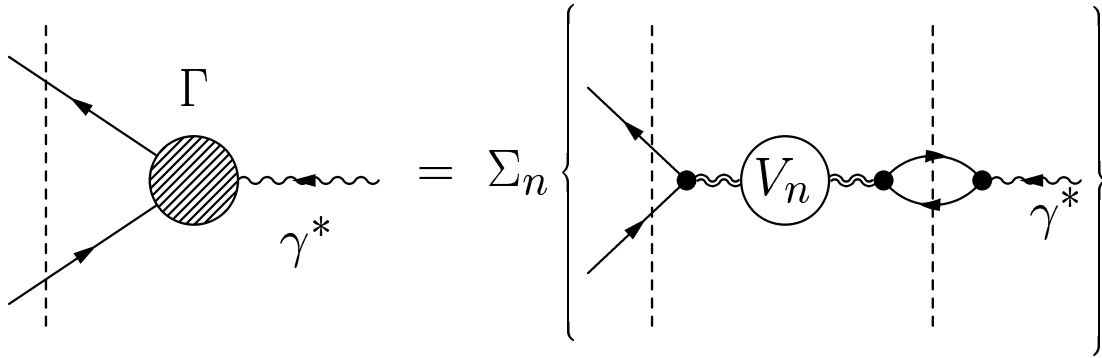
$$P_{\pi}^+ < k^+ < q^+ (\text{non-val.})$$

$$\times \Rightarrow k \text{ on its mass shell : } k_{on}^- = (m^2 + k_{\perp}^2)/k^+$$

★ First Problem: How to model the quark-photon vertex ? ★

★ ★ Second Problem: How to connect the Fock language with the Bethe-Salpeter one, e.g. how to describe the vertex amplitude  $\Lambda(k_i, P_h)$  in both the valence and the non-valence regions? ★ ★

★ In the limit  $m_\pi \rightarrow 0$ , the quark-photon vertex is dominated by the  $q\bar{q}$  production. In particular only the VMD mechanism is acting.



★ A Vector Meson Dominance approximation has been applied to the quark-photon vertex, when a  $q\bar{q}$  pair is produced

$$\Gamma^+(k, q) = \sqrt{2} \sum_{n, \lambda} \left[ \epsilon_\lambda \cdot \hat{V}_n(k, k - q) \right] \frac{\Lambda_n(k, P_n) [\epsilon_\lambda^+]^* f_{V_n}}{(q^2 - M_n^2 + i M_n \Gamma_n(q^2))}$$

- $f_{V_n}$  is the decay constant of the n-th vector meson into a virtual photon (to be calculated in our model !),  $M_n$  the mass,  $\Gamma_n(q^2) = \Gamma_n q^2 / M_n^2$  (for  $q^2 > 0$ ) the corresponding total decay width and  $\epsilon_\lambda$  is the VM polarization
- $\left[ \epsilon_\lambda \cdot \hat{V}_n(k, k - q) \right] \equiv$  Dirac structure of the VM Bethe-Salpeter amplitude.

$\Lambda_n(k, q) \equiv$  momentum-dependent part of the BS amplitude (to be approximated in our approach).

By eigenfunctions of a relativistic CQ square mass operator  
(Frederico, Pauli & Zhou, PRD 66 (2002) 116011)

## Pion EM Form Factor in the space- and time-like regions

The pion EM form factor can be extracted using the definitions

$$j_{TL}^\mu = \langle \pi \bar{\pi} | \bar{q}(0) \gamma^\mu q(0) | 0 \rangle = e (P_\pi^\mu - P_{\bar{\pi}}^\mu) F_\pi(q^2) \quad ,$$

$$j_{SL}^\mu = \langle \pi | \bar{q}(0) \gamma^\mu q(0) | \pi' \rangle = e (P_\pi^\mu + P_{\pi'}^\mu) F_\pi(q^2)$$

From i) the Mandelstam formula, ii) integrating over  $k^-$  taking into account only the poles of Dirac propagators and iii) putting  $m_\pi \rightarrow 0$  one obtains the following expression of the EM pion form factor

$$F_\pi(q^2) = \sum_n \frac{f_{Vn}}{q^2 - M_n^2 + iM_n\Gamma_n(q^2)} g_{Vn}^+(q^2)$$

calculated  $\downarrow$

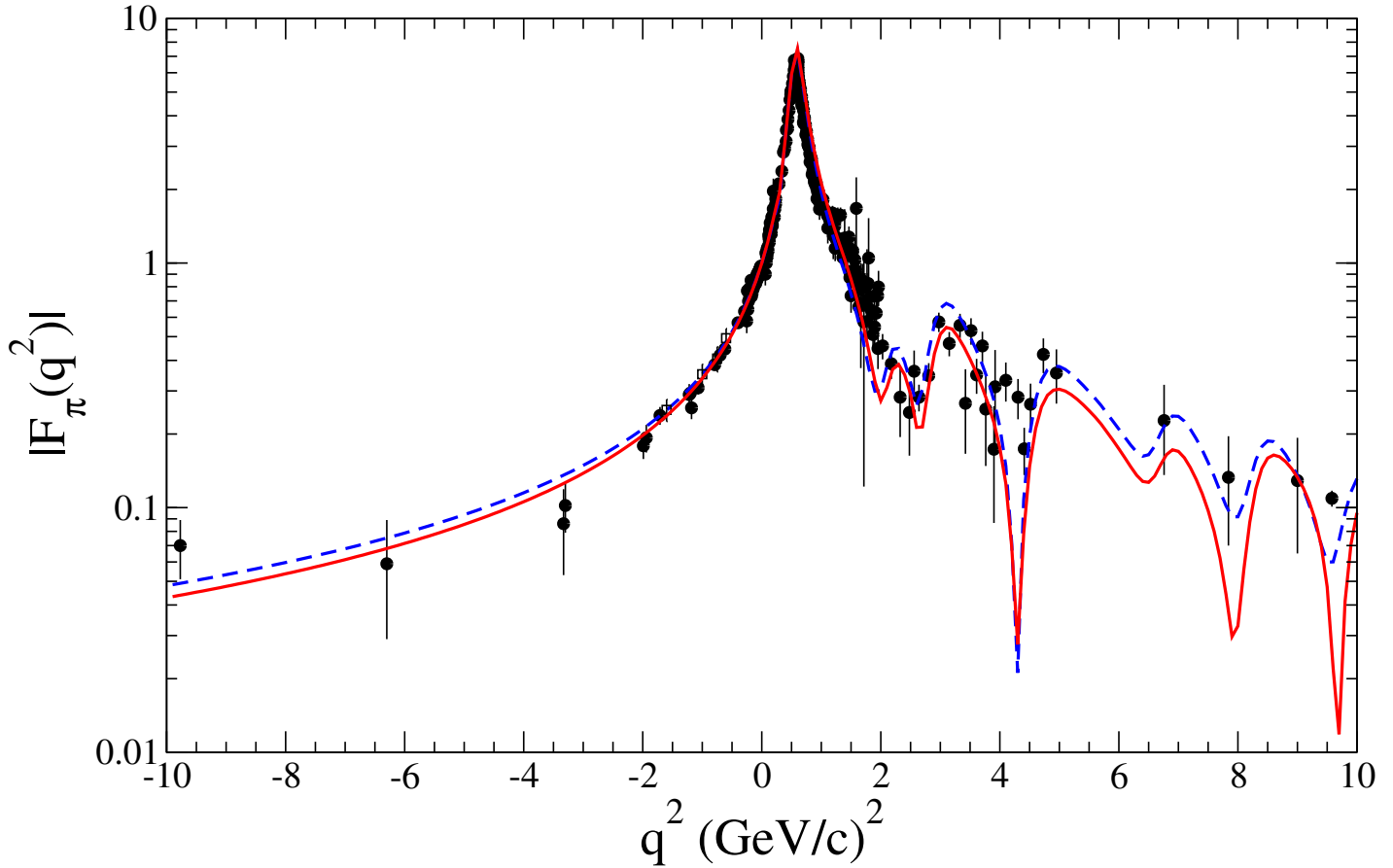
calculated  $\uparrow$

## Two adjusted parameters

- 1) The width,  $\Gamma_n$ , of the VM's with mass  $> 2.150 \text{ GeV}$ . The chosen value  $\Gamma_n = 0.15 \text{ GeV}$  is similar to the width of the first four VM's
- 2)  $w_{VM}$ , the weight the so-called instantaneous contributions.

# Pion EM Form Factor in the SL and TL regions

## Comparison with Exp. data



●: Data, R. Baldini et al. (EPJ. C11 (1999) 709, and Refs. therein.)

Solid line: calculation with the pion w.f. from the FPZ model for the Bethe-Salpeter amplitude in the valence region ( $w_{VM} = -0.7$ ).

Dashed line: the same as the solid line, but with the asymptotic pion w.f. ( $\Lambda_\pi(k; P_\pi) = 1$ )

$$\psi_\pi(k^+, \mathbf{k}_\perp; P_\pi^+, \mathbf{P}_{\pi\perp}) = \frac{m}{f_\pi} \frac{P_\pi^+}{[M_\pi^2 - M_0^2(k^+, \mathbf{k}_\perp; P_\pi^+, \mathbf{P}_{\pi\perp})]}$$

Our microscopical VMD depends upon one parameter and now is fixed !

## The EM Nucleon Form Factors

The Dirac structure of the quark-nucleon vertex is suggested, as in the case of the quark-pion vertex, by an effective Lagrangian (de Araujo et al PLB B478 (2001) 86)

$$\mathcal{L}_{eff}(x) = \frac{\epsilon_{abc}}{\sqrt{2}} \int d^4x_1 d^4x_2 d^4x_3 \mathcal{F}(x_1, x_2, x_3, x) \sum_{\tau_1, \tau_2, \tau_3} \times$$

$$\left[ m_N \alpha \bar{q}^a(x_1, \tau_1) i\tau_y \gamma^5 q_C^b(x_2, \tau_2) \bar{q}^c(x_3, \tau_3) - \frac{(1-\alpha)}{\sqrt{3}} \times \right.$$

$$\left. \bar{q}^a(x_1, \tau_1) \vec{\tau} i\tau_y \gamma^5 \gamma_\mu q_C^b(x_2, \tau_2) \cdot \bar{q}^c(x_3, \tau_3) \vec{\tau} (-i\partial^\mu) \right] \psi_N(x, \tau_N)$$

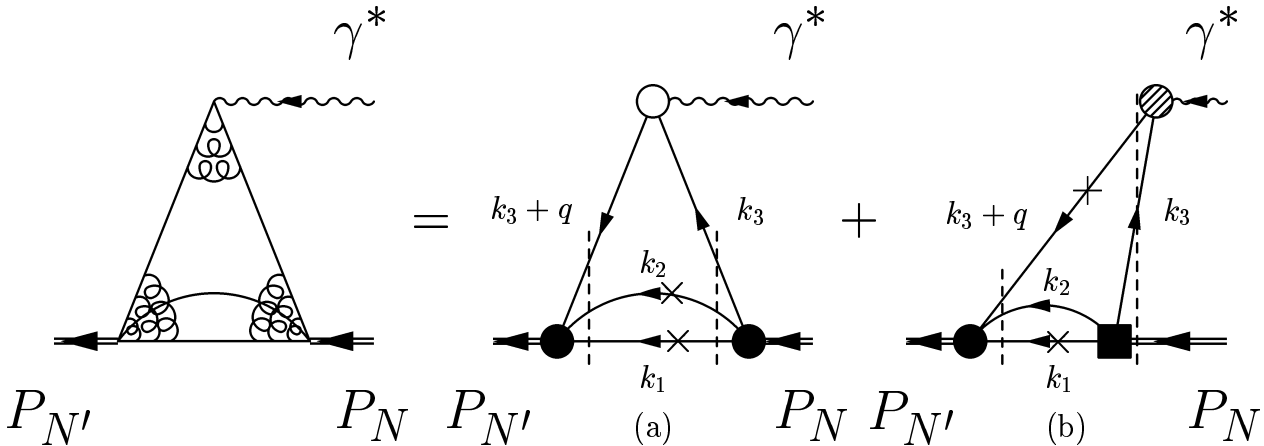
$$+ \dots$$

for the present time  $\alpha = 1$ , i.e. no derivative coupling

### Space-like

Triangle cont.

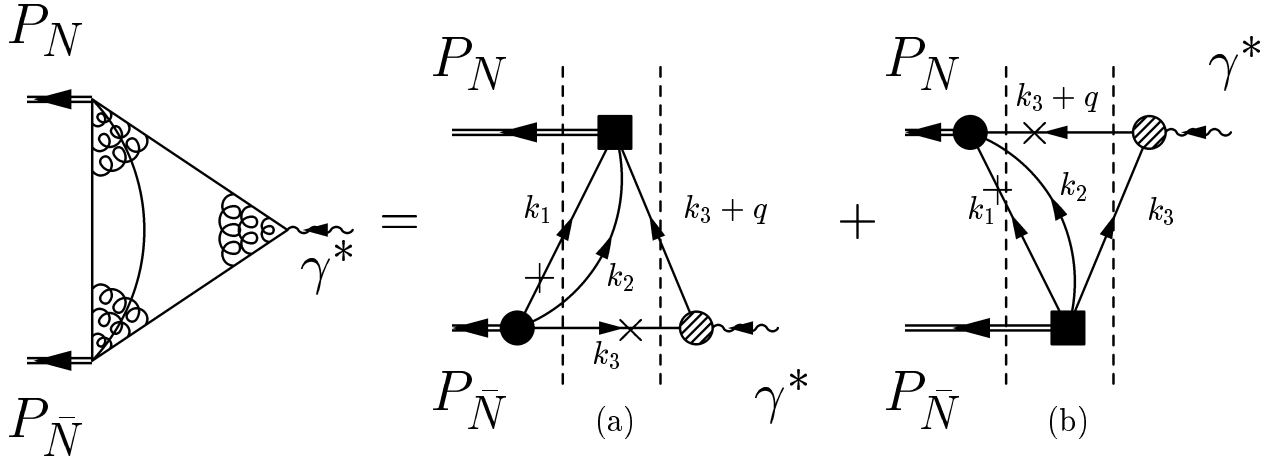
Pair cont. (Z-diag.)



$$(\text{val.}) \ 0 < k_{1(2)}^+ < P_N^+ \quad P_N^+ < k_1^+ < q^+ \ (\text{non-val.})$$

$$\times \Rightarrow \quad k_1 \text{ on the mass shell : } k_{1on}^- = (m^2 + k_{1\perp}^2)/k_1^+$$

## Time-like



The non valence contribution of the photon is involved:  $|q\bar{q}, q\bar{q}, q\bar{q}\rangle$

## Definition

The nucleon em form factors (Dirac and Pauli ff's) are introduced as usual from the matrix elements of the macroscopic em current

$$\begin{aligned}
 \langle N; \sigma', p' | j^\mu | p, \sigma; N \rangle &= \bar{U}_N(p', \sigma') \left[ -F_2(Q^2) \frac{p'^\mu + p^\mu}{2M_N} + \right. \\
 &\quad \left. (F_1(Q^2) + F_2(Q^2)) \gamma^\mu \right] U_N(p, \sigma) = \\
 &= 3 N_c \int \frac{d^4 k_1}{(2\pi)^4} \int \frac{d^4 k_2}{(2\pi)^4} \sum \left\{ \bar{\Phi}^{\sigma'}(k_1, k_2, k'_3, P'_N) \times \right. \\
 &\quad \left. S^{-1}(k_1) S^{-1}(k_2) \mathcal{I}^\mu(k_3, q) \Phi^\sigma(k_1, k_2, k_3, P_N) \right\}
 \end{aligned}$$

where  $\mathcal{I}^\mu(k_1, k_2, k_3, q)$  is the quark-photon vertex,  $\Phi_N^\sigma(k_1, k_2, k_3, p)$  the Bethe-Salpeter amplitude that contains a Dirac structure (highly non trivial...) and a dependence upon the four-momenta of the quarks.

# Quark-Photon Vertex

$$\mathcal{I}^\mu = \mathcal{I}_{IS}^\mu + \tau_z \mathcal{I}_{IV}^\mu$$

each term contains a contribution corresponding to a purely valence sector ( Space-like only) and a contribution corresponding to the pair production (or Z-diagram).

In turn, the Z-diagram contribution can be decomposed in a bare term + a Vector Meson Dominance term (according to the decomposition of the photon state in bare, hadronic [and leptonic] contributions), viz

$$\begin{aligned} \mathcal{I}_{IS(IV)}^\mu(k, q) = & \mathcal{N}_{IS(IV)} \theta(P_N^+ - k^+) \theta(k^+) \gamma^\mu + \\ & + \theta(q^+ + k^+) \theta(-k^+) \{ \mathbf{Z}_b \mathcal{N}_{IS(IV)} \gamma^\mu + \mathbf{Z}_V \Gamma_V^\mu[k, q, IS(IV)] \} \end{aligned}$$

with  $\mathcal{N}_{IS} = 1/6$  and  $\mathcal{N}_{IV} = 1/2$ . The constant  $\mathbf{Z}_b$  (bare term) and  $\mathbf{Z}_V$  (VMD term) are unknown renormalization constants to be extracted from the phenomenological analysis of the data.

# Momentum Dependence of the Bethe-Salpeter Amplitude

In the valence sector, where the spectator quarks are on their-own  $k^-$ -shell and the struck one is a quark, the momentum dependence of the Nucleon Bethe-Salpeter amplitude reduces to a 3-momentum dependence, due to the LF projection we have applied.

It is approximated through a Nucleon Wave Function a la Brodsky (PQCD inspired)

$$\mathcal{W}_N = \mathcal{N} \frac{1}{(\xi_1 \xi_2 \xi_3)^{0.13}} \frac{1}{[\beta^2 + M_0^2(1, 2, 3)]^{7/2}}$$

★  $\beta$  fixed through anomalous magnetic moments

Proton: 2.878 (Exp. 2.793)

Neutron : -1.859 (Exp. -1.913)

★ ★ The powers allow a falloff, a little bit faster than  $1/Q^4$  for the triangle contribution

In the non-valence sector, relevant for evaluating the Z-diagram contribution, the momentum dependence is approximated by

$$\Lambda^{SL} = [g_{12}]^{5/2} \frac{(k_1^+ + k_2^+)}{P_N'^+} g_{N\bar{3}} \left( \frac{P_N^+}{k_3^+} \right)^r$$

where  $g_{AB} = (m_A^2 + m_B^2) / [\beta^2 + M_0^2(A, B)]$  and  $r = 0.37$  for obtaining the charge radius of the proton correctly ( $\sim 0.9 fm$ , cf C. E. Hyde-Wright and K. de Jager, A. Rev. Nucl. and Part. Sci. **54**, 217 (2004)).

An analogous expression holds in the TL.

## Fixed parms

- quark mass adopted: 200 MeV
- VMD, up to 20 mesons for achieving a good convergence in the  $q^2$ -range investigated. The isovector part from the Pion analysis. The isoscalar part is an extension of the isovector one, with eigenvectors and eigenvalues from the Frederico, Pauli and Zhou model.

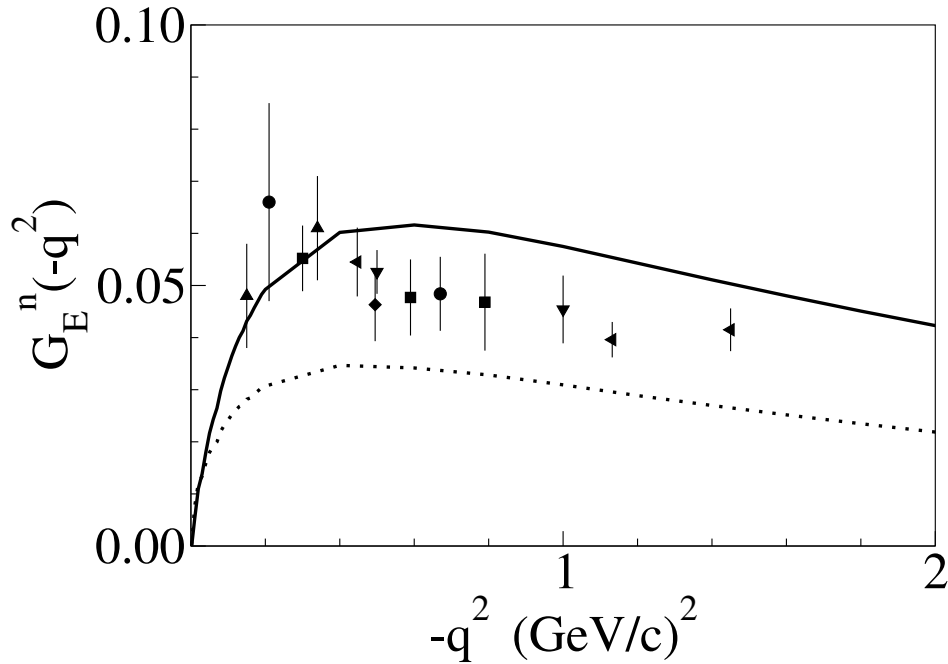
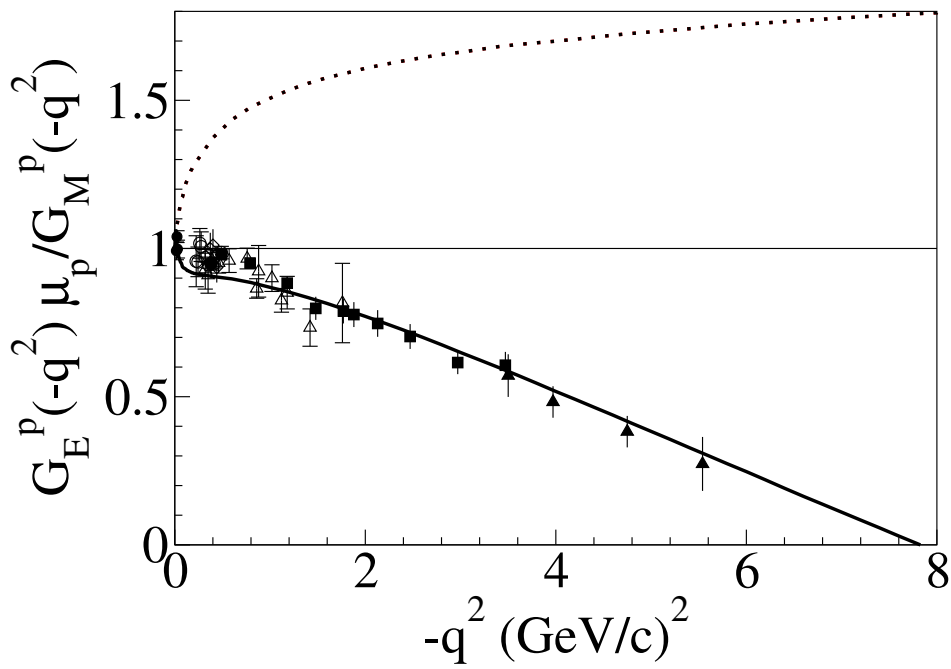
## Adjusted parms

- $Z_b$  and  $Z_V$ : renormalization constants for the pair production terms
- the power  $r$  in the non-valence vertex function

in the SL

- $G_E^p \mu_p / G_M^p$  and  $G_E^n$
- $G_M^p / G_D$  and  $G_M^n / G_D$

with  $G_D = 1/(1 - q^2/0.71)^2$

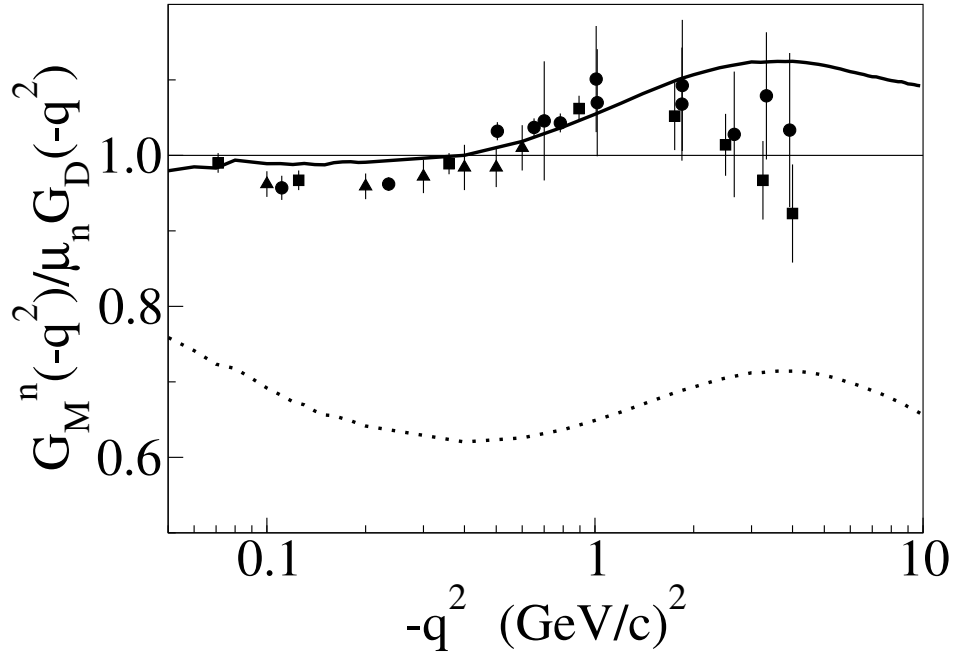
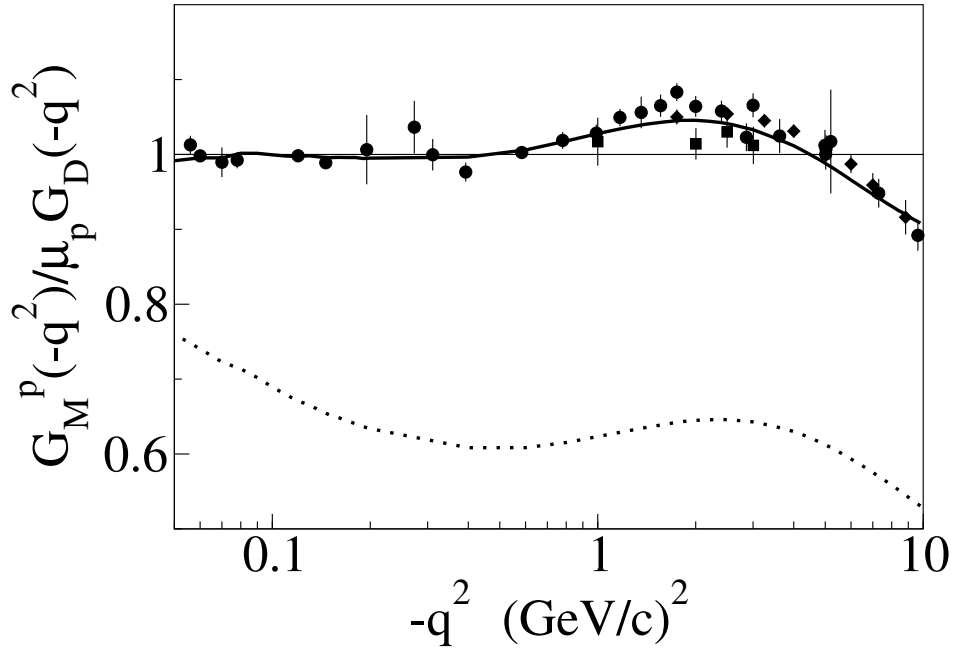


**Solid line:** full calculation  $\equiv \mathcal{F}_\Delta + Z_b \mathcal{F}_{bare} + Z_V \mathcal{F}_{VMD}$

**Dotted line:**  $\mathcal{F}_\Delta$  (elastic contribution only)

**Data:** [www.jlab.org/cseely/nucleons.html](http://www.jlab.org/cseely/nucleons.html) and Refs. therein.

The possible zero in  $G_E^p$  seems in strong relation to the Z-diagram contribution, i.e. higher Fock components of the proton state.



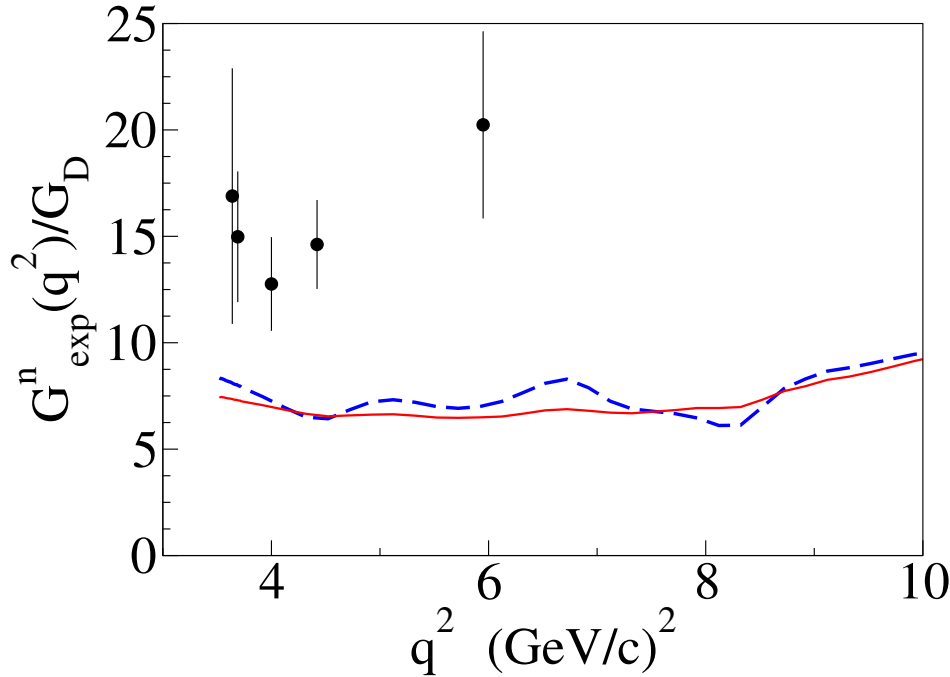
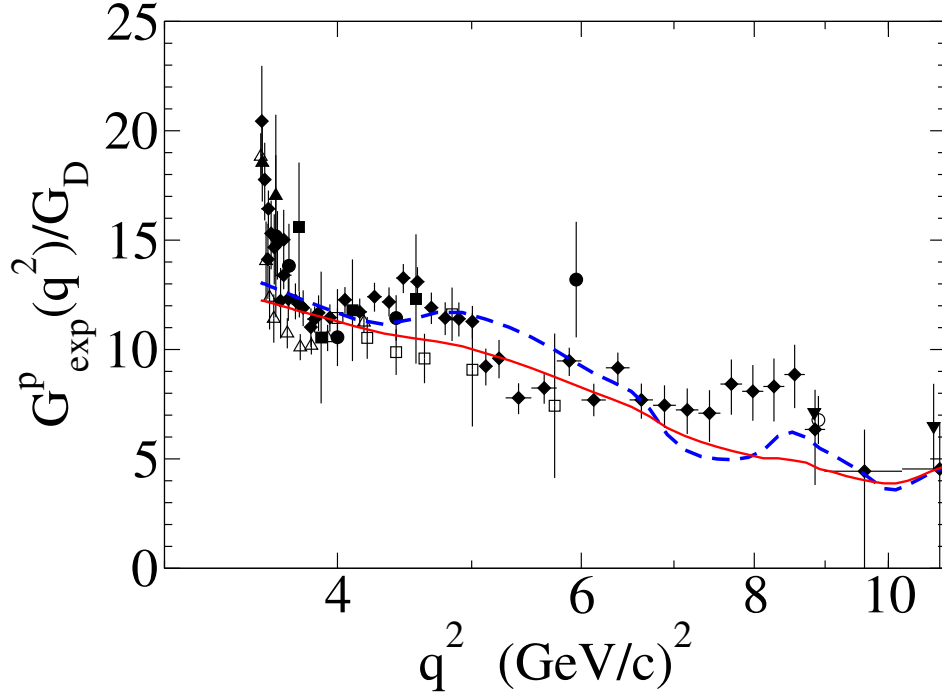
**Solid line:** full calculation  $\equiv \mathcal{F}_\Delta + Z_B \mathcal{F}_{bare} + Z_V \mathcal{F}_{VMD}$

**Dotted line:**  $\mathcal{F}_\Delta$  (elastic contribution only)

**Data:** [www.jlab.org/cseely/nucleons.html](http://www.jlab.org/cseely/nucleons.html) and Refs. therein.

Definition: Experimental TL form factors ( $\eta = 2M_N^2/q^2$ )

$$G_{exp}^{p(n)}(q^2) = \sqrt{|G_M^{p(n)}(q^2)|^2 + \eta |G_E^{p(n)}(q^2)|^2} / (1 + \eta)$$



**Solid line:** full calculation  $\equiv Z_b \mathcal{F}_{bare} + Z_V \mathcal{F}_{VMD}$  - **Dashed line:**  $Z_V \sim Z_b$

**Green:** BaBar, PRD 73, 012005 (2006), and refs. therein quoted; R. Baldini - S. Pacetti, priv. com.

## Conclusions & Perspectives

- We developed a microscopical, Poincaré covariant model for the hadron em form factors in both SL and TL region
- The Quark-photon vertex has been approximated by a microscopical VMD model plus a bare term
- The Z-diagram (higher Fock components) are essential for both Pion and Nucleon, in the reference frame adopted ( $q^+ \neq 0$ )
- Pion: Results yielded a first successful test for our model.
- Nucleon: a new insight in the challenging result from TJLAB: the possible zero in  $G_E^p$  seems in strong relation to the Z-diagram contribution, i.e. to the nonvalence component of the nucleon state. A great deal of info from the TL region: proton/neutron puzzle?

Next step, GPD's